Honors Stat Mech
Project 1

Physics 423 Spring 2017

Monte Carlo determination of π. Imagine we throw N darts into a 2×2 square with an inscribed circle of radius 1. Assume all darts land somewhere in the square, and the probability density that they land at a given (x, y) is uniform across the square. Then the probability p that a given dart lands within the circle is given by the ratio of the area of the circle to the area of the square, $p = \pi/4$. Define a random variable X with two values: 1 if the dart lands in the circle, and zero if it lands outside the circle. As N becomes large, the law of large numbers (LLN) says that the X sample average S_N ($S_N \equiv \sum_i X_i/N$, which is effectively the fraction of darts landing in the circle) will tend to $S_N \to \langle X \rangle = p$, allowing us to extract an approximate value for π. (Note that this procedure relates an *area under a curve* to a *probability.* The generalization of this example to a multidimensional dartboard with a complicated inscribed shape is the basis for Monte Carlo integration techniques.)

Write a program to perform the simulation described above. Automatically perform the simulation a large number of times for a given value of N. For your report, show that as N increases, the distribution of S_N obtained in this way tends to a particular normal distribution that depends on N. (For example, you could show a couple of histograms of results for different values of N, overlaid with a curve showing the probability density function of the relevant normal distribution.) Explain this with the central limit theorem and use it to argue that the error in your estimate of π decreases with a particular power of N.