LEI / LCC
Departamento de Informática
Universidade do Minho

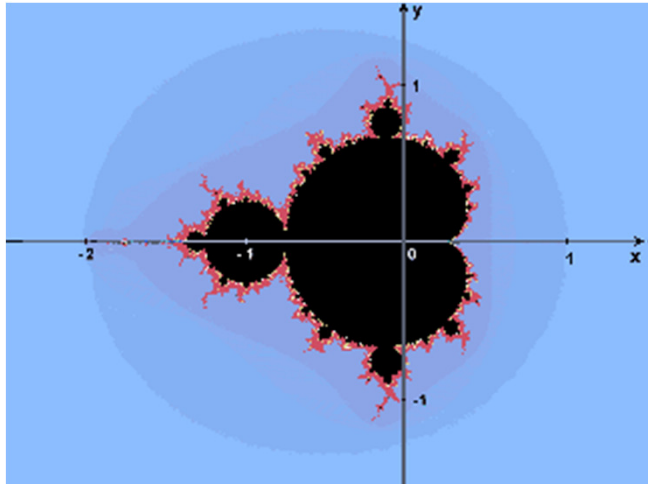
Fractais

Fractais

- **Definição:**
  - **Fractal** - *Term coined by Benoit Mandelbrot in 1975, referring to objects built using recursion, where some aspect of the limiting object is infinite and another is finite, and where at any iteration, some piece of the object is a scaled down version of the previous iteration*

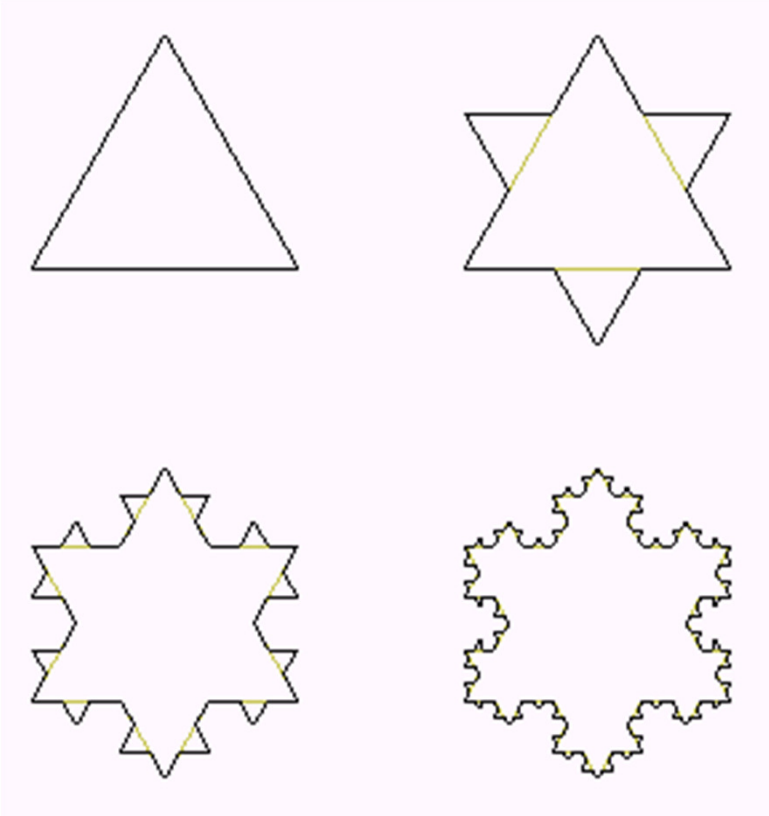
Fractais

- **Propriedades**:
  - Auto-semelhança
  - Construção Iterativa/Recursiva
  - Dimensão Fractal



Fractais

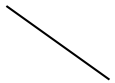
- **Propriedades:**
  - Auto-semelhança
  - Construção Iterativa/Recursiva



Koch Snowflake

Fractais

- **Propriedades: Dimensão Fractal**
  - dimensão: 0

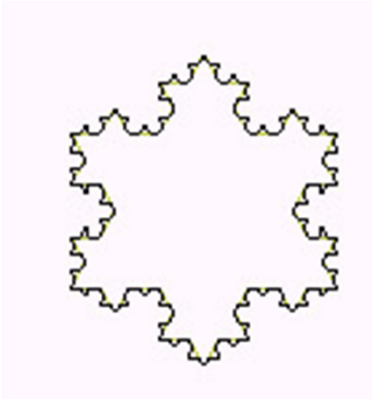


dimensão: 1

dimensão: 2

dimensão: 3

dimensão: ?



Fractais

- **Cálculo da dimensão**
  - figuras auto-semelhantes:

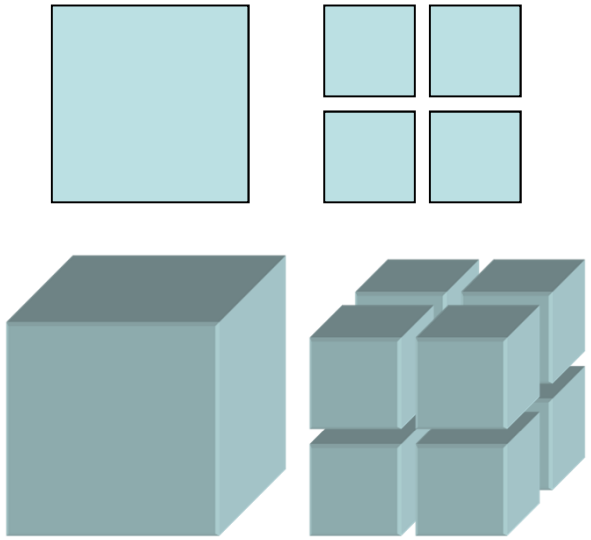


Figura	Dimensão	Cópias
segmento	1	$2 = 2^1$
quadrado	2	$4 = 2^2$
cubo	3	$8 = 2^3$

Fractais

- **Cálculo da dimensão: Dimensão de Hausdorff**

$$\frac{factor\_de\_multiplicação}{factor\_de\_contração} = 2^d$$

$$4/3 = 2^d$$

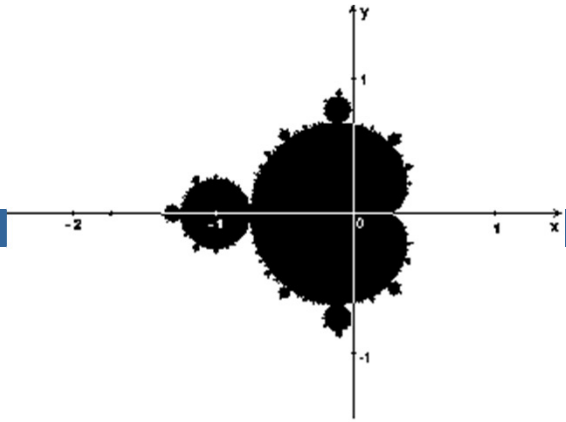
$$d = \ln(4/3)$$

$$d = 1.26$$

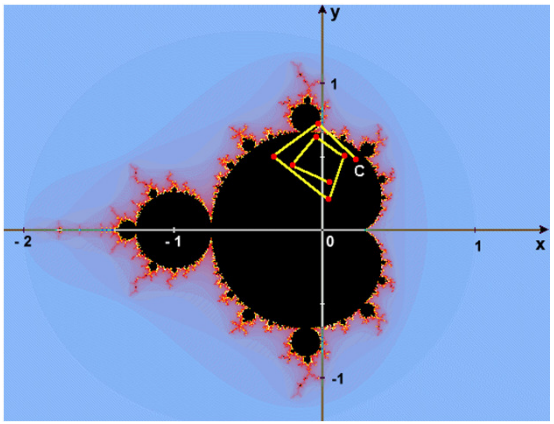
- Curva de Koch

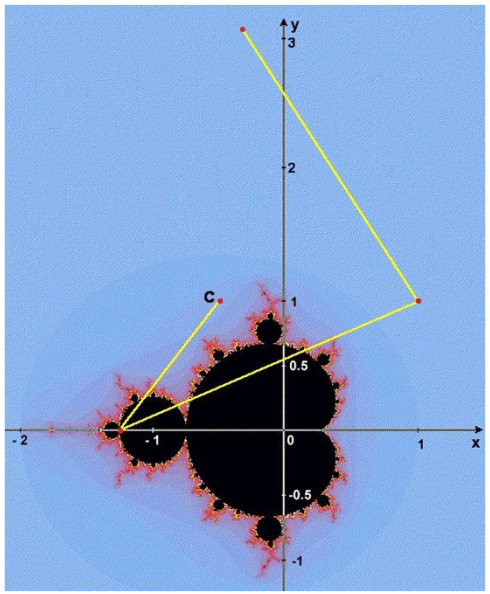
comp = 3 4

Fractais



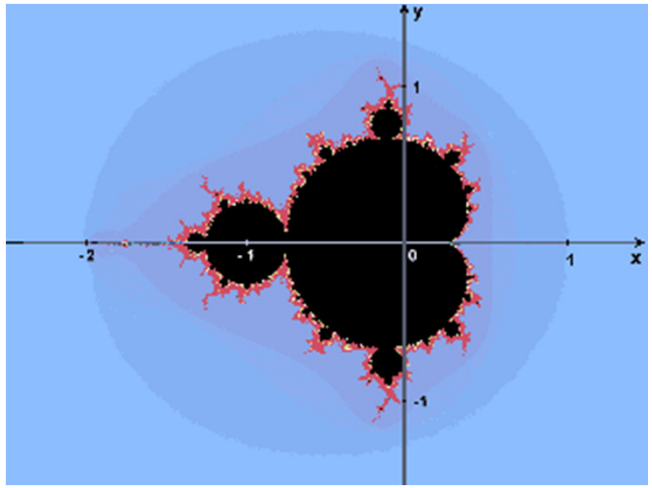
- **Mandelbrot**
  - $Z_n = Z_{n-1}^2 + c$
  - Se um ponto *c* não divergir em *n* iterações então pertence ao conjunto de Mandelbrot.





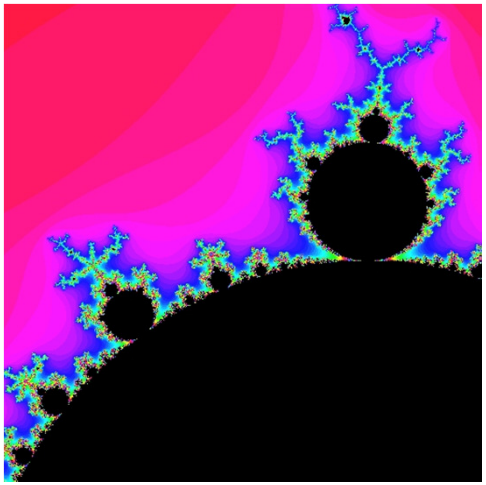
Fractais

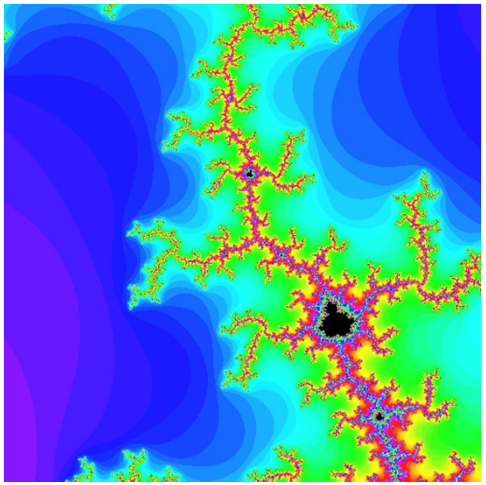
- Mandelbrot
  - A côr é definida pelo número de iterações que cada ponto necessita para divergir.
  - Por divergir entende-se que a distância do ponto Z_n ao centro é superior a d (normalmente d = 2).

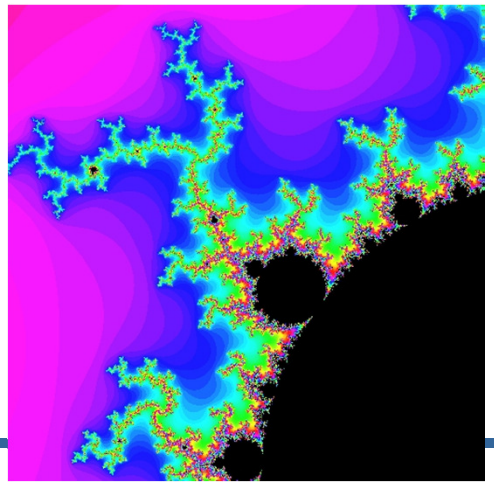


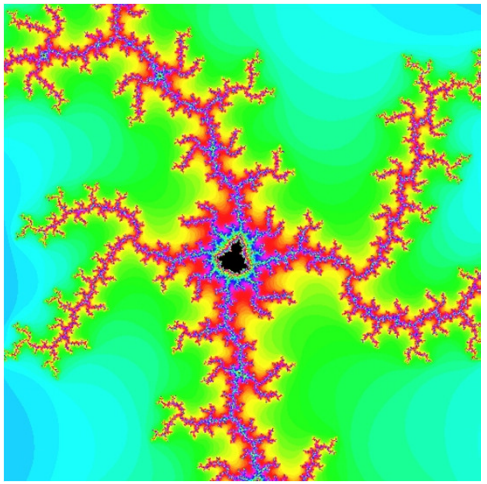
Fractais

- Mandelbrot





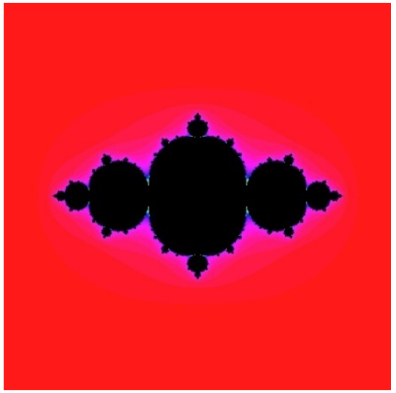




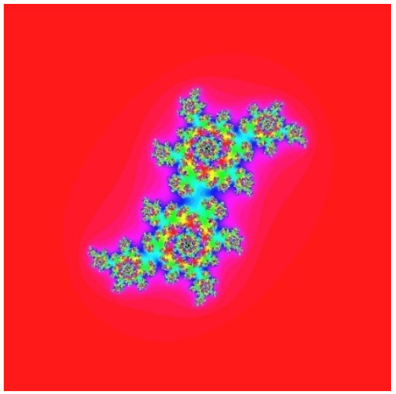
Fractais

- Julia
  - $Z_n = Z_{n-1}^2 + c$
  - Mesma formula que Mandelbrot, mas...
  - Z_0 é o ponto inicial
  - c é constante

c = -0.75 + 0i

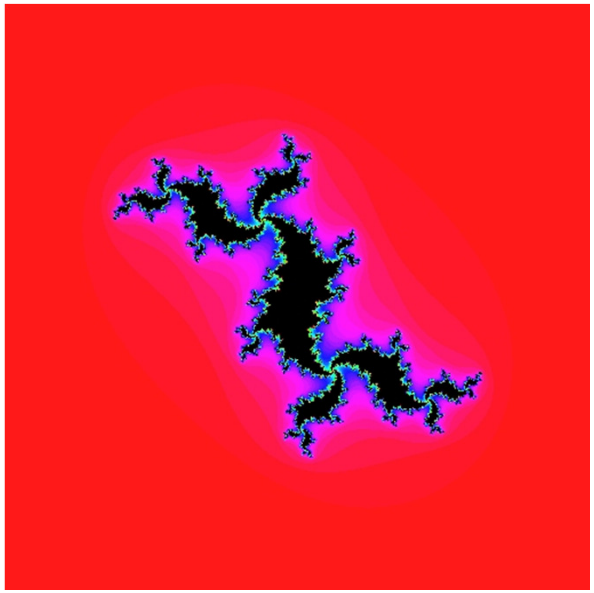


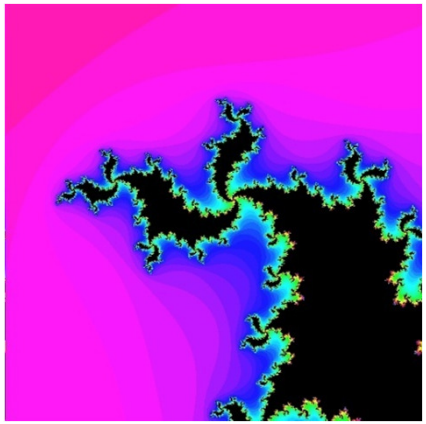
c = 0.1445 - 0.5977i

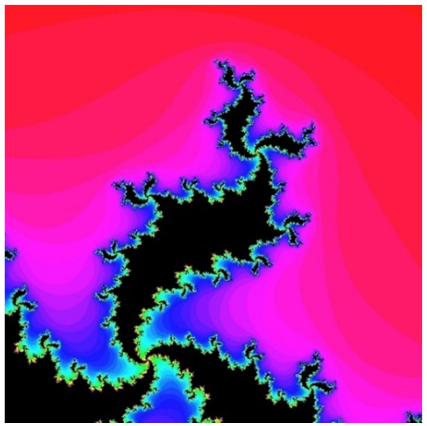


Fractais

- Julia





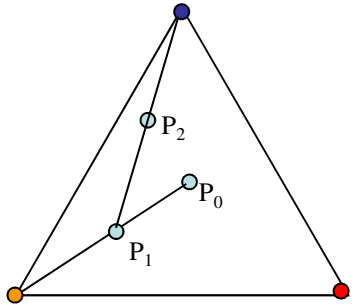


Fractais

- Números complexos
  - Os fractais Mandelbrot e Julia são definidos no plano dos números complexos.
    - $z = a + bi$
    - $z_1 * z_2 = (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i$
    - $z^2 = (a^2 - b^{2)} + (2ab)i$

Fractais

- Serpinski Gasket
  - Jogo do Caos
    - Definir um triangulo em que cada vértice tem uma cor
    - Seleccionar um ponto aleatoriamente
    - Escolher uma cor aleatoriamente
    - Mover o ponto para a posição intermédio entre o ponto e o vértice da mesma cor
    - Repetir os 3 passos anteriores *n* vezes

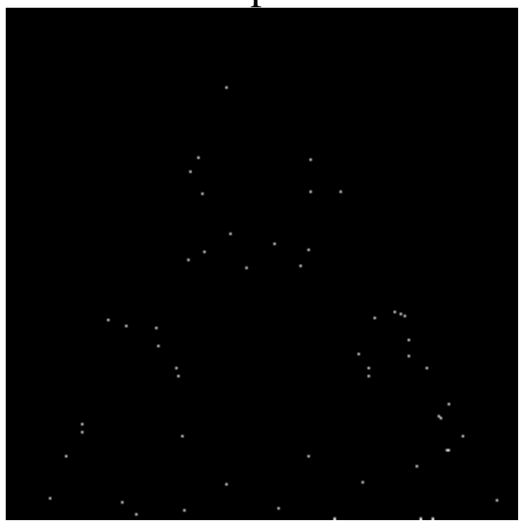


  - Qual o padrão formado?

Fractais

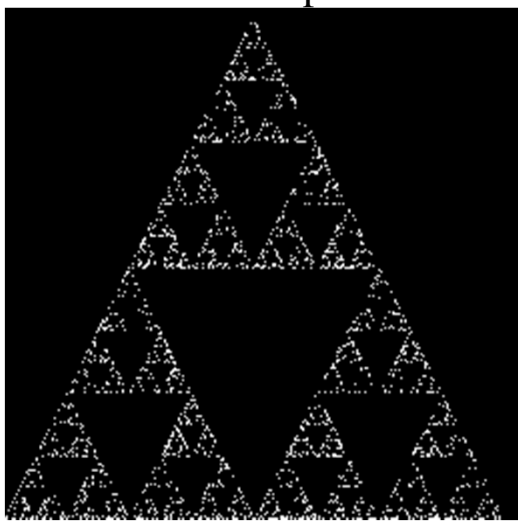
- Serpinski Gasket - Resultados

200 pontos



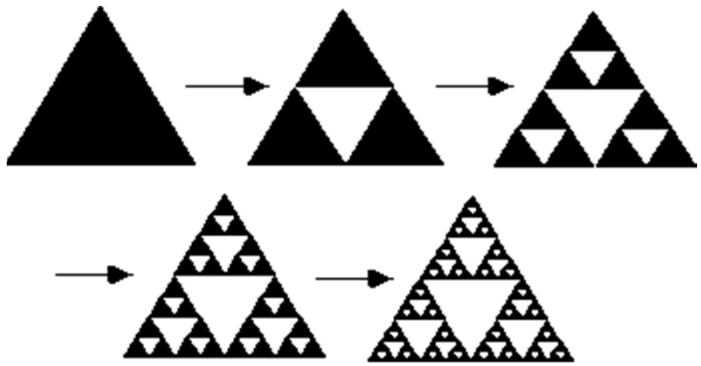
600 pontos

muitos pontos



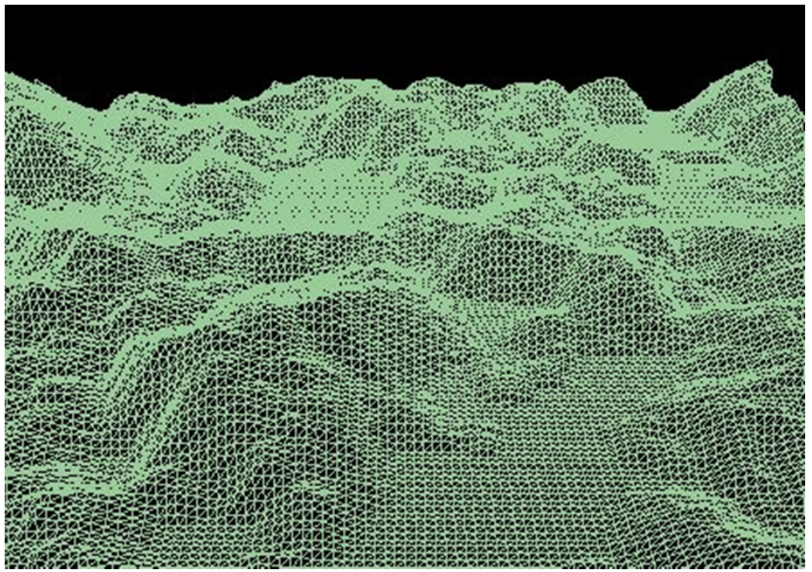
Fractais

- Serpinski Gasket
  - Construção iterativa/recursiva



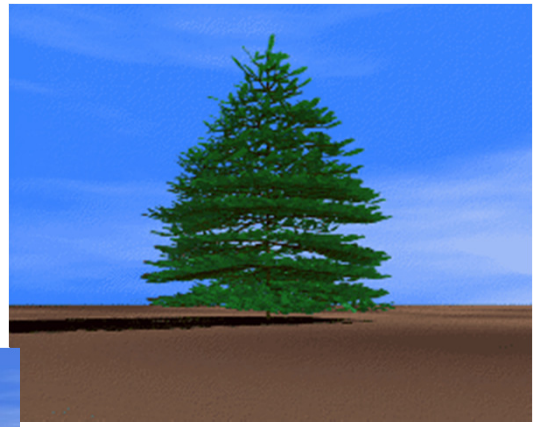
Fractais

- Modelação de Terrenos

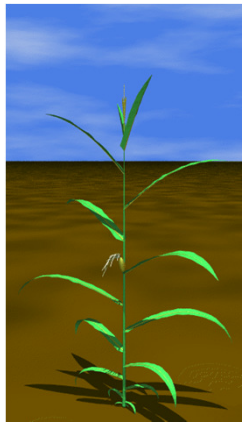


Fractais

- Modelação da Natureza
  - L - Systems



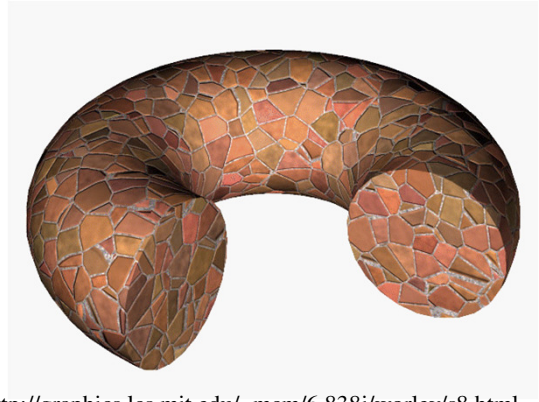
imagens obtidas em http://www.ifi.unizh.ch/staff/noser/Limages.html



Fractais

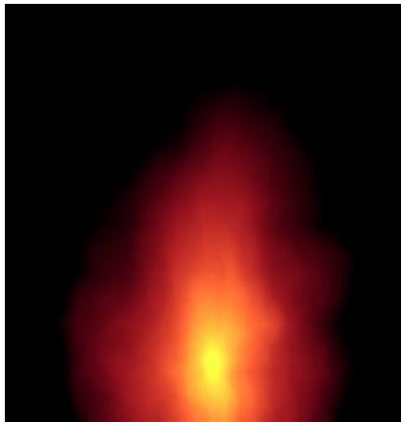
- Texturas

http://freespace.virgin.net/hugo.elias/models/m_perlin.htm



http://graphics.lcs.mit.edu/~mcm/6.838j/worley/s8.html

http://users.starpower.net/larch78/assignment5final.html



Fractais

- Referências
  - http://www.geocities.com/CapeCanaveral/2854/
  - http://www.hyperdictionary.com/computing/fractal
  - http://www.math.umass.edu/~mconnors/fractal/fractal.html