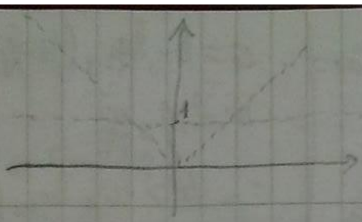
CAL Dúvidas – Teste

[1] $f(x) = \begin{cases} |x| & \text{se } x \in \mathbb{Q} \\ 1 & \text{se } x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$



c) $\lim_{x\to -1^-} f(x) = 1$ $\lim_{x\to -1^+} f(x) = 1$ $f(-1) = 1$

$\lim_{x\to -1} f(x) = 1$ A função f é contínua apenas em $x=-1$ e $x=1$

logo c)

[2] $\cos\left(\frac{\pi}{3} + \arcsen\left(-\frac{1}{2}\right)\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ logo é d)

[3] $\frac{sh(2x)}{2\,sh^2(x)} = \frac{\frac{e^{2x}-e^{-2x}}{2}}{2\times\frac{(e^{x}-e^{-x})^2}{4}} = \frac{e^{2x}-e^{-2x}}{(e^{x}-e^{-x})^2} = \frac{(e^{x})^2-(e^{-x})^2}{(e^{x}-e^{-x})^2} = \frac{(e^{x}-e^{-x})(e^{x}+e^{-x})}{(e^{x}-e^{-x})^2}$

$= \coth(x)$ logo a)

[4] $g(x) = f(x^2-1)$ $f(0)=1$ $f'(0)=-1$

$g'(1) = ?$ $g'(x) = 2x\, f'(x^2-1)$

$g'(1) = 2 f'(0) = -2$ e)

mas se tivesse duas retas com o mesmo declive?

$(1, g(1)) = (1, f(0)) = (1,1)$

$y-1 = -2(x-1) \Rightarrow y = -2x+3$

[5] $f(x) = x^5 + x^3 + 6$ $]-2,-1[$

$f(-2) = -32 - 8 + 6 < 0$
$f(-1) = -1 - 1 + 6 > 0$
f tem pelo menos, um zero em $]-2,-1[$

logo b)

prova: $f'(x) = 5x^4 + 3x^2 > 0, \forall x \in]-2,-1[$

$f'(x) = 0 \Leftrightarrow x^2\underbrace{(5x^2+3)}_{\text{positivo}} = 0 \Leftrightarrow x = 0$

[6] $F'(x) = (1+e^{x} - \ln(1+e^{x}))' = e^{x} - \frac{e^{x}}{1+e^{x}} = \frac{e^{x}+e^{2x}-e^{x}}{1+e^{x}}$

$\therefore \int \frac{e^{2x}}{1+e^{x}}\,dx = 1 + e^{x} - \ln(1+e^{x})$

[7] a) $\lim_{x\to 0} x \times \cos\frac{x+1}{x} = 0$ pq $\lim_{x\to 0} = 0$ e a função cosseno é limitada

b) $-1/6$ (pela regra de L'Hôpital (2x))

[8] $f(x) = \frac{\pi}{3} + 2 \times \arcsen\frac{1}{x}$

a) $Dom(f) = \{x \in \mathbb{R} : -1 \leq \frac{1}{x} \leq 1 \text{ e } x \neq 0\}$

$-1 \leq \frac{1}{x} \leq 1$ e $x \neq 0 \Leftrightarrow \frac{1}{x} \geq -1$ e $\frac{1}{x} \leq 1$, e $x \neq 0 \Leftrightarrow$

$\Leftrightarrow \boxed{\frac{1+x}{x} \geq 0}$ e $\boxed{\frac{1-x}{x} \leq 0}$ e $x \neq 0$

$\frac{1+x}{x}$

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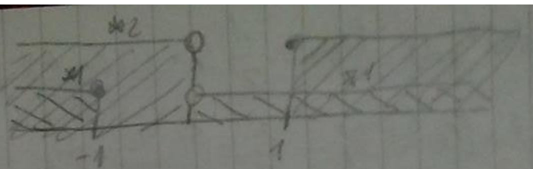
#1		-1		0	
$1+x$	$-$	0	$+$	$+$	$+$
x	$-$	$-$	$-$	0	$+$
	$+$	0	$-$	/	$+$

#2		0		1	
$1-x$	$+$	$+$	$+$	0	$-$
x	$-$	0	$+$	$+$	$+$
	$-$	/	$+$	0	$-$



Logo $x \leq -1$ ou $x \geq 1$ Por isso: $D_f =]-\infty, -1] \cup [1, +\infty[$

Para $x \in]-\infty, -1] \cup [1, +\infty[$,

$-\pi/2 \leq \operatorname{arcsen} \frac{1}{x} \leq \pi/2$ e $1/x \neq 0 \Leftrightarrow -\pi \leq 2 \operatorname{arcsen} \frac{1}{x} \leq \pi$ e $2 \operatorname{arcsen} \frac{1}{x} \neq 0$

$\Leftrightarrow -\frac{2}{3}\pi \leq \frac{\pi}{3} + 2\operatorname{arcsen} \frac{1}{x} \leq \frac{4}{3}\pi$ e $\frac{\pi}{3} + 2 \operatorname{arcsen} \frac{1}{x} \neq \frac{\pi}{3}$

$\operatorname{Im}(f) = [-2/3\pi, 4/3\pi] \setminus \{\pi/3\}$

b) $f^{-1}: [-2/3\pi, 4/3\pi] \setminus \{\pi/3\} \longrightarrow]-\infty, -1] \cup [1, +\infty[$
$x \longmapsto f^{-1}(x)$

$y = \pi/3 + 2 \operatorname{arcsen} \frac{1}{x} \Leftrightarrow \operatorname{arcsen} \frac{1}{x} = \frac{1}{2}\left(y - \frac{\pi}{3}\right) \Leftrightarrow \frac{1}{x} = \operatorname{sen}\left(\frac{y}{2} - \frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{1}{\operatorname{sen}(y/2 - \pi/6)}$

$\boxed{9}$ a) $f'(x) = (\operatorname{sh} \sqrt{x})' = (x^{1/2})' \operatorname{ch} \sqrt{x} = \frac{1}{2} x^{-1/2} \operatorname{ch} \sqrt{x} = \frac{\operatorname{ch} \sqrt{x}}{2\sqrt{x}}$

$f': \mathbb{R}^+ \longrightarrow \mathbb{R}$
$x \longmapsto f'(x) = \frac{\operatorname{ch} \sqrt{x}}{2\sqrt{x}}$

b) $y = \operatorname{sh}\sqrt{x} \Leftrightarrow y = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2} \Leftrightarrow e^{\sqrt{x}} - 2y - e^{-\sqrt{x}} = 0 \Leftrightarrow$

$\Leftrightarrow (e^{\sqrt{x}})^2 - 2y\, e^{\sqrt{x}} - 1 = 0 \Leftrightarrow e^{\sqrt{x}} = \frac{2y \pm \sqrt{4y^2+4}}{2} \Leftrightarrow$

$\Leftrightarrow e^{\sqrt{x}} = y \pm \sqrt{y^2+1}$ Como a exponencial $e^{\sqrt{x}} > 0$, $e^{\sqrt{x}} = y + \sqrt{y^2+1}$

$\Leftrightarrow \sqrt{x} = \ln(y + \sqrt{y^2+1}) \Leftrightarrow x = \ln^2(y + \sqrt{y^2+1})$

$\therefore f^{-1}(x) = \ln^2(x + \sqrt{x^2+1})$

c) $(f^{-1})'(x) = 2 \ln(x + \sqrt{x^2+1}) \times \frac{(x+\sqrt{x^2+1})'}{(x + \sqrt{x^2+1})^2} =$ passos intermédios $=$

$= 2 \times \frac{\ln(x + \sqrt{x^2+1})}{\sqrt{x^2+1}}$ $\quad (f^{-1})': \mathbb{R}^+ \longrightarrow \mathbb{R}$
$x \longmapsto (f^{-1})'(x) = 2 \times \frac{\ln(x + \sqrt{x^2+1})}{\sqrt{x^2+1}}$

$\boxed{9}$ a) $\int \frac{4x+2}{\sqrt{x^2+x+1}}\, dx = 2 \int (2x+1)(x^2+x+1)^{-1/2}\, dx =$

$= 2 \times \frac{(x^2+x+1)^{1/2}}{1/2} + C,\ 4\sqrt{x^2+x+1} + C$, com C uma constante real arbitrária

$$\int x^2 \ln(x^2+1)\,dx = \frac{1}{3}x^3\ln(x^2+1) - \int \frac{1}{3}x^3 \frac{2x}{x^2+1}\,dx =$$

$$= \frac{1}{3}x^3\ln(x^2+1) - \frac{2}{3}\int \frac{x^4}{x^2+1}\,dx$$

$$\frac{x^4}{x^2+1} = \frac{(x^2+1)^2 - 2x^2 - 1}{x^2+1} = x^2+1 - \frac{2(x^2+1)-1}{x^2+1} = x^2+1-2+\frac{1}{x^2+1} =$$

$$= x^2 - 1 + \frac{1}{x^2+1}$$

$$= \frac{1}{3}x^3\ln(x^2-1) - \frac{2}{3}\int\left(x^2 - 1 + \frac{1}{x^2+1}\right)dx =$$

$$= \frac{1}{3}x^3\ln(x^2-1) - \frac{2}{9}x^3 + \frac{2}{3}x - \frac{2}{3}\operatorname{arctg}(x^2+1) + c, \text{ con } c \in$$