



Exercício 5.1 Estabeleça as seguintes igualdades:

a) $\cos^2 x = \frac{\cos 2x + 1}{2}, \quad x \in \mathbb{R};$

b) $\sin^2 x = \frac{1 - \cos 2x}{2}, \quad x \in \mathbb{R}.$

Exercício 5.2 Calcule:

a) $\cos(\arccos(\frac{1}{8}));$

f) $\arcsen(\sin(-\frac{\pi}{6}));$

b) $\arctg(\tg(\frac{9\pi}{4}));$

g) $\arcsen(\sin \frac{23\pi}{6});$

c) $\arcsen(\sin(\frac{5\pi}{4}));$

h) $\arccos(\cos(-\frac{\pi}{3}));$

d) $\sin(\arcsen(-\frac{1}{2}));$

i) $\arctg(\tg \pi);$

e) $\sin(\arcsen(1) + \pi);$

j) $\tg(\arctg(-1)).$

Exercício 5.3 Deduza as seguintes igualdades em domínios que deverá especificar:

a) $\sin(\arccos x) = \sqrt{1 - x^2};$

d) $\tg(\arcsen x) = \frac{x}{\sqrt{1 - x^2}};$

b) $\tg(\arccos x) = \frac{\sqrt{1 - x^2}}{x};$

e) $\sin(\arctg x) = \frac{x}{\sqrt{1 + x^2}};$

c) $\cos(\arcsen x) = \sqrt{1 - x^2};$

f) $\cos(\arctg x) = \frac{1}{\sqrt{1 + x^2}}.$

Exercício 5.4 Recorde que $\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$ e que $\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$. Prove que:

a) $\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1;$

e) $\operatorname{sh}(x + y) = \operatorname{sh} x \operatorname{ch} y + \operatorname{ch} x \operatorname{sh} y;$

b) $\operatorname{ch} x + \operatorname{sh} x = e^x;$

f) $\operatorname{ch}(x + y) = \operatorname{ch} x \operatorname{ch} y + \operatorname{sh} x \operatorname{sh} y;$

c) $\operatorname{sh}(-x) = -\operatorname{sh} x;$

g) $\operatorname{th}^2 x + \frac{1}{\operatorname{ch}^2 x} = 1;$

d) $\operatorname{ch}(-x) = \operatorname{ch} x;$

h) $\operatorname{coth}^2 x - \frac{1}{\operatorname{sh}^2 x} = 1.$

Exercício 5.5 Verifique que:

- a) $\operatorname{argsh} x = \ln \left(x + \sqrt{x^2 + 1} \right), \quad x \in \mathbb{R};$
- b) $\operatorname{argch} x = \ln \left(x + \sqrt{x^2 - 1} \right), \quad x \in [1, +\infty[;$
- c) $\operatorname{argth} x = \ln \left(\sqrt{\frac{1+x}{1-x}} \right), \quad x \in]-1, 1[;$
- d) $\operatorname{argcoth} x = \ln \left(\sqrt{\frac{x+1}{x-1}} \right), \quad x \in \mathbb{R} \setminus]-1, 1[.$

Exercício 5.6 Seja $f : \mathbb{R} \longrightarrow \mathbb{R}$ a função definida por

$$f(x) = \begin{cases} 0 & \text{se } x \leq -1, \\ \operatorname{arcsen} x & \text{se } -1 < x < 1, \\ \frac{\pi}{2} \operatorname{sen} \left(\frac{\pi}{2} x \right) & \text{se } x \geq 1. \end{cases}$$

- a) Estude a continuidade da função f .
- b) Indique o contradomínio de f .
- c) Determine, caso existam, $\lim_{x \rightarrow -\infty} f(x)$ e $\lim_{x \rightarrow +\infty} f(x)$.

Exercício 5.7 Seja $f : \mathbb{R} \longrightarrow \mathbb{R}$ a função definida por

$$f(x) = \begin{cases} k \operatorname{arctg} \left(\frac{1}{x} \right) & \text{se } x > 0, \\ \frac{1}{x^2 + 1} & \text{se } x \leq 0. \end{cases}$$

- a) Determine k de modo que f seja contínua.
- b) Calcule $\lim_{x \rightarrow -\infty} f(x)$ e $\lim_{x \rightarrow +\infty} f(x)$.

Exercício 5.8 Resolva as seguintes equações:

- a) $e^x = e^{1-x};$
- b) $e^{2x} + 2e^x - 3 = 0;$
- c) $e^{3x} - 2e^{-x} = 0;$
- d) $\ln(x^2 - 1) + 2 \ln 2 = \ln(4x - 1).$