4.

Exercício 4.1

```
MatrizJacobiana[funcao_List?VectorQ, variaveis_List] :=
 D[funcao, {variaveis, 1}] /. MapThread[Rule, {variaveis, {##}}] &
Derivada[funcao_List?VectorQ, variaveis_List, ponto_List] :=
 Dot[MatrizJacobiana[funcao, variaveis][Sequence @@ ponto], {##}] &
```

a)

```
f[x_, y_] = {x, y, x y};
```

$$Jf(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ y & x \end{pmatrix}$$

b)

```
f[x_, y_] = {x Exp[y] + Cos[y], x, x + Exp[y]};
```

$$Jf(x,y) = \begin{pmatrix} e^y & e^y x - \mathrm{Sin}[y] \\ 1 & 0 \\ 1 & e^y \end{pmatrix}$$

c)

```
f[x_, y_] = {x y Exp[x y], x Sin[y], 5 x y^2};
```

$$Jf(x,y) = \begin{pmatrix} e^{x y} y + e^{x y} x y^2 & e^{x y} x + e^{x y} x^2 y \\ \mathrm{Sin}[y] & x\,\mathrm{Cos}[y] \\ 5 y^2 & 10 x y \end{pmatrix}$$

d)

```
f[x_, y_, z_] = {x - y, y + z};
```

$$Jf(x,y,z) = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

e)

```
f[x_, y_, z_] = {x + y + Exp[z], x^2 y};
```

$$Jf(x,y,z) = \begin{pmatrix} 1 & 1 & e^z \\ 2xy & x^2 & 0 \end{pmatrix}$$

Exercício 4.2

```
f[x_, y_, z_] = {x - y + z, x^2 y z, x y z};
g[x_, y_, z_] = {x y, y z, 2 x, x y z};
```

a)

$Df((-1,0,-1);(2,3,-1)) = \{-2, -3, 3\}$

$Dg((-1,0,-1);(2,3,-1)) = \{-3, -3, 4, 3\}$

b)

$Df(-1,0,-1)(x,y,z) = \{x - y + z, -y, y\}$

$Dg(-1,0,-1)(x,y,z) = \{-y, -y, 2x, y\}$

Exercício 4.3

```
f[x_, y_] = {3 x, x + 2 y};
```

a)

$$Jf(x,y) = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$$

b)

A função f é de classe C^1.

c)

$Df(1,2)(x,y) = \{3x, x + 2y\}$

d)

$Df(x_0,y_0)(x,y) = \{3x, x + 2y\}$

Exercício 4.4

```
f[x_, y_] = {2 x^2, 3 y, 2 x y};
```

a)

$$Jf(x,y) = \begin{pmatrix} 4x & 0 \\ 0 & 3 \\ 2y & 2x \end{pmatrix}$$

b)

A função f é de classe C^1.

$\{4x,\ 3y,\ 2x + 2y\}$

c)

$\{8,\ 9,\ 10\}$

Exercício 4.5

a)

```
f[x_, y_] = 2 x y / (x^2 + y^2) ^2;
```

$$\text{Hess } f(x,y) = \begin{pmatrix} \frac{24 y (x^3 - x y^2)}{(x^2+y^2)^4} & -\frac{6 (x^4 - 6 x^2 y^2 + y^4)}{(x^2+y^2)^4} \\ -\frac{6 (x^4 - 6 x^2 y^2 + y^4)}{(x^2+y^2)^4} & \frac{24 x y (-x^2+y^2)}{(x^2+y^2)^4} \end{pmatrix}$$

b)

```
f[x_, y_] = Cos[x y^2];
```

$$\text{Hess } f(x,y) = \begin{pmatrix} -y^4 \operatorname{Cos}[x y^2] & -2 y (x y^2 \operatorname{Cos}[x y^2] + \operatorname{Sin}[x y^2]) \\ -2 y (x y^2 \operatorname{Cos}[x y^2] + \operatorname{Sin}[x y^2]) & -2 x (2 x y^2 \operatorname{Cos}[x y^2] + \operatorname{Sin}[x y^2]) \end{pmatrix}$$

c)

```
f[x_, y_] = Exp[-x y^2] + y^3 x^4;
Print["Hess f(x,y) = ", D[f[x, y], {{x, y}, 2}] // Simplify // MatrixForm]
```

$$\text{Hess } f(x,y) = \begin{pmatrix} y^3 (12 x^2 + e^{-x y^2} y) & 2 e^{-x y^2} y (-1 + 6 e^{x y^2} x^3 y + x y^2) \\ 2 e^{-x y^2} y (-1 + 6 e^{x y^2} x^3 y + x y^2) & 2 e^{-x y^2} x (-1 + 3 e^{x y^2} x^3 y + 2 x y^2) \end{pmatrix}$$

d)

```
f[x_, y_] = 1 / (Cos[x]^2 + Exp[-y]);
```

$$\text{Hess } f(x,y) = \begin{pmatrix} \frac{2 e^{2 y} \left(e^{y} \cos[x]^4 - \sin[x]^2 + \cos[x]^2 \left(1+3 e^{y} \sin[x]^2\right)\right)}{\left(1+e^{y} \cos[x]^2\right)^3} & \frac{4 e^{2 y} \cos[x] \sin[x]}{\left(1+e^{y} \cos[x]^2\right)^3} \\ \frac{4 e^{2 y} \cos[x] \sin[x]}{\left(1+e^{y} \cos[x]^2\right)^3} & -\frac{e^{y} \left(-1+e^{y} \cos[x]^2\right)}{\left(1+e^{y} \cos[x]^2\right)^3} \end{pmatrix}$$

Exercício 4.6

```
g[x_, t_] = 2 + Exp[-t] Sin[x];
```

$$\frac{\partial g}{\partial t}(x,t) = -e^{-t} \sin[x]$$

$$\frac{\partial^2 g}{\partial x^2}(x,t) = -e^{-t} \sin[x]$$

Exercício 4.7

```
f[x, y, z, w] = Exp[x y z] Sin[x w];
```

$$f_{xzw} = e^{x y z}\, x\, y\, ((2 + x y z) \cos[w x] - w x \sin[w x])$$

$$f_{zwx} = e^{x y z}\, x\, y\, ((2 + x y z) \cos[w x] - w x \sin[w x])$$

Exercício 4.8

a)

```
D[2 x^3, y]
```

0

```
D[x^2 y + x, y]
```

x^2

A função f seria de classe C^2 mas $f_{xy} \neq f_{yx}$

b)

```
D[x Sin[y], y]
x Cos[y]
D[y Sin[x], y]
Sin[x]
```

A função f seria de classe C^2 mas $f_{xy} \neq f_{yx}$

Exercício 4.9

```
f[0, 0] = 0;
f[x_, y_] = x y^3 / (x^2 + y^2);
```

a)

$$f_x[0,0] = 0$$

$$f_x[x,y] = \frac{y^3\left(-x^2+y^2\right)}{\left(x^2+y^2\right)^2}, \quad (x,y) \neq (0,0)$$

$$f_y[0,0] = 0$$

$$f_y[x,y] = \frac{x\left(3x^2y^2+y^4\right)}{\left(x^2+y^2\right)^2}, \quad (x,y) \neq (0,0)$$

b)

$$f_{xy}[0,0] = 1$$

$$f_{yx}[0,0] = 0$$

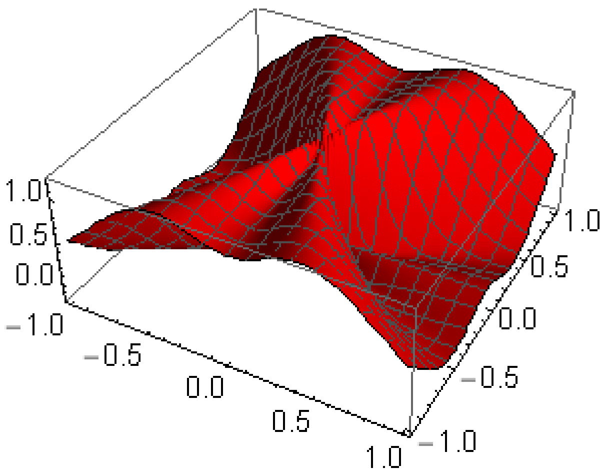
c)

$$f_{xy}[x,y] = \frac{-3x^4y^2+6x^2y^4+y^6}{\left(x^2+y^2\right)^3}, \quad (x,y) \neq (0,0)$$

```
Limit[f_xy[x, x], x → 0]
```

$$\frac{1}{2}$$

Conclui-se assim que f não é de classe C^2.



Exercício 4.10

```
u[x_, y_] = x y;
v[x_, y_] = Sin[x y];
w[x_, y_] = Exp[x];
f[x_, y_, z_] = x^2 y + y^2 z;
h[x_, y_] = f[u[x, y], v[x, y], w[x, y]]
Grad[h[x, y], {x, y}] // Simplify
```

$$\{x^2 y^3 \operatorname{Cos}[x y] + \operatorname{Sin}[x y] \left(2 x y^2 + 2 e^x y \operatorname{Cos}[x y] + e^x \operatorname{Sin}[x y]\right), x \left(x^2 y^2 \operatorname{Cos}[x y] + 2 x y \operatorname{Sin}[x y] + e^x \operatorname{Sin}[2 x y]\right)\}$$

Exercício 4.11

```
f[x_, y_, z_] = x^2 y - x z;
h[t_] = {a t^2, a t, t^3};
```

a)

$$Df(1,0,0)(1,2,2) = 0$$

b)

$$g'(t) = 5 a \left(-1 + a^2\right) t^4$$

$$g'(t) = 0 \Leftrightarrow a \in \{-1, 0, 1\}$$

Exercício 4.13

a)

```
u[x_, y_] = Log[Sin[x / y]]; x[t_] = 3 t^2; y[t_] = Sqrt[1 + t^2];
```

$$\frac{du}{dt} = \left(-\frac{3\,t^3}{\left(1+t^2\right)^{3/2}} + \frac{6\,t}{\sqrt{1+t^2}}\right) \mathrm{Cot}\left[\frac{3\,t^2}{\sqrt{1+t^2}}\right]$$

b)

```
w[r_, s_] = r^2 + s^2; r[p_, q_] = p q^2; s[p_, q_] = p^2 Sin[q];
```

$$\frac{\partial w}{\partial p} = 2\,p\left(q^4 + 2\,p^2\,\mathrm{Sin}[q]^2\right)$$

$$\frac{\partial w}{\partial q} = 4\,p^2\,q^3 + p^4\,\mathrm{Sin}[2\,q]$$

c)

```
z[x_, y_] = x^2 Sin[y]; x[s_, t_] = s^2 + t^2; y[s_, t_] = 2 s t;
```

$$\frac{\partial z}{\partial s} = 2\left(s^2+t^2\right)\left(t\left(s^2+t^2\right)\mathrm{Cos}[2\,s\,t] + 2\,s\,\mathrm{Sin}[2\,s\,t]\right)$$

$$\frac{\partial z}{\partial t} = 2\left(s^2+t^2\right)\left(s\left(s^2+t^2\right)\mathrm{Cos}[2\,s\,t] + 2\,t\,\mathrm{Sin}[2\,s\,t]\right)$$