# introdução aos sistemas dinâmicos

## tabela de primitivas

na tabela abaixo, omitiram-se sistematicamente as constantes arbitrárias de integração.

$$\int u^n du = \frac{u^{n+1}}{n+1}, \quad n \neq -1$$

$$\int \frac{du}{u} = \log|u|$$

$$\int \frac{du}{u \log u} = \log|\log u|$$

$$\int a^u \, du = \frac{a^u}{\log a}, \quad a > 0,\ a \neq 1$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right)$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a} \log\left|\frac{a+u}{a-u}\right|, \quad u^2 \neq a^2$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arccos}\left(\frac{a}{u}\right), \quad u > a > 0$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arcsen}\left(\frac{u}{a}\right), \quad u^2 < a^2$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \log(u + \sqrt{u^2+a^2})$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \log|u + \sqrt{u^2-a^2}|, \quad u^2 > a^2$$

$$\int u\, e^{au} \, du = \frac{au-1}{a^2} e^{au}$$

$$\int u^n \log(u) \, du = \frac{u^{n+1}}{n+1}\left(\log(u) - \frac{1}{n+1}\right), \quad u > 0,\ n \neq -1$$

$$\int \operatorname{sen}^2(u) \, du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen}(2u)$$

$$\int \operatorname{sen}^n(u) \, du = -\frac{\operatorname{sen}^{n-1}(u)\cos(u)}{n} + \frac{n-1}{n}\int \operatorname{sen}^{n-2}(u) \, du$$

$$\int \cos^2(u) \, du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen}(2u)$$

$$\int \cos^n(u) \, du = \frac{\cos^{n-1}(u)\operatorname{sen}(u)}{n} + \frac{n-1}{n}\int \cos^{n-2}(u) \, du$$

$$\int u \operatorname{sen}(u) \, du = \operatorname{sen}(u) - u\cos(u)$$

$$\int u^n \operatorname{sen}(u) \, du = -u^n \cos(u) + n\int u^{n-1}\cos(u) \, du$$

$$\int u \cos(u) \, du = \cos(u) + u\operatorname{sen}(u)$$

$$\int u^n \cos(u) \, du = u^n \operatorname{sen}(u) - n\int u^{n-1}\operatorname{sen}(u) \, du$$

$$\int \tan(u)\,du \;=\; -\log|\cos(u)| \qquad \int \sec(u)\,du \;=\; \log|\sec(u)+\tan(u)|$$

$$\int \cot(u)\,du \;=\; \log|\operatorname{sen}(u)| \qquad \int \csc(u)\,du \;=\; -\log|\csc(u)+\cot(u)|$$

$$\int \sec^2(u)\,du \;=\; \tan(u) \qquad \int \csc^2(u)\,du \;=\; -\cot(u)$$

$$\int \tan^2(u)\,du \;=\; \tan(u)-u \qquad \int \cot^2(u)\,du \;=\; -\cot(u)-u$$

$$\int \operatorname{sen}(au)\operatorname{sen}(bu)\,du \;=\; -\frac{\operatorname{sen}\big((a+b)u\big)}{2(a+b)}+\frac{\operatorname{sen}\big((a-b)u\big)}{2(a-b)}\,,\quad a^2\neq b^2$$

$$\int \cos(au)\cos(bu)\,du \;=\; \frac{\operatorname{sen}\big((a+b)u\big)}{2(a+b)}+\frac{\operatorname{sen}\big((a-b)u\big)}{2(a-b)}\,,\quad a^2\neq b^2$$

$$\int \operatorname{sen}(au)\cos(bu)\,du \;=\; -\frac{\cos\big((a+b)u\big)}{2(a+b)}-\frac{\cos\big((a-b)u\big)}{2(a-b)}\,,\quad a^2\neq b^2$$

$$\int \mathrm{e}^{au}\cos(nu)\,du=\frac{\mathrm{e}^{au}\big(a\,\cos(nu)+n\operatorname{sen}(nu)\big)}{a^2+n^2}$$

$$\int \mathrm{e}^{au}\operatorname{sen}(nu)\,du=\frac{\mathrm{e}^{au}\big(a\operatorname{sen}(nu)-n\,\cos(nu)\big)}{a^2+n^2}$$

**Observação:** Se $u$ é uma função de $x$, $u=g(x)$, tem-se

$$\int f(u)\,du=\int f(g(x))\,g'(x)\,dx.$$