7.

Exercício 7.1

```
f[x_, y_] = 1 - x^2 / 2 - y^2 / 2;
pontos = Flatten[Table[{i, j}, {i, 1 / 8, 1, 1 / 4}, {j, 1 / 8, 1, 1 / 4}], 1];
VolumeAprox = Plus @@ f @@@ pontos / 16;
VolumeEx = Integrate[f[x, y], {x, 0, 1}, {y, 0, 1}];
Print["Valor aproximado para o volume: ", VolumeAprox, " ≃ ", N[VolumeAprox]];
Print["Valor exato do volume: ", VolumeEx, " ≃ ", N[VolumeEx]];
```

Valor aproximado para o volume: $\frac{43}{64} \simeq 0.671875$

Valor exato do volume: $\frac{2}{3} \simeq 0.666667$

Exercício 7.2

a)

$$\frac{31}{12}$$

b)

$$-1 + (-1 + e)\, e$$

c)

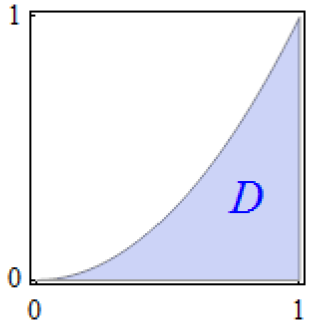
$$\frac{7 \operatorname{Sin}[1]}{9}$$

d)

$$-2 + \operatorname{Log}[16]$$

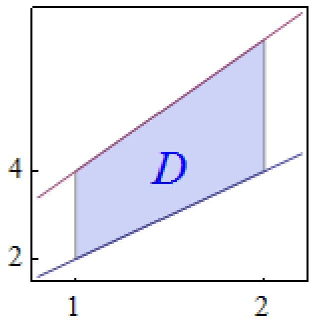
Exercício 7.3

a)



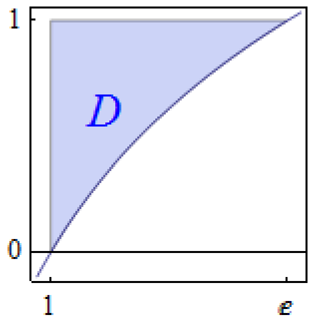
$$\int_0^1 \int_0^{x^2} dydx = \frac{1}{3}$$

b)



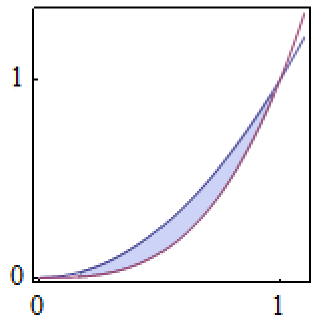
$$\int_1^2 \int_{2x}^{3x+1} dydx = \frac{5}{2}$$

c)



$$\int_0^1 \int_1^{e^y} (x+y)\, dxdy = \frac{1}{4} \left(-1 + e^2\right)$$

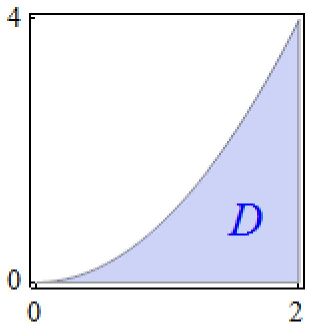
d)



$$\int_0^1 \int_{x^3}^{x^2} ydydx = \frac{1}{35}$$

Exercício 7.4

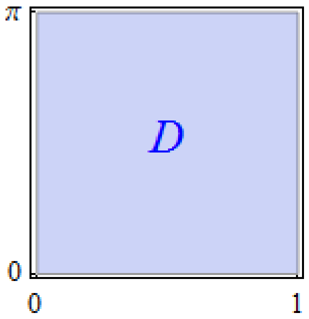
a)



$$\int_0^2 \int_0^{x^2} xydydx = \frac{16}{3}$$

$$\int_0^4 \int_{\sqrt{y}}^{2} xydxdy = \frac{16}{3}$$

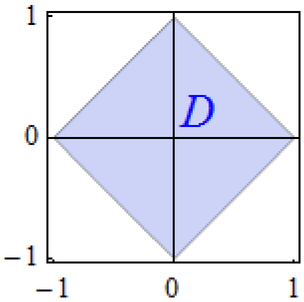
b)



$$\int_0^1\int_0^{\pi} x\ \mathrm{Sin}[x+y]\,dy\,dx = 2\ (-1 + \mathrm{Cos}[1] + \mathrm{Sin}[1])$$

$$\int_0^{\pi}\int_0^1 x\ \mathrm{Sin}[x+y]\,dx\,dy = 2\ (-1 + \mathrm{Cos}[1] + \mathrm{Sin}[1])$$

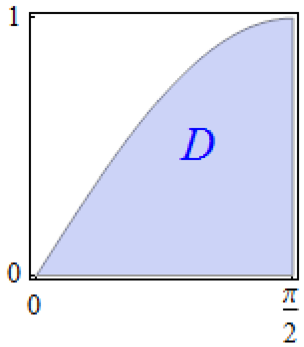
c)



$$\int_{-1}^{0}\int_{-x-1}^{x+1} \mathrm{Exp}[x+y]\,dy\,dx + \int_0^1\int_{x-1}^{-x+1} \mathrm{Exp}[x+y]\,dy\,dx = -\frac{1}{e} + e$$

$$\int_{-1}^{1}\int_{y-1}^{1-y} \mathrm{Exp}[x+y]\,dx\,dy + \int_{-1}^{1}\int_{-y-1}^{y+1} \mathrm{Exp}[x+y]\,dx\,dy = -\frac{1}{e} + e$$

d)

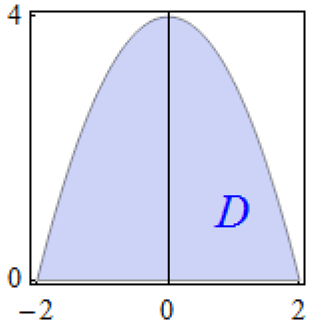


$$\int_0^{\frac{\pi}{2}}\int_0^{\mathrm{Sin}[x]} x^2 + y^2\,dy\,dx = -\frac{16}{9} + \pi$$

$$\int_0^1\int_{\mathrm{ArcSin}[y]}^{\frac{\pi}{2}} x^2 + y^2\,dx\,dy = -\frac{16}{9} + \pi$$

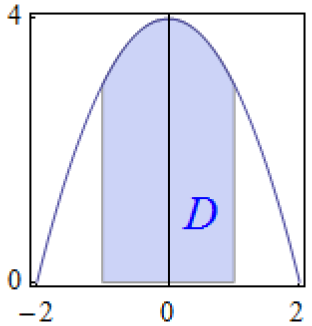
Exercício 7.5

a)



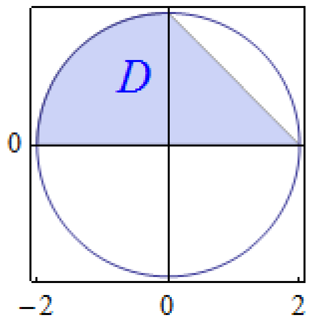
$$\int_{-2}^{2}\int_{0}^{4-x^2} f[x,y]\,dy\,dx = \int_{0}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x,y]\,dx\,dy$$

b)



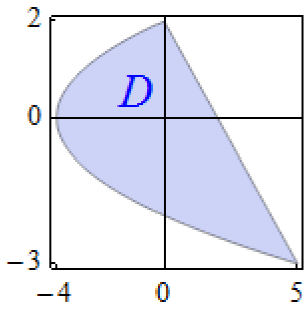
$$\int_{-1}^{1}\int_{0}^{4-x^2} f[x,y]\,dy\,dx = \int_{3}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x,y]\,dx\,dy + \int_{0}^{3}\int_{-1}^{1} f[x,y]\,dx\,dy$$

c)



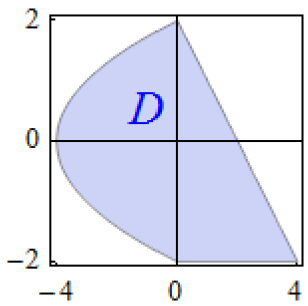
$$\int_{0}^{2}\int_{-\sqrt{4-y}}^{2-y} f[x,y]\,dx\,dy = \int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} f[x,y]\,dy\,dx + \int_{0}^{2}\int_{0}^{2-x} f[x,y]\,dy\,dx$$

d)



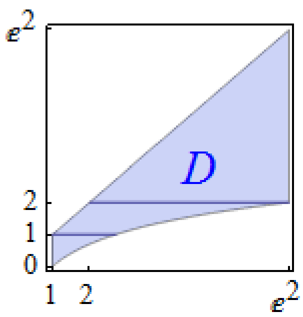
$$\int_{-3}^{2}\int_{-4+y^2}^{2-y} f[x,y]\,dx\,dy = \int_{-4}^{0}\int_{-\sqrt{x+4}}^{\sqrt{x+4}} f[x,y]\,dy\,dx + \int_{0}^{5}\int_{-\sqrt{x+4}}^{2-x} f[x,y]\,dy\,dx$$

e)



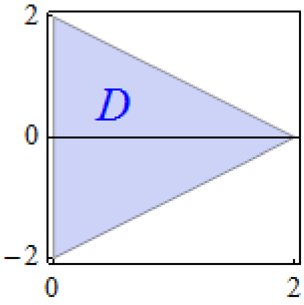
$$\int_{-2}^{2}\int_{-4+y^2}^{2-y} f[x,y]\,dx\,dy = \int_{-4}^{0}\int_{-\sqrt{x+4}}^{\sqrt{x+4}} f[x,y]\,dy\,dx + \int_{0}^{4}\int_{-2}^{2-x} f[x,y]\,dy\,dx$$

f)



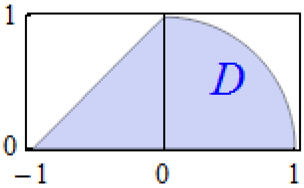
$$\int_{1}^{e^2}\int_{\mathrm{Log}[x]}^{x} f[x,y]\,dy\,dx = \int_{0}^{1}\int_{0}^{\mathrm{Exp}[y]} f[x,y]\,dx\,dy + \int_{1}^{2}\int_{y}^{\mathrm{Exp}[y]} f[x,y]\,dx\,dy + \int_{2}^{e^2}\int_{y}^{e^2} f[x,y]\,dx\,dy$$

g)



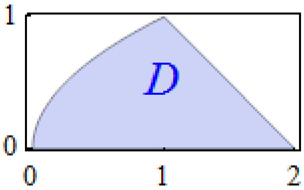
$$\int_{-2}^{2}\int_{0}^{-|y|+2} f[x,y]\,dx\,dy=\int_{0}^{2}\int_{x-2}^{2-x} f[x,y]\,dy\,dx$$

h)



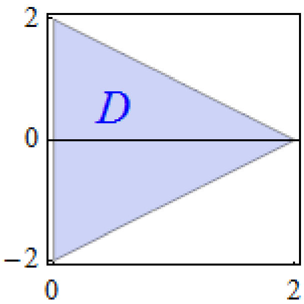
$$\int_{0}^{1}\int_{y-1}^{\sqrt{1-y^2}} f[x,y]\,dx\,dy=\int_{-1}^{0}\int_{0}^{x+1} f[x,y]\,dy\,dx+\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} f[x,y]\,dy\,dx$$

i)



$$\int_{0}^{1}\int_{0}^{\sqrt{x}} f[x,y]\,dy\,dx+\int_{1}^{2}\int_{0}^{-x+2} f[x,y]\,dy\,dx=\int_{0}^{1}\int_{y^2}^{2-y} f[x,y]\,dx\,dy$$

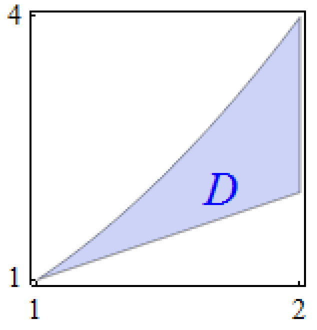
j)



$$\int_{-2}^{0}\int_{0}^{y+2} f[x,y]\,dx\,dy+\int_{0}^{2}\int_{0}^{-y+2} f[x,y]\,dx\,dy=\int_{0}^{2}\int_{x-2}^{2-x} f[x,y]\,dy\,dx$$

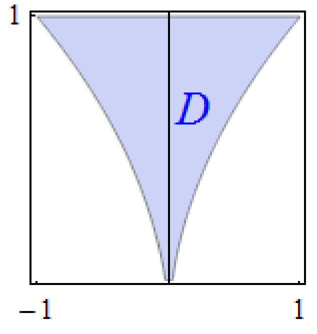
Exercício 7.6

a)



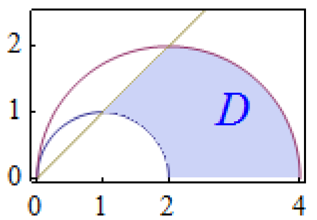
$$\text{Área} = \frac{5}{6}$$

b)



$$\text{Área} = \frac{2}{3}$$

Exercício 7.7



$$\text{Área} = \frac{3\,(2+\pi)}{4}$$

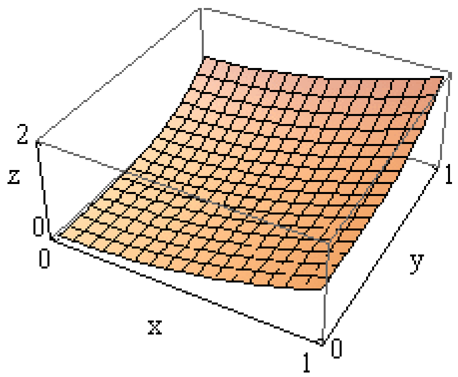
Exercício 7.8

$$\text{Área da circunferência} = 4\int_0^r \int_0^{\sqrt{r^2-x^2}} dy\,dx = \pi\, r^2$$

$$\text{Área da elipse} = 4\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy\,dx = a\, b\, \pi$$

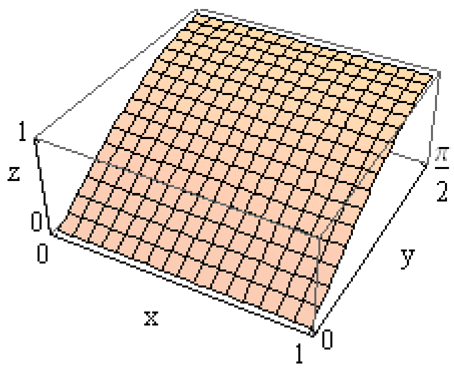
Exercício 7.9

a)



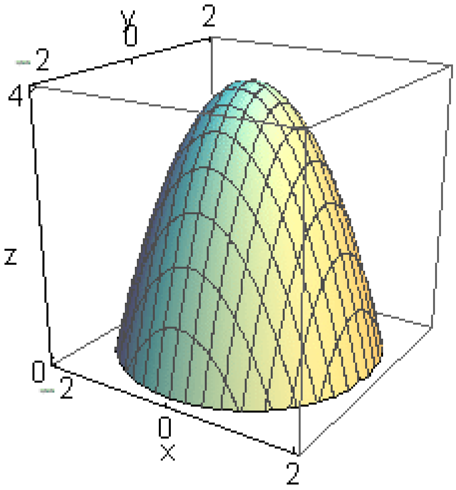
$$\text{Volume} = \frac{8}{15}$$

b)



$$\text{Volume} = 1$$

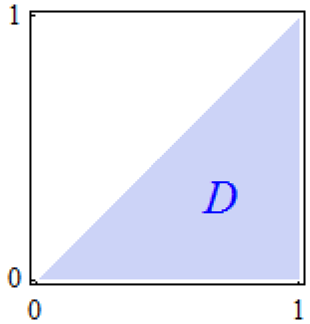
c)



Volume=8 π

Exercício 7.10

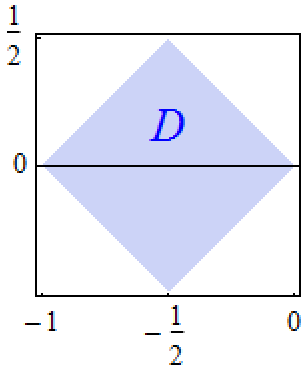
a)



$$\frac{1}{2}$$

Exercício 7.11

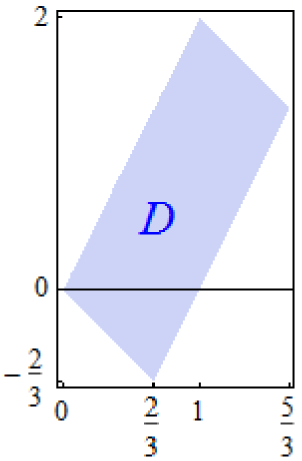
a)




```
Integrate[x^2 + y^2, {x, -1, -1/2}, {y, -1 - x, 1 + x}] + Integrate[x^2 + y^2, {x, -1/2, 0}, {y, x, -x}]
```

$$\frac{1}{6}$$

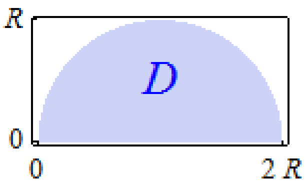
b)



$$\frac{1}{3}$$

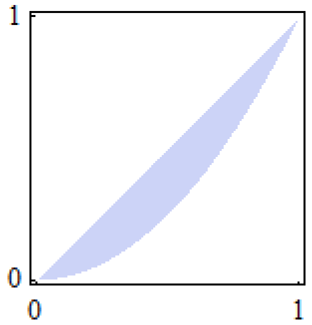
Exercício 7.12

a)



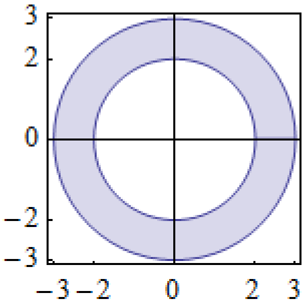
$$\frac{3\,\pi\,R^4}{4}$$

b)



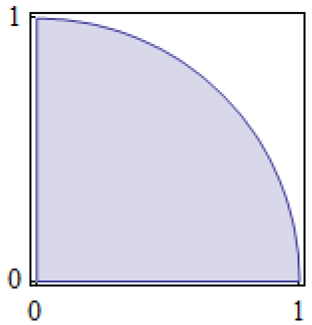
$$\frac{1}{6}$$

Exercício 7.13



$$\frac{38\,\pi}{3}$$

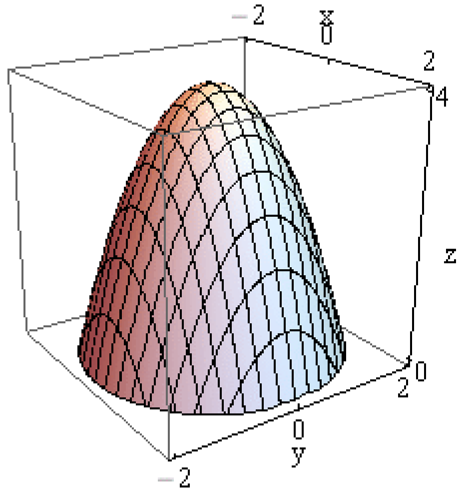
Exercício 7.14



$$\frac{1}{24}$$

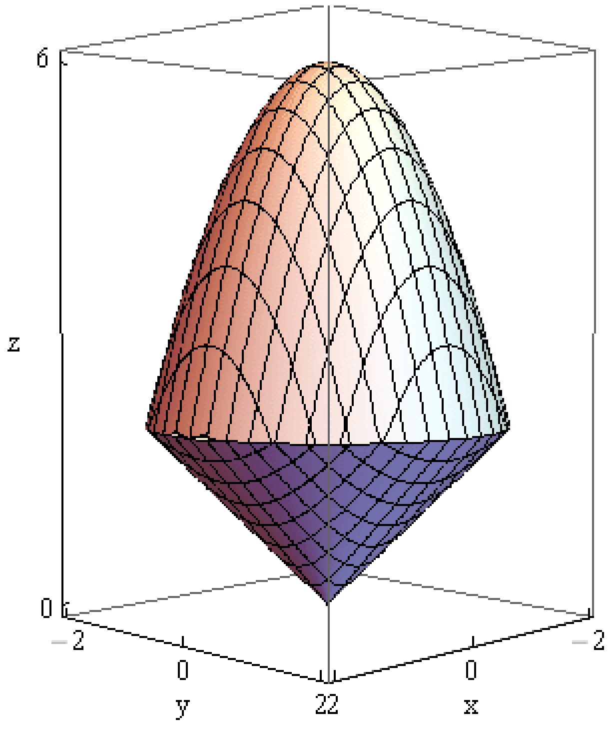
Exercício 7.15

a)



8π

b)



$\frac{32\pi}{3}$