FORMULÁRIO

Integrais

$$\int \cos(nx)\cos(mx)dx = \frac{\sin((n+m)x)}{2(n+m)} + \frac{\sin((n-m)x)}{2(n-m)},\ n^2 \neq m^2$$
$$\int \sin(nx)\sin(mx)dx = -\frac{\sin((n+m)x)}{2(n+m)} - \frac{\sin((n-m)x)}{2(n-m)},\ n^2 \neq m^2$$
$$\int \sin(nx)\cos(mx)dx = -\frac{\cos((n+m)x)}{2(n+m)} - \frac{\cos((n-m)x)}{2(n-m)},\ n^2 \neq m^2$$

Problemas com EDPs

$$(i) \quad \begin{cases} u_t = \alpha^2 u_{xx}, & 0 < x < L,\ t > 0 \\ u(0,t) = u(L,t) = 0, & t \geq 0 \\ u(x,0) = f(x), & 0 \leq x \leq L, \end{cases} \tag{1}$$

$$u(x,t) = \sum_{n=1}^{+\infty} b_n e^{-(\frac{\alpha\pi n}{L})^2 t} \sin(\frac{n\pi x}{L})$$
$$f(x) = \sum_{n=1}^{+\infty} b_n \sin(\frac{n\pi x}{L})$$

$$(ii) \quad \begin{cases} u_{tt} = c^2 u_{xx}, & 0 < x < L,\ t > 0 \\ u(0,t) = u(L,t) = 0, & t \geq 0 \\ u(x,0) = f(x), & 0 \leq x \leq L. \\ u_t(x,0) = g(x), & 0 \leq x \leq L, \end{cases} \tag{2}$$

$$u(x,t) = \sum_{n=1}^{+\infty} \left(a_n \cos(\frac{n\pi ct}{L}) + b_n \frac{L}{\pi cn} \sin(\frac{n\pi ct}{L}) \right) \sin(\frac{n\pi x}{L})$$
$$f(x) = \sum_{n=1}^{+\infty} a_n \sin(\frac{n\pi x}{L})$$
$$g(x) = \sum_{n=1}^{+\infty} b_n \sin(\frac{n\pi x}{L})$$

$$(iii) \quad \begin{cases} u_{xx} + u_{yy} = 0, & 0 < x < a,\ 0 < y < b \\ u(x,0) = u(x,b) = 0, & 0 \leq x \leq a \\ u(0,y) = f(y), & 0 \leq y \leq b \\ u(a,y) = g(y), & 0 \leq y \leq b \end{cases} \tag{3}$$

$$u(x,y) = \sum_{n=1}^{+\infty} \frac{\sin(\frac{n\pi y}{b})}{\sinh(\frac{n\pi a}{b})} \left[b_n \sinh(\frac{n\pi x}{b}) - a_n \sinh(\frac{n\pi}{b})(x-a) \right]$$
$$f(x) = \sum_{n=1}^{+\infty} a_n \sin(\frac{n\pi x}{b})$$
$$g(x) = \sum_{n=1}^{+\infty} b_n \sin(\frac{n\pi x}{b})$$