8.

Exercício 8.1

a)
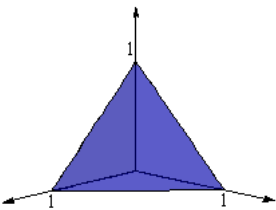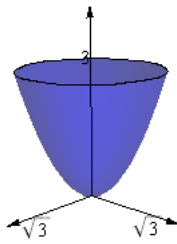

24

b)

$$\frac{1}{2} \, (-1 + e)^2$$

c)

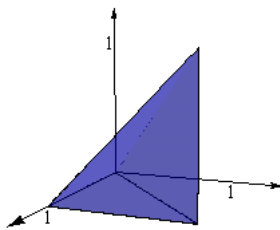$$\frac{1}{120}$$

d)

0

Exercício 8.2

$$\int_{-\sqrt{a}}^{\sqrt{a}} \int_{-\sqrt{a-x^2}}^{\sqrt{a-x^2}} \int_{0}^{a-x^2-y^2} dz\,dy\,dx = \frac{a^2\,\pi}{2}$$

Exercício 8.3

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-2\sqrt{4-x^2-y^2}}^{2\sqrt{4-x^2-y^2}} dz\,dy\,dx = \frac{64\,\pi}{3}$$

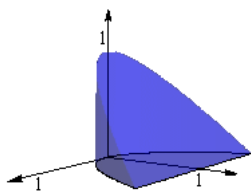
Exercício 8.4

a)

$$\int_{0}^{1} \int_{z}^{1} \int_{y}^{1} f[x,\,y,\,z]\,dx\,dy\,dz$$

b)

$$\int_{0}^{1} \int_{0}^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f[x,\,y,\,x]\,dx\,dy\,dz$$

Exercício 8.5

1

Exercício 8.6
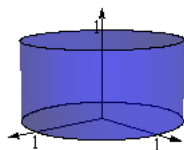
$$\int_0^1 \int_0^{2\pi} \int_0^1 z\, r^3\, dz\, d\theta\, dr = \frac{\pi}{4}$$

Exercício 8.7
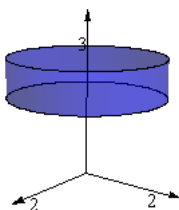
$$\int_0^2 \int_0^{2\pi} \int_2^3 z\, r\, \mathrm{Exp}\left[r^2\right] dz\, d\theta\, dr = \frac{5}{2}\left(-1 + e^4\right)\pi$$

Exercício 8.8
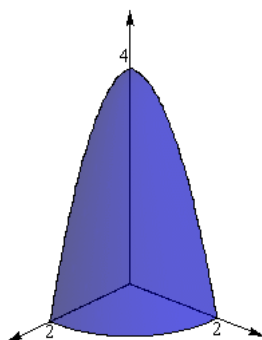
$$\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} \left(r^2\,\mathrm{Cos}[\theta] + r^2\,\mathrm{Sin}[\theta]\right) dz\, dr\, d\theta = \frac{128}{15}$$

Exercício 8.9
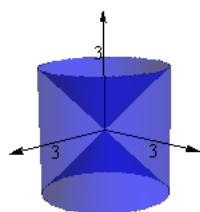
$$\int_0^{2\pi} \int_0^3 \int_{-r}^r r\, dz\, dr\, d\theta = 36\pi$$

Exercício 8.10
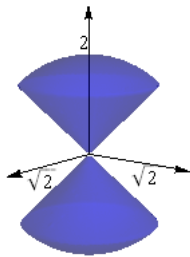
$$\int_0^{\frac{\pi}{4}} \int_0^2 \int_0^{2\pi} \rho^2 \, \text{Sin}[\phi] \, d\theta \, d\rho \, d\phi \; + \; \int_{\fra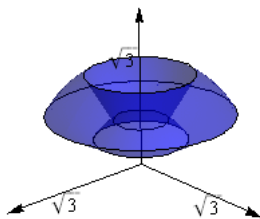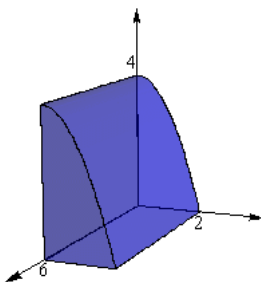c{3\pi}{4}}^{\pi} \int_0^2 \int_0^{2\pi} \rho^2 \, \text{Sin}[\phi] \, d\theta \, d\rho \, d\phi \; = \; 2 \int_0^{\frac{\pi}{4}} \int_0^2 \int_0^{2\pi} \rho^2 \, \text{Sin}[\phi] \, d\theta \, d\rho \, d\phi \; = \; -\frac{16}{3} \left(-2 + \right.$$

Exercício 8.11

$$\int_1^2 \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Exp}\left[\rho^3\right] \rho^2 \, \text{Sin}[\phi] \, d\phi \, d\theta \, d\rho \; = \; \frac{1}{3} \left(-1 + \sqrt{3}\right) e \left(-1 + e^7\right) \pi$$

Exercício 8.12

$$\int_0^6 \int_0^2 \int_0^{4-y^2} 1 \, dz \, dy \, dx \; = \; 32$$

Exercício 8.13

a)

$$\int_0^{2\pi} \int_0^{\pi} \int_{\sqrt{3}}^3 \rho^2 \, \text{Sin}[\phi] \, d\rho \, d\phi \, d\theta \; = \; -4 \left(-9 + \sqrt{3}\right) \pi$$

b)

$$\int_0^{2\pi} \int_0^{\sqrt{6}} \int_{r^2}^{12-r^2} r \, dz \, dr \, d\theta \; = \; 36 \pi$$

c)

$$\int_0^{2\pi} \int_0^2 \int_{\frac{r^2}{4}}^1 r\, dz\, dr\, d\theta \; = \; 2\pi$$

d)

$$\int_{-2}^2 \int_{-\sqrt{9-\frac{9\,z^2}{4}}}^{\sqrt{9-\frac{9\,z^2}{4}}} \int_{y^2+\frac{9\,z^2}{4}}^9 1\, dx\, dy\, dz \; = \; 27\,\pi$$

e)

$$\int_0^{2\pi} \int_1^2 \int_0^{r^2} r\, dz\, dr\, d\theta \; = \; \frac{15\,\pi}{2}$$

Exercício 8.14

$$\frac{\pi}{3}$$

Exercício 8.15

a)

$$\frac{\pi}{4}$$

b)

$$\left(\text{Cos}[1] - \text{Cos}\left[\frac{4+\pi}{4}\right]\right)\text{Log}[2]\,\text{Sin}[1]$$

Exercício 8.16

$$2 \int_0^{2\pi} \int_0^\pi \int_0^r \sqrt{r^2 - \rho^2}\;\rho^2\,\text{Sin}[\phi]\, d\rho\, d\phi\, d\theta \; = \; \frac{\pi^2\,r^4}{2}$$