# 7.

#### Exercício 7.1

a)

$$\frac{31}{12}$$

b)

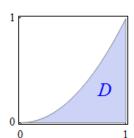
c)

$$\frac{7\sin[1]}{9}$$

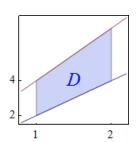
d)

#### Exercício 7.2

a)

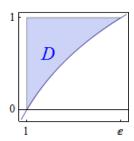


$$\int_0^1 \int_0^{\mathbf{x}^2} d\mathbf{y} d\mathbf{x} = \frac{1}{3}$$



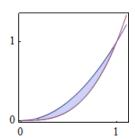
$$\int_{1}^{2} \int_{2}^{3} \frac{x+1}{x} dx dx = \frac{5}{2}$$

c)



$$\int_0^1 \int_1^{e^y} (x+y) \, dx dy = \frac{1}{4} \left(-1 + e^2\right)$$

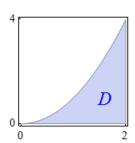
d)



$$\int_0^1 \int_{\mathbf{x}^3}^{\mathbf{x}^2} \mathbf{y} d\mathbf{y} d\mathbf{x} = \frac{1}{35}$$

## Exercício 7.3

a)



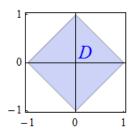
$$\int_0^2 \int_0^{x^2} xy dy dx = \frac{16}{3}$$

$$\int_0^4 \int_{\sqrt{y}}^2 xy dx dy = \frac{16}{3}$$

$$\int_{0}^{1} \int_{0}^{\pi} x \ \text{Sin}[x+y] \, dy dx = 2 \ (-1 + \text{Cos}[1] + \text{Sin}[1])$$

$$\int_0^\pi \int_0^1 x \ \text{Sin} \, [\, x \! + \! y \,] \, \text{d}x \, \text{d}y \! = \! 2 \ (\, \text{-1} + \text{Cos} \, [\, 1 \,] \, + \, \text{Sin} \, [\, 1 \,] \, )$$

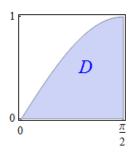
c)



$$\int_{-1}^{0}\int_{-\mathbf{x}-1}^{\mathbf{x}+1} \texttt{Exp}\left[\,\mathbf{x}+\mathbf{y}\,\right]\,\mathrm{d}\mathbf{y}\mathrm{d}\mathbf{x} + \int_{0}^{1}\int_{\mathbf{x}-1}^{-\mathbf{x}+1} \texttt{Exp}\left[\,\mathbf{x}+\mathbf{y}\,\right]\,\mathrm{d}\mathbf{y}\mathrm{d}\mathbf{x} = -\,\frac{1}{\mathbb{e}}\,+\,\mathbb{e}$$

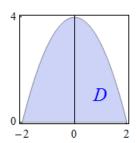
$$\int_{-1}^{1} \int_{y-1}^{1-y} \text{Exp}\left[\,x+y\,\right] \, \mathrm{d}x \, \mathrm{d}y + \int_{-1}^{1} \int_{-y-1}^{y+1} \text{Exp}\left[\,x+y\,\right] \, \mathrm{d}x \, \mathrm{d}y = -\,\frac{1}{e} \, + \, e$$

d)



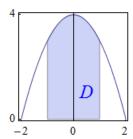
$$\int_0^{\frac{\pi}{2}} \! \int_0^{\mathtt{Sin}[\mathbf{x}]} \mathbf{x}^2 \! + \! y^2 \mathtt{d}y \mathtt{d}x \! = \! -\frac{16}{9} + \pi$$

$$\int_{0}^{1} \int_{\text{ArcSin}[y]}^{\frac{\pi}{2}} x^{2} + y^{2} dx dy = -\frac{16}{9} + \pi$$



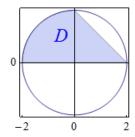
$$\int_{-2}^{2} \int_{0}^{4-x^{2}} f[x,y] dy dx = \int_{0}^{4} \int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x,y] dx dy$$

b)



$$\int_{-1}^{1} \int_{0}^{4-x^{2}} f[x,y] dy dx = \int_{3}^{4} \int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x,y] dx dy + \int_{0}^{3} \int_{-1}^{1} f[x,y] dx dy$$

c)



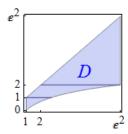
$$\int_{0}^{2} \int_{-\sqrt{4-y}}^{2-y} f[x,y] dxdy = \int_{-2}^{0} \int_{0}^{\sqrt{4-x^{2}}} f[x,y] dydx + \int_{0}^{2} \int_{0}^{2-x} f[x,y] dydx$$

d)

$$\int_{-3}^{2} \int_{-4+y^{2}}^{2-y} f[x,y] dxdy = \int_{-4}^{0} \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f[x,y] dydx + \int_{0}^{5} \int_{-\sqrt{x+4}}^{2-x} f[x,y] dydx$$

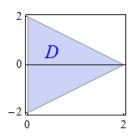
$$\int_{-2}^{2} \int_{-4+y^{2}}^{2-y} f[x,y] dxdy = \int_{-4}^{0} \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f[x,y] dydx + \int_{0}^{4} \int_{-2}^{2-x} f[x,y] dydx$$

f)



$$\int_{1}^{e^{2}} \int_{\text{Log}[x]}^{x} f[x,y] dy dx = \int_{0}^{1} \int_{0}^{\text{Exp}[y]} f[x,y] dx dy + \int_{1}^{2} \int_{y}^{\text{Exp}[y]} f[x,y] dx dy + \int_{2}^{e^{2}} \int_{y}^{e^{2}} f[x,y] dx dy$$

g)



$$\int_{-2}^{2} \int_{0}^{-|y|+2} f[x,y] dxdy = \int_{0}^{2} \int_{x-2}^{2-x} f[x,y] dydx$$

h)

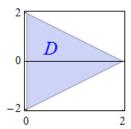
$$\int_{0}^{1} \int_{y-1}^{\sqrt{1-y^2}} f[x,y] dxdy = \int_{-1}^{0} \int_{0}^{x+1} f[x,y] dydx + \int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} f[x,y] dydx$$

i)

$$\begin{array}{c|cccc}
1 & & & \\
D & & & \\
0 & & 1 & & 2
\end{array}$$

$$\int_{0}^{1} \int_{0}^{\sqrt{x}} f[x,y] dy dx + \int_{1}^{2} \int_{0}^{-x+2} f[x,y] dy dx = \int_{0}^{1} \int_{y^{2}}^{2-y} f[x,y] dx dy$$

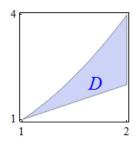
j)



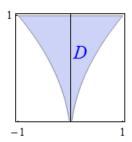
$$\int_{-2}^{0} \int_{0}^{y+2} f[x,y] dxdy + \int_{0}^{2} \int_{0}^{-y+2} f[x,y] dxdy = \int_{0}^{2} \int_{x-2}^{2-x} f[x,y] dydx$$

#### Exercício 7.5

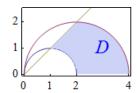
a)



$$\text{Área} = \frac{5}{6}$$



$$\text{Área} = \frac{2}{3}$$



$$Area = \frac{3(2+\pi)}{4}$$

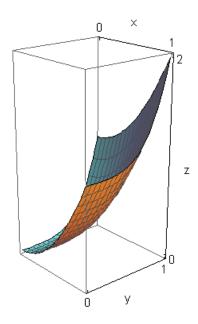
## Exercício 7.7

Área da circunferência= $4\int_0^r\int_0^{\sqrt{r^2-x^2}}\mathrm{d}y\mathrm{d}x$ = $\pi$   $r^2$ 

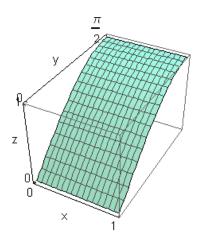
Área da elipse=
$$4\int_0^a\int_0^{\frac{b}{a}\sqrt{a^2-x^2}}\mathrm{d}y\mathrm{d}x$$
=ab $\pi$ 

#### Exercício 7.8

a)

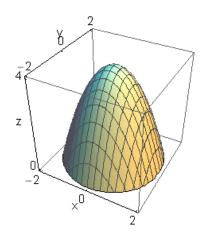


$$Volume = \frac{8}{15}$$



Volume=1

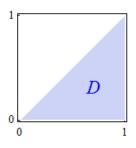
c)



 ${\tt Volume=8}\;\pi$ 

## Exercício 7.9

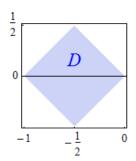
a)



 $\frac{1}{2}$ 

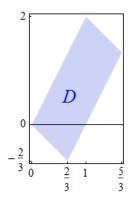
## Exercício 7.10

a)



 $\frac{1}{6}$ 

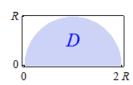
b)



 $\frac{1}{3}$ 

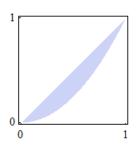
## Exercício 7.11

a)



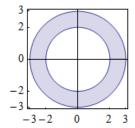
 $\frac{3~\pi~R^4}{4}$ 

b)



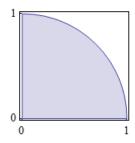
 $\frac{1}{6}$ 

Exercício 7.12



$$\frac{38 \pi}{3}$$

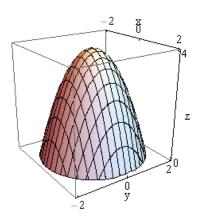
Exercício 7.13



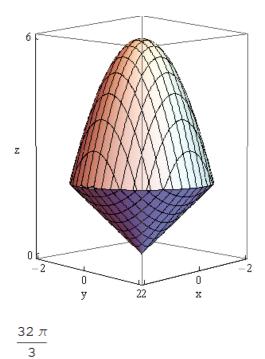
 $\frac{1}{24}$ 

Exercício 7.14

a)



8 π



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