Introduction to Vectors

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Vectors in $\mathbb{R}^N$

Theorem 0.1

$$\mathbb{R}^n = \begin{bmatrix} X_i \\ \vdots \\ X_n \end{bmatrix} \mid X_i, \ldots X_n \in \mathbb{R}$$

Sometimes to better understand vectors in $\mathbb{R}^n$, we can use points to visualize them.

$$\vec{x} = \begin{bmatrix} X_i \\ \vdots \\ X_n \end{bmatrix} \text{ is equivalent to } (x_i, \ldots, x_n)$$

Example 0.2 *Vector* $\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ *as point* $(1, 2)$

Vectors always start at the origin and end at the points it's defined by.

Theorem 0.3 *The set of vectors of the form*

$$\vec{x} = \begin{bmatrix} s \\ t \\ 0 \end{bmatrix}, s, t \in \mathbb{R}$$

are the set of all points of the form $(s, t, 0)$*. Hence the plane in* $\mathbb{R}^3$

Definition Let $\vec{x} = \begin{bmatrix} X_i \\ \vdots \\ X_n \end{bmatrix}, \vec{y} = \begin{bmatrix} Y_i \\ \vdots \\ Y_n \end{bmatrix} \in \mathbb{R}$, we say that $\vec{x}$ and $\vec{y}$ are **equal** and write $\vec{x} = \vec{y}$ if $x_i = y_i$ for all $i <= x <= y$.

Definition

We define addition by $\vec{x} + \vec{y} \;\; = \begin{bmatrix} X_i & + & Y_1 \\ & \vdots & \\ X_n & + & Y_n \end{bmatrix}$

For any real scalar $t \in \mathbb{R}$, we define scalar multiplication by:

$$t\vec{x} = \begin{bmatrix} tx_i \\ \vdots \\ tx_n \end{bmatrix}$$

Example 0.4 -

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Example 0.5 -

$3 \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$

Example 0.6 -

$(-1) \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} + (\sqrt{2}) \cdot \begin{bmatrix} 2 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} tbd \\ tbd \end{bmatrix}$