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Subject: MTH 331 Discrete Mathematics Assignment 1.

# Assignment 1

1. 5 Examples of sets:
(a) a set of names of ~~musics~~ songs in a playlist
(b) A set of 5 shapes
(c) A set of natural odd numbers from 1 to 100 [included]
(d) A set of all positive integers
(e) A set of all months in a year.

2. 5 non-sets:
(a) A bag of marbles
(b) A team of players
(c) A heap of sand
(d) A traffic jam
(e) A recipe

3. Set builder form:
(a) {2, 4, 6, 8, 10}
SB form: A = {x : x is a positive even integer between 1 and 10 (included)}
(b) {2, 3, 5, 7, 11}

(c) {January, June, July}
SB form: A = {x : x is ~~a~~ months of the year starting with the alphabet "J"}
(d) {a, e, i, o, u}
SB form: A = {x : x is ~~a~~ a list of all vowels in the english alphabet}
(e) {Tuesday, Thursday}
SB form: A = {x : x is the days of the week starting with alphabet "T"}

(f) $\{1,4,9,16,25\}$

SB form : $A = \{x : x$ is a square of the first 5 positive natural numbers$\}$

(g) $\{5,10,15,20,25,30\}$

SB form : $A = \{x : x$ is the first six positive multiples of 5 $\}$

4. $R = \{(a,a),(b,c),(a,b)\}$

To make R reflexive, add the ordered pairs $(b,b)$ & $(c,c)$.

Now $R = \{(a,a),(b,b),(c,c),(b,c),(a,b)\}$

To make R transitive, we need to add $(b,a)$ to R.

The result R which is both reflexive & transitive is:

$R = \{(a,a),(b,b),(c,c),(a,b),(b,c),(a,b)\}$

5. $A = \{1,2,3,4,5\}$

$R = \{(a,b) \mid |a-b|$ is even$\}$

For a relation to be an equivalence relation, it has to satisfy 3 conditions : reflexivity, symmetry and transitivity:

(i) Reflexivity : For any a in A, we get $|a-a| = 0$, which is an even number. therefore, $(a,a)$ is in R for all a in A. ∴ R ⇒ reflexive

(ii) Symmetry : For any $(a,b)$ in R, we ~~we~~ have $|a-b|$ is even. This also means that $|b-a|$ is also even. so $(b,a)$ is also in R. Hence, R is symmetric.

(iii) Transitivity : For any $(a,b)$ and $(b,c)$ in R, we have $|a-b|$ & $|b-c|$ are even. This implies that $|a-c|$ is also even. hence, $(a,c)$ is in R. so R is transitive.

6. (a) $5x \equiv 6 \pmod 8$

$5x - 6 = (\text{multiple of } 8)$

$5x - 6 = 24$

$5x = 30$

$\boxed{x = 6}$

(b) $150x \equiv 35 \pmod{31}$

$150x - 35 = (\text{multiple of } 31)$

$150x - 35 = 31k$

7 p: Jupiter is a planet
q: India is an island

(i) $-p \Rightarrow$ Jupiter is not a planet
(ii) $p \vee -q \Rightarrow$ Jupiter is a planet or India is not an island
(iii) $-p \vee q \Rightarrow$ Jupiter is not a planet or India is an island.
(iv) $p \to -q \Rightarrow$ If Jupiter is a planet, then India is not an island
(v) $p \leftrightarrow q \Rightarrow$ ~~can~~ Jupiter is a planet if and only if India is an Island.

8. $p \to$ 19 is a prime number
$q \to$ all angles in a triangle are equal

(i) $-p \wedge q$
(ii) $p \vee -q$
(iii) $p \wedge q$
(iv) $-p$

9. (i) $-p \wedge -q$

| $p$ | $q$ | $-p$ | $-q$ | $-p \wedge -q$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |

(ii) $-(p \wedge -q)$

| $p$ | $q$ | $-q$ | $p \wedge -q$ | $-(p \wedge -q)$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |

(iii) $(p \vee q) \vee -q$

| $p$ | $q$ | $-q$ | $p \vee q$ | $(p \vee q) \vee -q$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |

(iv) $(-p \rightarrow r) \wedge (p \leftrightarrow q)$

| $p$ | $q$ | $r$ | $-p$ | $-p \rightarrow r$ | $p \leftrightarrow q$ | $(-p \rightarrow r) \wedge (p \leftrightarrow q)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 |

10. (i) $(p \land q) \land -(p \lor q)$

| $p$ | $q$ | $p \land q$ | $p \lor q$ | $-(p \lor q)$ | $(p \land q) \land -(p \lor q)$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |

$\therefore$ Contradiction

(ii) $((p \lor q) \land -p) \rightarrow q$

| $p$ | $q$ | $-p$ | $p \lor q$ | $(p \lor q) \land -p$ | $((p \lor q) \land -p) \rightarrow q$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 |

$\therefore$ Tautology

(iii) $(p \rightarrow q) \leftrightarrow (-p \rightarrow q)$

| $p$ | $q$ | $-p$ | $p \rightarrow q$ | $-p \rightarrow q$ | $(p \rightarrow q) \leftrightarrow (-p \rightarrow q)$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | $\cancel{0}$ 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

$\therefore$ Contingency

(iv) $((p \to q) \wedge (q \to r)) \to (p \to r)$

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $p \to r$ | $((p\to q)\wedge(q\to r))$ | $((p\to q)\wedge(q\to r))\to(p\to r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | $\cancel{0}$ 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | $\cancel{0}$ 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

$\therefore$ Tautology

11. De Morgan's law states that the negation of a conjunction is logically equivalent to the disjunction of the negations of the individual statements, and vice versa.

$-(p \wedge q) \equiv -p \vee -q$ (or)
$-(p \vee q) \equiv -p \wedge -q$

| $p$ | $q$ | $p \wedge q$ | $-(p \wedge q)$ | $-p$ | $-q$ | $-p \vee -q$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |

$\therefore -(p \wedge q) \equiv -p \vee -q$