9.) $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in A: \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} \alpha - \beta \\ 2\alpha + \beta \\ \beta \\ \alpha + \beta \end{pmatrix}$

$$\begin{array}{cc} \alpha & \beta \end{array}$$
$$\left(\begin{array}{cc|c} 1 & -1 & x \\ 2 & 1 & y \\ 0 & 1 & z \\ 1 & 1 & t \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x \\ 0 & 3 & y-2x \\ 0 & 1 & z \\ 0 & 2 & t-x \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x \\ 0 & 1 & z \\ 0 & 3 & y-2x \\ 0 & 2 & t-x \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x \\ 0 & 1 & z \\ 0 & 0 & y-2x-3z \\ 0 & 0 & t-x-2z \end{array}\right)$$

$c(A)=2$ para que o sistema seja possível, temos que ter $c(A|b)=2$

$y-2x-3z=0 \Rightarrow y=2x+3z$

$t-x-2z=0 \Rightarrow t=x+2z$

$\begin{cases} \alpha-\beta=x \\ \beta=z \end{cases} \quad \begin{cases} \alpha=x+z \\ \beta=z \end{cases}$

$\begin{pmatrix} x+z \\ 2x+2z \\ 0 \\ x+z \end{pmatrix} + \begin{pmatrix} -z \\ z \\ z \\ z \end{pmatrix} = \begin{pmatrix} x \\ 2x+3z \\ z \\ x+2z \end{pmatrix}$ $\qquad \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in A : A = \left\{ \begin{pmatrix} x \\ 2x+3z \\ z \\ x+2z \end{pmatrix} : x,z \in \mathbb{R} \right\}$

$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in B : \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 2\beta \\ 2\beta \\ \alpha+2\beta \\ \alpha+2\beta \end{pmatrix}$

$\Rightarrow \begin{cases} 2\beta = x \\ 2\beta = y \\ \alpha+2\beta = z \\ \alpha+2\beta = t \end{cases} \qquad B = \left\{ \begin{pmatrix} x \\ x \\ z \\ z \end{pmatrix} : x,z \in \mathbb{R} \right\}$

$\boxed{A \cap B}$ $\quad a \in A: \; a = \begin{pmatrix} x \\ 2x+3z \\ z \\ x+2z \end{pmatrix}$

$a \in B: \; a = \begin{pmatrix} x \\ x \\ z \\ t \end{pmatrix}$

$\Rightarrow \begin{cases} x = 2x+3z \\ z = x+2z \end{cases} \quad \begin{cases} -x = 3z \\ z = -3z+2z \end{cases}$

$\begin{cases} x=0 \\ 2z=0 \end{cases} \qquad A \cap B = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

10.)

a)

i) $x - y = 0 \Rightarrow x = y$ $\qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y \\ y \\ -2y \end{pmatrix} \in V_3$

$2y + z = 0 \Rightarrow z = -2y$

ii) $y + z = 0 \Rightarrow y = -z \quad y = 0$ $\qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \in V_2$

$y - z = 0 \Rightarrow -2z = 0 \quad z = 0$

b.) i) $a \in V_2 \quad \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$

satisfaz as condições dos 2 vetores

$\Rightarrow \quad x = 0$

$z = 0$

$a \in V_4 \quad \begin{pmatrix} x \\ -x \\ z \end{pmatrix}$

$a \in V_2 \cap V_4 = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

logo $V_2 \cap V_4$ é subespaço $\mathbb{R}^3$.
(A intersecção é sempre um subespaço).

ii)

$V_3 \quad \begin{pmatrix} a \\ a \\ -2a \end{pmatrix}$

$V_2 \quad \begin{pmatrix} b \\ a \\ a \end{pmatrix}$

$$\begin{pmatrix} a \\ a \\ -2a \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a+b \\ a \\ -2a \end{pmatrix} \notin V_2 \cup V_3$$

$$\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \notin V_2 \cup V_3$$

logo $V_2 \cup V_3$ não é um subespaço de $\mathbb{R}^3$.

iii) $V_2 \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$

$V_1 \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

$V_2 \in V_1$ e assim $V_2 \cup V_1 = V_1$ e de acordo com o enunciado $V_1$ é subespaço logo $V_2 \cup V_1$ é subespaço de $\mathbb{R}^3$.