Problem Set 5
Due date: 10.07.2025

Problems

(1) Let $(M^3, g(t))$ be a Ricci flow with $\mathrm{Ric}(g(0)) > 0$. Prove that the solution becomes singular in finite-time.

(2) Recall the normalized Ricci flow

$$\partial_t g_{ij} = -2R_{ij} + \frac{2}{n} r g_{ij} \tag{0.1}$$

where $r = \dfrac{\int_M R d\mu}{\int_M d\mu}$ is the average scalar curvature.

- Find the ODE which the eigenvalues $\lambda \geq \mu \geq \nu$ satisfy along the normalized Ricci flow in dimension 3.
- We'll prove in the lecture that $\tilde{R}_{\max}$ which is the maximum of the scalar curvature along the normalized Ricci flow is bounded. Use this to prove that the normalized Ricci flow on M^3 with the initial metric having positive Ricci curvature exists for all time.
  (**Hint:**First prove that along the unnormalized flow we must have $\int_0^T R_{\max} = \infty$ and then look at the normalized version of this equation.)