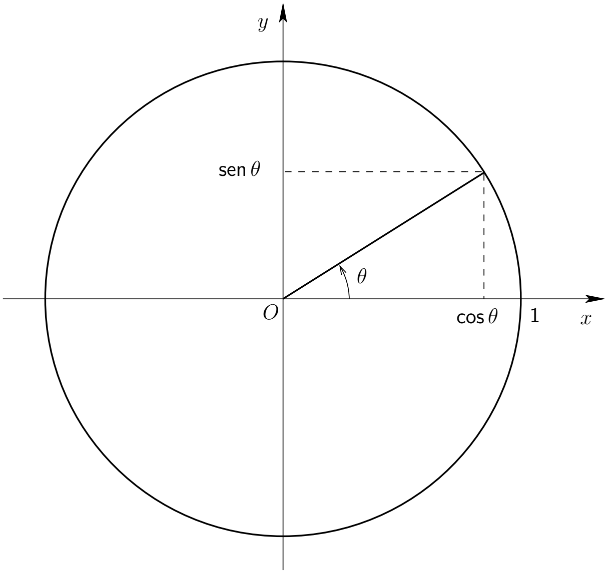
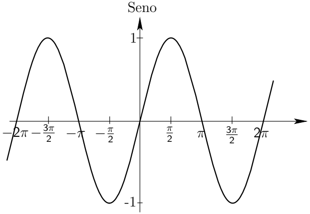
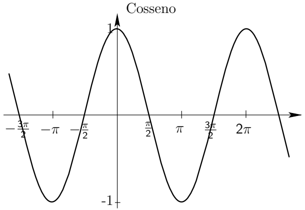
Funções trigonométricas

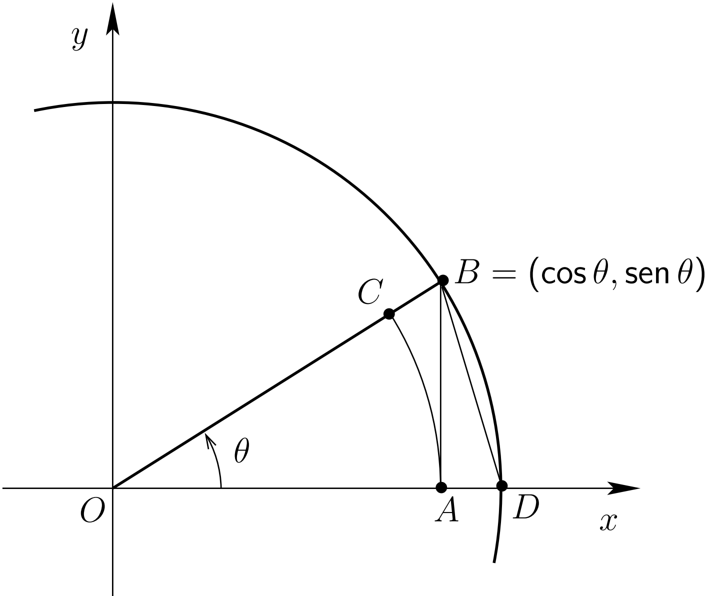


Gráficos das funções trigonométricas





$$\lim_{\theta \to 0} \frac{\operatorname{sen} \theta}{\theta} = 1$$



Funções trigonométricas

Tangente

$$\mathrm{tg} : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : \ k \in \mathbb{Z} \right\} \longrightarrow \mathbb{R} \quad \text{tal que } \mathrm{tg}\, x = \frac{\mathrm{sen}\, x}{\cos x}$$

Cotangente

$$\mathrm{cotg} : \mathbb{R} \setminus \{ k\pi : \ k \in \mathbb{Z} \} \longrightarrow \mathbb{R} \quad \text{tal que } \mathrm{cotg}\, x = \frac{\cos x}{\mathrm{sen}\, x}$$

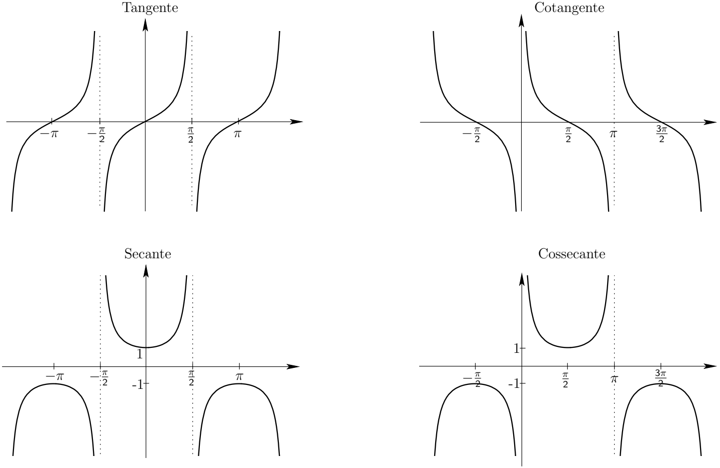
Secante

$$\sec : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : \ k \in \mathbb{Z} \right\} \longrightarrow \mathbb{R} \quad \text{tal que } \sec x = \frac{1}{\cos x}$$

Cossecante

$$\mathrm{cosec} : \mathbb{R} \setminus \{ k\pi : \ k \in \mathbb{Z} \} \longrightarrow \mathbb{R} \quad \text{tal que } \mathrm{cosec}\, x = \frac{1}{\mathrm{sen}\, x}$$

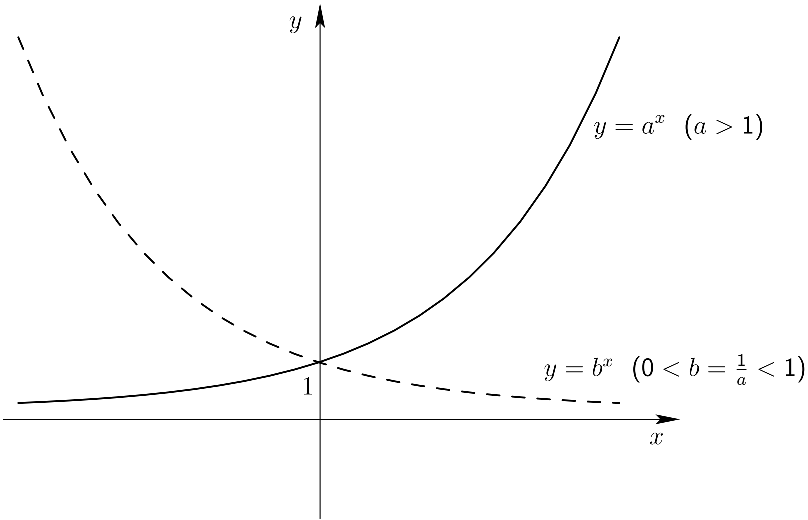
Gráficos das funções trigonométricas



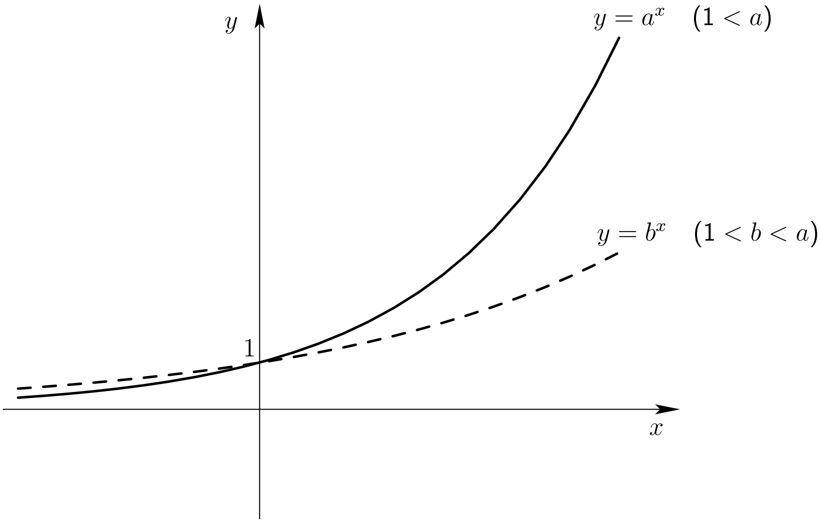
Algumas propriedades das funções trigonométricas

1. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}^2 a + \cos^2 a = 1$;
2. $\forall\, a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$;
3. $\forall\, a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$;
4. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(-a) = -\operatorname{sen} a \quad$ (sen é ímpar);
5. $\forall\, a \in \mathbb{R} \quad \cos(-a) = \cos a \quad$ (cos é par);
6. $\forall\, a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \operatorname{sen} a \quad$ e $\quad \operatorname{sen}(\frac{\pi}{2} - a) = \cos a$;
7. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(a + 2\pi) = \operatorname{sen} a \quad$ (sen tem período 2π);
8. $\forall\, a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a \quad$ (cos tem período 2π);
9. $\forall\, a,\ b \in \mathbb{R} \quad \operatorname{sen}(a + b) = \operatorname{sen} a\, \cos b + \operatorname{sen} b\, \cos a$;
10. $\forall\, a,\ b \in \mathbb{R} \quad \cos(a + b) = \cos a\, \cos b - \operatorname{sen} b\, \operatorname{sen} a$;
11. $\forall\, a,\ b \in \mathbb{R} \quad \cos a - \cos b = -2 \operatorname{sen} \dfrac{a-b}{2} \operatorname{sen} \dfrac{a+b}{2}$;
12. $\forall\, a,\ b \in \mathbb{R} \quad \operatorname{sen} a - \operatorname{sen} b = 2 \operatorname{sen} \dfrac{a-b}{2} \cos \dfrac{a+b}{2}$.

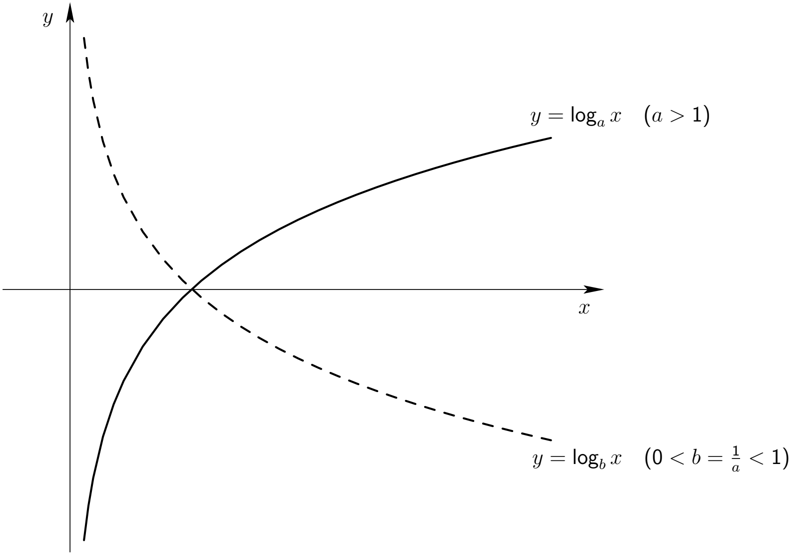
Funções exponenciais



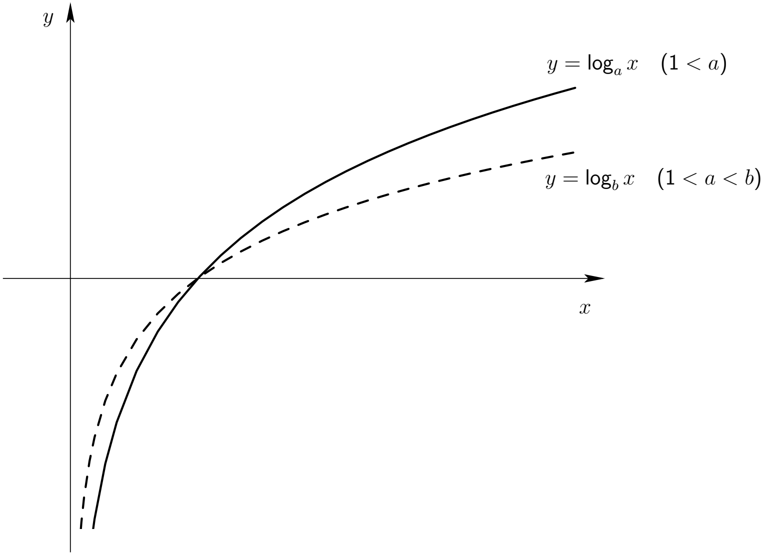
Funções exponenciais



Funções logaritmos



Funções logaritmos



Funções hiperbólicas

Seno hiperbólico

$$\begin{aligned} \mathsf{sh}: \quad \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x - e^{-x}}{2} \end{aligned}$$

Cosseno hiperbólico

$$\begin{aligned} \mathsf{ch}: \quad \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x + e^{-x}}{2} \end{aligned}$$

Tangente hiperbólica

$$\begin{aligned} \mathsf{th}: \quad \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{\mathsf{sh}\, x}{\mathsf{ch}\, x} \end{aligned}$$

Cotangente hiperbólica

$$\begin{aligned} \mathsf{coth}: \quad \mathbb{R} \setminus \{0\} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\mathsf{th}\, x} \end{aligned}$$

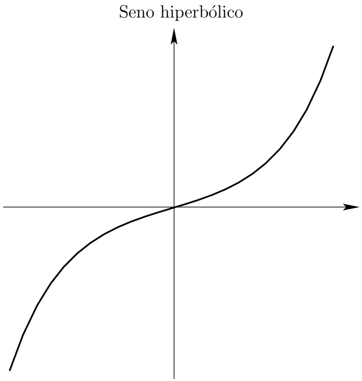
Secante hiperbólica

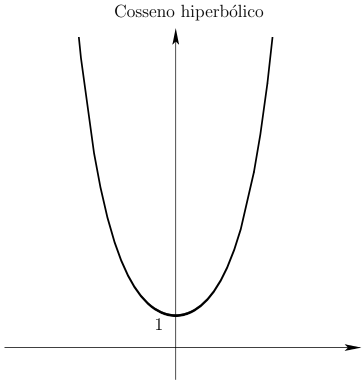
$$\begin{aligned} \mathsf{sech}: \quad \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\mathsf{ch}\, x} \end{aligned}$$

Cossecante hiperbólica

$$\begin{aligned} \mathsf{cosech}: \quad \mathbb{R} \setminus \{0\} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\mathsf{sh}\, x} \end{aligned}$$

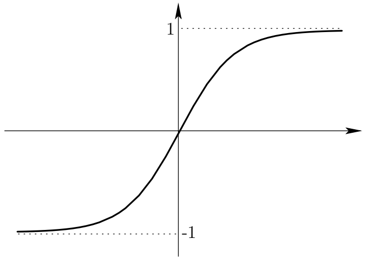
Funções hiperbólicas



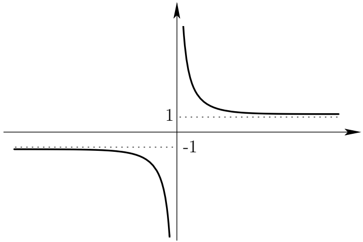


Funções hiperbólicas

Tangente hiperbólica

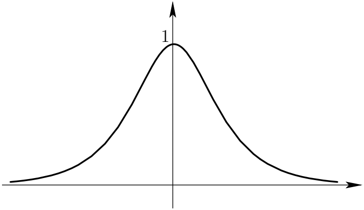


Cotangente hiperbólica

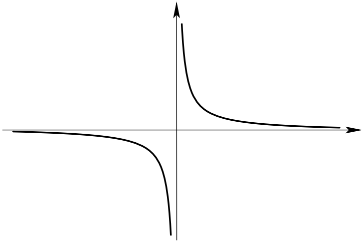


Funções hiperbólicas

Secante hiperbólica



Cossecante hiperbólica



Funções hiperbólicas - propriedades

1. $\forall\, a \in \mathbb{R} \qquad \operatorname{ch}^2 a - \operatorname{sh}^2 a = 1;$

2. $\forall\, a \in \mathbb{R} \qquad \operatorname{th}^2 a + \operatorname{sech}^2 a = 1;$

3. $\forall\, a \in \mathbb{R} \setminus \{0\} \qquad \operatorname{coth}^2 a - \operatorname{cosech}^2 a = 1;$

4. $\forall\, a \in \mathbb{R} \qquad \operatorname{sh}(-a) = -\operatorname{sh} a$ (a função sh é ímpar);

5. $\forall\, a \in \mathbb{R} \qquad \operatorname{ch}(-a) = \operatorname{ch} a$ (a função ch é par);

6. $\forall\, a,\, b \in \mathbb{R} \qquad \operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a;$

7. $\forall\, a,\, b \in \mathbb{R} \qquad \operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} b \operatorname{sh} a;$

8. $\forall\, n \in \mathbb{N} \quad \forall\, a \in \mathbb{R} \qquad (\operatorname{ch} a + \operatorname{sh} a)^n = \operatorname{ch}(na) + \operatorname{sh}(na).$

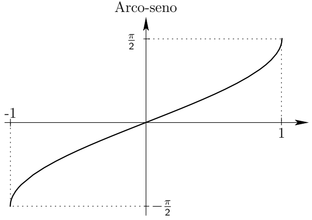
Funções trigonométricas inversas

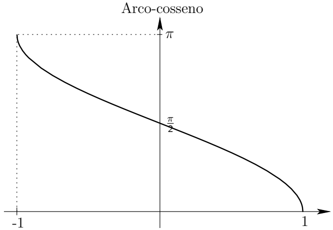
Arco-seno

$$\begin{array}{rccc} \text{arcsen}: & [-1,1] & \longrightarrow & \left[-\frac{\pi}{2},\frac{\pi}{2}\right] \\ & x & \longmapsto & \left(\text{sen}|_{[-\frac{\pi}{2},\frac{\pi}{2}]}\right)^{-1}(x) \end{array}$$

Arco-cosseno

$$\begin{array}{rccc} \text{arccos}: & [-1,1] & \longrightarrow & [0,\pi] \\ & x & \longmapsto & \left(\cos|_{[0,\pi]}\right)^{-1}(x) \end{array}$$



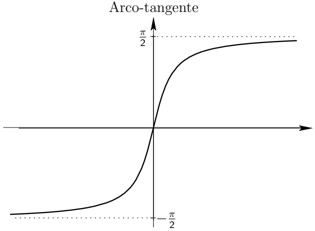


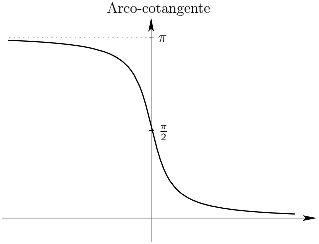
Arco-tangente

$$\begin{array}{rccl} \operatorname{arctg}: & \mathbb{R} & \longrightarrow & \left]-\frac{\pi}{2},\frac{\pi}{2}\right[\\ & x & \longmapsto & \left(\operatorname{tg}_{|_{\left]-\frac{\pi}{2},\frac{\pi}{2}\right[}}\right)^{-1}(x) \end{array}$$

Arco-cotangente

$$\begin{array}{rccl} \operatorname{arcotg}: & \mathbb{R} & \longrightarrow &]0,\pi[\\ & x & \longmapsto & \left(\operatorname{cotg}_{|_{]0,\pi[}}\right)^{-1}(x) \end{array}$$



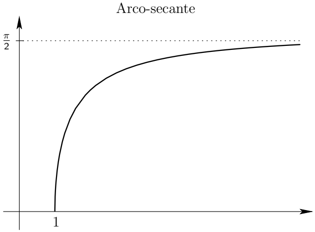


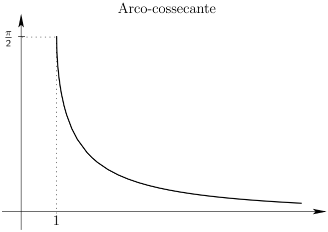
Arco-secante

$$\begin{array}{rccc} \text{arcsec}: & [1,+\infty[& \longrightarrow & [\,0,\frac{\pi}{2}[\\ & x & \longmapsto & \left(\sec_{|_{[0,\frac{\pi}{2}[}}\right)^{-1}(x) \end{array}$$

Arco-cossecante

$$\begin{array}{rccc} \text{arcosec}: & [1,+\infty[& \longrightarrow &]0,\frac{\pi}{2}] \\ & x & \longmapsto & \left(\text{cosec}_{|_{]0,\frac{\pi}{2}]}}\right)^{-1}(x) \end{array}$$





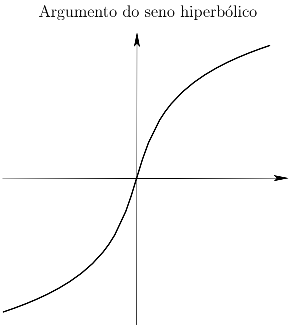
Funções hiperbólicas inversas

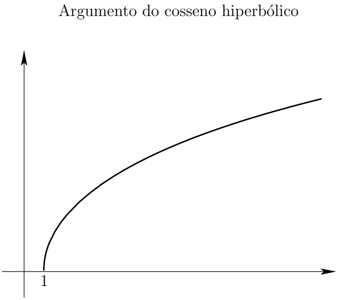
Argumento do seno hiperbólico

$$\begin{array}{rccc} \mathsf{argsh}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & (\mathsf{sh})^{-1}(x) \end{array}$$

Argumento do cosseno hiperbólico

$$\begin{array}{rccc} \mathsf{argch}: & [1,+\infty[& \longrightarrow & \mathbb{R}_0^+ \\ & x & \longmapsto & \left(\mathsf{ch}_{|_{\mathbb{R}_0^+}}\right)^{-1}(x) \end{array}$$





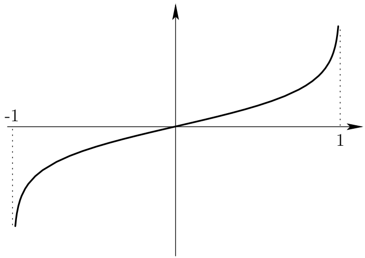
Argumento da tangente hiperbólica

$$\begin{array}{rccc} \mathsf{argth}: &]-1,1[& \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \mathsf{th}^{-1}(x) \end{array}$$

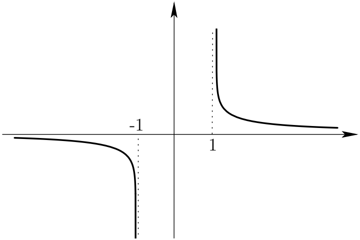
Argumento da cotangente hiperbólica

$$\begin{array}{rccc} \mathsf{argcoth}: & \mathbb{R}\setminus[-1,1] & \longrightarrow & \mathbb{R}\setminus\{0\} \\ & x & \longmapsto & \mathsf{coth}^{-1}(x) \end{array}$$

Argumento da tangente hiperbólica



Argumento da cotangente hiperbólica



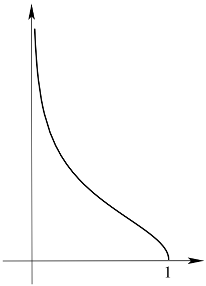
Argumento da secante hiperbólica

$$\begin{array}{rccc} \mathsf{argsech}: &]0,1] & \longrightarrow & \mathbb{R}_0^+ \\ & x & \longmapsto & \left(\mathsf{sec}_{|_{\mathbb{R}_0^+}}\right)^{-1}(x) \end{array}$$

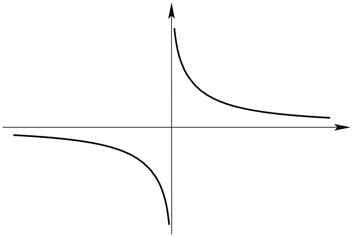
Argumento da cossecante hiperbólica

$$\begin{array}{rccc} \mathsf{argcosech}: & \mathbb{R}\setminus\{0\} & \longrightarrow & \mathbb{R}\setminus\{0\} \\ & x & \longmapsto & \mathsf{cosech}^{-1}(x) \end{array}$$

Argumento da secante hiperbólica



Argumento da cossecante hiperbólica



Derivadas das funções trigonométricas e das funções hiperbólicas

$$\operatorname{sen}' x = \cos x \qquad \cos' x = -\operatorname{sen} x$$

$$\operatorname{tg}' x = \sec^2 x \qquad \operatorname{cotg}' x = -\operatorname{cosec}^2 x$$

$$\sec' x = \sec x \ \operatorname{tg} x \qquad \operatorname{cosec}' x = -\operatorname{cosec} x \ \operatorname{cotg} x$$

$$\operatorname{sh}' x = \operatorname{ch} x \qquad \operatorname{ch}' x = \operatorname{sh} x$$

$$\operatorname{th}' x = \operatorname{sech}^2 x \qquad \operatorname{coth}' x = -\operatorname{cosech}^2 x$$

$$\operatorname{sech}' x = -\operatorname{sech} x \ \operatorname{th} x \qquad \operatorname{cosech}' x = -\operatorname{cosech} x \ \operatorname{coth} x$$

Derivadas das funções trigonométricas inversas e das funções hiperbólicas inversas

$$\operatorname{arcsen}' x = \frac{1}{\sqrt{1-x^2}} \qquad \operatorname{arccos}' x = \frac{-1}{\sqrt{1-x^2}}$$

$$\operatorname{arctg}' x = \frac{1}{1+x^2} \qquad \operatorname{arcotg}' x = \frac{-1}{1+x^2}$$

$$\operatorname{arcsec}' x = \frac{1}{x\sqrt{x^2-1}} \qquad \operatorname{arcosec}' x = \frac{-1}{x\sqrt{x^2-1}}$$

$$\operatorname{argsh}' x = \frac{1}{\sqrt{1+x^2}} \qquad \operatorname{argch}' x = \frac{1}{\sqrt{x^2-1}}$$

$$\operatorname{argth}' x = \frac{1}{1-x^2} \qquad \operatorname{argcoth}' x = \frac{1}{1-x^2}$$

$$\operatorname{argsech}' x = \frac{-1}{x\sqrt{1-x^2}} \qquad \operatorname{argcosech}' x = \frac{-1}{x\sqrt{1+x^2}}$$