Recurso - fevereiro de 2013

1) $x' = -2x + t, \ t \in \mathbb{R}$

(a) $x' + 2x = t, \ t \in \mathbb{R}$

$\Leftrightarrow \underbrace{\dfrac{dx}{dy}}_{} + \underbrace{2}_{P(y)}x = \underbrace{t}_{Q(y)}$,

EDO de 1ª ordem linear pois é da forma $\dfrac{dx}{dy} + P(y)\,x = \underbrace{Q(y)}_{c^{te} = t}$

(b) $\dfrac{dx}{dy} + \underbrace{2}_{P(y)}x = \underbrace{t}_{Q(y)}$

$$x(y) = e^{-\int P(y)\,dy}\left[\int e^{\int P(y)dy}\, Q(y)\, dy + C\right]$$

$$= e^{-\int 2\,dy}\left[\int e^{\int 2\,dy}\, t\, dy + C\right]$$

$$= e^{-2y}\left[\int e^{2y}\, t\, dy + C\right]$$

$$= e^{-2y}\left[\frac{t}{2}\int e^{2y}.2\, dy + C\right]$$

$$= e^{-2y}\cdot\frac{t}{2}\cdot e^{2y} + C$$

$$= \frac{t}{2} + C \quad (t \in \mathbb{R} \text{ e } C \ c^{te} \text{ arbitrária})$$

$$x(y) = \frac{t}{2} + C$$

(c) $x(0) = \dfrac{3}{2} \Leftrightarrow C = \dfrac{3}{2}$

$x_p(t) = \dfrac{t}{2} + \dfrac{3}{2}$

$$\lim_{t \to +\infty} x_p(t) = \lim_{t \to +\infty}\left(\frac{t}{2} + \frac{3}{2}\right) = \underline{\underline{+\infty}}$$

2) $x'' + 2x' + x = \cos(2t), \ t \in \mathbb{R}$

a) $x'' + 2x' + x = 0$

Soluções do tipo $x(t) = e^{mt}$

$x'(t) = me^{mt}$

$x''(t) = m^2 e^{mt}$

Substituindo na EDO:

$$m^2 + 2m + 1 = 0$$

$$\Leftrightarrow m = \frac{-2 \pm \sqrt{4 - 4\times1\times1}}{2} \Leftrightarrow m = \frac{-2 \pm \sqrt{0}}{2} \Leftrightarrow m = -1 \text{ (multiplicidade 2)}$$

$x_H(t) = c_1 e^{-t} + c_2 t\, e^{-t}$, com c_1, c_2 c^{tes} arbitrárias

b) Conjunto CI $= \{\cos(2t), \operatorname{sen}(2t)\}$

$x_p(t) = A\cos(2t) + B\operatorname{sen}(2t)$

$x'_p(t) = -2A\operatorname{sen}(2t) + 2B\cos(2t)$

$x''_p(t) = -4A\cos(2t) - 4B\operatorname{sen}(2t)$

Substituindo ...

$$-4A\cos(2t) - 4B\,\text{sen}(2t) + 2\left(-2A\,\text{sen}(2t) + 2B\cos(2t)\right) + A\cos(2t) + B\,\text{sen}(2t) = \cos(2t)$$

$$\Leftrightarrow \underline{-4A\cos(2t)} - 4B\,\text{sen}(2t) - 4A\,\text{sen}(2t) + 4B\cos(2t) + \underline{A\cos(2t)} + B\,\text{sen}(2t) = \cos(2t)$$

$$\Leftrightarrow -3A\cos(2t) - 3B\,\text{sen}(2t) - 4A\,\text{sen}(2t) + 4B\cos(2t) = \cos(2t)$$

$$\Leftrightarrow \cos(2t)(-3A + 4B) + \text{sen}(2t)(-3B - 4A) = \cos(2t)$$

$$\begin{cases} -3A + 4B = 1 \\ -3B - 4A = 0 \end{cases} \Leftrightarrow \begin{cases} 4B = 1 + 3A \\ \text{———} \end{cases} \Leftrightarrow \begin{cases} B = \dfrac{1+3A}{4} \\ -3\left(\dfrac{1+3A}{4}\right) - 4A = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{———} \\ -\dfrac{3+3A}{4} - 4A = 0 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ -3+3A-16A = 0 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ -13A = 3 \end{cases} \Leftrightarrow \begin{cases} B = 11/26 \\ A = 3/13 \end{cases}$$

$$x_p(t) = \frac{3}{13}\cos(2t) + \frac{11}{26}\,\text{sen}(2t)$$

(c) $x(t) = c_1 e^{-t} + c_2 t e^{-t} + \dfrac{3}{13}\cos(2t) + \dfrac{11}{26}\,\text{sen}(2t)$, c_1, c_2 c^{tes} arbitrárias