3.8

$$\begin{cases} \lambda x_1 + x_2 + x_3 = 1 \\ x_1 + \lambda x_2 + x_3 = \lambda \\ x_1 + x_2 + \lambda x_3 = \lambda^2 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & \lambda & \lambda^2 \\ 1 & \lambda & 1 & \lambda \\ \lambda & 1 & 1 & 1 \end{array}\right) \xrightarrow[L_3 \leftarrow L_3 - \lambda L_1]{L_2 \leftarrow L_2 - L_1} \left(\begin{array}{ccc|c} 1 & 1 & \lambda & \lambda^2 \\ 0 & \lambda-1 & 1-\lambda & \lambda-\lambda^2 \\ 0 & 1-\lambda & 1-\lambda^2 & 1-\lambda^3 \end{array}\right)$$

$$\xrightarrow[L_3 \leftarrow L_3 + L_2]{} \left(\begin{array}{ccc|c} 1 & 1 & \lambda & \lambda^2 \\ 0 & \lambda-1 & 1-\lambda & \lambda-\lambda^2 \\ 0 & 0 & 2-\lambda-\lambda^2 & 1+\lambda-\lambda^2-\lambda^3 \end{array}\right)$$

$2-\lambda-\lambda^2 = -(\lambda+2)(\lambda-1)$ Zeros: -2 e 1

$1+\lambda-\lambda^2-\lambda^3 = -(\lambda+1)^2(\lambda-1)$ Zeros: -1 e 1

$\lambda = -2 \Rightarrow c(A) = 2 < 3 = c(A|b) \rightarrow$ Sistema Impossível

$\lambda = 1 \Rightarrow c(A) = c(A|b) = 1 \rightarrow$ Sistema Possível Indeterminado

Restantes casos $\rightarrow$ Possível determinado

3.9

a)
$$\left(\begin{array}{cccc|c} 2 & 6 & -1 & 1 & -3 \\ 1 & -1 & 1 & -1 & 2 \\ -1 & -3 & 3 & 2 & 9 \end{array}\right) \xrightarrow[L_2 \leftrightarrow L_1]{} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 2 & 6 & -1 & 1 & -3 \\ -1 & -3 & 3 & 2 & 9 \end{array}\right) \xrightarrow[L_3 \leftarrow L_3 + L_1]{L_2 \leftarrow L_2 - 2L_1} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 8 & -3 & 3 & -7 \\ 0 & -4 & 4 & 1 & 11 \end{array}\right)$$

$$\xrightarrow[L_3 \leftarrow L_3 + \frac{1}{2} L_2]{} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 8 & -3 & 3 & -7 \\ 0 & 0 & 5/2 & 5/2 & 15/2 \end{array}\right)$$

$$\begin{cases} x_1 - x_2 + x_3 - x_4 = 2 \\ 8x_2 - 3x_3 + 3x_4 = -7 \\ 5/2\, x_3 + 5/2\, x_4 = 15/2 \end{cases} \Leftrightarrow \begin{cases} x_1 = -\frac{3}{4} + 5/4\, x_4 \\ x_2 = \frac{1}{4} - \frac{3}{4} x_4 \\ x_3 = 3 - x_4 \end{cases}$$

Tomando $x_4 = \alpha$, $\alpha \in \mathbb{R}$, tem-se
$$\begin{cases} x_1 = -3/4 + 5/4\,\alpha \\ x_2 = 1/4 - 3/4\,\alpha \\ x_3 = 3 - \alpha \\ x_4 = \alpha \end{cases}$$
e

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \underbrace{\begin{pmatrix} -3/4 \\ 1/4 \\ 3 \\ 0 \end{pmatrix}}_{\text{sol. partic.}} + \alpha \underbrace{\begin{pmatrix} 5/4 \\ -3/4 \\ -1 \\ 1 \end{pmatrix}}_{\text{sol. sist. homogéneo "associado"}}$$

3.9 b)
$$\begin{cases} x_1 - x_3 + 3x_4 = 4 \\ 2x_1 + 2x_2 + x_4 = -3 \\ -3x_1 - 2x_2 + x_3 + 4x_4 = -1 \\ -x_1 + 2x_2 + 2x_3 + 16x_4 = -15 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 4 \\ 2 & 2 & 0 & 1 & -3 \\ -3 & -2 & 1 & 4 & -1 \\ -1 & 2 & 3 & 16 & -15 \end{array}\right) \xrightarrow[\substack{L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 + 3L_1 \\ L_4 \leftarrow L_4 + L_1}]{} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 4 \\ 0 & 2 & 2 & -5 & -11 \\ 0 & -2 & -2 & 13 & 11 \\ 0 & 2 & 2 & 19 & -11 \end{array}\right) \xrightarrow[\substack{L_3 \leftarrow L_3 + L_2 \\ L_4 \leftarrow L_4 - L_2}]{} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 4 \\ 0 & 2 & 2 & -5 & -11 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 24 & 0 \end{array}\right)$$

$$\xrightarrow[L_4 \leftarrow L_4 - 3L_3]{} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 4 \\ 0 & 2 & 2 & -5 & -11 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{cases} x_1 - x_3 + 3x_4 = 4 \\ 2x_2 + 2x_3 - 5x_4 = -11 \\ 8x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 4 + x_3 \\ x_2 = -\frac{11}{2} - x_3 \\ x_4 = 0 \end{cases}$$

Tomando $x_3 = \alpha$, $\alpha \in \mathbb{R}$, tem-se
$$\begin{cases} x_1 = 4 + \alpha \\ x_2 = -\frac{11}{2} - \alpha \\ x_3 = \alpha \\ x_4 = 0 \end{cases} \text{ e } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 \\ -11/2 \\ 0 \\ 0 \end{pmatrix}}_{\text{sol. partic.}} + \alpha \underbrace{\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}}_{\substack{\text{sol.} \\ \text{sist.} \\ \text{hom.} \\ \text{"associado"}}}$$

3.10 a)
$$\begin{cases} x_2 + 2x_3 + x_4 = 0 \\ x_1 + 2x_2 + 2x_3 = 0 \\ -2x_1 + 4x_2 + 2x_3 + 3x_4 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 2 & 0 \\ -2 & 4 & 2 & 3 \\ 0 & 1 & 2 & 1 \end{pmatrix} \xrightarrow[L_2 \leftarrow L_2 + 2L_1]{} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 8 & 6 & 3 \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow[L_3 \leftarrow L_3 - \frac{1}{8}L_2]{} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 8 & 6 & 3 \\ 0 & 0 & 5/4 & 5/8 \end{pmatrix} \quad \begin{cases} x_1 + 2x_2 + 2x_3 = 0 \\ 8x_2 + 6x_3 + 3x_4 = 0 \\ 5/4\, x_3 + 5/8\, x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = x_4 \\ x_2 = 0 \\ x_3 = -\frac{1}{2} x_4 \end{cases}$$

Tomando $x_4 = \alpha$, $\alpha \in \mathbb{R}$, tem-se
$$\begin{cases} x_1 = \alpha \\ x_2 = 0 \\ x_3 = -1/2\, \alpha \\ x_4 = \alpha \end{cases} \text{ e } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ -1/2 \\ 1 \end{pmatrix}$$

3.10 b) $\begin{cases} 3x_1+2x_2+x_3-x_4-x_5=0 \\ x_1-x_2-x_3-x_4+2x_5=0 \\ -x_1+2x_2+3x_3+x_4-x_5=0 \end{cases}$

$$\begin{pmatrix} 1 & -1 & -1 & -1 & 2 \\ -1 & 2 & 3 & 1 & -1 \\ 3 & 2 & 1 & -1 & -1 \end{pmatrix} \xrightarrow[L_3 \leftarrow L_3-3L_1]{L_2 \leftarrow L_2+L_1} \begin{pmatrix} 1 & -1 & -1 & -1 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 5 & 4 & 2 & -7 \end{pmatrix} \xrightarrow{L_3 \leftarrow L_3-5L_1} \begin{pmatrix} 1 & -1 & -1 & -1 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -6 & 2 & -12 \end{pmatrix}$$

$$\begin{cases} x_1-x_2-x_3-x_4+2x_5=0 \\ x_2+2x_3+x_5=0 \\ -3x_3+x_4-6x_5=0 \end{cases} \Leftrightarrow \begin{cases} x_1=2x_3+3x_5 \\ x_2=-2x_3-x_5 \\ x_4=3x_3+6x_5 \end{cases}$$

Tomando $x_3=\alpha$ e $x_5=\beta$, $\alpha,\beta \in IR$, tem-se

$$\begin{cases} x_1=2\alpha+3\beta \\ x_2=-2\alpha-\beta \\ x_3=\alpha \\ x_4=3\alpha+6\beta \\ x_5=\beta \end{cases} \quad \text{e} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ -2 \\ 1 \\ 3 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ -1 \\ 0 \\ 6 \\ 1 \end{pmatrix}$$

3.11 $\begin{cases} x_1-2x_3=0 \\ x_2+x_3-x_4=0 \end{cases} \Leftrightarrow \begin{cases} x_1=2x_3 \\ x_2=x_4-x_3 \end{cases}$

Tomando $x_3=\alpha$ e $x_4=\beta$, $\alpha,\beta \in IR$, tem-se

$$\begin{cases} x_1=2\alpha \\ x_2=\beta-\alpha \\ x_3=\alpha \\ x_4=\beta \end{cases} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

estes vectores geram as soluções e são l.i. logo formam uma base do subespaço das soluções do sistema, tendo-se então, dimensão 2.

3.13

a) $A=\begin{pmatrix} 0 & 1 & -2 \\ 1 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{pmatrix} \xrightarrow[\substack{L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - L_1}]{} \begin{pmatrix} 1 & 2 & 4 \\ 0 & -2 & -6 \\ 0 & -2 & -1 \\ 0 & 1 & -2 \end{pmatrix}$

$\xrightarrow[\substack{L_3 \leftarrow L_3 - L_2 \\ L_4 \leftarrow L_4 + \frac{1}{2}L_2}]{} \begin{pmatrix} 1 & 2 & 4 \\ 0 & -2 & -6 \\ 0 & 0 & 5 \\ 0 & 0 & -5 \end{pmatrix} \xrightarrow[L_4 \leftarrow L_4 + L_3]{} \begin{pmatrix} 1 & 2 & 4 \\ 0 & -2 & -6 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad \begin{cases} x_1 + 2x_2 + 4x_3 = 0 \\ -2x_2 - 6x_3 = 0 \\ 5x_3 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$ Apenas tem a solução nula (trivial)

Base de $N(A)$: $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ e $\dim(N(A)) = 0$

$\| $

$3 - 3$

$\downarrow n \quad \downarrow c(A)$

b) $B = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 0 \\ -2 & 4 & 2 & 3 \end{pmatrix} \xrightarrow[L_1 \leftrightarrow L_2]{} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ -2 & 4 & 2 & 3 \end{pmatrix} \xrightarrow[L_3 \leftarrow L_3 + 2L_1]{}$

$\rightarrow \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 6 & 3 \end{pmatrix} \xrightarrow[L_3 \leftarrow L_3 - 8L_2]{} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -10 & -5 \end{pmatrix}$ $\quad \begin{cases} x_1 + 2x_2 + 2x_3 = 0 \\ x_2 + 2x_3 + x_4 = 0 \\ -10x_3 - 5x_4 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x_1 = x_4 \\ x_2 = 0 \\ x_3 = -x_4/2 \end{cases}$ Tomando $x_4 = \alpha$, $\alpha \in \mathbb{R}$, tem-se

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \alpha \underbrace{\begin{pmatrix} 1 \\ 0 \\ -1/2 \\ 1 \end{pmatrix}}_{\text{s.l.i.}}$ Base de $N(B)$: $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1/2 \\ 1 \end{pmatrix} \right\}$

$\dim(N(B)) = 1$

$\|$

$4 - 3$

$\downarrow n \quad \downarrow c(A)$

c) $C = \begin{pmatrix} 0 & 1 & 1 & 1 & -2 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & -2 \\ 1 & -2 & -3 & -1 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 0 & 2 & -2 \\ 1 & -2 & -3 & -1 & 5 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} \xrightarrow[L_2 \leftarrow L_2 - L_1]{} \begin{pmatrix} 1 & 1 & 0 & 2 & -2 \\ 0 & -3 & -3 & -3 & 7 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\xrightarrow[\substack{L_3 \leftarrow L_3 + \frac{1}{3}L_2 \\ L_4 \leftarrow L_4 + \frac{1}{3}L_2}]{} \begin{pmatrix} 1 & 1 & 0 & 2 & -2 \\ 0 & -3 & -3 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 10/3 \end{pmatrix} \xrightarrow[L_4 \leftarrow L_4 - 10L_3]{} \begin{pmatrix} 1 & 1 & 0 & 2 & -2 \\ 0 & -3 & -3 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad \begin{cases} x_1 + x_2 + 2x_4 - 2x_5 = 0 \\ -3x_2 - 3x_3 - 3x_4 + 7x_5 = 0 \\ \frac{1}{3}x_5 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x_1 = -x_2 - 2x_4 \\ x_3 = -x_2 - x_4 \\ x_5 = 0 \end{cases}$ Tomando $x_2 = \alpha$ e $x_4 = \beta$, $\alpha, \beta \in \mathbb{R}$ tem-se $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$

Base de $N(C)$: $\{(-1\ 1\ -1\ 0\ 0)^T, (-2\ 0\ -1\ 1\ 0)^T\}$

$\dim(N(C)) = 2$ $= (5 - 3$, i.e., $n - c(A))$