

SINGAPORE POLYTECHNIC**2023/2024 SEMESTER ONE MID-SEMESTER TEST****ADVANCED MATHEMATICS 2 (EP0602)**

Time allowed: 1 hr 30 min

2nd/ 3rd Year Full-Time

School of Architecture & the Built Environment (ABE)
DCE, DCEB

School of Chemical and Life Sciences (CLS)
DAPC, DCHE, DFST

School of Electrical and Electronic Engineering (EEE)
DASE, DCPE, DEB, DEEE

School of Mechanical Engineering (MAE)
DARE, DME, DMRO

School of Business (SB)
DAC, DBA, DBFK

School of Computing (SOC)
DAAA, DISM, DIT

Instructions:

1. The Singapore Polytechnic Examination rules are to be complied with.
2. This paper consists of **2 printed** pages, including the cover page. There are **4** questions and you are required to answer **ALL**.
3. You are allowed **ONE** A4-sized handwritten formula sheet for reference. Ensure that you do not have any other reference material or notes with you.
4. Write all solutions and answers in the answer booklet provided.
5. Unless otherwise stated, all decimal answers given should be correct to **three** significant figures.
6. Except for sketches, graphs and diagrams, no solution is to be written in pencil. Failure to do so will result in loss of marks.

1. (a) Determine whether each of the following series converges by using an appropriate test. Specify the test used. (13 marks)

(i)
$$\sum_{k=1}^{\infty} \frac{1}{e^{(1.1)^k} - 3^k}$$

(ii)
$$\sum_{k=1}^{\infty} \frac{(k+2)^3 - 5k^2 + 2}{3^k}$$

- (b) Use Mathematical Induction to prove the following statement: (12 marks)

For all $n \in \mathbb{N}$, $n^3 + 2n$ is divisible by 3.

2. (a) Find the first two non-zero terms of the Maclaurin series for (12 marks)

$$f(x) = \tan(x)$$

- (b) Hence or otherwise, estimate the value of $\int_0^{0.4} \tan(x^2) dx$ to four significant figures. (8 marks)

3. (a) For the following power series, find the radius of convergence without checking the endpoints: (20 marks)

$$\sum_{k=1}^{\infty} \left(\frac{k!}{k^k} \right) (x-2)^k$$

- (b) The following inequality is known as *Stirling's approximation*: (10 marks)

$$\ln \frac{k! e^k}{k^k} > 1, \quad k > 1$$

Evaluate the convergence of the endpoints of the series in Part (a) using Stirling's approximation and thus state the interval of convergence.

4. (a) Determine if the following series (20 marks)

$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{2^k}{3^k + 5^k} + \frac{\ln k}{k^3} \right)$$

converges absolutely, converges conditionally or diverges.

- (b) In the course notes, we have seen the theorem: (5 marks)

If the series $\sum |u_k|$ converges, then $\sum u_k$ converges.

Using the comparison test, prove this statement.

- End of Paper -