06-04-17
Quinta-feira
Prática

Exercício 3

$|\psi\rangle = a_\uparrow |\uparrow\rangle + a_\downarrow |\downarrow\rangle$
KET

$P_\uparrow = |a_\uparrow|^2$ $\qquad$ $P_\uparrow + P_\downarrow = 1$

$P_\downarrow = |a_\downarrow|^2$ $\qquad$ $|a_\uparrow|^2 + |a_\downarrow|^2 = 1$

$$\sum_i |a_i|^2 = 1$$

$$|\psi\rangle = \sum_i a_i |i\rangle$$

Resolução

fotão c/ polarização horizontal
fotão c/ polarização vertical

a) $|\psi_1\rangle = \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle)$

$$\left|\tfrac{1}{2}\right|^2 + \left|\tfrac{1}{2}\right|^2 + \left|\tfrac{1}{2}\right|^2 + \left|\tfrac{1}{2}\right|^2 = 4 \times \left|\tfrac{1}{2}\right|^2 =$$

$$= 4 \times \frac{1}{4} = 1$$

Sim, este estado está normalizado.

b)

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} + \frac{1}{2} = 1 \checkmark$$

Sim, está normalizado

c) $$|\psi_3\rangle = \frac{1}{2}|HH\rangle + \frac{\sqrt{3}}{2}(|VH\rangle + |VV\rangle)$$

$$\left|\frac{1}{2}\right|^2 + \left|\frac{\sqrt{3}}{2}\right|^2 + \left|\frac{\sqrt{3}}{2}\right|^2 = \frac{1}{4} + \frac{3}{4} + \frac{3}{4} = \frac{7}{4} \neq 1$$

Não está normalizado.

d) $$|\psi_4\rangle = \cos\alpha|HH\rangle + \operatorname{sen}\alpha|VV\rangle$$

$$|\cos\alpha|^2 + |\operatorname{sen}\alpha|^2 = \cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

Sim, está normalizado.

Exercício 7

a) $$|\psi_1\rangle = \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle)$$

$$|1\rangle = a|H\rangle + b|V\rangle$$
$$|2\rangle = c|H\rangle + d|V\rangle$$

$$\underbrace{|1\rangle \otimes |2\rangle \equiv |1\rangle|2\rangle \equiv |12\rangle}_{\text{3 maneiras de escrever a mesma coisa}}$$

4.

$$|A\uparrow\rangle,\ |A\downarrow\rangle,\ |A\leftarrow\rangle,\ |A\rightarrow\rangle,$$
$$|B\uparrow\rangle,\ |B\downarrow\rangle,\ |B\leftarrow\rangle,\ |B\rightarrow\rangle$$

5.

$$|\psi\rangle = 0.1|000\rangle + 0.3535(1+i)|001\rangle + 0.2|010\rangle - 0.1|100\rangle + 0.5|011\rangle - 0.361|101\rangle + 0.55|111\rangle$$

a)

$P_{000} = |0.1|^2 = 0.01$

$P_{001} = |0.3535 + 0.3535i|^2 = 0.25$

$P_{010} = |0.2|^2 = 0.04$

$P_{100} = |0.1|^2 = 0.01$

$P_{011} = |0.5|^2 = 0.25$

$P_{101} = |0.361|^2 = 0{,}13$

$\boxed{P_{111} = |0.55|^2 = 0.30}$

É o estado $|111\rangle$.

b)

$200 \times 0.04 = 8$

c) 0 ou 20 (ou é obtido ou não é)
(ou o sistema colapsa e é zero, o que se repete nos 20, ou tem sucesso nos 20)

d)

$$|0{,}1|^2 + |0.3535 + 0.3535i|^2 + |0.2|^2 + |0.5|^2$$
$$= 0{,}55$$

e) Sim, $|110\rangle$.

Exercício 6

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$

$$\begin{cases} ac = \frac{1}{\sqrt{2}} \\ ad = 0 \\ bc = 0 \\ bd = \frac{1}{\sqrt{2}} \end{cases}$$

$d = 0$ e $b = 0$

mas $bd = \frac{1}{\sqrt{2}}$, logo o sistema é impossível, ou seja, é um estado entrelaçado

$$= (a|H\rangle + b|V\rangle) \otimes (c|H\rangle + d|V\rangle) =$$

$$= ac\underbrace{|H\rangle|H\rangle}_{|HH\rangle} + ad\underbrace{|H\rangle|V\rangle}_{|HV\rangle} + bc\underbrace{|V\rangle|H\rangle}_{|VH\rangle} + bd\underbrace{|V\rangle|V\rangle}_{|VV\rangle}$$

se conseguir encontrar algo que satisfaça não estão entrelaçados

$$\begin{cases} ac = \frac{1}{2} \\ ad = \frac{1}{2} \\ bc = \frac{1}{2} \\ bd = \frac{1}{2} \end{cases}$$

$$a = b = c = d = \frac{1}{\sqrt{2}}$$

logo não estão entrelaçados.

b)

$$\begin{cases} ac = \frac{1}{2} \\ ad = \frac{1}{2} \\ bc = \frac{1}{2} \\ bd = -\frac{1}{2} \end{cases}$$

$b = \frac{1}{\sqrt{2}}$ $\quad d = \frac{1}{\sqrt{2}}$

$c = -\frac{1}{\sqrt{2}}$ $\quad a = \frac{1}{\sqrt{2}}$

mas $ac = \frac{1}{2}$ não funciona.

logo estão entrelaçados.

c) $|\psi_3\rangle = \frac{1}{2}|HH\rangle + \frac{\sqrt{3}}{2}|VH\rangle + \frac{\sqrt{3}}{2}|VV\rangle$

$$\begin{cases} ac = \frac{1}{2} \\ ad = 0 \\ bc = \frac{\sqrt{3}}{2} \\ bd = \frac{\sqrt{3}}{2} \end{cases}$$

Também não tem solução e não pode ser descrito como duas partículas individuais.
Estão entrelaçados.

d) $|\psi_4\rangle = \cos\alpha|HH\rangle + \text{sen}\,\alpha|VV\rangle$

$$\begin{cases} ac = \cos\alpha \\ ad = 0 \\ bc = 0 \\ bd = \text{sen}\,\alpha \end{cases}$$

Não está entrelaçado a menos que $\alpha = 0$
$\text{sen}\,\alpha = 0$ e $\cos\alpha = 1$
$b = 0, c = 1, d = 0$ e $a = 1$.

e) $|\psi_5\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$

$$\begin{cases} ac = 0 \\ ad = \frac{1}{\sqrt{2}} \\ bc = -\frac{1}{\sqrt{2}} \\ bd = 0 \end{cases}$$

Não estão entrelaçados.
(Impossível)

⑧

$$|S_z\rangle = |+_z\rangle = \underbrace{\frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)}_{\text{combinação linear em } x}$$

$$P_{\rightarrow} = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} = 0{,}5$$

⑨

$$|S_z\rangle = \sqrt{\frac{2}{3}}\,|+_z\rangle - \sqrt{\frac{1}{3}}\,|-_z\rangle$$

a) $$|S_z\rangle = \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)\right) - \sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}\left(|\rightarrow\rangle - |\leftarrow\rangle\right)\right)$$

$$= \frac{1}{\sqrt{3}}|\rightarrow\rangle + \frac{1}{\sqrt{3}}|\leftarrow\rangle - \frac{1}{\sqrt{6}}|\rightarrow\rangle + \frac{1}{\sqrt{6}}|\leftarrow\rangle$$

$$= \left(\underset{\times\sqrt{2}}{\frac{1}{\sqrt{3}}} - \frac{1}{\sqrt{6}}\right)|\rightarrow\rangle + \left(\underset{\times\sqrt{2}}{\frac{1}{\sqrt{3}}} + \frac{1}{\sqrt{6}}\right)|\leftarrow\rangle$$

$$= \frac{\sqrt{2}-1}{\sqrt{6}}|\rightarrow\rangle + \frac{\sqrt{2}+1}{\sqrt{6}}|\leftarrow\rangle$$

$$P_{+x} = \left|\frac{\sqrt{2}-1}{\sqrt{6}}\right|^2 = \frac{2 - 2\sqrt{2} + 1}{6} = \frac{3 - 2\sqrt{2}}{6}$$

$$= \frac{1}{2} - \frac{\sqrt{2}}{3}$$

$$\simeq 0{,}029$$

Exercício 9

$z \quad |+z\rangle \quad |-z\rangle$

$x \quad |+x\rangle \quad |-x\rangle$

$y \quad |+y\rangle \quad |-y\rangle$

a)

$$|+z\rangle = \frac{1}{\sqrt{2}}\left(|+x\rangle + |-x\rangle\right)$$

$$\equiv |\uparrow\rangle$$

$$|-z\rangle = \frac{1}{\sqrt{2}}\left(|+x\rangle - |-x\rangle\right)$$

$$\equiv |\downarrow\rangle$$

$$|S_z\rangle = \sqrt{\frac{2}{3}}|+z\rangle - \sqrt{\frac{1}{3}}|-z\rangle$$

$$= \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}\left(|+x\rangle + |-x\rangle\right)\right) - \sqrt{\frac{1}{3}}\left(\frac{1}{\sqrt{2}}\left(|+x\rangle - |-x\rangle\right)\right)$$

$$= \frac{\sqrt{3}}{3}|+x\rangle + \frac{\sqrt{3}}{3}|-x\rangle - \frac{\sqrt{6}}{6}|+x\rangle + \frac{\sqrt{6}}{6}|-x\rangle$$

$$= \left(\frac{\sqrt{3}}{3} - \frac{\sqrt{6}}{6}\right)|+x\rangle + \left(\frac{\sqrt{3}}{3} + \frac{\sqrt{6}}{6}\right)|-x\rangle$$

$$P_{+x} = \left|\frac{\sqrt{2}-1}{\sqrt{6}}\right|^2 = \frac{2 - 2\sqrt{2} + 1}{6} = \frac{3 - 2\sqrt{2}}{6}$$

$$= \frac{1}{2} - \frac{\sqrt{2}}{3}$$

$$= 0{,}029$$

b) $|S_z\rangle = \sqrt{\frac{2}{3}}\,|+_z\rangle - \sqrt{\frac{1}{3}}\,|-_z\rangle$

$$= \sqrt{\frac{2}{3}} \times \frac{1}{\sqrt{2}}\left(|\nearrow\rangle + |\swarrow\rangle\right) - \sqrt{\frac{1}{3}} \times \frac{1}{i\sqrt{2}}\left(|\nearrow\rangle - |\swarrow\rangle\right)$$

$$= \frac{1}{\sqrt{3}}|\nearrow\rangle + \frac{1}{\sqrt{3}}|\swarrow\rangle - \frac{1}{i\sqrt{6}}|\nearrow\rangle + \frac{1}{i\sqrt{6}}|\swarrow\rangle$$

$$= \left(\frac{1}{\sqrt{3}} - \frac{1}{i\sqrt{6}}\right)|\nearrow\rangle + \left(\frac{1}{\sqrt{3}} + \frac{1}{i\sqrt{6}}\right)|\swarrow\rangle$$

$$P_{+y} = \left|\underset{\times\sqrt{2}i}{\frac{1}{\sqrt{3}}} - \frac{1}{i\sqrt{6}}\right|^2$$

$$= \left|\frac{\sqrt{2}i - 1}{i\sqrt{6}}\right|^2 = \frac{1+2}{6} = \frac{3}{6} = 0.5$$

(12)

$|\psi\rangle = |000\rangle$

$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$CNOT(|0\rangle|0\rangle) = |0\rangle|0\rangle$

$CNOT(|1\rangle|0\rangle) = |1\rangle|1\rangle$

$CNOT(|0\rangle|1\rangle) = |0\rangle|1\rangle$

$CNOT(|1\rangle|1\rangle) = |1\rangle|0\rangle$

e)

$$\text{CNOT}\,|\varphi'\rangle = \frac{1}{2}\left(|000\rangle + |110\rangle + |001\rangle + |111\rangle\right)$$

$$P_{000} = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$$

Exercício 11

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|101\rangle + |010\rangle\right)$$

$$H_2|\psi\rangle = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\left(|101\rangle + |111\rangle + |000\rangle - |010\rangle\right)$$

$$= \frac{1}{2}\left(|101\rangle + |111\rangle + |000\rangle - |010\rangle\right)$$

b) $|S_z\rangle = \sqrt{\frac{2}{3}}\,|+_z\rangle - \sqrt{\frac{1}{3}}\,|-_z\rangle$

$= \sqrt{\frac{2}{3}} \times \frac{1}{\sqrt{2}}\left(|\nearrow\rangle + |\swarrow\rangle\right) - \sqrt{\frac{1}{3}} \times \frac{1}{i\sqrt{2}}\left(|\nearrow\rangle - |\swarrow\rangle\right)$

$= \frac{1}{\sqrt{3}}|\nearrow\rangle + \frac{1}{\sqrt{3}}|\swarrow\rangle - \frac{1}{i\sqrt{6}}|\nearrow\rangle + \frac{1}{i\sqrt{6}}|\swarrow\rangle$

$= \left(\frac{1}{\sqrt{3}} - \frac{1}{i\sqrt{6}}\right)|\nearrow\rangle + \left(\frac{1}{\sqrt{3}} + \frac{1}{i\sqrt{6}}\right)|\swarrow\rangle$

$P_{+y} = \left|\underset{\times\sqrt{2}i}{\frac{1}{\sqrt{3}}} - \frac{1}{i\sqrt{6}}\right|^2$

$= \left|\frac{\sqrt{2}i - 1}{i\sqrt{6}}\right|^2 = \frac{1+2}{6} = \frac{3}{6} = 0.5$

12

$|\psi\rangle = |000\rangle$

$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$CNOT(|0\rangle|0\rangle) = |0\rangle|0\rangle$

$CNOT(|1\rangle|0\rangle) = |1\rangle|1\rangle$

$CNOT(|0\rangle|1\rangle) = |0\rangle|1\rangle$

$CNOT(|1\rangle|1\rangle) = |1\rangle|0\rangle$

a)

$$H_1|000\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$$

$$H_2H_1|000\rangle = \frac{1}{\sqrt{2}} \times \left(\frac{1}{\sqrt{2}}(|000\rangle + |010\rangle + |100\rangle + |110\rangle\right)$$

$$= \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |110\rangle)$$

$$H_3H_2H_1 = \frac{1}{\sqrt{2}} \times \frac{1}{2}(|000\rangle + |001\rangle + \underline{|010\rangle} + \underline{|011\rangle} + |100\rangle + |101\rangle + \underline{|110\rangle} + \underline{|111\rangle})$$

$$P_{000} = \left|\frac{1}{2\sqrt{2}}\right|^2 = \frac{1}{8} = 0{,}125$$

b)

$$|\psi'\rangle = \underbrace{\frac{1}{2}}_{\text{Valor necessário para normalizar}}(|000\rangle + |100\rangle + |001\rangle + |101\rangle)$$

Relações de Broglie: $p = \hbar k$; $E = \hbar\omega$

Exercício (13)

$$p^2 = 2mE_c + \frac{E_c^2}{c^2} \quad , \quad p^2 = 2mE_c$$

a) $v = 1000\,km/h = 1000000\,m/h = 1\times10^6\,m/h$

$$= \frac{10^6}{3600}\,m/s = \frac{2500}{9}\,m/s$$

$\rightarrow$ momento

$$\boxed{p = m \times v \Leftrightarrow}$$

$$\boxed{\begin{array}{l} m_e = 9{,}11\times10^{-31}\,kg \\ \hbar = 6.63\times10^{-34}\,J.s \end{array}}$$

$$p = 9{,}11\times10^{-31} \times \frac{2500}{9} = 2{,}53\times10^{-28}\,kg.m/s$$

$$\boxed{\lambda = \frac{h}{p}} = \frac{6.63\times10^{-34}}{2.53\times10^{-28}} = 2{,}62\times10^{-6}\,m = 2{,}62\,\mu m$$

b)

$E_c = 10\,keV = 10^4 \times \boxed{1{,}6\times10^{-19}}$ $\rightarrow$ carga do eletrão

$$= 1{,}6\times10^{-15}\,J$$

$$p = \sqrt{2\times9.11\times10^{-31}\times1{,}6\times10^{-15} + \frac{(1.6\times10^{-15})^2}{(3\times10^8)^2}}$$

$$= 5{,}43\times10^{-23}$$

$\rightarrow$ velocidade da luz

$$\lambda = \frac{6.63\times10^{-34}}{5.43\times10^{-23}} = 1{,}22\times10^{-11}\,m$$

$$= 0{,}122\times10^{-10}\,m$$

$$= 0{,}122\,\text{Å} = 0{,}0122\,nm$$

Exercício 14

$$\lambda = 5890\ \text{Å} = 5890 \times 10^{-10}\ m$$

$$\lambda = \frac{h}{p} \Leftrightarrow 5890 \times 10^{-10} = \frac{6{,}63 \times 10^{-34}}{p}$$

$$p = \frac{6{,}63 \times 10^{-34}}{5890 \times 10^{-10}}$$

$$\Leftrightarrow p \simeq 1{,}13 \times 10^{-27}$$

$$p^2 = 2m\,Ec \Leftrightarrow \frac{(1{,}13 \times 10^{-27})^2}{2 \times 9{,}11 \times 10^{-31}} = Ec$$

$$\Leftrightarrow Ec = 7{,}01 \times 10^{-25}\ J$$

$$= \frac{7{,}01 \times 10^{-25}}{1{,}6 \times 10^{-19}} = 4{,}38 \times 10^{-6}\ eV$$

Exercício 15

$$\lambda = 2.0\ \text{Å} = 2.0 \times 10^{-10}\ m$$

a) $$p = \frac{h}{\lambda} = \frac{6{,}63 \times 10^{-34}}{2.0 \times 10^{-10}} = 3{,}315 \times 10^{-24}\ kg\,m/s$$

b) $$\boxed{Ec = \frac{p^2}{2m}}$$

Para o eletrão: $Ec = \dfrac{(3{,}315 \times 10^{-24})^2}{2 \times 9{,}11 \times 10^{-31}} = 6.031 \times 10^{-18}\ J$

Para o fotão: (não tem massa $m=0$)

$$p^2 = \cancel{2mEc} + \frac{Ec^2}{c^2}$$

$$\Leftrightarrow p^2 = \frac{Ec^2}{c^2} \Leftrightarrow p^2 \times c^2 = Ec^2$$

$$\Leftrightarrow Ec = p \times c$$

logo $Ec = 3{,}315 \times 10^{-24} \times 3 \times 10^{8} = 9{,}945 \times 10^{-16}$ J

e) A $E_f > E_e$.

Exercício 16

$$\frac{\lambda_p}{\lambda_e} = 1{,}813 \times 10^{-4} \quad p = \frac{h}{\lambda} \Leftrightarrow \lambda = \frac{h}{p} \quad v_p = 3v_e$$

$$\frac{\frac{h}{p_p}}{\frac{h}{p_e}} = \frac{p_e}{p_p} = \frac{m_e \times v_e}{m_p \times v_p} = \frac{m_e \times \cancel{v_e}}{m_p \times 3\cancel{v_e}}$$

$$\frac{9{,}11 \times 10^{-31}}{m_p \times 3} = 1{,}813 \times 10^{-4} \Leftrightarrow m_p = 1{,}67494 \times 10^{-27}$$

(Neutrão)

Exercício (17)

a) $E_c = \frac{3}{2} k_B T$ $\quad$ $T = 300\,k$
$k_B = 1.38 \times 10^{-23}\, J/k$

$$E_c = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}\, J$$

$$E_c = \frac{6.21 \times 10^{-21}}{1.6 \times 10^{-19}} = 0{,}0388\, eV$$

b) $\boxed{\lambda = \frac{h}{p}}$ $\quad$ $\boxed{m_n = 1{,}67 \times 10^{-27}\, kg}$

$$p = \sqrt{2m \times E_c} = \sqrt{2 \times 1{,}67 \times 10^{-27} \times 6{,}21 \times 10^{-21}}$$

$$= 4{,}55 \times 10^{-24}$$

$$\lambda = \frac{6{,}63 \times 10^{-34}}{4.55 \times 10^{-24}} = 1 \times 10^{-10}\, m = 1\, \mathring{A}$$

Exercício (18)

$E_c = m$

$$\lambda = 1.7898 \times 10^{-6}\, \mathring{A} = 1.7898 \times 10^{-6} \times 10^{-10}$$
$$= 1.7898 \times 10^{-16}\, m$$

Exercício 19

$$E_c = 50\,GeV$$
$$= 5\times10^{10} \times \underbrace{1{,}6\times10^{-19}}_{\text{carga do eletrão}}$$
$$= 8\times10^{-9}\,J$$

$$p = \sqrt{2\times 9{,}11\times10^{-31}\times 8\times10^{-9} + \frac{(8\times10^{-9})^2}{(3\times10^8)^2}}$$
$$p \simeq 2{,}67\times10^{-17}$$

$$\lambda = \frac{6{,}63\times10^{-34}}{2{,}67\times10^{-17}} = 2{,}48\times10^{-17}\,m$$
$$= 2.5\times10^{-17}\,m$$

Exercício 21

$$\psi(x) = A\,\text{sen}\left(\frac{m\pi}{a}x\right)$$

a) $\int |\psi(x)|^2 dx = 1$

$$\int_0^a |\psi(x)|^2 dx = \int_0^a A^2 \text{sen}^2\left(\frac{m\pi}{a}x\right) dx$$
$$= A^2 \int_0^a \text{sen}^2\left(\frac{m\pi}{a}x\right) dx$$
$$= A^2 \left[\frac{x}{2} - \frac{1}{4m\pi}\,\text{sen}\left(\frac{2m\pi}{a}x\right)\right]_0^a = A^2 \times \frac{a}{2}$$

$$A^2 \times \frac{a}{2} = 1$$
$$\Leftrightarrow A^2 = \frac{2}{a}$$
$$\Leftrightarrow A = \sqrt{\frac{2}{a}}$$

b)

$$\psi(x) = A \operatorname{sen}\left(\frac{n\pi}{a}x\right)$$

$$= \sqrt{\frac{2}{a}} \operatorname{sen}\left(\frac{n\pi}{a}x\right)$$

$$P_n\left(\frac{a}{2}\right) = \left|\psi\left(\frac{a}{2}\right)\right|^2 = \frac{2}{a} \times \operatorname{sen}^2\left(\frac{n\pi}{\cancel{a}} \times \frac{\cancel{a}}{2}\right)$$

$$= \frac{2}{a} \times \operatorname{sen}^2\left(\frac{n\pi}{2}\right)$$

Se $n = 1$ $\quad P_1\left(\frac{a}{2}\right) = \frac{2}{a}$

Se $n = 2$ $\quad P_2\left(\frac{a}{2}\right) = 0$

Se $n = 3$ $\quad P_3\left(\frac{a}{2}\right) = \frac{2}{a}$

Conclusão:

Se n ímpar $\quad P_n\left(\frac{a}{2}\right) = \frac{2}{a}$ (Antinodos)

Se n par $\quad P_n\left(\frac{a}{2}\right) = 0$ (Nodos)

c) $p^2 = 2m E_c \Leftrightarrow E_c = \dfrac{p^2}{2m}$

$\Leftrightarrow E_c = \dfrac{\hbar^2 k^2}{2m} \Leftrightarrow$ número de onda $= \dfrac{\pi n}{a}$

$\Leftrightarrow E_c = \dfrac{\hbar^2 \times \left(\frac{\pi n}{a}\right)^2}{2m} \Leftrightarrow E_c = \dfrac{\hbar^2 \times \pi^2 \times n^2}{2 m a^2}$

Exercício 22

$P = e^{-2\alpha L}$

L = comprimento da barreira $= 1mm = 1\times10^{-9}$

$$\alpha = \sqrt{\frac{2m(U-E)}{\hbar^2}}$$

$$P = e^{-2\times\alpha\times10^{-9}} \simeq 9{,}39202\times10^{-8}$$

$$\alpha = \sqrt{\frac{2\times9.11\times10^{-31}(4{,}8\times10^{-19} - 8\times10^{-20})}{\left(\frac{6.63\times10^{-34}}{2\pi}\right)^2}} = 8{,}09041\times10^{9}$$

$E = 0{,}5eV = 0{,}5\times1{,}6\times10^{-19} = 8\times10^{-20}\ J$

$U = 3eV = 3\times1{,}6\times10^{-19} = 4{,}8\times10^{-19}\ J$

a)

$$|\psi\rangle = |000\rangle$$

$$H_1|000\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |100\rangle\right)$$

$$H_2H_1|000\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|100\rangle + \frac{1}{\sqrt{2}}|010\rangle + \frac{1}{\sqrt{2}}|110\rangle\right)$$

$$H_3H_2H_1|000\rangle = \frac{1}{\sqrt{2}} \times \frac{1}{2}\left(|000\rangle + |100\rangle + |010\rangle + |110\rangle + |001\rangle + |101\rangle + |011\rangle + |111\rangle\right)$$

b)

$$|\psi'\rangle = \frac{1}{2}\left(|000\rangle + |100\rangle + |001\rangle + |101\rangle\right)$$

c)

$$CNOT\,|\psi'\rangle = \frac{1}{2}\left(|000\rangle + |110\rangle + |001\rangle + |111\rangle\right)$$

$$P_{000} = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$$

b) $|^+z\rangle = \frac{1}{\sqrt{2}}(|^+y\rangle + |^-y\rangle)$

$|^-z\rangle = \frac{1}{i\sqrt{2}}(|^+y\rangle - |^-y\rangle)$

$$|S_z\rangle = \sqrt{\frac{2}{3}}|^+z\rangle - \sqrt{\frac{1}{3}}|^-z\rangle$$

$$= \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(|^+y\rangle + |^-y\rangle)\right) - \sqrt{\frac{1}{3}}\left(\frac{1}{i\sqrt{2}}(|^+y\rangle - |^-y\rangle)\right)$$

$$= \sqrt{\frac{2}{6}}|^+y\rangle - \frac{1}{i}\sqrt{\frac{1}{6}}|^+y\rangle + \sqrt{\frac{2}{6}}|^-y\rangle + \frac{1}{i}\sqrt{\frac{1}{6}}|^-y\rangle$$

$$= \frac{\sqrt{2}+i}{\sqrt{6}}|^+y\rangle + \frac{\sqrt{2}-i}{\sqrt{6}}|^-y\rangle$$

→ módulo do nº complexo

$$P_{+y} = \left|\frac{\sqrt{2}+i}{\sqrt{6}}\right|^2 = \frac{2+1}{6} = \frac{1}{2} = 0,5$$

Exercício 12

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$CNOT(|0\rangle|0\rangle) = |0\rangle|0\rangle$ $\quad CNOT(|0\rangle|1\rangle) = |0\rangle|1\rangle$

$CNOT(|1\rangle|0\rangle) = |1\rangle|1\rangle$ $\quad CNOT(|1\rangle|1\rangle) = |1\rangle|0\rangle$