3

Exercício 3.1

```
f[x_, y_] = x^2 y;
```

$\partial_x f(0,0) = \quad 0$

$\partial_x f(x_0, y_0) = \quad 2\, x_0\, y_0$

$\partial_y f(1,2) = \quad 1$

$\partial_y f(x_0, y_0) = \quad x_0^2$

Exercício 3.2

```
f[x_, y_] = 3 x^2 + 2 y^2;
```

$\partial_x f = \quad 6\, x$

$\partial_y f = \quad 4\, y$

```
f[x_, y_] = Sin[x^2 - 3 x y];
```

$\partial_x f = \quad (2\, x - 3\, y)\, \mathrm{Cos}[x^2 - 3\, x\, y]$

$\partial_y f = \quad -3\, x\, \mathrm{Cos}[x^2 - 3\, x\, y]$

```
f[x_, y_] = x^2 y^2 Exp[2 x y];
```

$\partial_x f = \quad 2\, e^{2\, x\, y}\, x\, y^2 + 2\, e^{2\, x\, y}\, x^2\, y^3$

$\partial_y f = \quad 2\, e^{2\, x\, y}\, x^2\, y + 2\, e^{2\, x\, y}\, x^3\, y^2$

```
f[x_, y_] = Exp[x] Log[x y];
```

$$\partial_x f = \frac{e^x}{x} + e^x \, \mathrm{Log}[x\, y]$$

$$\partial_y f = \frac{e^x}{y}$$

```
f[x_, y_] = Exp[Sin[x Sqrt[y]]];
```

$$\partial_x f = e^{\mathrm{Sin}\left[x \sqrt{y}\right]} \sqrt{y}\; \mathrm{Cos}\left[x \sqrt{y}\right]$$

$$\partial_y f = \frac{e^{\mathrm{Sin}\left[x \sqrt{y}\right]}\, x\, \mathrm{Cos}\left[x \sqrt{y}\right]}{2 \sqrt{y}}$$

$$\mathtt{f[x\_,\ y\_]} = \frac{x^2 + y^2}{x^2 - y^2};$$

$$\partial_x f = -\frac{4\, x\, y^2}{(x^2 - y^2)^2}$$

$$\partial_y f = \frac{4\, x^2\, y}{(x^2 - y^2)^2}$$

```
f[x_, y_] = x Cos[x] Cos[y];
```

$\partial_x f =$ Cos[y] (Cos[x]-x Sin[x])

$\partial_y f =$ -x Cos[x] Sin[y]

$$\mathtt{f[x\_,\ y\_]} = \mathrm{ArcTan}\left[x^2\, y^3\right];$$

$$\partial_x f = \frac{2\, x\, y^3}{1 + x^4\, y^6}$$

$$\partial_y f = \frac{3\, x^2\, y^2}{1 + x^4\, y^6}$$

```
f[x_, y_] = x + x y^2 + Log[Sin[x^2 + y]];
```

$\partial_x f =$ $1 + y^2 + 2\, x\, \mathrm{Cot}[x^2 + y]$

$\partial_y f =$ $2\, x\, y + \mathrm{Cot}[x^2 + y]$

```
f[x_, y_, z_] = z Exp[x^2 + y^2];
```

$\partial_x f =$ $2\, e^{x^2+y^2}\, x\, z$

$\partial_y f =$ $2\, e^{x^2+y^2}\, y\, z$

$\partial_z f =$ $e^{x^2+y^2}$

```
f[x_, y_, z_] = Log[Exp[x] + z^y];
```

$\partial_x f =$ $\dfrac{e^x}{e^x + z^y}$

$\partial_y f =$ $\dfrac{z^y\, \mathrm{Log}[z]}{e^x + z^y}$

$\partial_z f =$ $\dfrac{y\, z^{-1+y}}{e^x + z^y}$

```
f[x_, y_, z_] = (x y^3 + Exp[z])/(x^3 y - Exp[z]);
```

$\partial_x f =$ $-\dfrac{y\, (2\, x^3\, y^3 + e^z\, (3\, x^2 + y^2))}{(e^z - x^3\, y)^2}$

$\partial_y f =$ $\dfrac{2\, x^4\, y^3 - e^z\, x\, (x^2 + 3\, y^2)}{(e^z - x^3\, y)^2}$

$\partial_z f =$ $\dfrac{e^z\, x\, y\, (x^2 + y^2)}{(e^z - x^3\, y)^2}$

Exercício 3.3

a)

$\partial_x f(0,0) = \quad 0$

$\partial_y f(0,0) = \quad 0$

b)

$\partial_x f(0,0) = \quad 0$

$\partial_y f(0,0) = \quad 0$

Exercício 3.4

a)

```
f[x_, y_] = Exp[x y];
```

$x\partial_x f = \quad e^{x y}\, x\, y$

$y\partial_y f = \quad e^{x y}\, x\, y$

b)

```
f[x_, y_] = Log[x^2 + y^2 + x y];
```

$$x\partial_x f = \quad \frac{x\,(2\,x + y)}{x^2 + x\,y + y^2}$$

$$y\partial_y f = \quad \frac{y\,(x + 2\,y)}{x^2 + x\,y + y^2}$$

$x\partial_x f \ + \ y\partial_y f \ = \ 2$

c)

```
f[x_, y_, z_] = x + (x - y)/(y - z);
```

$\partial_x f = 1 + \frac{1}{y - z}$

$\partial_y f = \frac{-x + z}{(y - z)^2}$

$\partial_z f = \frac{x - y}{(y - z)^2}$

$\partial_x f + \partial_y f + \partial_z f = 1$

Exercício 3.5

```
Ddirecao[f_, P_, u_] := Limit[(f @@ (P + h u) - f @@ P)/h, h → 0];
```

a)

```
f[x_, y_] = x^2 y + x; P = {1, 0}; u = {1, 1};
Ddirecao[f, P, Normalize[u]]
```

$\sqrt{2}$

b)

```
f[x_, y_] = x^2 Sin[2 y]; P = {1, π/2}; u = {3, -4};
Ddirecao[f, P, Normalize[u]]
```

$\frac{8}{5}$

c)

```
f[x_, y_, z_] = x^2 + y^2 + z^2; P = {1, 2, 3}; u = {1, 1, 1};
Ddirecao[f, P, Normalize[u]]
```

$4\sqrt{3}$

Exercício 3.6

a)

```
Grad[x Exp[-x + y], {x, y}]
```

$\{e^{-x+y} - e^{-x+y} x,\ e^{-x+y} x\}$

b)

```
Grad[x Exp[-x^2 - y^2 - z^2], {x, y, z}]
```

$$\left\{e^{-x^2-y^2-z^2} - 2\,e^{-x^2-y^2-z^2}\,x^2,\ -2\,e^{-x^2-y^2-z^2}\,x\,y,\ -2\,e^{-x^2-y^2-z^2}\,x\,z\right\}$$

c)

```
Grad[x y z/(x^2 + y^2 + z^2 + 1), {x, y, z}] // Simplify
```

$$\left\{\frac{y\,z\,(1-x^2+y^2+z^2)}{(1+x^2+y^2+z^2)^2},\ \frac{x\,z\,(1+x^2-y^2+z^2)}{(1+x^2+y^2+z^2)^2},\ \frac{x\,y\,(1+x^2+y^2-z^2)}{(1+x^2+y^2+z^2)^2}\right\}$$

d)

```
Grad[z^2 Exp[x] Cos[y], {x, y, z}] // Simplify
```

$$\{e^x\,z^2\,\mathrm{Cos}[y],\ -e^x\,z^2\,\mathrm{Sin}[y],\ 2\,e^x\,z\,\mathrm{Cos}[y]\}$$

Exercício 3.7

```
planoTangente[f_, a_, b_] :=
  f[a, b] + Derivative[1, 0][f][a, b] (x - a) + Derivative[0, 1][f][a, b] (y - b);
f[x_, y_] = x^2 + y^3;
```

$z = -11 + 6\,x + 3\,y$

Exercício 3.8

```
f[x_, y_] = x^2 + y^2;
```

$z = 0$

```
g[x_, y_] = -x^2 - y^2 + x y^3;
```

$z = 0$

Exercício 3.9

```
f[x_, y_] = Exp[x - y];
```

$z = 1 + x - y$

```
Solve[z == planoTangente[f, 1, 1] && x == 0 && y == 0, {x, y, z}]
```

$\{\{x \to 0,\ y \to 0,\ z \to 1\}\}$

Exercício 3.10

Ver diapositivos

Exercício 3.11

Ver diapositivos

Exercício 3.12

a)

```
f[x_, y_] = x^2 y/(x^2 + y^2);
f[0, 0] = 0;
Simplify[Ddirecao[f, {0, 0}, Normalize@{u1, u2}], Element[u1 | u2, Reals]]
```

$$\frac{u1^2\ u2}{\left(u1^2+u2^2\right)^{3/2}}$$

```
Simplify[Ddirecao[f, {x, y}, Normalize@{u1, u2}], Element[u1 | u2, Reals]]
```

$$\frac{x\ \left(2\ u1\ y^3+u2\ \left(x^3-x\ y^2\right)\right)}{\sqrt{u1^2+u2^2}\ \left(x^2+y^2\right)^2}$$

b)

f não é diferenciável na origem

Exercício 3.13

a)

```
f[x_, y_] = (x^2 + y^2) Sin[1/Sqrt[x^2 + y^2]];
f[0, 0] = 0;
```

$\partial_x f(0,0) = 0$

$\partial_y f(0,0) = 0$

b)

$$\partial_x f(x,y) = -\frac{x\ \mathrm{Cos}\left[\frac{1}{\sqrt{x^2+y^2}}\right]}{\sqrt{x^2+y^2}} + 2\ x\ \mathrm{Sin}\left[\frac{1}{\sqrt{x^2+y^2}}\right]$$

$$\partial_y f(x,y) = -\frac{y\ \mathrm{Cos}\left[\frac{1}{\sqrt{x^2+y^2}}\right]}{\sqrt{x^2+y^2}} + 2\ y\ \mathrm{Sin}\left[\frac{1}{\sqrt{x^2+y^2}}\right]$$