Basis and Subspaces

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Theorem 1.1 *.5*
If $\mathbb{B} = \{\vec{v}_1, \ldots, \vec{v}_k\}$ is a basis for a subset S of $\mathbb{R}^n$, then every vector $\vec{x} \in \mathbb{S}$, can be written as a unique linear combination of $\vec{v}_1, \ldots, \vec{v}_k$.
Proof left as exercise

Definition
In $\mathbb{R}^n$, let $\vec{e}_i$ represent the vector whose i'th component is 1 and all other components are 0. The set $\{\vec{e}_1, \ldots, \vec{e}_n\}$ is called the **standard** basis for $\mathbb{R}^n$

Example 1.1 *.1*
Standard Basis for $\mathbb{R}^2$ *is* $\{\vec{e}_1, \vec{e}_2\} = \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$

Example 1.2 *.2*
Standard Basis for $\mathbb{R}^3$ *is* $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

Definition
A basis is a different coordinate system.

Example 1.3 *.3*
Prove that $\mathbb{B} = \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}$ *is a basis for* $\mathbb{R}^2$
Solution: By the theorem, we have that $\mathbb{B}$ is linearly independent since neither vector is a scalar multiple of the other.
To show spanning:
Let $\vec{x} = \begin{bmatrix}\vec{x}_1\\\vec{x}_2\end{bmatrix} \in \mathbb{R}^2$. *Consider*

$$\begin{bmatrix}\vec{x}_1\\\vec{x}_2\end{bmatrix} = C_1 \cdot \begin{bmatrix}1\\1\end{bmatrix} + C_2 \cdot \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}C_1 + C_2\\C_1 + 2C_2\end{bmatrix}$$

$X_1 = C_1 + C_2, X_2 = C_1 + 2C_2$. *Solving,* $C_2 = x_2 - x_1$ *and* $C_1 = 2X_1 - X_2$. $\therefore$ *every* $\vec{x} \in \mathbb{R}^2$ *can be written as a linear combination of the vectors in* $\mathbb{B}$.

Surfaces in Higher Dimensions

We can extend over geometrical concepts of lines and planes to $\mathbb{R}^n$ with $\vec{m} \neq \vec{0}$ for n > 3.

Definition

Let $\vec{m}, \vec{b} \in \mathbb{R}^n$, with $\vec{m} \neq \vec{0}$. We call the set with vector equation $\vec{x} = c_1 \cdot \vec{m} + \vec{b}, c_1 \in \mathbb{R}$ a line in $\mathbb{R}^n$ which passes through $\vec{1}$.