Universidade do Minho
Dep. de Matemática

Cálculo I

2009/2010

Folha 5

Exercício 5.1 Estabeleça as seguintes igualdades:

a) $\cos^2 x = \dfrac{\cos 2x + 1}{2}, \quad x \in \mathbb{R};$

b) $\operatorname{sen}^2 x = \dfrac{1 - \cos 2x}{2}, \quad x \in \mathbb{R}.$

Exercício 5.2 Calcule:

a) $\operatorname{sen}(\operatorname{arcsen}(-1/2));$

b) $\operatorname{arcsen}(\operatorname{sen}(7\pi/6));$

c) $\cos(\arccos(\sqrt{3}/2));$

d) $\arccos(\cos(-\pi/3));$

e) $\operatorname{arctg}(\operatorname{tg}(-\pi/4));$

f) $\operatorname{tg}(\operatorname{arctg}(-1)).$

Exercício 5.3 Deduza as seguintes igualdades em domínios que deverá especificar:

a) $\operatorname{sen}(\arccos x) = \sqrt{1-x^2};$

b) $\operatorname{tg}(\arccos x) = \dfrac{\sqrt{1-x^2}}{x};$

c) $\cos(\operatorname{arcsen} x) = \sqrt{1-x^2};$

d) $\operatorname{tg}(\operatorname{arcsen} x) = \dfrac{x}{\sqrt{1-x^2}};$

e) $\operatorname{sen}(\operatorname{arctg} x) = \dfrac{x}{\sqrt{1+x^2}};$

f) $\cos(\operatorname{arctg} x) = \dfrac{1}{\sqrt{1+x^2}}.$

Exercício 5.4 Resolva as seguintes equações:

a) $e^x = e^{1-x};$

b) $e^{2x} + 2e^x - 3 = 0;$

c) $e^{3x} - 2e^{-x} = 0;$

d) $\ln(x^2 - 1) + 2\ln 2 = \ln(4x - 1).$

Exercício 5.5 Calcule, onde existirem, as derivadas das funções definidas por:

a) $f(x) = \operatorname{ch}(3x);$

b) $f(x) = \operatorname{sh}(x^2 + 1);$

c) $f(x) = x^2 \operatorname{sh}^3 x;$

d) $f(x) = \ln(\operatorname{ch}(x+1)).$

Exercício 5.6 Recorde que sh $x = \frac{e^x - e^{-x}}{2}$ e ch $x = \frac{e^x + e^{-x}}{2}$. Prove que:

a) $\text{ch}^2 x - \text{sh}^2 x = 1$;

b) $\text{ch } x + \text{sh } x = e^x$;

c) $\text{sh}(-x) = -\text{sh } x$;

d) $\text{ch}(-x) = \text{ch } x$;

e) $\text{sh}(x + y) = \text{sh } x \text{ ch } y + \text{ch } x \text{ sh } y$;

f) $\text{ch}(x + y) = \text{ch } x \text{ ch } y + \text{sh } x \text{ sh } y$;

g) $\text{th}^2 x + \dfrac{1}{\text{ch}^2 x} = 1$;

h) $\text{coth}^2 x - \dfrac{1}{\text{sh}^2 x} = 1$.

Exercício 5.7 Calcule, onde existirem, as derivadas das funções definidas por:

a) $f(x) = \text{ch}(3x)$;

b) $f(x) = \text{sh}(x^2 + 1)$;

c) $f(x) = x^2 \text{ sh}^3 x$;

d) $f(x) = \ln(\text{ch}(x + 1))$.

Exercício 5.8 Verifique que:

a) $\text{argsh } x = \ln\left(x + \sqrt{x^2 + 1}\right), \quad x \in \mathbb{R}$;

b) $\text{argch } x = \ln\left(x + \sqrt{x^2 - 1}\right), \quad x \in [1, +\infty[$;

c) $\text{argth } x = \ln\left(\sqrt{\frac{1+x}{1-x}}\right), \quad x \in\ ]-1, 1[$;

d) $\text{argcoth } x = \ln\left(\sqrt{\frac{x+1}{x-1}}\right), \quad x \in \mathbb{R} \backslash]-1, 1[$.

Exercício 5.9 Mostre que:

a) $\text{argsh}' x = \dfrac{1}{\sqrt{x^2 + 1}}, \quad x \in \mathbb{R}$;

b) $\text{argch}' x = \dfrac{1}{\sqrt{x^2 - 1}}, \quad x \in\ ]1, +\infty[$;

c) $\text{argth}' x = \dfrac{1}{1 - x^2}, \quad |x| < 1$;

d) $\text{argcoth}' x = \dfrac{1}{1 - x^2}, \quad |x| > 1$.