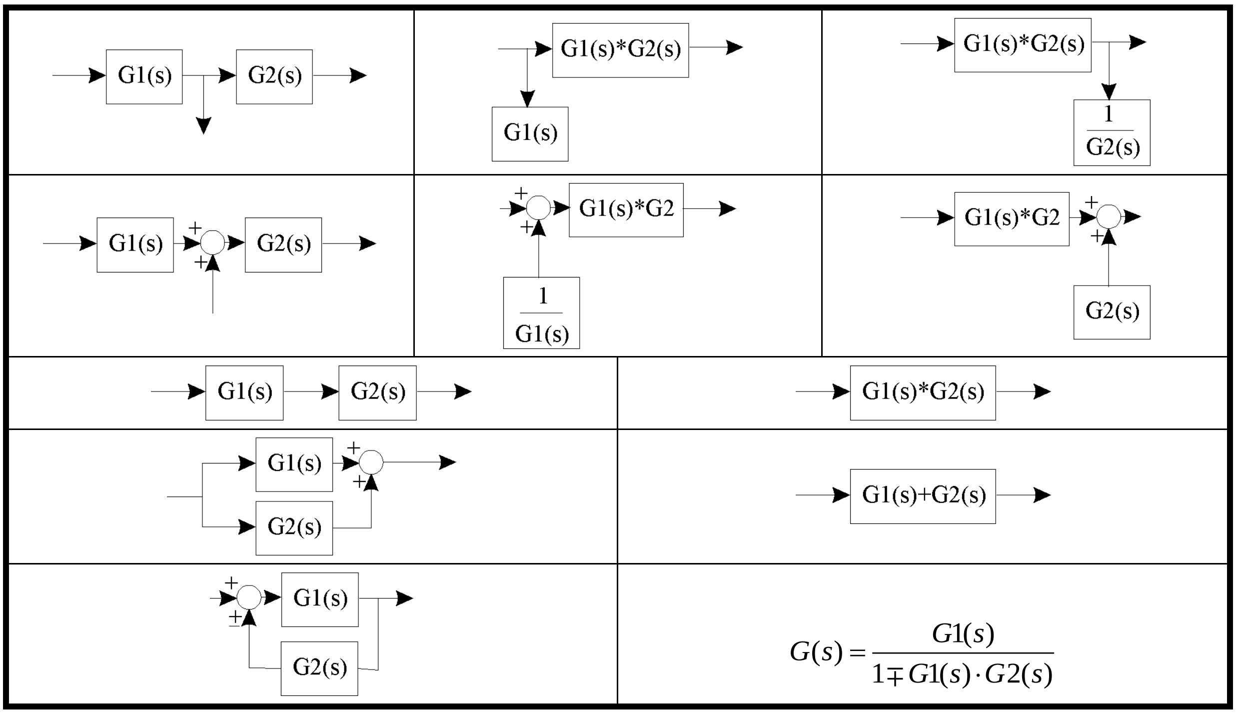
Departamento de Ingeniería de Sistemas y Automática

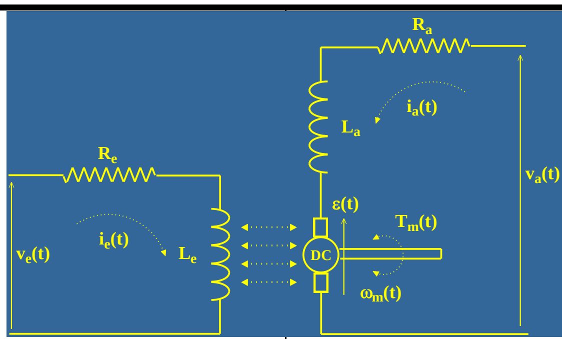
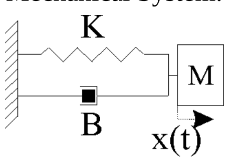
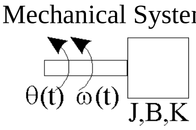
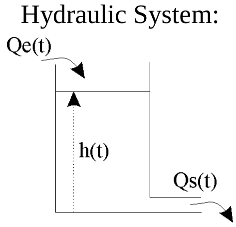
Selected Tables and Formulas for 1st Partial Exam

Block Diagram Simplification Rules



$$G(s)=\frac{G1(s)}{1\mp G1(s)\cdot G2(s)}$$

Physical System Models

Passive Electrical Systems:	$V_R(t)=R\cdot i(t)$ $Vc(t)=\dfrac{1}{C}\int_0^t i(\tau)d\tau$ $V_L(t)=L\cdot\dfrac{di(t)}{dt}$		
Translational Mechanical System: 	$\sum F(t)=M\cdot\dfrac{d^2x(t)}{dt^2}+B\cdot\dfrac{dx(t)}{dt}+K\cdot x(t)$ $\sum F(t)=M\cdot\dfrac{dv(t)}{dt}+B\cdot v(t)$	$T_m(t)=k\cdot i_e(t)\cdot i_a(t)$	$\varepsilon(t)=\psi\cdot\omega_m(t)$
Rotational Mechanical System 	$\sum T(t)=J\cdot\dfrac{d^2\theta(t)}{dt^2}+B\cdot\dfrac{d\theta(t)}{dt}+K\cdot\theta(t)$ $\sum T(t)=J\cdot\dfrac{d\omega(t)}{dt}+B\cdot\omega(t)$	Hydraulic System: 	$A\cdot\dfrac{dh(t)}{dt}=q_e(t)-q_s(t)$ $q_s(t)=K\cdot\sqrt{h(t)}$

Steady-state Errors

Type \ Error	r(t)=A → e_p	r(t)=At → e_v	r(t)= $At^2/2$ → e_a
0	$\dfrac{A}{1+Kp}$; $Kp=\lim\limits_{s\to 0} G_{open-loop}(s)$	∞	∞
1	0	$\dfrac{A}{Kv}$; $Kv=\lim\limits_{s\to 0} s\cdot G_{open-loop}(s)$	∞
2	0	0	$\dfrac{A}{Ka}$; $Ka=\lim\limits_{s\to 0} s^2\cdot G_{open-loop}(s)$

NOTE: It is forbidden to write any comment or additional formula on this summary neither to use any other document during the exams.

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First-order System

$$G_1(s)=\frac{K_{est}}{1+\tau\cdot s} \quad ; \quad p=-\frac{1}{\tau} \quad ; \quad te_{98\%}=4\cdot\tau$$

Second-order System

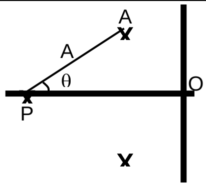
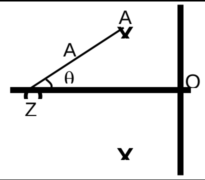
$$G_2(s)=\frac{K_{est}\cdot\left(\sigma^2+\omega_p^2\right)}{\left(s+\sigma\right)^2+\omega_p^2}=\frac{K_{est}\cdot\omega_n{}^2}{s^2+2\cdot\xi\cdot\omega_n\cdot s+\omega_n{}^2}$$

$$\xrightarrow{complex-poles} p_{1,2}=-\omega_n\cdot\cos\phi\pm\omega_n\cdot\sin\phi\cdot j=-\xi\cdot\omega_n\pm\omega_n\cdot\sqrt{1-\xi^2}\cdot j=-\sigma\pm\omega_p\cdot j$$

$$\xrightarrow{real-poles} p_{1,2}=-\xi\cdot\omega_n\pm\omega_n\cdot\sqrt{\xi^2-1}$$

$$\begin{cases} t_{e98\%}=\dfrac{4}{\sigma}=\dfrac{4}{\xi\cdot\omega_n} \\ tp=\dfrac{\pi}{\omega_p}=\dfrac{\pi}{\omega_n\cdot\sqrt{1-\xi^2}} \\ \delta=e^{-\sigma\cdot tp}=e^{-\frac{\pi\cdot\sigma}{\omega_p}}=e^{-\frac{\pi\cdot\xi}{\sqrt{1-\xi^2}}}=e^{-\frac{\pi\cdot\cos\phi}{\sin\phi}}=e^{-\frac{\pi}{tg\phi}} \end{cases}$$

High-order System

	$G(s)=\dfrac{K_{est}\cdot\left(\sigma^2+\omega_p{}^2\right)}{\left[\left(s+\sigma\right)^2+\omega_p{}^2\right]}\dfrac{1}{\left(1+\tau_p\cdot s\right)}$	$t'_p=\dfrac{\pi+\theta_p}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\dfrac{\overline{OP}}{\overline{AP}}\times 100$
	$G(s)=\dfrac{K_{est}\cdot\left(\sigma^2+\omega_p{}^2\right)}{\left[\left(s+\sigma\right)^2+\omega_p{}^2\right]}\left(1+\tau_z\cdot s\right)$	$t'_p=\dfrac{\pi-\theta_z}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\dfrac{\overline{AZ}}{\overline{OZ}}\times 100$
	$G(s)=\dfrac{K_{est}\cdot\left(\sigma^2+\omega_p{}^2\right)}{\left[\left(s+\sigma\right)^2+\omega_p{}^2\right]}\cdot\dfrac{\prod_i\left(1+\tau_{zi}\cdot s\right)}{\prod_j\left(1+\tau_{pj}\cdot s\right)}$	$t'_p=\dfrac{\pi-\sum_i\theta_{zi}+\sum_j\theta_{pj}}{\omega_p}$	$\delta'=e^{-\sigma\cdot t'_p}\dfrac{\prod_i\overline{AZ}}{\prod_i\overline{OZ}}\cdot\dfrac{\prod_j\overline{OP}}{\prod_j\overline{AP}}\cdot 100$

System simplification by reducing the transfer functions into an equivalent one

$G(s)=K\dfrac{\prod_{i=1}^{m}\left(s+z_i\right)}{\prod_{i=1}^{n}\left(s+p_i\right)} \qquad (n\geq m)$	Poles and zeros that are closed can be cancelled if $\lvert P-Z\rvert < P/10$ Far poles can be eliminates if $P > 10\ \sigma_{dominant}$ Steady-state gain is invariant: $\lim_{s\to 0}G(s)=\lim_{s\to 0}G_{eq}(s)$

Laplace Transform Table

Unit Impulse u(t)=δ(t)	1
Unit Step u(t)=1	$\dfrac{1}{s}$
Unit Ramp u(t)=t	$\dfrac{1}{s^2}$
$e^{-a\cdot t}$	$\dfrac{1}{s+a}$
$e^{-\sigma\cdot t}\cdot\sin\left(\omega\cdot t\right)$	$\dfrac{\omega}{\left(s+\sigma\right)^2+\omega^2}$
$e^{-\sigma\cdot t}\cdot cos\left(\omega\cdot t\right)$	$\dfrac{s+\sigma}{\left(s+\sigma\right)^2+\omega^2}$

$$y(t)=\sum_i A_i\cdot e^{-\sigma_i\cdot t}+\sum_i\left[B_i\cdot e^{-\sigma_i\cdot t}\cdot\cos\left(w_i\cdot t\right)+C_i\cdot e^{-\sigma_i\cdot t}\cdot\sin\left(w_i\cdot t\right)\right] \qquad Y(s)=\sum_i\frac{A_i}{s+\sigma_i}+\sum_i\left[\frac{B_i\cdot\left(s+\sigma_i\right)+C_i\cdot w_i}{\left(s+\sigma_i\right)^2+w_i^2}\right]$$

$$y(t)=\sum_i r_i\cdot e^{p_i\cdot t}+\sum_i 2\cdot e^{a_i\cdot t}\cdot\left[\alpha_i\cdot\cos\left(w_i\cdot t\right)-\beta_i\cdot\sin\left(w_i\cdot t\right)\right] \leftrightarrow \begin{cases} s=p_i\leftrightarrow r_i \\ s=a_i\pm w_i\cdot j\leftrightarrow\alpha_i\pm\beta_i\cdot j \end{cases}$$

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