Complex Practice

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Proposition 38.1 *If* $Z_1 = R_1(\cos(\theta) + i\sin(\theta))$ *and* $Z_1 = R_1(\cos(\theta_2) + i\sin(\theta_2))$ *which are two complex numbers in polar form, then*

$$Z_1 \cdot Z_2 = R_1 \cdot R_2(\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2)))$$

Example 38.2 *From 37.4*

$$\begin{aligned}
&(\sqrt{6} + \sqrt{2i}) \cdot (-3 \cdot\sqrt{2} + 3 \cdot\sqrt{6i}) \\
&= (2 \cdot\sqrt{2} \cdot (\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}))(6 \cdot\sqrt{2} \cdot (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})) \\
&= (2 \cdot\sqrt{2})(6 \cdot\sqrt{2})(\cos(\frac{\pi}{6} + \frac{2\pi}{3}) + i\sin(\frac{\pi}{6} + \frac{2\pi}{3}) \\
&= (24)(\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6})) \\
&= -12 \cdot\sqrt{3} + 12i
\end{aligned}$$

Example 38.3 *If* $Z = r(cos(\theta + i\sin(\theta)))$
Calculate IZ.

$$\begin{aligned}
IZ &= (\cos\frac{\pi}{2} + \sin\frac{\pi}{2})(r(\cos\theta + \sin\theta) \\
&= r(\cos(\theta + \frac{\pi}{2}) + i\sin(\theta + \frac{\pi}{2})
\end{aligned}$$

Proof of PCMN

$$\begin{aligned}Z_1 \cdot Z_2 &= (r_1(\cos\theta_1 + i\sin\theta_1))(r_2(\cos\theta_2 + i\sin\theta_2))\\ &= (r_1 \cdot r_2)(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\\ &= (r_1 \cdot r_2)(\cos(\theta_1+\theta_2)) + (i\sin(\theta_1+\theta_2))\end{aligned}$$

Theorem 38.4 *Demoives Theorem (DMT)*
If $\theta \in \mathbb{R}, n \in \mathbb{Z}$ Then

$$(\cos\theta + i\sin\theta)^n = \cos(n\cdot\theta) + i\sin(n\cdot\theta)$$

Corollary 38.5 .
IF

$$Z = R(cos\theta + isin(\theta)$$

Then

$$Z = R(cos(n\cdot\theta) + isin(n\cdot\theta))$$

Example 38.6 *Evaluate* $(1 - \frac{1}{\sqrt{3}}i)^{10}$

$$\begin{aligned}1 - \frac{1}{\sqrt{3}} &= \cos(0) \therefore \theta = 0\\ \frac{-1}{\sqrt{3}} &= \sin(\frac{-\pi}{3}) \therefore \theta = \frac{-\pi}{3}\end{aligned}$$

$(\cos(0) + i\sin(\frac{-\pi}{3}))^{10}$

$$\begin{aligned}&= \cos(10\cdot 0) + i\sin(10\cdot\frac{-\pi}{3})\\ &= 1 + i\sin(\frac{-10\pi}{3})\\ &= 1 + i\sin(\frac{2\pi}{3})\\ &= 1 + i\frac{\sqrt{3}}{2}\end{aligned}$$