19/11/13

Correcção do 1º Teste - 16/11/13

Teste A

Grupo I -

1) $f: [0, \pi] \to \mathbb{R}$, $f(x) = \cos x$

Resposta b) f é uma função injectiva e não sobrejectiva

2) $f(x) = \begin{cases} 0 & , \text{se } x \leq -1 \\ \arcsin x & , \text{se } -1 < x < 1 \\ \frac{\pi}{2} \operatorname{sen}\left(\frac{\pi}{2} x\right) & , \text{se } x \geq 1 \end{cases}$

Resposta b) f é uma função contínua em $\mathbb{R} \setminus \{-1\}$.

Teste B

Grupo I -

1) $f: [0, \pi] \to [-1, 1]$, $f(x) = \cos x$

Resposta a) f é uma função bijectiva.

2) $f(x) = \begin{cases} 0 & , \text{se } x \leq -1 \\ \arcsin x & , \text{se } -1 < x < 1 \\ \pi \operatorname{sen}(\pi x) & , \text{se } x \geq 1 \end{cases}$

Resposta d) f é uma função contínua em $\mathbb{R} \setminus \{-1, 1\}$

Teste A

3) $f(x) = \begin{cases} x \cos \frac{1}{x}, & \text{se } x \neq 0 \\ 0, & \text{se } x = 0 \end{cases}$

$\lim_{x \to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{x \cos 1/x}{x}$

- $\lim_{x \to 0^-} \cos \frac{1}{x} \to$ Não existe
- $\lim_{x \to 0^+} \cos \frac{1}{x} \to$ Não existe

Resposta d) não existe $f'(0)$.

4) $f, g: \mathbb{R} \to \mathbb{R}$, $f(x) = g(5x - x^2)$,
$g(4) = g'(4) = 2$

$f(x) = g(5x - x^2)$
$f'(x) = g'(5x - x^2)(5 - 2x)$
$f'(1) = g'(4) \times 3 = 6$
$f(1) = g(4) = 2$

$y - 2 = -\frac{1}{6}(x - 1)$

$\Leftrightarrow y = -\frac{1}{6}x + \frac{13}{6}$

Resposta d) $y = -\frac{1}{6}x + \frac{13}{6}$

5) $3x - \text{sen}^2(x) - 2 = 0$

$f(x) = 3x - \text{sen}^2(x) - 2$
$\left.\begin{array}{l} f(0) = -2 < 0 \\ f(\pi) = 3\pi - 2 > 0 \end{array}\right\}$ Existe, pelo menos, um zero em $[0, \pi]$
$f'(x) = 3 - 2 \,\text{sen}\, x \cos x > 0$
$\hookrightarrow$ f só tem um zero em $[0, \pi]$

Resposta b) tem uma única solução no intervalo $[0, \pi]$.

6) $f(x) = -\text{sen}\, x$
$\int -\text{sen}\, x \, dx = \cos x + C$
Logo, as respostas a) $F(x) = -\cos x$ e c) $F(x) = -\text{sen}\, x$ são falsas.

$F(x) = \cos^3 x + \cos x \,\text{sen}^2 x$
$= \cos x (\cos^2 x + \text{sen}^2 x)$
$= \cos x$

Resposta b) $F(x) = \cos^3 x + \cos x \,\text{sen}^2 x$

Teste B

3) $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & \text{se } x \neq 0 \\ 0, & \text{se } x = 0 \end{cases}$

$\lim_{x \to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0} \frac{x^2 \cos 1/x}{x} =$

$= \lim_{x \to 0} x \cos \frac{1}{x} = \lim_{x \to 0} 0 \cos \frac{1}{0} = 0$,

porque $\lim_{x \to 0} 0 = 0$ e a função cosseno é limitada.

Resposta a) existe $f'(0)$.

4) $f, g: \mathbb{R} \to \mathbb{R}$, $f(x) = g(x^2 - 5x)$,
$g(-4) = g'(-4) = 2$

$f(x) = g(x^2 - 5x)$
$f'(x) = g'(x^2 - 5x)(2x - 5)$
$f'(1) = g'(-4) \times (-3) = -6$
$f(1) = g(-4) = 2$

$y - 2 = -\frac{1}{-6}(x - 1)$

$\Leftrightarrow y = \frac{1}{6}x + \frac{11}{6}$

Resposta c) $y = \frac{1}{6}x + \frac{11}{6}$

5) $3x - \text{sen}^2(x) - 2 = 0$

Resposta a) tem uma única solução no intervalo $[0, \pi]$.

6) $f(x) = \cos x$
$\int \cos x \, dx = \text{sen}\, x + C$
Logo, as respostas a) $F(x) = -\cos x$ e c) $F(x) = -\text{sen}\, x$ são falsas.

$F(x) = \text{sen}^3 x + \text{sen}\, x \cos^2 x$
$= \text{sen}\, x (\text{sen}^2 x + \cos^2 x)$
$= \text{sen}\, x$

Resposta d) $F(x) = \text{sen}^3 x + \text{sen}\, x \cos^2 x$

Grupo II -

7) Teste A:

a) $\lim_{x \to +\infty} \frac{x - \cos x}{x + \cos x} = \lim_{x \to +\infty} \frac{1 - 1/x \cos x}{1 + 1/x \cos x} = \frac{1-0}{1-0} = 1$

- $\lim_{x \to +\infty} \frac{1}{x} \cos x = 0$, porque $\lim_{x \to +\infty} \frac{1}{x} = 0$ e a função cosseno é limitada.

Teste B: $\lim_{x \to +\infty} \frac{x - \operatorname{sen} x}{x + \operatorname{sen} x} = \lim_{x \to +\infty} \frac{1 - 1/x \operatorname{sen} x}{1 + 1/x \operatorname{sen} x} = \frac{1-0}{1-0} = 1$

- $\lim_{x \to +\infty} \frac{1}{x} \operatorname{sen} x = 0$, porque $\lim_{x \to +\infty} \frac{1}{x} = 0$ e a função seno é limitada.

b) Teste A: $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{1 - \cos x} \left(\frac{0}{0}\right)$

Pela regra de l'Hôpital, $\lim_{x \to 0} \left(\frac{e^x - e^{-x} - 2x}{1 - \cos x}\right)' = \lim_{x \to 0} \frac{e^x - e^{-x} - 2}{\operatorname{sen} x} \left(\frac{0}{0}\right)$

Pela regra de l'Hôpital, $\lim_{x \to 0} \left(\frac{e^x - e^{-x} - 2}{\operatorname{sen} x}\right)' = \lim_{x \to 0} \frac{e^x - e^{-x}}{\cos x} = \lim_{x \to 0} \frac{e^0 - e^{-0}}{\cos 0} =$

$= \frac{1-1}{1} = 0$

A regra de l'Hôpital permite concluir que $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{1 - \cos x} = 0$.

Teste B: $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{1 - \cos x} = 0$

8) Teste A: $f(x) = -\pi + \operatorname{arctg}(2x - 3)$

a) - $\operatorname{Dom}(f) = \mathbb{R}$
- Para $x \in \mathbb{R}$, $-\frac{\pi}{2} < \operatorname{arctg}(2x-3) < \frac{\pi}{2} \Leftrightarrow -\frac{3}{2}\pi < -\pi + \operatorname{arctg}(2x-3) < -\frac{\pi}{2}$

$\operatorname{Im}(f) = \left]-\frac{3}{2}\pi, -\frac{\pi}{2}\right[$

b) $y = -\pi + \operatorname{arctg}(2x-3) \Leftrightarrow \operatorname{arctg}(2x-3) = y + \pi$
$\Leftrightarrow 2x - 3 = \operatorname{tg}(y + \pi)$
$\Leftrightarrow 2x - 3 = \operatorname{tg}(y)$
$\Leftrightarrow x = \frac{\operatorname{tg}(y) + 3}{2}$

$f^{-1}: \left]-3/2\,\pi, -\pi/2\right[\longrightarrow \mathbb{R}$
$x \longmapsto f^{-1}(x) = \frac{\operatorname{tg}(x) + 3}{2}$

$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$

c) $f(x) = -\pi + \operatorname{arctg}(2x-3)$

$$f'(x) = \frac{(2x-3)'}{1+(2x-3)^2} = \frac{2}{4x^2-12x+10} = \frac{1}{2x^2-6x+5}$$

$$f^{-1}: \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f'(x) = \frac{1}{2x^2-6x+5}$$

Teste B: $\pi + \operatorname{arctg}(2x+3)$

a) • $\operatorname{Dom}(f) = \mathbb{R}$

• Para $x \in \mathbb{R}$, $-\frac{\pi}{2} < \operatorname{arctg}(2x+3) < \frac{\pi}{2} \Leftrightarrow \frac{\pi}{2} < \pi + \operatorname{arctg}(2x+3) < \frac{3}{2}\pi$

$\operatorname{Im}(f) = \left]\frac{\pi}{2}, \frac{3}{2}\pi\right[$

b) $y = \pi + \operatorname{arctg}(2x+3) \Leftrightarrow \operatorname{arctg}(2x+3) = y - \pi$

$\Leftrightarrow 2x+3 = \operatorname{tg}(y-\pi)$

$\Leftrightarrow 2x+3 = \operatorname{tg}(y)$

$\Leftrightarrow x = \frac{\operatorname{tg}(y)-3}{2}$

$$f^{-1}: \left]\pi/2, 3/2\,\pi\right[\rightarrow \mathbb{R}$$
$$x \longmapsto f^{-1}(x) = \frac{\operatorname{tg}(x)-3}{2}$$

$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

c) $f(x) = \pi + \operatorname{arctg}(2x+3)$

$$f'(x) = \frac{(2x+3)'}{1+(2x+3)^2} = \frac{2}{4x^2+12x+10} = \frac{1}{2x^2+6x+5}$$

9) Teste A:

a) $$\int \frac{1+\operatorname{arctg}^2 x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{1}{1+x^2}\operatorname{arctg}^2 x\,dx$$
$$= \operatorname{arctg} x + \frac{1}{3}\operatorname{arctg}^3 x + C, \text{ com } C \in \mathbb{R}$$

b) $\int x \operatorname{sh} x\,dx = x \operatorname{ch} x - \int \operatorname{ch} x\,dx = x \operatorname{ch} x - \operatorname{sh} x + C$, com $C \in \mathbb{R}$

Teste B:

a) $$\int \frac{1+\operatorname{arctg}^3 x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{1}{1+x^2}\operatorname{arctg}^3 x\,dx$$
$$= \operatorname{arctg} x + \frac{1}{4}\operatorname{arctg}^4 x + C, \text{ com } C \in \mathbb{R}$$

b) $\int x \operatorname{ch} x\,dx = x \operatorname{sh} x - \int \operatorname{sh} x\,dx = x \operatorname{sh} x - \operatorname{ch} x + C$, com $C \in \mathbb{R}$