

# **Tópicos de Física Moderna (LEI)**

**2011 / 2012 – 2º semestre**

## The SI Base Units

| Quantity                  | Name     | Symbol | Definition   |
|---------------------------|----------|--------|--|
| length                    | meter    | m      | "... the length of the path traveled by light in vacuum in 1/299,792,458 of a second." (1983)  |
| mass                      | kilogram | kg     | "... this prototype [a certain platinum–iridium cylinder] shall henceforth be considered to be the unit of mass." (1889)   |
| time                      | second   | s      | "... the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom." (1967)   |
| electric current          | ampere   | A      | "... that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length." (1946) |
| thermodynamic temperature | kelvin   | K      | "... the fraction 1/273.16 of the thermodynamic temperature of the triple point of water." (1967)  |
| amount of substance       | mole     | mol    | "... the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12." (1971)   |
| luminous intensity        | candela  | cd     | "... the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of 1/683 watt per steradian." (1979)  |



## Some SI Derived Units

| Quantity                                  | Name of Unit                           | Symbol                                |                                       |
|---|--|---------------------------------------|---------------------------------------|
| area                                      | square meter                           | $\text{m}^2$                          |                                       |
| volume                                    | cubic meter                            | $\text{m}^3$                          |                                       |
| frequency                                 | hertz                                  | Hz                                    | $\text{s}^{-1}$                       |
| mass density (density)                    | kilogram per cubic meter               | $\text{kg}/\text{m}^3$                |                                       |
| speed, velocity                           | meter per second                       | $\text{m}/\text{s}$                   |                                       |
| angular velocity                          | radian per second                      | $\text{rad}/\text{s}$                 |                                       |
| acceleration                              | meter per second per second            | $\text{m}/\text{s}^2$                 |                                       |
| angular acceleration                      | radian per second per second           | $\text{rad}/\text{s}^2$               |                                       |
| force                                     | newton                                 | N                                     | $\text{kg} \cdot \text{m}/\text{s}^2$ |
| pressure                                  | pascal                                 | Pa                                    | $\text{N}/\text{m}^2$                 |
| work, energy, quantity of heat            | joule                                  | J                                     | $\text{N} \cdot \text{m}$             |
| power                                     | watt                                   | W                                     | $\text{J}/\text{s}$                   |
| quantity of electric charge               | coulomb                                | C                                     | $\text{A} \cdot \text{s}$             |
| potential difference, electromotive force | volt                                   | V                                     | $\text{W}/\text{A}$                   |
| electric field strength                   | volt per meter (or newton per coulomb) | $\text{V}/\text{m}$                   | $\text{N}/\text{C}$                   |
| electric resistance                       | ohm                                    | $\Omega$                              | $\text{V}/\text{A}$                   |
| capacitance                               | farad                                  | F                                     | $\text{A} \cdot \text{s}/\text{V}$    |
| magnetic flux                             | weber                                  | Wb                                    | $\text{V} \cdot \text{s}$             |
| inductance                                | henry                                  | H                                     | $\text{V} \cdot \text{s}/\text{A}$    |
| magnetic flux density                     | tesla                                  | T                                     | $\text{Wb}/\text{m}^2$                |
| magnetic field strength                   | ampere per meter                       | $\text{A}/\text{m}$                   |                                       |
| entropy                                   | joule per kelvin                       | $\text{J}/\text{K}$                   |                                       |
| specific heat                             | joule per kilogram kelvin              | $\text{J}/(\text{kg} \cdot \text{K})$ |                                       |
| thermal conductivity                      | watt per meter kelvin                  | $\text{W}/(\text{m} \cdot \text{K})$  |                                       |
| radiant intensity                         | watt per steradian                     | $\text{W}/\text{sr}$                  |                                       |

## SI Prefixes\*

| Factor    | Prefix | Symbol | Factor     | Prefix | Symbol |
|-----------|--------|--------|------------|--------|--------|
| $10^{24}$ | yotta  | Y      | $10^{-1}$  | deci   | d      |
| $10^{21}$ | zetta  | Z      | $10^{-2}$  | centi  | c      |
| $10^{18}$ | exa    | E      | $10^{-3}$  | milli  | m      |
| $10^{15}$ | peta   | P      | $10^{-6}$  | micro  | $\mu$  |
| $10^{12}$ | tera   | T      | $10^{-9}$  | nano   | n      |
| $10^9$    | giga   | G      | $10^{-12}$ | pico   | p      |
| $10^6$    | mega   | M      | $10^{-15}$ | femto  | f      |
| $10^3$    | kilo   | k      | $10^{-18}$ | atto   | a      |
| $10^2$    | hecto  | h      | $10^{-21}$ | zepto  | z      |
| $10^1$    | deka   | da     | $10^{-24}$ | yocto  | y      |



| Constant                                      | Symbol       | Computational Value  | Best (1998) Value   |                          |
|---|--------------|--|---------------------|--------------------------|
|   |              |  | Value <sup>a</sup>  | Uncertainty <sup>b</sup> |
| Speed of light in a vacuum                    | $c$          | $3.00 \times 10^8$ m/s                                     | 2.997 924 58        | exact                    |
| Elementary charge                             | $e$          | $1.60 \times 10^{-19}$ C                                   | 1.602 176 462       | 0.039                    |
| Gravitational constant                        | $G$          | $6.67 \times 10^{-11}$ m <sup>3</sup> /s <sup>2</sup> · kg | 6.673               | 1500                     |
| Universal gas constant                        | $R$          | 8.31 J/mol · K   | 8.314 472           | 1.7                      |
| Avogadro constant                             | $N_A$        | $6.02 \times 10^{23}$ mol <sup>-1</sup>                    | 6.022 141 99        | 0.079                    |
| Boltzmann constant                            | $k$          | $1.38 \times 10^{-23}$ J/K                                 | 1.380 650 3         | 1.7                      |
| Stefan–Boltzmann constant                     | $\sigma$     | $5.67 \times 10^{-8}$ W/m <sup>2</sup> · K <sup>4</sup>    | 5.670 400           | 7.0                      |
| Molar volume of ideal gas at STP <sup>d</sup> | $V_m$        | $2.27 \times 10^{-2}$ m <sup>3</sup> /mol                  | 2.271 098 1         | 1.7                      |
| Permittivity constant                         | $\epsilon_0$ | $8.85 \times 10^{-12}$ F/m                                 | 8.854 187 817 62    | exact                    |
| Permeability constant                         | $\mu_0$      | $1.26 \times 10^{-6}$ H/m                                  | 1.256 637 061 43    | exact                    |
| Planck constant                               | $h$          | $6.63 \times 10^{-34}$ J · s                               | 6.626 068 76        | 0.078                    |
| Electron mass <sup>c</sup>                    | $m_e$        | $9.11 \times 10^{-31}$ kg                                  | 9.109 381 88        | 0.079                    |
|   |              | $5.49 \times 10^{-4}$ u                                    | 5.485 799 110       | 0.0021                   |
| Proton mass <sup>c</sup>                      | $m_p$        | $1.67 \times 10^{-27}$ kg                                  | 1.672 621 58        | 0.079                    |
|   |              | 1.0073 u   | 1.007 276 466 88    | $1.3 \times 10^{-4}$     |
| Ratio of proton mass to electron mass         | $m_p/m_e$    | 1840   | 1836.152 667 5      | 0.0021                   |
| Electron charge-to-mass ratio                 | $e/m_e$      | $1.76 \times 10^{11}$ C/kg                                 | 1.758 820 174       | 0.040                    |
| Neutron mass <sup>c</sup>                     | $m_n$        | $1.68 \times 10^{-27}$ kg                                  | 1.674 927 16        | 0.079                    |
|   |              | 1.0087 u   | 1.008 664 915 78    | $5.4 \times 10^{-4}$     |
| Hydrogen atom mass <sup>c</sup>               | $m_{1H}$     | 1.0078 u   | 1.007 825 031 6     | 0.0005                   |
| Deuterium atom mass <sup>c</sup>              | $m_{2H}$     | 2.0141 u   | 2.014 101 777 9     | 0.0005                   |
| Helium atom mass <sup>c</sup>                 | $m_{4He}$    | 4.0026 u   | 4.002 603 2         | 0.067                    |
| Muon mass                                     | $m_\mu$      | $1.88 \times 10^{-28}$ kg                                  | 1.883 531 09        | 0.084                    |
| Electron magnetic moment                      | $\mu_e$      | $9.28 \times 10^{-24}$ J/T                                 | 9.284 763 62        | 0.040                    |
| Proton magnetic moment                        | $\mu_p$      | $1.41 \times 10^{-26}$ J/T                                 | 1.410 606 663       | 0.041                    |
| Bohr magneton                                 | $\mu_B$      | $9.27 \times 10^{-24}$ J/T                                 | 9.274 008 99        | 0.040                    |
| Nuclear magneton                              | $\mu_N$      | $5.05 \times 10^{-27}$ J/T                                 | 5.050 783 17        | 0.040                    |
| Bohr radius                                   | $a$          | $5.29 \times 10^{-11}$ m                                   | 5.291 772 083       | 0.0037                   |
| Rydberg constant                              | $R$          | $1.10 \times 10^7$ m <sup>-1</sup>                         | 1.097 373 156 854 8 | $7.6 \times 10^{-6}$     |
| Electron Compton wavelength                   | $\lambda_C$  | $2.43 \times 10^{-12}$ m                                   | 2.426 310 215       | 0.0073                   |

### Some Distances from Earth

|  |                         |  |                        |
|--|-------------------------|--|------------------------|
| To the Moon*                           | $3.82 \times 10^8$ m    | To the center of our galaxy            | $2.2 \times 10^{20}$ m |
| To the Sun*                            | $1.50 \times 10^{11}$ m | To the Andromeda Galaxy                | $2.1 \times 10^{22}$ m |
| To the nearest star (Proxima Centauri) | $4.04 \times 10^{16}$ m | To the edge of the observable universe | $\sim 10^{26}$ m       |

\*Mean distance.

### The Sun, Earth, and the Moon

| Property                              | Unit              | Sun                        | Earth                        | Moon                  |
|---------------------------------------|-------------------|----------------------------|------------------------------|-----------------------|
| Mass                                  | kg                | $1.99 \times 10^{30}$      | $5.98 \times 10^{24}$        | $7.36 \times 10^{22}$ |
| Mean radius                           | m                 | $6.96 \times 10^8$         | $6.37 \times 10^6$           | $1.74 \times 10^6$    |
| Mean density                          | kg/m <sup>3</sup> | 1410                       | 5520                         | 3340                  |
| Free-fall acceleration at the surface | m/s <sup>2</sup>  | 274                        | 9.81                         | 1.67                  |
| Escape velocity                       | km/s              | 618                        | 11.2                         | 2.38                  |
| Period of rotation <sup>a</sup>       | —                 | 37 d at poles <sup>b</sup> | 26 d at equator <sup>b</sup> | 27.3 d                |
| Radiation power <sup>c</sup>          | W                 | $3.90 \times 10^{26}$      |                              |                       |

<sup>a</sup>Measured with respect to the distant stars.

<sup>b</sup>The Sun, a ball of gas, does not rotate as a rigid body.

<sup>c</sup>Just outside Earth's atmosphere solar energy is received, assuming normal incidence, at the rate of 1340 W/m<sup>2</sup>.

### Some Properties of the Planets

|   | Mercury | Venus             | Earth  | Mars   | Jupiter   | Saturn     | Uranus              | Neptune    | Pluto |
|---|---------|-------------------|--------|--------|-----------|------------|---------------------|------------|-------|
| Mean distance from Sun,<br>10 <sup>6</sup> km     | 57.9    | 108               | 150    | 228    | 778       | 1430       | 2870                | 4500       | 5900  |
| Period of revolution, y                           | 0.241   | 0.615             | 1.00   | 1.88   | 11.9      | 29.5       | 84.0                | 165        | 248   |
| Period of rotation, <sup>a</sup> d                | 58.7    | −243 <sup>b</sup> | 0.997  | 1.03   | 0.409     | 0.426      | −0.451 <sup>b</sup> | 0.658      | 6.39  |
| Orbital speed, km/s                               | 47.9    | 35.0              | 29.8   | 24.1   | 13.1      | 9.64       | 6.81                | 5.43       | 4.74  |
| Inclination of axis to orbit                      | <28°    | ≈3°               | 23.4°  | 25.0°  | 3.08°     | 26.7°      | 97.9°               | 29.6°      | 57.5° |
| Inclination of orbit to<br>Earth's orbit          | 7.00°   | 3.39°             |        | 1.85°  | 1.30°     | 2.49°      | 0.77°               | 1.77°      | 17.2° |
| Eccentricity of orbit                             | 0.206   | 0.0068            | 0.0167 | 0.0934 | 0.0485    | 0.0556     | 0.0472              | 0.0086     | 0.250 |
| Equatorial diameter, km                           | 4880    | 12 100            | 12 800 | 6790   | 143 000   | 120 000    | 51 800              | 49 500     | 2300  |
| Mass (Earth = 1)                                  | 0.0558  | 0.815             | 1.000  | 0.107  | 318       | 95.1       | 14.5                | 17.2       | 0.002 |
| Density (water = 1)                               | 5.60    | 5.20              | 5.52   | 3.95   | 1.31      | 0.704      | 1.21                | 1.67       | 2.03  |
| Surface value of g, <sup>c</sup> m/s <sup>2</sup> | 3.78    | 8.60              | 9.78   | 3.72   | 22.9      | 9.05       | 7.77                | 11.0       | 0.5   |
| Escape velocity, <sup>c</sup> km/s                | 4.3     | 10.3              | 11.2   | 5.0    | 59.5      | 35.6       | 21.2                | 23.6       | 1.1   |
| Known satellites                                  | 0       | 0                 | 1      | 2      | 60 + ring | 31 + rings | 21 + rings          | 11 + rings | 1     |



# Conversion Factors

## Plane Angle

|                                   | ° | '                      | "                   | RADIAN                 | rev                    |
|-----------------------------------|---|------------------------|---------------------|------------------------|------------------------|
| 1 degree = 1                      |   | 60                     | 3600                | $1.745 \times 10^{-2}$ | $2.778 \times 10^{-3}$ |
| 1 minute = $1.667 \times 10^{-2}$ |   | 1                      | 60                  | $2.909 \times 10^{-4}$ | $4.630 \times 10^{-5}$ |
| 1 second = $2.778 \times 10^{-4}$ |   | $1.667 \times 10^{-2}$ | 1                   | $4.848 \times 10^{-6}$ | $7.716 \times 10^{-7}$ |
| 1 RADIAN = 57.30                  |   | 3438                   | $2.063 \times 10^5$ | 1                      | 0.1592                 |
| 1 revolution = 360                |   | $2.16 \times 10^4$     | $1.296 \times 10^6$ | 6.283                  | 1                      |

## Length

|                              | cm | METER                                    | km                     | in.                                       | ft                     | mi                     |
|------------------------------|----|--|------------------------|---|------------------------|------------------------|
| 1 centimeter = 1             |    | $10^{-2}$                                | $10^{-5}$              | 0.3937                                    | $3.281 \times 10^{-2}$ | $6.214 \times 10^{-6}$ |
| 1 METER = 100                |    | 1  | $10^{-3}$              | 39.37                                     | 3.281                  | $6.214 \times 10^{-4}$ |
| 1 kilometer = $10^5$         |    | 1000                                     | 1                      | $3.937 \times 10^4$                       | 3281                   | 0.6214                 |
| 1 inch = 2.540               |    | $2.540 \times 10^{-2}$                   | $2.540 \times 10^{-5}$ | 1   | $8.333 \times 10^{-2}$ | $1.578 \times 10^{-5}$ |
| 1 foot = 30.48               |    | 0.3048                                   | $3.048 \times 10^{-4}$ | 12  | 1                      | $1.894 \times 10^{-4}$ |
| 1 mile = $1.609 \times 10^5$ |    | 1609                                     | 1.609                  | $6.336 \times 10^4$                       | 5280                   | 1                      |
| 1 angström = $10^{-10}$ m    |    | 1 fermi = $10^{-15}$ m                   |                        | 1 fathom = 6 ft                           |                        | 1 rod = 16.5 ft        |
| 1 nautical mile = 1852 m     |    | 1 light-year = $9.461 \times 10^{12}$ km |                        | 1 Bohr radius = $5.292 \times 10^{-11}$ m |                        | 1 mil = $10^{-3}$ in.  |
| = 1.151 miles = 6076 ft      |    | 1 parsec = $3.084 \times 10^{13}$ km     |                        | 1 yard = 3 ft                             |                        | 1 nm = $10^{-9}$ m     |

## Mass

Quantities in the colored areas are not mass units but are often used as such. For example, when we write 1 kg “=” 2.205 lb, this means that a kilogram is a *mass* that *weighs* 2.205 pounds at a location where  $g$  has the standard value of  $9.80665 \text{ m/s}^2$ .

|  | g | KILOGRAM                | slug                    | u                      | oz                      | lb                      | ton                     |
|--|---|-------------------------|-------------------------|------------------------|-------------------------|-------------------------|-------------------------|
| 1 gram = 1                                   |   | 0.001                   | $6.852 \times 10^{-5}$  | $6.022 \times 10^{23}$ | $3.527 \times 10^{-2}$  | $2.205 \times 10^{-3}$  | $1.102 \times 10^{-6}$  |
| 1 KILOGRAM = 1000                            |   | 1                       | $6.852 \times 10^{-2}$  | $6.022 \times 10^{26}$ | 35.27                   | 2.205                   | $1.102 \times 10^{-3}$  |
| 1 slug = $1.459 \times 10^4$                 |   | 14.59                   | 1                       | $8.786 \times 10^{27}$ | 514.8                   | 32.17                   | $1.609 \times 10^{-2}$  |
| 1 atomic mass unit = $1.661 \times 10^{-24}$ |   | $1.661 \times 10^{-27}$ | $1.138 \times 10^{-28}$ | 1                      | $5.857 \times 10^{-26}$ | $3.662 \times 10^{-27}$ | $1.830 \times 10^{-30}$ |
| 1 ounce = 28.35                              |   | $2.835 \times 10^{-2}$  | $1.943 \times 10^{-3}$  | $1.718 \times 10^{25}$ | 1                       | $6.250 \times 10^{-2}$  | $3.125 \times 10^{-5}$  |
| 1 pound = 453.6                              |   | 0.4536                  | $3.108 \times 10^{-2}$  | $2.732 \times 10^{26}$ | 16                      | 1                       | 0.0005                  |
| 1 ton = $9.072 \times 10^5$                  |   | 907.2                   | 62.16                   | $5.463 \times 10^{29}$ | $3.2 \times 10^4$       | 2000                    | 1                       |

1 metric ton = 1000 kg

# Conversion Factors

## Power

|                                     | Btu/h | ft · lb/s | hp                     | cal/s                  | kW                     | WATT   |
|-------------------------------------|-------|-----------|------------------------|------------------------|------------------------|--------|
| 1 British thermal unit per hour = 1 |       | 0.2161    | $3.929 \times 10^{-4}$ | $6.998 \times 10^{-2}$ | $2.930 \times 10^{-4}$ | 0.2930 |
| 1 foot-pound per second = 4.628     |       | 1         | $1.818 \times 10^{-3}$ | 0.3239                 | $1.356 \times 10^{-3}$ | 1.356  |
| 1 horsepower = 2545                 |       | 550       | 1                      | 178.1                  | 0.7457                 | 745.7  |
| 1 calorie per second = 14.29        |       | 3.088     | $5.615 \times 10^{-3}$ | 1                      | $4.186 \times 10^{-3}$ | 4.186  |
| 1 kilowatt = 3413                   |       | 737.6     | 1.341                  | 238.9                  | 1                      | 1000   |
| 1 WATT = 3.413                      |       | 0.7376    | $1.341 \times 10^{-3}$ | 0.2389                 | 0.001                  | 1      |

## Force

Force units in the colored areas are now little used. To clarify: 1 gram-force (= 1 gf) is the force of gravity that would act on an object whose mass is 1 gram at a location where  $g$  has the standard value of  $9.80665 \text{ m/s}^2$ .

|  | dyne | NEWTON                 | lb                     | pdl                    | gf                     | kgf                    |
|--|------|------------------------|------------------------|------------------------|------------------------|------------------------|
| 1 dyne = 1                             |      | $10^{-5}$              | $2.248 \times 10^{-6}$ | $7.233 \times 10^{-5}$ | $1.020 \times 10^{-3}$ | $1.020 \times 10^{-6}$ |
| 1 NEWTON = $10^5$                      |      | 1                      | 0.2248                 | 7.233                  | 102.0                  | 0.1020                 |
| 1 pound = $4.448 \times 10^5$          |      | 4.448                  | 1                      | 32.17                  | 453.6                  | 0.4536                 |
| 1 poundal = $1.383 \times 10^4$        |      | 0.1383                 | $3.108 \times 10^{-2}$ | 1                      | 14.10                  | $1.410 \times 10^2$    |
| 1 gram-force = 980.7                   |      | $9.807 \times 10^{-3}$ | $2.205 \times 10^{-3}$ | $7.093 \times 10^{-2}$ | 1                      | 0.001                  |
| 1 kilogram-force = $9.807 \times 10^5$ |      | 9.807                  | 2.205                  | 70.93                  | 1000                   | 1                      |

## Pressure

|  | atm | dyne/cm <sup>2</sup> | inch of water          | cm Hg                  | PASCAL              | lb/in. <sup>2</sup>    | lb/ft <sup>2</sup>     |
|--|-----|----------------------|------------------------|------------------------|---------------------|------------------------|------------------------|
| 1 atmosphere = 1   |     | $1.013 \times 10^6$  | 406.8                  | 76                     | $1.013 \times 10^5$ | 14.70                  | 2116                   |
| 1 dyne per centimeter <sup>2</sup> = $9.869 \times 10^{-7}$          |     | 1                    | $4.015 \times 10^{-4}$ | $7.501 \times 10^{-5}$ | 0.1                 | $1.405 \times 10^{-5}$ | $2.089 \times 10^{-3}$ |
| 1 inch of water <sup>a</sup> at 4°C = $2.458 \times 10^{-3}$         |     | 2491                 | 1                      | 0.1868                 | 249.1               | $3.613 \times 10^{-2}$ | 5.202                  |
| 1 centimeter of mercury <sup>a</sup> at 0°C = $1.316 \times 10^{-2}$ |     | $1.333 \times 10^4$  | 5.353                  | 1                      | 1333                | 0.1934                 | 27.85                  |
| 1 PASCAL = $9.869 \times 10^{-6}$                                    |     | 10                   | $4.015 \times 10^{-3}$ | $7.501 \times 10^{-4}$ | 1                   | $1.450 \times 10^{-4}$ | $2.089 \times 10^{-2}$ |
| 1 pound per inch <sup>2</sup> = $6.805 \times 10^{-2}$               |     | $6.895 \times 10^4$  | 27.68                  | 5.171                  | $6.895 \times 10^3$ | 1                      | 144                    |
| 1 pound per foot <sup>2</sup> = $4.725 \times 10^{-4}$               |     | 478.8                | 0.1922                 | $3.591 \times 10^{-2}$ | 47.88               | $6.944 \times 10^{-3}$ | 1                      |

<sup>a</sup>Where the acceleration of gravity has the standard value of  $9.80665 \text{ m/s}^2$ .

1 bar =  $10^6 \text{ dyne/cm}^2$  = 0.1 MPa

1 millibar =  $10^3 \text{ dyne/cm}^2$  =  $10^2 \text{ Pa}$

1 torr = 1 mm Hg



# Conversion Factors

## Energy, Work, Heat

Quantities in the colored areas are not energy units but are included for convenience. They arise from the relativistic mass-energy equivalence formula  $E = mc^2$  and represent the energy released if a kilogram or unified atomic mass unit (u) is completely converted to energy (bottom two rows) or the mass that would be completely converted to one unit of energy (rightmost two columns).

|   | Btu   | erg                     | ft · lb                 | hp · h                  | JOULE                   | cal                     | kW · h                  | eV                     | MeV                    | kg                      | u                      |
|---|-------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|------------------------|------------------------|-------------------------|------------------------|
| 1 British thermal unit = 1              |       | $1.055 \times 10^{10}$  | 777.9                   | $3.929 \times 10^{-4}$  | 1055                    | 252.0                   | $2.930 \times 10^{-4}$  | $6.585 \times 10^{21}$ | $6.585 \times 10^{15}$ | $1.174 \times 10^{-14}$ | $7.070 \times 10^{12}$ |
| 1 erg = $10^{-11}$                      | 9.481 | 1                       | $7.376 \times 10^{-8}$  | $3.725 \times 10^{-14}$ | $10^{-7}$               | $2.389 \times 10^{-8}$  | $2.778 \times 10^{-14}$ | $6.242 \times 10^{11}$ | $6.242 \times 10^5$    | $1.113 \times 10^{-24}$ | 670.2                  |
| 1 foot-pound = $10^{-3}$                | 1.285 | $1.356 \times 10^7$     | 1                       | $5.051 \times 10^{-7}$  | 1.356                   | 0.3238                  | $3.766 \times 10^{-7}$  | $8.464 \times 10^{18}$ | $8.464 \times 10^{12}$ | $1.509 \times 10^{-17}$ | $9.037 \times 10^9$    |
| 1 horsepower-hour = 2545                | 2.685 | $1.980 \times 10^{13}$  | $1.980 \times 10^6$     | 1                       | $2.685 \times 10^6$     | 6.413                   | 0.7457                  | $1.676 \times 10^{25}$ | $1.676 \times 10^{19}$ | $2.988 \times 10^{-11}$ | $1.799 \times 10^{16}$ |
| 1 JOULE = $10^{-4}$                     | 9.481 | $10^7$                  | 0.7376                  | $3.725 \times 10^{-7}$  | 1                       | 0.2389                  | $2.778 \times 10^{-7}$  | $6.242 \times 10^{18}$ | $6.242 \times 10^{12}$ | $1.113 \times 10^{-17}$ | $6.702 \times 10^9$    |
| 1 calorie = $10^{-3}$                   | 3.968 | $4.1868 \times 10^7$    | 3.088                   | $1.560 \times 10^{-6}$  | 4.1868                  | 1                       | $1.163 \times 10^{-6}$  | $2.613 \times 10^{19}$ | $2.613 \times 10^{13}$ | $4.660 \times 10^{-17}$ | $2.806 \times 10^{10}$ |
| 1 kilowatt-hour = 3413                  | 3.600 | $2.655 \times 10^{13}$  | $2.655 \times 10^6$     | 1.341                   | $3.600 \times 10^6$     | 8.600                   | 1                       | $2.247 \times 10^{25}$ | $2.247 \times 10^{19}$ | $4.007 \times 10^{-11}$ | $2.413 \times 10^{16}$ |
| 1 electron-volt = $10^{-22}$            | 1.519 | $1.602 \times 10^{-12}$ | $1.182 \times 10^{-19}$ | $5.967 \times 10^{-26}$ | $1.602 \times 10^{-19}$ | $3.827 \times 10^{-20}$ | $4.450 \times 10^{-26}$ | 1                      | $10^{-6}$              | $1.783 \times 10^{-36}$ | $1.074 \times 10^{-9}$ |
| 1 million electron-volts = $10^{-16}$   | 1.519 | $1.602 \times 10^{-6}$  | $1.182 \times 10^{-13}$ | $5.967 \times 10^{-20}$ | $1.602 \times 10^{-13}$ | $3.827 \times 10^{-14}$ | $4.450 \times 10^{-20}$ | $10^{-6}$              | 1                      | $1.783 \times 10^{-30}$ | $1.074 \times 10^{-3}$ |
| 1 kilogram = $10^{13}$                  | 8.521 | $8.987 \times 10^{23}$  | $6.629 \times 10^{16}$  | $3.348 \times 10^{10}$  | $8.987 \times 10^{16}$  | $2.146 \times 10^{16}$  | $2.497 \times 10^{10}$  | $5.610 \times 10^{35}$ | $5.610 \times 10^{29}$ | 1                       | $6.022 \times 10^{26}$ |
| 1 unified atomic mass unit = $10^{-13}$ | 1.415 | $1.492 \times 10^{-3}$  | $1.101 \times 10^{-10}$ | $5.559 \times 10^{-17}$ | $1.492 \times 10^{-10}$ | $3.564 \times 10^{-11}$ | $4.146 \times 10^{-17}$ | $9.320 \times 10^8$    | 932.0                  | $1.661 \times 10^{-27}$ | 1                      |

## Mathematical Formulas\*

### Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Binomial Theorem

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (x^2 < 1)$$

### Products of Vectors

Let  $\theta$  be the smaller of the two angles between  $\vec{a}$  and  $\vec{b}$ .  
Then

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = a_x b_x + a_y b_y + a_z b_z = ab \cos \theta$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k}$$

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

### Trigonometric Identities

### Derivatives and Integrals

$$\frac{d}{dx} \sin x = \cos x \qquad \int \sin x \, dx = -\cos x$$

$$\frac{d}{dx} \cos x = -\sin x \qquad \int \cos x \, dx = \sin x$$

$$\frac{d}{dx} e^x = e^x \qquad \int e^x \, dx = e^x$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

### Cramer's Rule

Two simultaneous equations in unknowns  $x$  and  $y$ ,

$$a_1 x + b_1 y = c_1 \quad \text{and} \quad a_2 x + b_2 y = c_2,$$

have the solutions

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$$

and

$$\begin{vmatrix} a_1 & c_1 \end{vmatrix}$$



## Geometry

Circle of radius  $r$ : circumference  $= 2\pi r$ ; area  $= \pi r^2$ .

Sphere of radius  $r$ : area  $= 4\pi r^2$ ; volume  $= \frac{4}{3}\pi r^3$ .

Right circular cylinder of radius  $r$  and height  $h$ :  
area  $= 2\pi r^2 + 2\pi rh$ ; volume  $= \pi r^2 h$ .

Triangle of base  $a$  and altitude  $h$ : area  $= \frac{1}{2}ah$ .

## Quadratic Formula

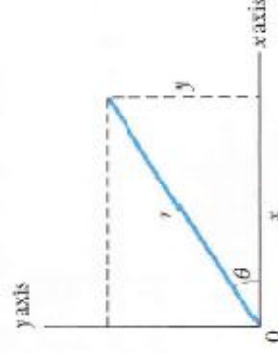
If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

## Trigonometric Functions of Angle $\theta$

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

$$\sec \theta = \frac{r}{x} \quad \csc \theta = \frac{r}{y}$$



## Pythagorean Theorem

In this right triangle,

$$a^2 + b^2 = c^2$$



## Triangles

Angles are  $A, B, C$

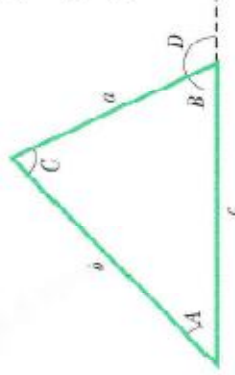
Opposite sides are  $a, b, c$

Angles  $A + B + C = 180^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Exterior angle  $D = A + C$



## Mathematical Signs and Symbols

$=$  equals

$\approx$  equals approximately

$\sim$  is the order of magnitude of

$\neq$  is not equal to

$\equiv$  is identical to, is defined as

$>$  is greater than ( $\gg$  is much greater than)

$<$  is less than ( $\ll$  is much less than)

$\geq$  is greater than or equal to (or, is no less than)

$\leq$  is less than or equal to (or, is no more than)

$\pm$  plus or minus

$\propto$  is proportional to

$\Sigma$  the sum of

$x_{\text{avg}}$  the average value of  $x$

## Trigonometric Identities

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sin \theta \cos \theta = \tan \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$