Primitivas Imediatas

Na lista de primitivas que se segue, $f : I \longrightarrow \mathbb{R}$ é uma função derivável no intervalo I e $\mathcal{C}$ denota uma constante real arbitrária.

$$\int a\,dx = ax + \mathcal{C} \quad (a \in \mathbb{R})$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + \mathcal{C}$$

$$\int f'(x)\,\cos(f(x))\,dx = \operatorname{sen}(f(x)) + \mathcal{C}$$

$$\int f'(x)\sec^2(f(x))\,dx = \operatorname{tg}(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{tg}(f(x))\,dx = -\ln|\cos(f(x))| + \mathcal{C}$$

$$\int f'(x)\,\operatorname{ch}(f(x))\,dx = \operatorname{sh}(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{cosech}^2(f(x))\,dx = \operatorname{th}(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{th}(f(x))\,dx = \ln(\cosh(f(x))) + \mathcal{C}$$

$$\int \frac{f'(x)}{\sqrt{1-f^2(x)}}\,dx = \operatorname{arcsen}(f(x)) + \mathcal{C}$$

$$\int \frac{f'(x)}{\sqrt{f^2(x)+1}}\,dx = \operatorname{argsh}(f(x)) + \mathcal{C}$$

$$\int f'(x)\,f^{\alpha}(x)\,dx = \frac{f^{\alpha+1}(x)}{\alpha+1} + \mathcal{C} \quad (\alpha \neq -1)$$

$$\int a^{f(x)}\,f'(x)\,dx = \frac{a^{f(x)}}{\ln a} + \mathcal{C} \quad (a \in \mathbb{R}^+\backslash\{1\})$$

$$\int f'(x)\,\operatorname{sen}(f(x))\,dx = -\cos(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{cosec}^2(f(x))\,dx = -\operatorname{cotg}(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{cotg}(f(x))\,dx = \ln|\operatorname{sen}(f(x))| + \mathcal{C}$$

$$\int f'(x)\,\operatorname{sh}(f(x))\,dx = \operatorname{ch}(f(x)) + \mathcal{C}$$

$$\int f'(x)\operatorname{sech}^2(f(x))\,dx = -\coth(f(x)) + \mathcal{C}$$

$$\int f'(x)\coth(f(x))\,dx = \ln|\operatorname{sh}(f(x))| + \mathcal{C}$$

$$\int \frac{f'(x)}{1+f^2(x)}\,dx = \operatorname{arctg}(f(x)) + \mathcal{C}$$

$$\int \frac{f'(x)}{\sqrt{f^2(x)-1}}\,dx = \operatorname{argch}(f(x)) + \mathcal{C}$$

Algumas regras de derivação

(estamos a omitir os domínios de definição das funções)

$C' = 0, \quad C$ constante

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

$(g \circ f)'(x) = g'(f(x))f'(x))$

$(e^x)' = e^x$

$(a^x)' = a^x \ln a$

$\operatorname{sen}' x = \cos x$

$\operatorname{tg}' x = \sec^2 x$

$\sec' x = \sec x \operatorname{tg} x$

$\operatorname{sh}' x = \operatorname{ch} x$

$\operatorname{th}' x = \operatorname{sech}^2 x$

$\operatorname{sech}' x = -\operatorname{sech} x \operatorname{th} x$

$\operatorname{arcsen}' x = \dfrac{1}{\sqrt{1-x^2}}$

$\operatorname{arctg}' x = \dfrac{1}{1+x^2}$

$\operatorname{arcsec}' x = \dfrac{1}{x\sqrt{x^2-1}}$

$\operatorname{argsh}' x = \dfrac{1}{\sqrt{1+x^2}}$

$\operatorname{argth}' x = \dfrac{1}{1-x^2}$

$\operatorname{argsech}' x = \dfrac{-1}{x\sqrt{1-x^2}} \quad (x < 1)$

$(x^{\alpha})' = \alpha\, x^{\alpha-1}, \quad (\alpha \in \mathbb{R})$

$\left(\dfrac{f}{g}\right)'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$\left(f^{-1}\right)'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$\ln' x = \dfrac{1}{x}$

$\log_a' x = \dfrac{1}{x \ln a}$

$\cos' x = -\operatorname{sen} x$

$\operatorname{cotg}' x = -\operatorname{cosec}^2 x$

$\operatorname{cosec}' x = -\operatorname{cosec} x \operatorname{cotg} x$

$\operatorname{ch}' x = \operatorname{sh} x$

$\coth' x = -\operatorname{cosech}^2 x$

$\operatorname{cosech}' x = -\operatorname{cosech} x \coth x$

$\operatorname{arccos}' x = \dfrac{-1}{\sqrt{1-x^2}}$

$\operatorname{arcotg}' x = \dfrac{-1}{1+x^2}$

$\operatorname{arcosec}' x = \dfrac{-1}{x\sqrt{x^2-1}}$

$\operatorname{argch}' x = \dfrac{1}{\sqrt{x^2-1}}$

$\operatorname{argcoth}' x = \dfrac{1}{1-x^2}$

$\operatorname{argcosech}' x = \dfrac{-1}{x\sqrt{1+x^2}}$

Algumas propriedades das funções trigonométricas

1. $\forall a \in \mathbb{R} \quad \operatorname{sen}^2 a + \cos^2 a = 1$

2. $\forall a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$

3. $\forall a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$

4. $\forall a \in \mathbb{R} \quad \operatorname{sen}(-a) = -\operatorname{sen} a$ (sen é ímpar)

5. $\forall a \in \mathbb{R} \quad \cos(-a) = \cos a$ (cos é par)

6. $\forall a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \operatorname{sen} a$ e $\operatorname{sen}(\frac{\pi}{2} - a) = \cos a$

7. $\forall a \in \mathbb{R} \quad \operatorname{sen}(a + 2\pi) = \operatorname{sen} a$ (sen tem período 2π)

8. $\forall a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a$ (cos tem período 2π)

9. $\forall a,\ b \in \mathbb{R} \quad \operatorname{sen}(a+b) = \operatorname{sen} a \cos b + \operatorname{sen} b \cos a$

10. $\forall a,\ b \in \mathbb{R} \quad \cos(a+b) = \cos a \cos b - \operatorname{sen} b \operatorname{sen} a$

11. $\forall a,\ b \in \mathbb{R} \quad \cos a - \cos b = -2 \operatorname{sen} \frac{a-b}{2} \operatorname{sen} \frac{a+b}{2}$

12. $\forall a,\ b \in \mathbb{R} \quad \operatorname{sen} a - \operatorname{sen} b = 2 \operatorname{sen} \frac{a-b}{2} \cos \frac{a+b}{2}$