Selected Tables and Formulas for 2nd Partial Exam

First-order System

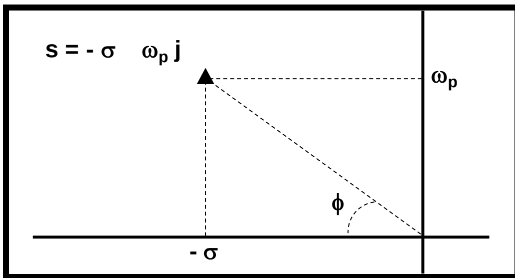
$$G_1(s)=\frac{K_{est}}{1+\tau\cdot s} \quad ; \quad p=-\frac{1}{\tau} \quad ; \quad te_{98\%}=4\cdot\tau$$

Second-order System

$$G_2(s)=\frac{K_{est}\cdot\left(\sigma^2+\omega_p^2\right)}{\left(s+\sigma\right)^2+\omega_p^2}=\frac{K_{est}\cdot\omega_n^{\,2}}{s^2+2\cdot\xi\cdot\omega_n\cdot s+\omega_n^{\,2}} \quad ; \quad \begin{array}{l} p1=-\sigma+\omega_p\cdot j=-\xi\cdot\omega_n+\omega_n\cdot\sqrt{\xi^2-1} \\ p2=-\sigma-\omega_p\cdot j=-\xi\cdot\omega_n-\omega_n\cdot\sqrt{\xi^2-1} \end{array}$$

$$tp=\frac{\pi}{\omega_p}=\frac{\pi}{\omega_n\cdot\sqrt{1-\xi^2}} \quad ; \quad te_{98\%}=\frac{4}{\sigma}=\frac{4}{\xi\cdot\omega_n} \quad ; \quad \delta=e^{-\sigma\cdot tp}=e^{-\frac{\pi\cdot\sigma}{\omega_p}}=e^{-\frac{\pi\cdot\xi}{\sqrt{1-\xi^2}}}$$

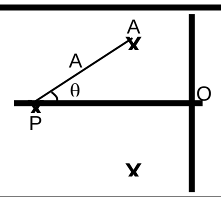
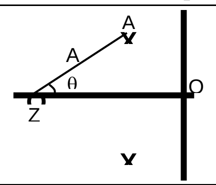
Relations between the complex s-plane and the time-domain dynamic response.

	Settling time	Time to Peak	Overshoot
	$t_{s(98\%)}=\frac{4}{\sigma}$	$t_p=\frac{\pi}{\omega_p}$	$\delta=e^{\frac{-\sigma\cdot\pi}{\omega_p}}=e^{\frac{-\pi}{tg\,\phi}}$

Steady-state Errors (in magnitude)

Error / Type	r(t)=A → e_p	r(t)=At → e_v	r(t)= At^2/2 → e_a
0	$\frac{A}{1+Kp}$; $Kp=\lim\limits_{s\to 0}G_{OpenLoop}(s)$	∞	∞
1	0	$\frac{A}{Kv}$; $Kv=\lim\limits_{s\to 0}\ s\cdot G_{OpenLoop}(s)$	∞
2	0	0	$\frac{A}{Ka}$; $Ka=\lim\limits_{s\to 0}\ s^2\cdot G_{OpenLoop}(s)$

High-order System

	$G(s)=\frac{K_{est}\cdot\left(\sigma^2+\omega_p^{\,2}\right)}{\left[\left(s+\sigma\right)^2+\omega_p^{\,2}\right]}\frac{1}{\left(1+\tau_p\cdot s\right)}$	$t'_p=\frac{\pi+\theta_p}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\frac{\overline{OP}}{\overline{AP}}\times 100$
	$G(s)=\frac{K_{est}\cdot\left(\sigma^2+\omega_p^{\,2}\right)}{\left[\left(s+\sigma\right)^2+\omega_p^{\,2}\right]}\left(1+\tau_z\cdot s\right)$	$t'_p=\frac{\pi-\theta_z}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\frac{\overline{AZ}}{\overline{OZ}}\times 100$
	$G(s)=\frac{K_{est}\cdot\left(\sigma^2+\omega_p^{\,2}\right)}{\left[\left(s+\sigma\right)^2+\omega_p^{\,2}\right]}\cdot\frac{\prod_i\left(1+\tau_{zi}\cdot s\right)}{\prod_j\left(1+\tau_{pj}\cdot s\right)}$	$t'_p=\frac{\pi-\sum_i\theta_{zi}+\sum_j\theta_{pj}}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\frac{\prod_i\overline{AZ}}{\prod_i\overline{OZ}}\cdot\frac{\prod_j\overline{OP}}{\prod_j\overline{AP}}\cdot 100$

NOTE
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Departamento de Ingeniería de Sistemas y Automática

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Laplace Transform Table

$$y(t)=\sum_i A_i\cdot e^{-\sigma_i\cdot t}+\sum_i\left[B_i\cdot e^{-\sigma_i\cdot t}\cdot\cos(w_i\cdot t)+C_i\cdot e^{-\sigma_i\cdot t}\cdot\sin(w_i\cdot t)\right]\qquad Y(s)=\sum_i\frac{A_i}{s+\sigma_i}+\sum_i\left[\frac{B_i\cdot(s+\sigma_i)+C_i\cdot w_i}{(s+\sigma_i)^2+w_i^2}\right]$$

$$y(t)=\sum_i r_i\cdot e^{p_i\cdot t}+\sum_i 2\cdot e^{a_i\cdot t}\cdot\left[\alpha_i\cdot\cos(w_i\cdot t)-\beta_i\cdot\sin(w_i\cdot t)\right]\ \leftrightarrow\ \begin{cases}s=p_i\leftrightarrow r_i\\ s=a_i\pm w_i\cdot j\leftrightarrow\alpha_i\pm\beta_i\cdot j\end{cases}$$

Module (Magnitude) and Argument (Angular) Criteria

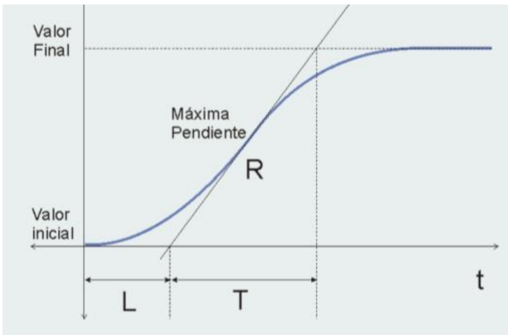
$$1+\frac{K_{RL}\cdot\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}=0\rightarrow\frac{K_{RL}\cdot\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}=-1\rightarrow -K_{RL}=\frac{\prod_{j=1}^{n}(s+p_j)}{\prod_{i=1}^{m}(s+z_i)}\rightarrow\begin{cases}K_{RL}=\dfrac{\prod_{j=1}^{n}|s+p_j|}{\prod_{i=1}^{m}|s+z_j|}\\ 180^\circ\pm q\cdot 360^\circ=\sum_{j=1}^{n}\tan^{-1}(s+p_j)-\sum_{i=1}^{m}\tan^{-1}(s+z_j)\end{cases}$$

PID controllers/regulators

$G_{PID}(s)=K_r(1+T_d s)\left(1+\frac{1}{T_i s}\right)$	$K_r=\frac{K_{LDR}}{K_G\cdot K_H\cdot T_d}$	$T_d=\frac{1}{z_d}$	$T_i=\frac{1}{z_i}$
$G_{PID}(s)=K_r'\left(1+T_d' s+\frac{1}{T_i' s}\right)$	$K_r'=K_r\cdot\frac{T_i+T_d}{T_i}$	$T_d'=\frac{T_d\cdot T_i}{T_d+T_i}$	$T_i'=T_i+T_d$

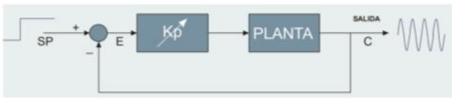
Step-response Ziegler-Nichols Method or Open-loop Ziegler-Nichols Method

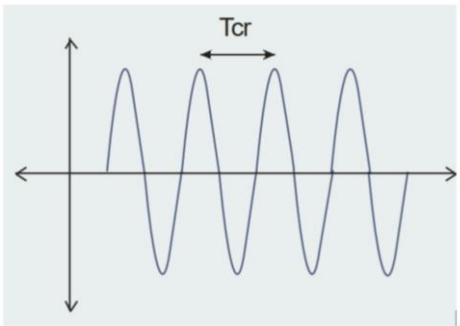
TIPO DE CONTROLADOR	Kp	Ti	Td
P	$\frac{1}{R\cdot L}$		
PI	$\frac{0,9}{R\cdot L}$	$3L$	
PID	$\frac{1,2}{R\cdot L}$	$2L$	$0,5L$



Critical Oscillation Ziegler-Nichols Method or Closed-loop Ziegler-Nichols Method

TIPO DE CONTROLADOR	Kp	Ti	Td
P	$0,5Kcr$		
PI	$0,45Kcr$	$\frac{Tcr}{1,2}$	
PID	$0,6Kcr$	$\frac{Tcr}{2}$	$\frac{Tcr}{8}$





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