

1-

P	Q	$\neg Q$	$P \Rightarrow Q$	$\neg(P \Rightarrow Q)$	$(P \wedge \neg Q)$	Result
V	V	F	V	F	F	V
V	F	V	F	V	V	V
F	V	F	V	F	F	V
F	F	V	V	F	F	V

test

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow Q$
V	V	V	V
V	F	F	V
F	V	V	V
F	F	V	F

non Contr. non test.

P	$\neg P$	$P \Rightarrow \neg P$
V	F	F
V	F	F
F	V	V
F	V	V

Contr.

$$(P \Leftrightarrow \neg P)$$

P	$\neg P$	$P \Leftrightarrow \neg P$
V	F	F
V	F	F
F	V	F
F	V	F

Contr.

$$(P \Leftrightarrow \neg P) \Leftrightarrow (Q \wedge \neg Q)$$

Q	$\neg Q$	P	$\neg P$	$(P \Leftrightarrow \neg P)$	$Q \wedge \neg Q$	$(P \Leftrightarrow \neg P) \Leftrightarrow (Q \wedge \neg Q)$
V	F	V	F	F	F	V
F	V	V	F	F	F	V
V	F	F	V	F	F	V
F	V	F	V	F	F	V

test.

$$2 - \exists x \forall y \quad x > y^2 \Rightarrow x^2 > y^2$$

$$\forall x \exists y \quad x > y^2 \Rightarrow x^2 \leq y^2$$

3 -

$$x \leq x < 9,5$$

$$\begin{array}{cc} \text{V} & \text{F} \\ A \Rightarrow B & \Rightarrow \text{P.F.} \end{array}$$

$$\begin{array}{cc} \text{F} & \text{F} \\ A \Rightarrow B & \Rightarrow \text{P.V.} \end{array}$$

$$\exists x \quad A(x) \wedge B(x) \rightarrow \text{P.F.}$$

IV - $A = \{1, 3, \{1, 3\}, \{2\}\} \cup \mathbb{Z}$

$$A \subseteq \mathbb{Z} \rightarrow \text{F}$$

$$A \cap \mathbb{Z} = \mathbb{Z} \rightarrow \text{V}$$

$$\{1, 3\} \in A \rightarrow \text{F}$$

$$\{1, 3\} \in A \rightarrow \text{V}$$

$$\{1, 2\} \subseteq A \rightarrow \text{F}$$

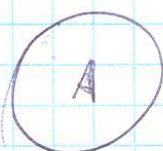
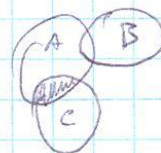
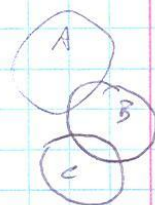
V $A \cap B \cap C = \emptyset$

$$1. \forall x \in A \exists y \in B \Rightarrow x \notin C \rightarrow \text{V}$$

$$2. \forall x \in A \exists y \in B \Rightarrow x \in C \rightarrow \text{F}$$

$$3. A \cap C = A \setminus B \rightarrow \text{F}$$

$$4. A \cap B \subseteq A \cap C \rightarrow \text{V}$$



VI

$$A = \{ X \subseteq \mathbb{R} \mid \exists x, y \in \mathbb{N} \quad X = \{x, y\} \}$$

$$B = \{ x \in \mathbb{N} \mid 1 \leq x^2 \leq 3 \}$$

~~1, 2, 3~~

$$A = \{1, 1\}$$

a) $B = \{1, 2, 3\}$

b) $\mathcal{P}(B) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\} \}$

c) $A \cap \mathcal{P}(B) = \{ \{1, 2\}, \{2, 3\}, \{1, 3\} \}$

d) $\mathcal{P}(A) \cap \mathcal{P}(B) = \{ \phi, \{1, 1\}, \{1, 2\}, \{2, 3\}, \{1, 3\} \}$

VII

$$\forall A, B \quad A \times B = B \times A \rightarrow F$$

$$\forall A, B \quad A \times B \neq B \times A \rightarrow V$$

$$\forall A, B \quad (A \times A) \setminus (B \times B) = (A \setminus B) \times (A \setminus B) \rightarrow F$$

$$\exists A, B \quad (A \times A) \setminus (B \times B) = (A \setminus B) \times (A \setminus B) \rightarrow V$$

VIII

$$(P \vee Q) \Rightarrow R$$

P	Q	R	$P \vee Q$	$P \vee Q \Rightarrow R$
V	V	V	V	V
V	V	F	V	F
V	F	V	V	V
V	F	F	V	F
F	V	V	V	V
F	V	F	V	F
F	F	V	F	V
F	F	F	F	V

E' logicamente equivalente

$$(P \Rightarrow R) \wedge (Q \Rightarrow R)$$

P	Q	R	$P \Rightarrow R$	$Q \Rightarrow R$	$P \Rightarrow R \wedge Q \Rightarrow R$
V	V	V	V	V	V
V	V	F	F	F	F
V	F	V	V	V	V
V	F	F	F	V	F
F	V	V	V	V	V
F	V	F	V	F	F
F	F	V	V	V	V
F	F	F	V	V	V

IX

$$A \cup B = A \cap B \Rightarrow$$

$$\Rightarrow x \in (A \cup B) = x \in (A \cap B)$$

$$\Rightarrow x \in A \vee x \in B = x \in A \wedge x \in B$$

$$\Rightarrow (x \in A \wedge x \in A) \equiv (x \in B \wedge x \in B)$$

$$\Rightarrow x \in A = x \in B \Rightarrow A = B$$

X

$$A \setminus (B \cap C) \Rightarrow x \in A \wedge x \notin (B \cap C) \Rightarrow$$

$$\Rightarrow x \in A \wedge \neg x \in (B \cap C) \Rightarrow x \in A \wedge (x \notin B \vee x \notin C) \Rightarrow$$

$$\Rightarrow (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C) \Rightarrow x \in (A \setminus B) \vee x \in (A \setminus C) \Rightarrow$$

$$\Rightarrow A \setminus B \cup A \setminus C$$