

Folha 6



## Exercício 6.1 Calcule:

1) 
$$\int (3x^2 - 2x^5) dx$$
;

$$2) \quad \int (\sqrt{x} + 2)^2 \, dx;$$

3) 
$$\int (2x+10)^{20} dx$$
;

4) 
$$\int x^2 e^{x^3} dx;$$

5) 
$$\int x^4 (x^5 + 10)^9 dx$$
;

6) 
$$\int \frac{2x+1}{x^2+x+3} \, dx;$$

$$7) \quad \int \sqrt{2x+1} \, dx;$$

8) 
$$\int \frac{x}{3-x^2} \, dx;$$

$$9) \quad \int \frac{1}{4-3x} \, dx;$$

10) 
$$\int \frac{1}{e^{3x}} \, dx;$$

$$11) \quad \int \frac{-7}{\sqrt{1-5x}} \, dx;$$

$$12) \quad \int \frac{\sqrt{1+3\ln x}}{x} \, dx;$$

13) 
$$\int x \, \sin x^2 \, dx;$$

14) 
$$\int \frac{1}{x(\ln^2 x + 1)} dx;$$

15) 
$$\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2} dx;$$

16) 
$$\int \operatorname{sen}(\pi - 2x) \, dx;$$

17) 
$$\int \operatorname{th} x \, dx$$
;

18) 
$$\int \operatorname{sen} x \, \cos x \, dx;$$

19) 
$$\int \operatorname{sen}(2x) \cos x \, dx;$$

20) 
$$\int \sin^2 x \, dx;$$

21) 
$$\int \operatorname{sen}^2 \frac{x}{2} \cos^2 \frac{x}{2} dx;$$

$$22) \quad \int \cos^3 x \, dx;$$

$$23) \quad \int \frac{x}{x^2 - 1} \, dx;$$

$$24) \quad \int \frac{x}{\sqrt{x^2 - 1}} \, dx;$$

25) 
$$\int \frac{1}{x} \, \operatorname{sen}(\ln x) \, dx;$$

$$26) \quad \int \frac{-3}{x \left(\ln x\right)^3} \, dx;$$

$$27) \quad \int \frac{e^x}{1 + e^{2x}} \, dx;$$

28) 
$$\int \frac{e^x}{1 - 2e^x} dx;$$

29) 
$$\int \frac{1}{\cos^2(7x)} dx;$$

30) 
$$\int \left(\sqrt{2x-1} - \sqrt{1+3x}\right) dx;$$

31) 
$$\int \frac{1}{x} (1 + \ln^2 x) dx$$
;

32) 
$$\int \frac{2 + \sqrt{\arctan(2x)}}{1 + 4x^2} dx$$
;

33) 
$$\int \frac{e^{\operatorname{arctg} x}}{1+x^2} dx;$$

$$34) \quad \int \frac{\sin x}{\sqrt{1+\cos x}} \, dx.$$

## Exercício 6.2 Calcule:

a)  $\int \ln x \, dx$ ;

 $j) \int e^x \cos x \, dx;$ 

b)  $\int x \, \operatorname{sen}(2x) \, dx;$ 

k)  $\int \operatorname{arcsen} x \, dx$ ;

c)  $\int \operatorname{arctg} x \, dx$ ;

1)  $\int e^{\sin x} \sin x \cos x \, dx;$ 

d)  $\int x \cos x \, dx;$ 

m)  $\int \frac{\arcsin\sqrt{x}}{\sqrt{x}} dx;$ 

e)  $\int \ln(1-x) dx$ ;

n)  $\int x \arctan x \, dx$ ;

f)  $\int x \ln x \, dx$ ;

o)  $\int x^2 \ln x \, dx$ ;

g)  $\int x^2 \sin x \, dx$ ;

p)  $\int \operatorname{sen}(\ln x) dx$ ;

h)  $\int x \, \sin x \, \cos x \, dx;$ 

q)  $\int \operatorname{ch} x \operatorname{sen}(3x) dx$ ;

i)  $\int \ln^2 x \, dx$ ;

r)  $\int x^3 e^{x^2} dx.$ 

## Exercício 6.3 Usando o método de substituição, calcule:

a) 
$$\int x (x+3)^{1/3} dx$$
;

e) 
$$\int \frac{e^{2x}}{3 + e^x} \, dx;$$

b) 
$$\int \frac{1}{\sin x} dx$$
;

f) 
$$\int \frac{x^2}{\sqrt{1-x^2}} dx$$
;

c) 
$$\int \frac{x}{\sqrt{2-3x}} \, dx;$$

g) 
$$\int \frac{\sqrt{x}}{x - \sqrt[3]{x}} \, dx;$$

$$\mathrm{d)} \quad \int \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx;$$

h) 
$$\int \sqrt{1+x^2} \, dx.$$

Exercício 6.4 Calcule:

a) 
$$\int \frac{2x^2 + x + 1}{(x - 1)(x + 1)^2} dx$$
;

g) 
$$\int \frac{27}{x^4 - 3x^3} \, dx$$
;

b) 
$$\int \frac{3x^2 - 4x - 1}{(x^2 - 1)(x - 2)} dx;$$

h) 
$$\int \frac{x^4 - 8}{x^3 - 2x^2} dx$$
;

c) 
$$\int \frac{2x^2 - x - 2}{x^2(x - 2)} dx$$
;

i) 
$$\int \frac{x+3}{(x-2)(x^2-2x+5)} dx$$
;

d) 
$$\int \frac{2x^3 + 5x^2 + 6x + 2}{x(x+1)^3} dx$$
;

$$j) \int \frac{x+1}{x(x^2+1)^2} \, dx;$$

e) 
$$\int \frac{x^2 - x + 2}{x(x^2 - 1)} dx$$
;

k) 
$$\int \frac{x+2}{2x(x-1)^2(x^2+1)} dx;$$

f) 
$$\int \frac{4x^2 + x + 1}{x^3 - x} dx$$
;

1) 
$$\int \frac{3x^3 + x^2 - x - 1}{x^2(x^2 - 1)} \, dx.$$

Exercício 6.5 Calcule:

a) 
$$\int \frac{1}{(2+\sqrt{x})^7 \sqrt{x}} dx;$$

e) 
$$\int \frac{1}{\cos^2 x \, \sin^2 x} \, dx;$$

b) 
$$\int \mathsf{tg}^2 x \, dx$$
;

f) 
$$\int \cos^2 x \, \sin^2 x \, dx$$
;

c) 
$$\int \frac{x + (\operatorname{arcsen}(3x))^2}{\sqrt{1 - 9x^2}} dx;$$

g) 
$$\int \frac{1}{1+e^x} \, dx;$$

d) 
$$\int \frac{x e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}} dx;$$

$$h) \int \frac{1}{x^2 \sqrt{4 - x^2}} \, dx.$$

Exercício 6.6 Sendo  $f: \mathbb{R} \longrightarrow \mathbb{R}$  definida por  $f(x) = x^2 \operatorname{sen} x$ , calcule a primitiva de f cujo gráfico passa pelo ponto  $\left(\frac{\pi}{2}, \pi\right)$ .

Exercício 6.7 Em cada alínea, determine a única função  $f: \mathbb{R} \longrightarrow \mathbb{R}$ , duas vezes derivável, tal que:

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a) 
$$f''(x) = 4x - 1$$
,  $x \in \mathbb{R}$ ,  $f(1) = 3$  e  $f'(2) = -2$ ;

b) 
$$f''(x) = \sin x \cos x$$
,  $x \in \mathbb{R}$ ,  $f(0) = 0$  e  $f'(0) = 1$ .