Integration by Parts [IBP]

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Section 7.1

Let u, v be functions of x.
By product rule, the derivative of UV = $\frac{du}{dx} \cdot V + \frac{dv}{dx} \cdot U$
Now let's integrate: $\int \frac{d}{dx}(UV) \cdot dx = \int \frac{du}{dx} \cdot V \cdot dx + \int U \cdot \frac{du}{dx} \cdot dx$

This results in the following **formula** which is used for integration by parts.

$$UV = \int U \cdot dv + \int U \cdot du$$

Strategies for when to choose U & V

- Choose U to be the simpler function when it's differentiated
- Pick DV (derivative of V) to be the harder function, that you know how to integrate

Good strategies for picking an integration technique, use **"I LATE"**
Goes from hardest to easiest.

- I = inverse trig functions
- L = Log functions
- A = Algebraic functions/ all polynomials
- T = Trig functions
- E = Exponential functions

Example 0.1 *1*

$$\begin{aligned} \int x^2 \cdot lnx \cdot dx &= \frac{x^3}{3} \cdot lnx - \int \frac{x^3}{3} \cdot \frac{1}{x} \cdot dx \\ &= \frac{x^3}{3} \cdot lnx - \int \frac{x^2}{3} \cdot dx \\ &= \frac{x^3}{3} \cdot lnx - \frac{x^3}{4} + C \end{aligned}$$

$$Let \quad u = lnx. \quad Let \quad dv = x^2 dx$$
$$du = \frac{1}{x}dx = lnx. \quad v = \frac{x^3}{3}dx$$

Example 0.2 *2*

$$\begin{aligned} \int lnx \cdot dx &= x \cdot lnx - \int \frac{x^3}{3} \cdot x \\ &= x \cdot lnx - \int dx \\ &= x \cdot lnx - x + C \end{aligned}$$

$$Let \quad u = lnx. \quad Let \quad dv = dx$$
$$du = \frac{1}{x}dx = lnx. \quad v = x$$

Example 0.3 *3*

$$\int x^2 \cdot \cos x \cdot dx = x^2 \cdot \sin x - \int 2x \sin x \cdot dx$$

Let $u = x^2$. *Let* $dv = \cos x dx$

$du = 2xdx \quad v = \sin x dx$

Do IBP again!