HKN ECE210 SP18 Review

Utsav Kawrani - kawrani2@illinois.edu
Yda Hoffer-Sohn - yhh2@illinois.edu
David Yan - davidzy2@illinois.edu
Yurui Cao - yuruic2@illinois.edu

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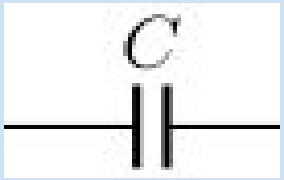
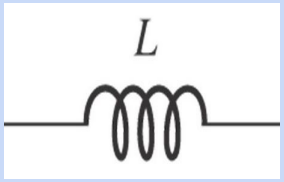
The Basics: Electrical Loads

Circuit Element	Voltage-Current Law	Circuit Symbol
Resistor	$v = iR$	
Capacitor	$i = C\frac{dv}{dt}$	
Inductor	$v = L\frac{di}{dt}$	

Complex numbers

Rectangular Form: a + jb

- Good for adding and subtracting
- To convert: $|A| = \sqrt{a^2 + b^2}$ $\tan^{-1}\frac{b}{a}$

Polar Form: Ae^{jw}

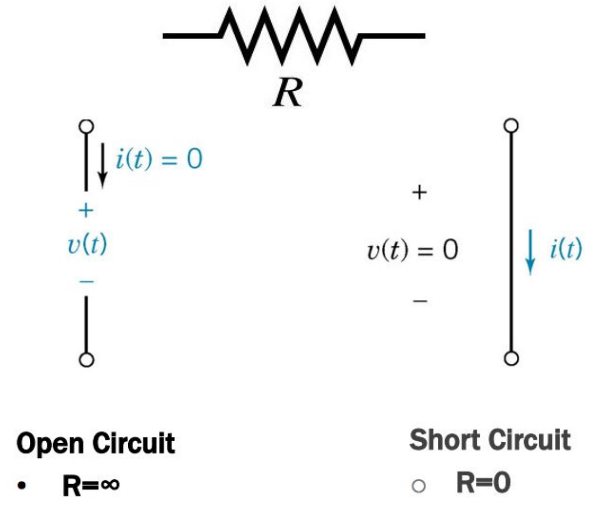
- Good for multiplying and dividing
- Convert with Euler's identity
- Other useful identities

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Special Cases of Resistors

- We shall consider two special cases of the resistor:
  - A short-circuit/**short** is a resistor with **zero** resistance.
  - A open-circuit/**open** is a resistor with **infinite** resistance.
- Their special symbols are shown here:



Special Cases of Capacitors and Inductors

- Recall the I-V characteristic equation of capacitors and inductors:

$$i = C\frac{\mathrm{dv}}{\mathrm{d}t} \qquad v = L\frac{\mathrm{di}}{\mathrm{d}t}$$

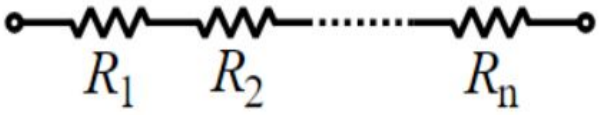
Special Cases of Capacitors and Inductors

- At D.C.:
  - Capacitors:
    - $\frac{dv}{dt} = 0$, $i = C\frac{dv}{dt} = 0$
    - act as open-circuits
  - Inductors:
    - $\frac{di}{dt} = 0$, $v = L\frac{di}{dt} = 0$
    - act as short-circuits

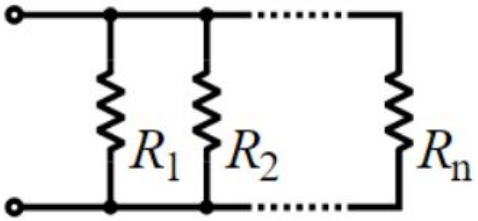
Resistor Combinations

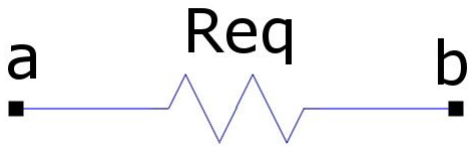
- When a set of resistors carry the same current through a single branch, they are in *series*.
- When a set of resistors support the same voltage between the same pair of nodes, they are in *parallel*.
- Simplify circuits by finding equivalent resistance

$$R_{eq} = R_1 + R_2 + \ ... + R_n$$

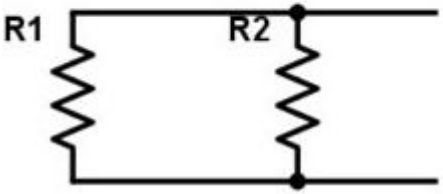


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

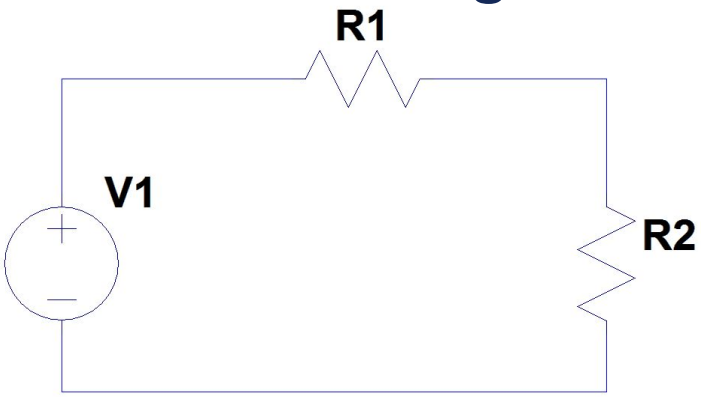




$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$



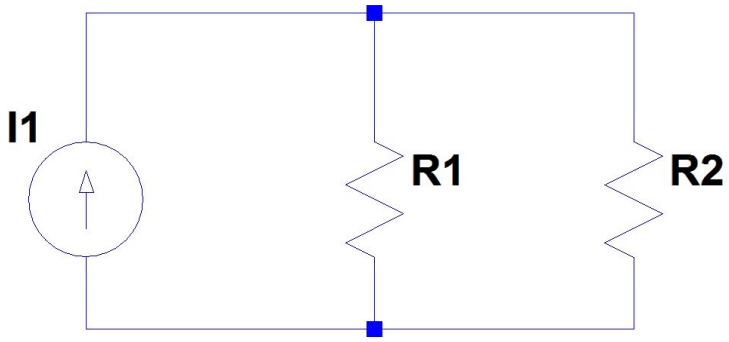
Current & Voltage Divider



$$V_{R1} = V_1 * \frac{R1}{R1+R2}$$

In general...

$$V_{Rn} = V_s * \frac{Rn}{\sum R}$$



$$I_{R1} = I_s * \frac{R2}{R1+R2}$$

In general...

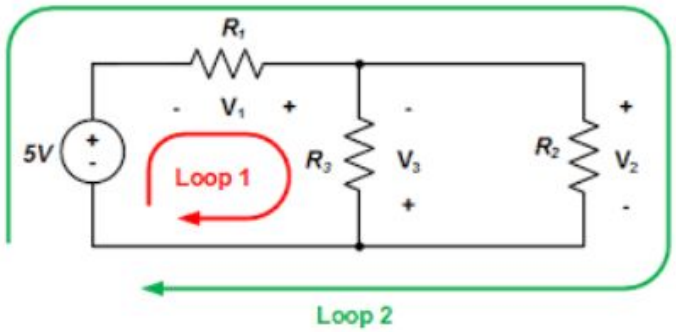
$$I_{Rn} = I_s * \frac{R_{eff}}{R_n+R_{eff}}$$

Where R_{eff} is the effective resistance of all the resistors except the one being inspected

Kirchhoff's Voltage Law

- Kirchhoff's Voltage Law (KVL): The sum of all voltage drops equals the sum of all voltage rises around a closed loop.

$$\sum V_{Rises} = \sum V_{Drops}$$



Loop 1: $5 + V_1 + V_3 = 0$

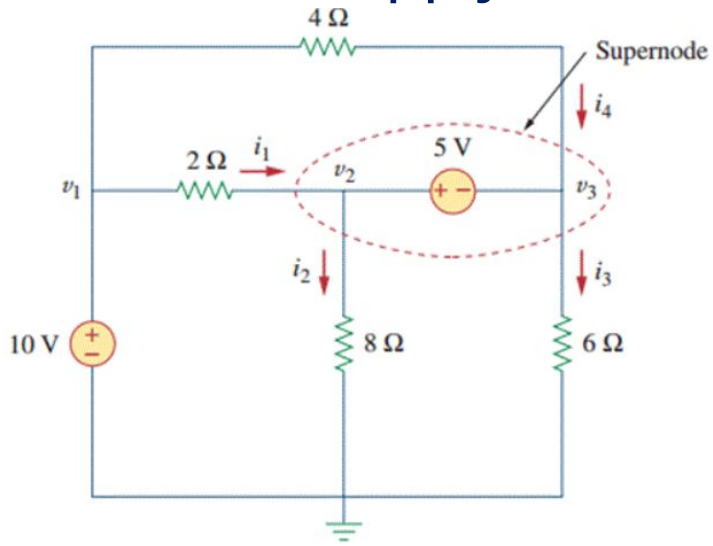
Loop 2: $5 + V_1 = V_2$

Kirchhoff's Current Law

- KCL: The sum of all currents entering a particular node is equal to the sum of all currents exiting that particular node.

$$\sum_{Entering} I = \sum_{Exiting} I$$

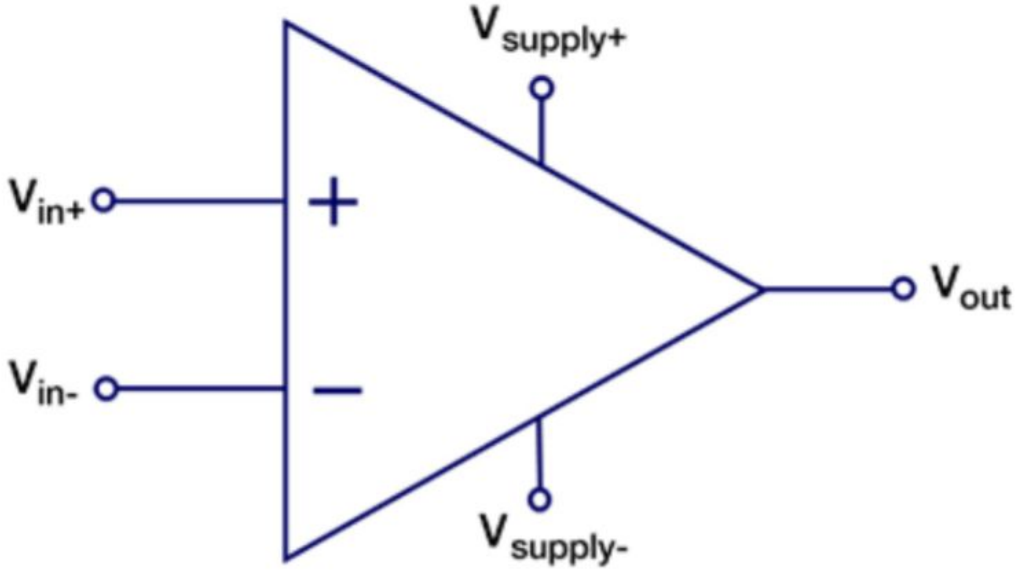
- We can also apply KCL at *super nodes:*



$$i_1 + i_4 = i_2 + i_3$$

Ideal Op-amp Model

$$v_- = v_+$$
$$i_+ = 0$$
$$i_- = 0$$

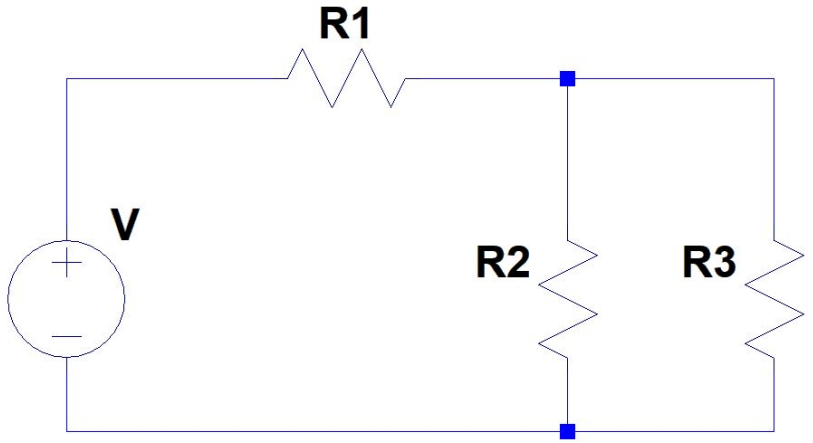


Node-Voltage Method

Using the fact that voltage is the same in a node

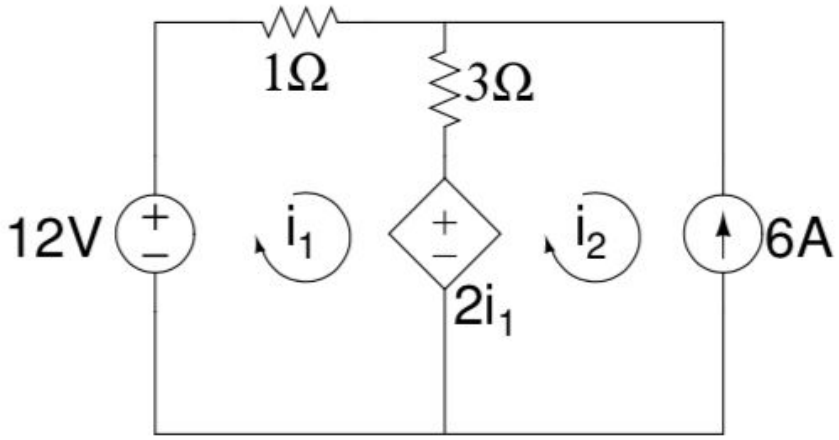
1. Assign relative ground node
2. Assign variables to nodes, assign currents
3. Set up KCL at nodes
4. Solve systems of equations

$$I_n = \frac{V_{START} - V_{END}}{R_n}$$



Loop-Current Method

- Systematic application of KVL and Ohm's law to solve for unknown currents and voltages
- Method:
  - For an N loop circuit, assign N loop current variables
  - carefully label resistors
  - KVL for each loop



Principle of Superposition

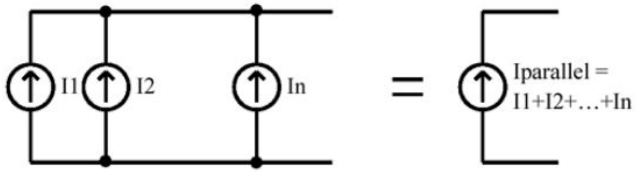
- In a **resistive** circuit, any electrical response can be expressed as a **weighted superposition** of the responses due to individual independent sources

- Method:
  - For circuit w/ N independent sources, redraw the circuit N times, each time suppressing all but one source
  - Calculate the value of the unknown quantity based on each single source
  - Add up the values found using individual sources to get actual value

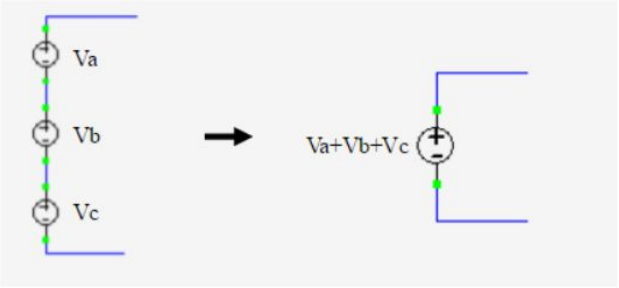
Source Combinations

- Voltage sources in series can be added to produce one equivalent voltage source.
- Current Sources in parallel can be added to produce one equivalent current source.

$$I_{out} = I_1 + I_2 + \cdots + I_n$$

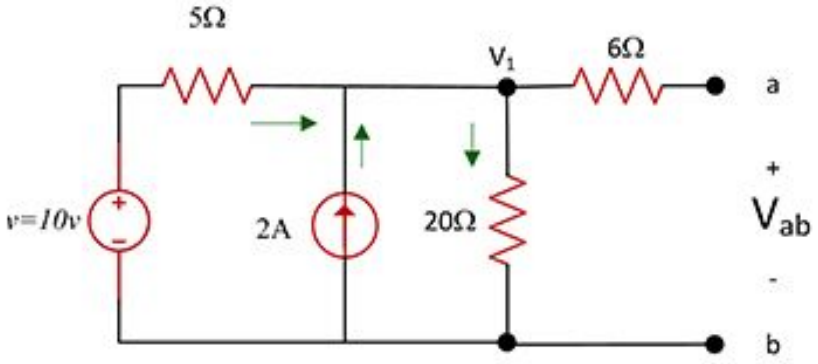


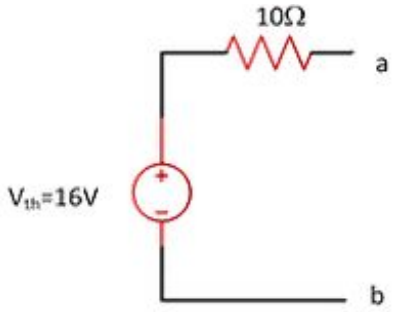
$$V_{out} = V_1 + V_2 + \cdots + V_n$$

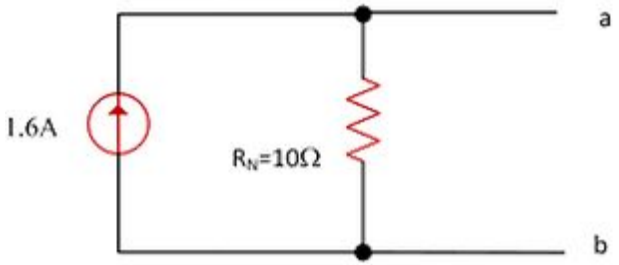


Thevenin and Norton Equivalents

- Thevenin Equivalent: Voltage source and resistor in series
- Norton Equivalent: Current source and resistor in parallel
- Every resistive network can be expressed as either its Thevenin *and/or* its Norton equivalents.







Equivalent circuit with only independent sources

1) **Thevenin Voltage**: Leave the output terminal open
2) **Norton Current**: Connect a short between the output terminals.
3) **Thevenin/Norton Resistance**: Suppress all sources. Calculate the equivalent resistance "looking in" from the output terminals.

$$V_{oc} = V_T \qquad I_{sc} = I_N \qquad R_N = R_T$$

Note: You only need to do two of the three steps above. One you know any of the two, the third can be calculated by rearranging the equation:

So pick the easier two of the three steps!

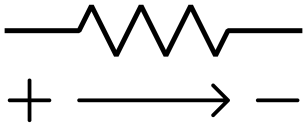
$$V_T = I_N R_T$$

Equivalent Circuits with Dependent Sources.

- The steps for calculating the thevenin voltage and norton current remain the same.
- However, source suppression no longer works for calculating the Thevenin/Norton Resistance.
- Instead, we must use the *test signal method.*
  - Connect a 1A independent current source to the output terminals of the circuit.
  - Suppress all independent sources.
  - Apply circuit analysis techniques to determine the voltage across this current source.
  - That voltage has the same value as the Thevenin/Norton resistance, just in ohms instead of volts, of course.
- The relation $V_T = I_N R_T$ still holds, so the test signal method is unnecessary if you already know the Thevenin voltage and Norton current. Again, you should choose to do the two steps that are easiest for the particular circuit you are given.

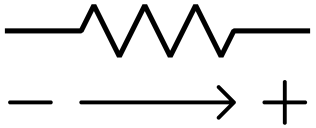
Sign Conventions

Standard Flow:



$V = I * R$ $P = I * V$

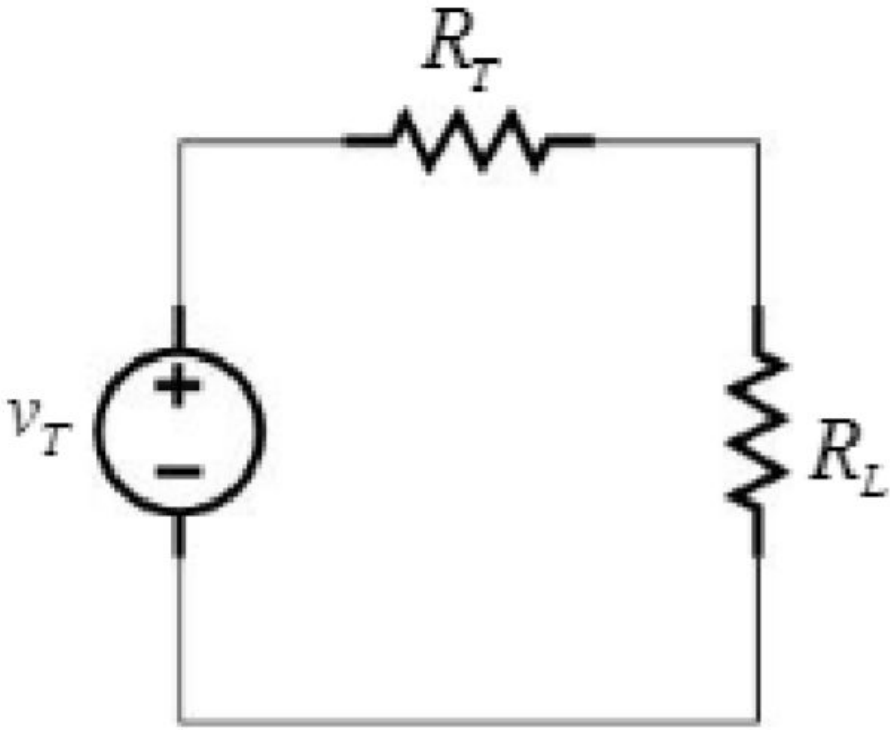
Nonstandard Flow:



$V = -I * R$ $P = -I * V$

Absorbing power if: $P > 0$ Injecting power if: $P < 0$

Available Power (Maximum Power)



Set $R_L = R_T$

$$P_a = V_T^2/4R_T$$

RL, RC, and RLC circuits

- So far, we have solved linear resistive networks. The math involved setting up a system of equations.
- RL and RC circuits require setting up and solving a first-order ODE
- RLC circuits require setting up and solving a second-order ODE. You will never need to actually solve it, but you simply need to know that the equation describing a circuit with both a capacitor and an inductor is a second order ODE.

First Order Differential Equations

Given equation, where α and β are constants

$$\frac{dy}{dt} + \alpha y(t) = \beta$$

Then, $y(t) = A + Be^{-\alpha t}$, where *A* and *B* are also constants

Get *A* by finding the limit as $t \to \infty$, and *B* from initial conditions

The limit $t \to \infty$ is the steady state

Time constant: $\tau = \frac{1}{\alpha}$

1. Homogeneous solution – the exponential term
2. Particular solution – the constant

Zero Input and Zero State

- Zero input is what would the voltage or current across an inductor or capacitor be if it was connected only to a resistor
  - Only depends on initial condition!
  - $$\frac{dy}{dt} + \alpha y(t) = 0$$
- Zero state is what would the voltage or current across an inductor or capacitor be if it's initial charge/current was 0
  - Only depends on source!
  - v(0-) = 0 or i(0-) = 0

Zero Input and Zero State

For an inductor

Zero Input: $i_{zi}(t) = i(0)e^{-\frac{t}{\tau}}$

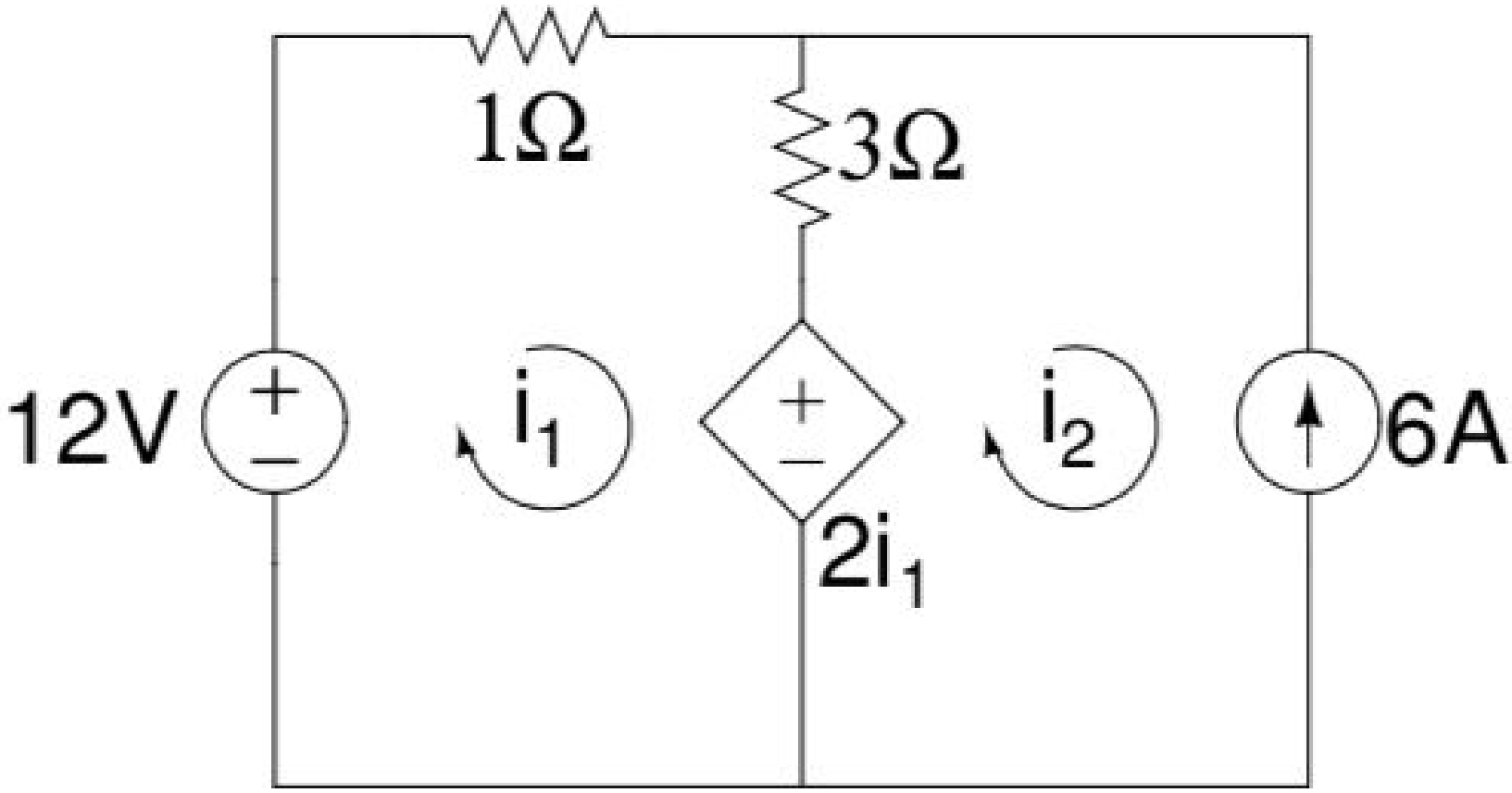
Zero State: $i_{zs}(t) = i_s - i_s e^{-\frac{t}{\tau}}$

For a capacitor

Zero Input: $V_{zi}(t) = V(0)e^{-\frac{t}{\tau}}$

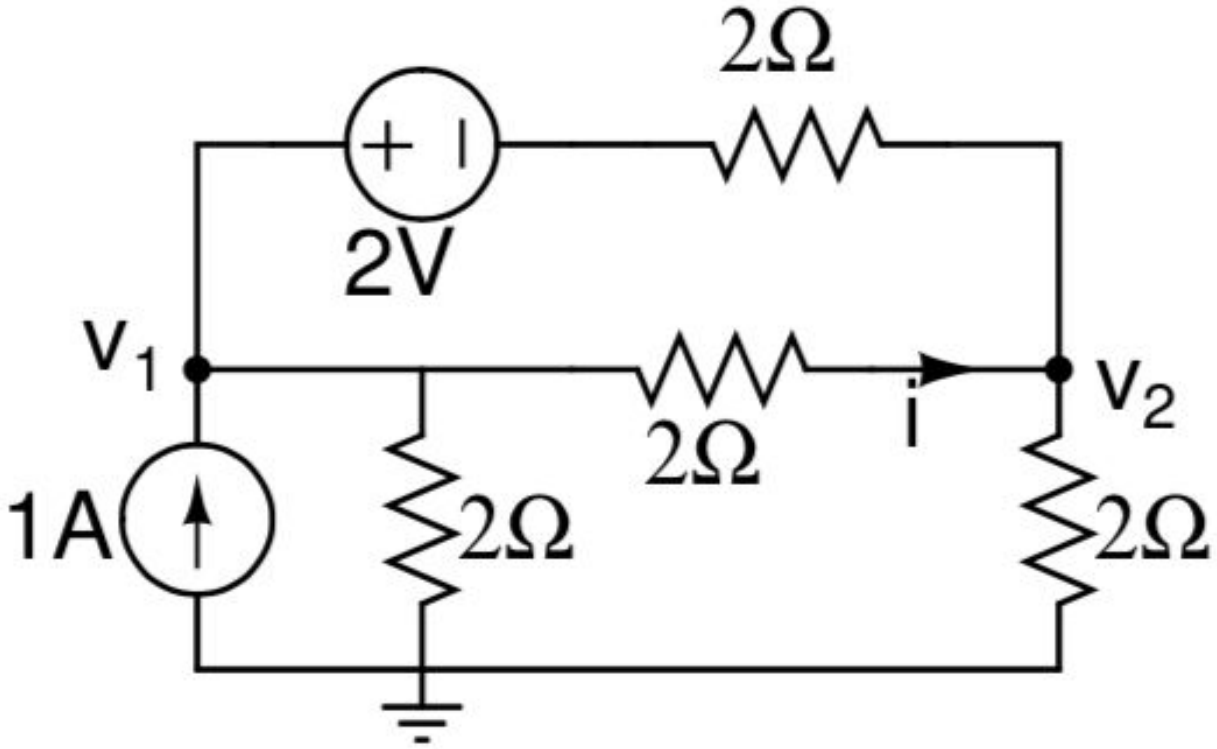
Zero State: $V_{zs}(t) = V_s - V_s e^{-\frac{t}{\tau}}$

Fall 2014 Exam 1,
Question 2, part (a):



Find i_1 and i_2

Fall 2014 Exam 1,
Question 2, part (b):

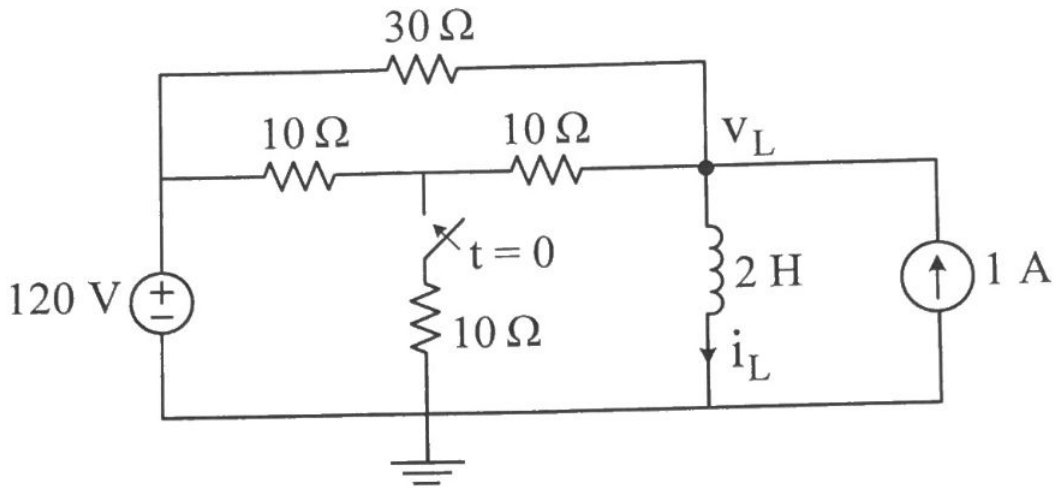


Find v_1, v_2, and i

Fall 2016

Problem 4 (25 points)

The circuit shown below is in DC steady-state before the switch flips at $t = 0$.



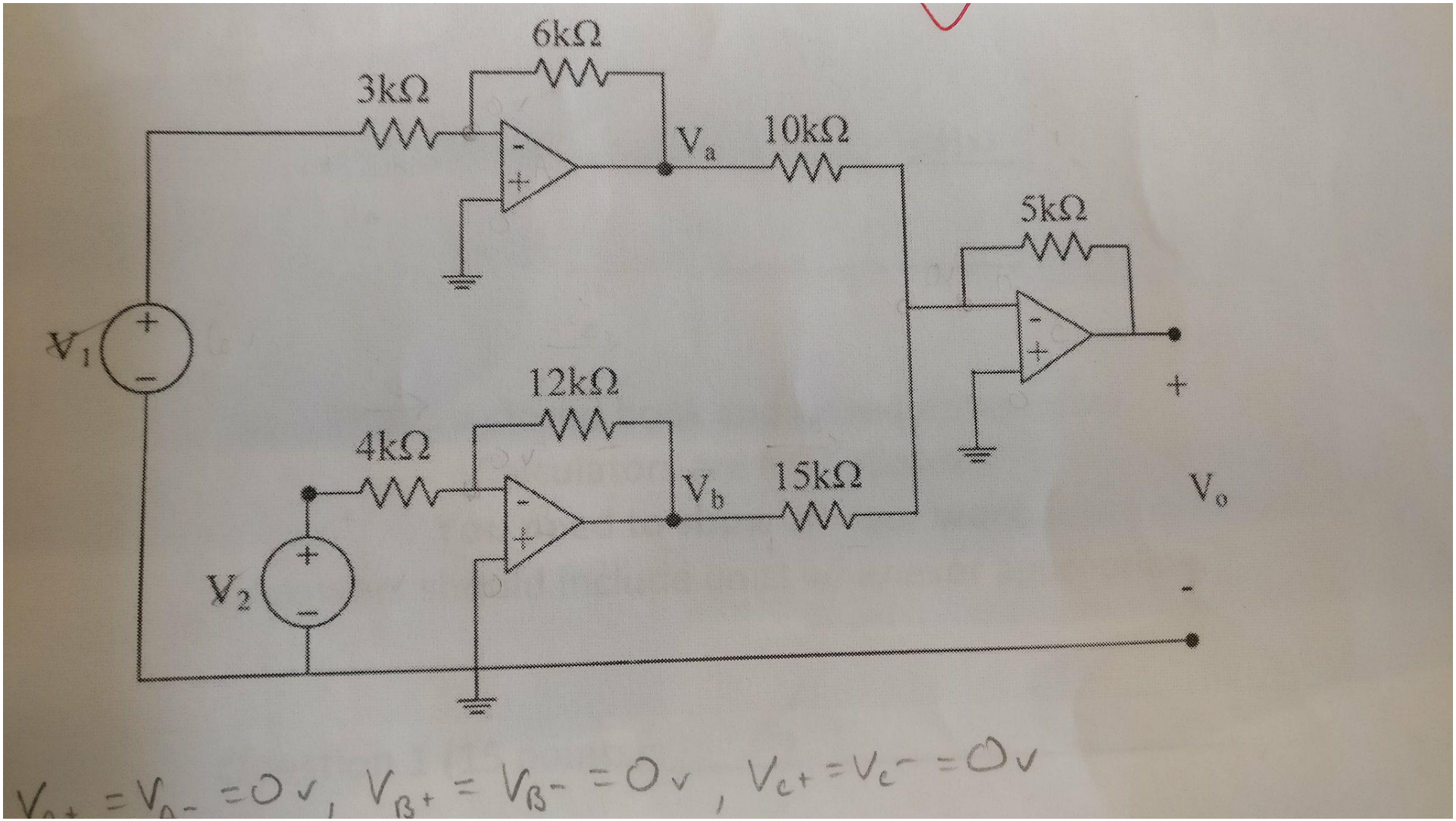
a) Find $i_L(0^-)$ and $i(t \to \infty)$. Also find $v_L(0^-)$ and $v_L(t \to \infty)$. Explain your work.

b) Find the time constant τ for $t > 0$ of this circuit.

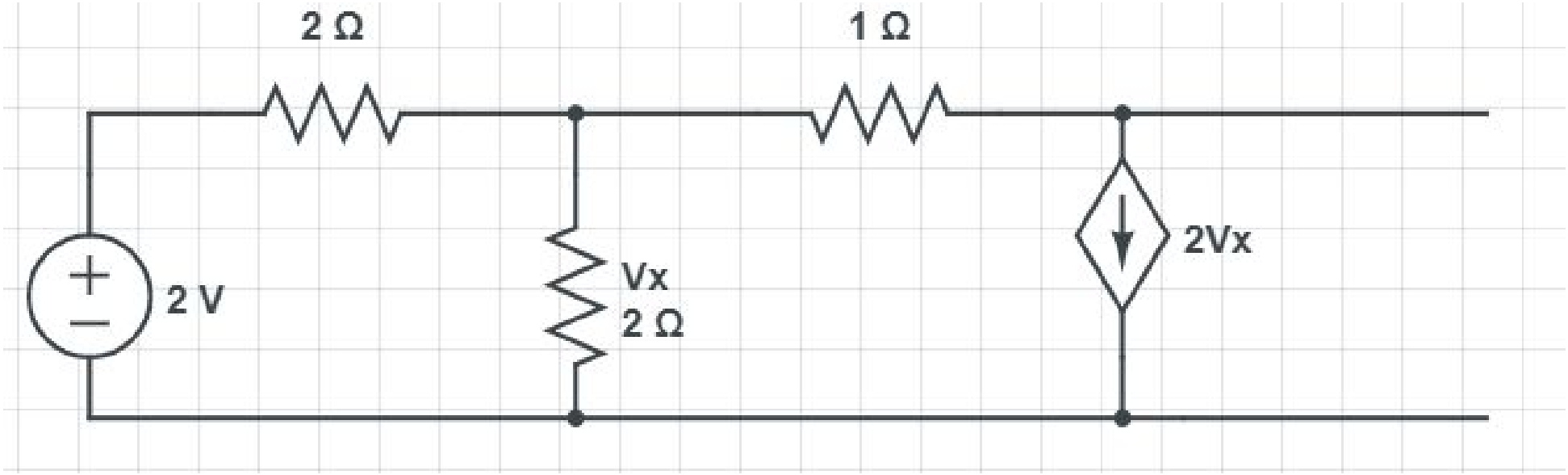
c) Find $i_L(t)$ and $v_L(t)$.

d) Sketch $i_L(t)$ and $v_L(t)$. Identify discontinuities (if any).

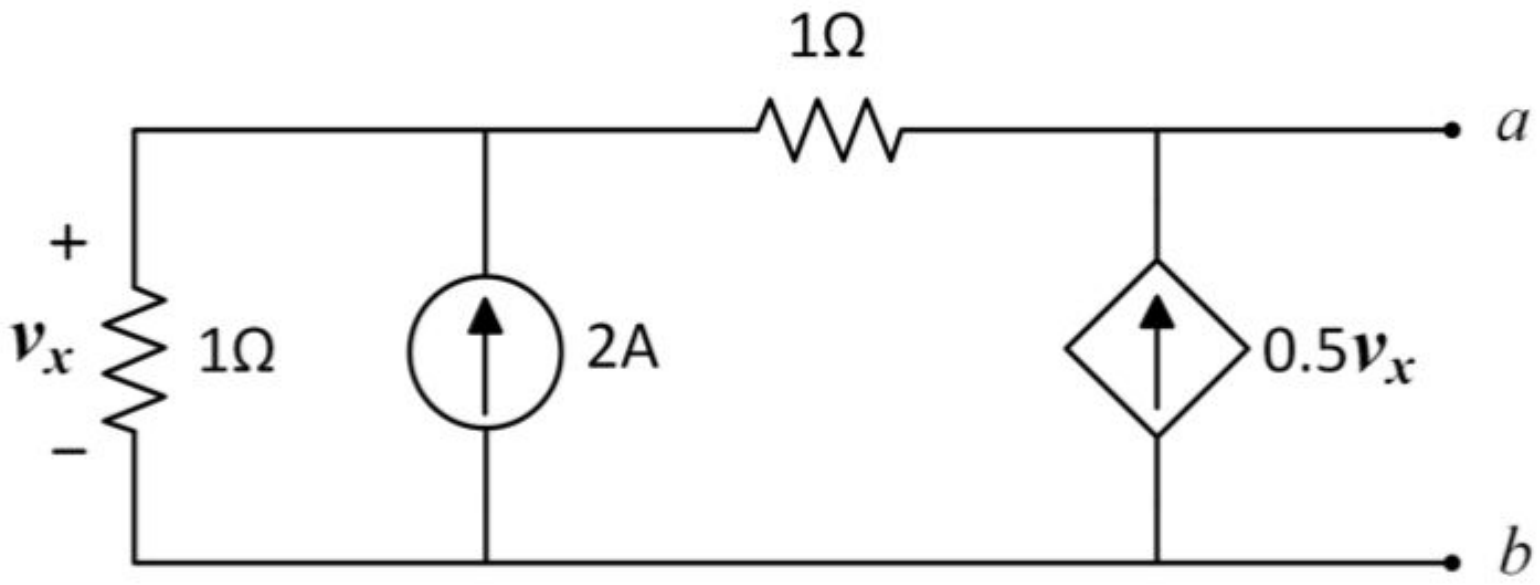
Q. For V1 = 6V and V2 = 2V, determine Va,Vb, and Vo. Assume circuit behaves linearly and make use of ideal op-amp approximations.



Summer 2017

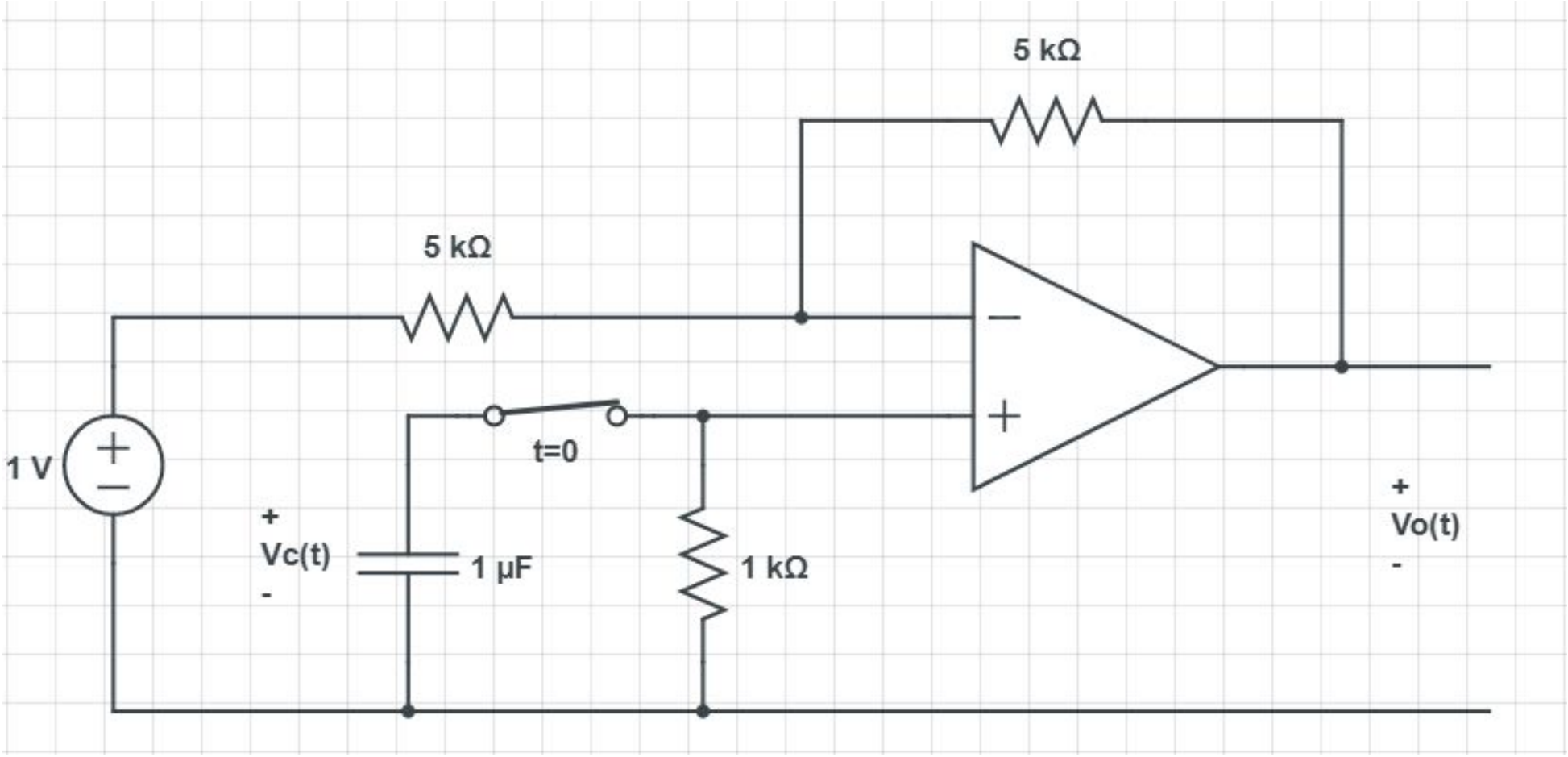


Fall 2014 Exam 1,
Question 3, part (a):



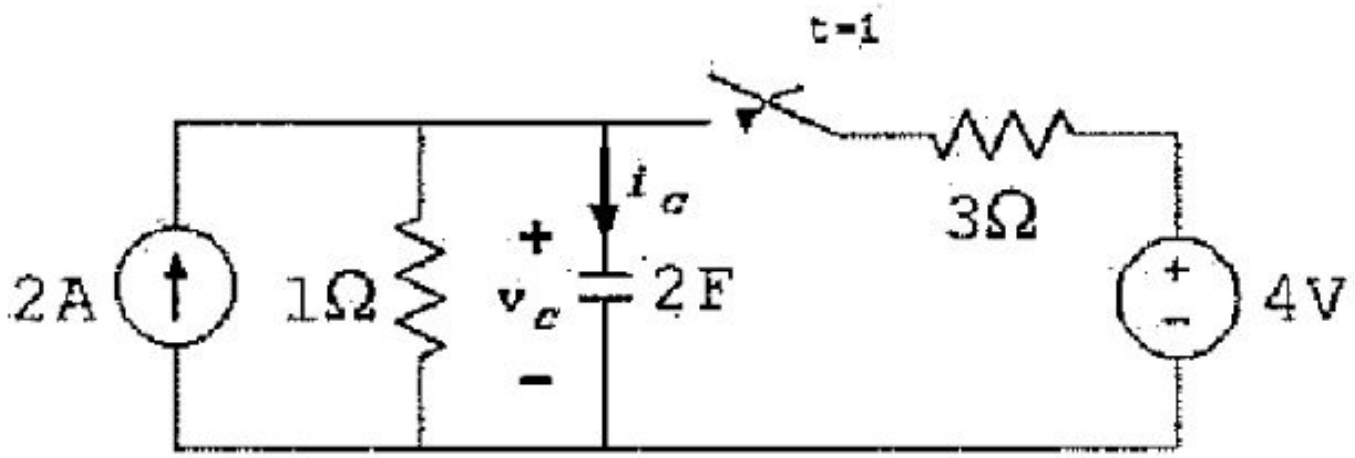
Find V_T, R_T, and available power

Homework Question



Spring 2017

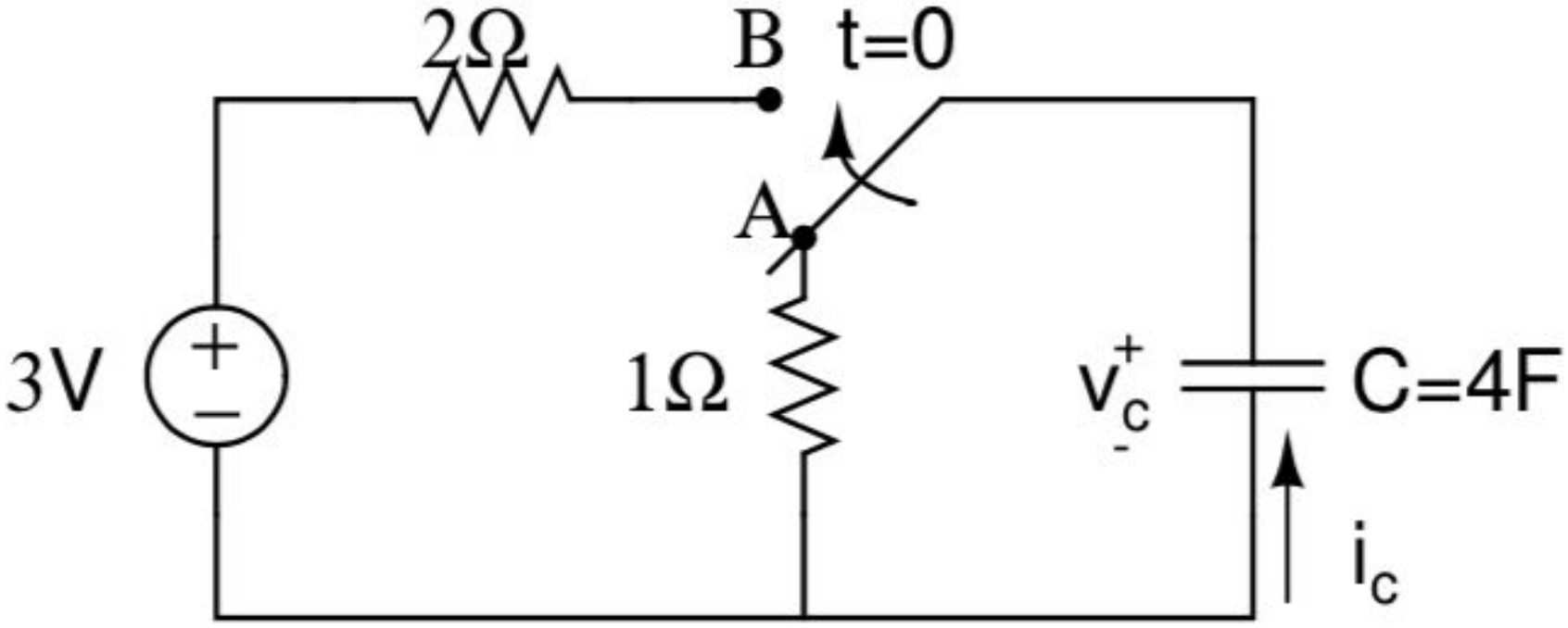
4. (25 pts) Consider the circuit shown below, in which the switch has been open for a long time, and it is closed at time $t = 1$. Obtain $v_c(0^-)$, $v_c(0^+)$, $v_c(1^-)$, $v_c(1^+)$, $i_c(0^-)$, $i_c(0^+)$, $i_c(1^-)$, $i_c(1^+)$, $v_c(\infty)$, and for $t > 2$, find the time constant τ, the zero-state voltage $v_{c,ZS}(t)$, the zero-input voltage $v_{c,ZI}(t)$, and the voltage $v_c(t)$.



For $t > 2$s

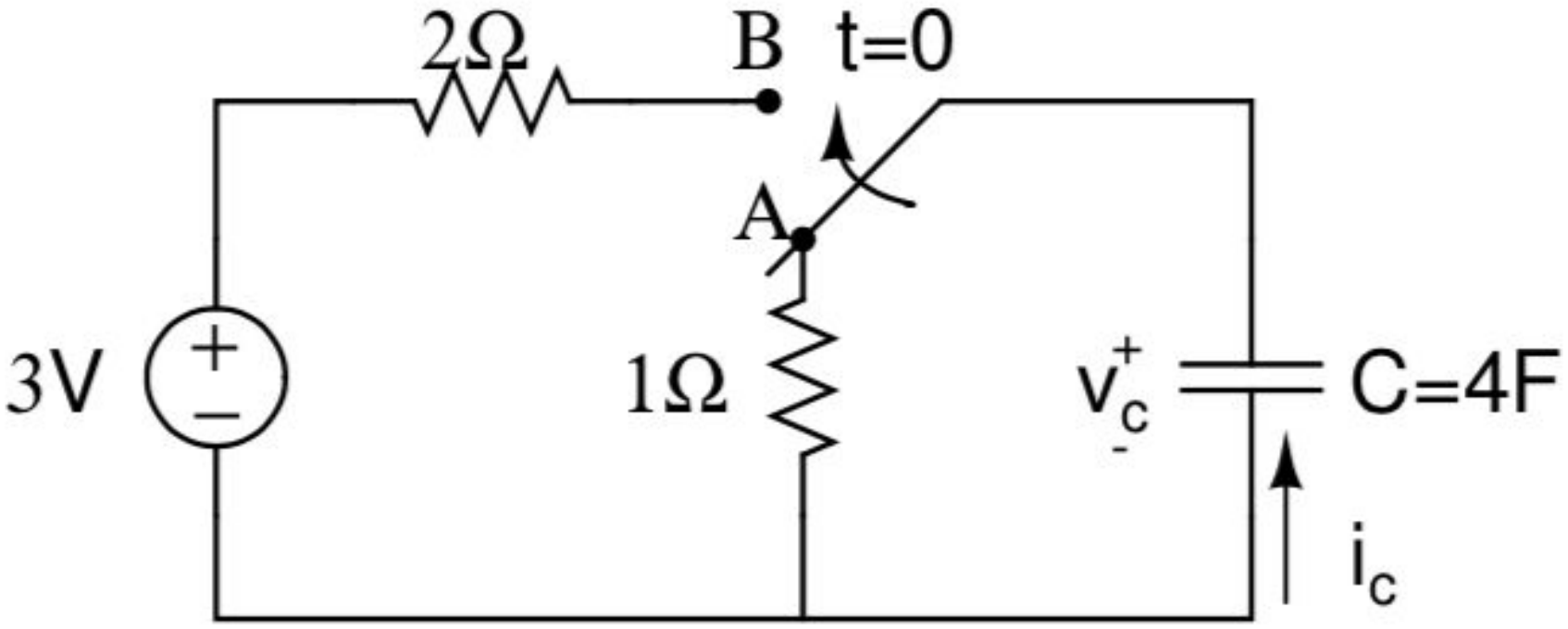
Fall 2014 Exam 1,
Question 4:

Switch in position A for a long time. T=0, move switch to position B.

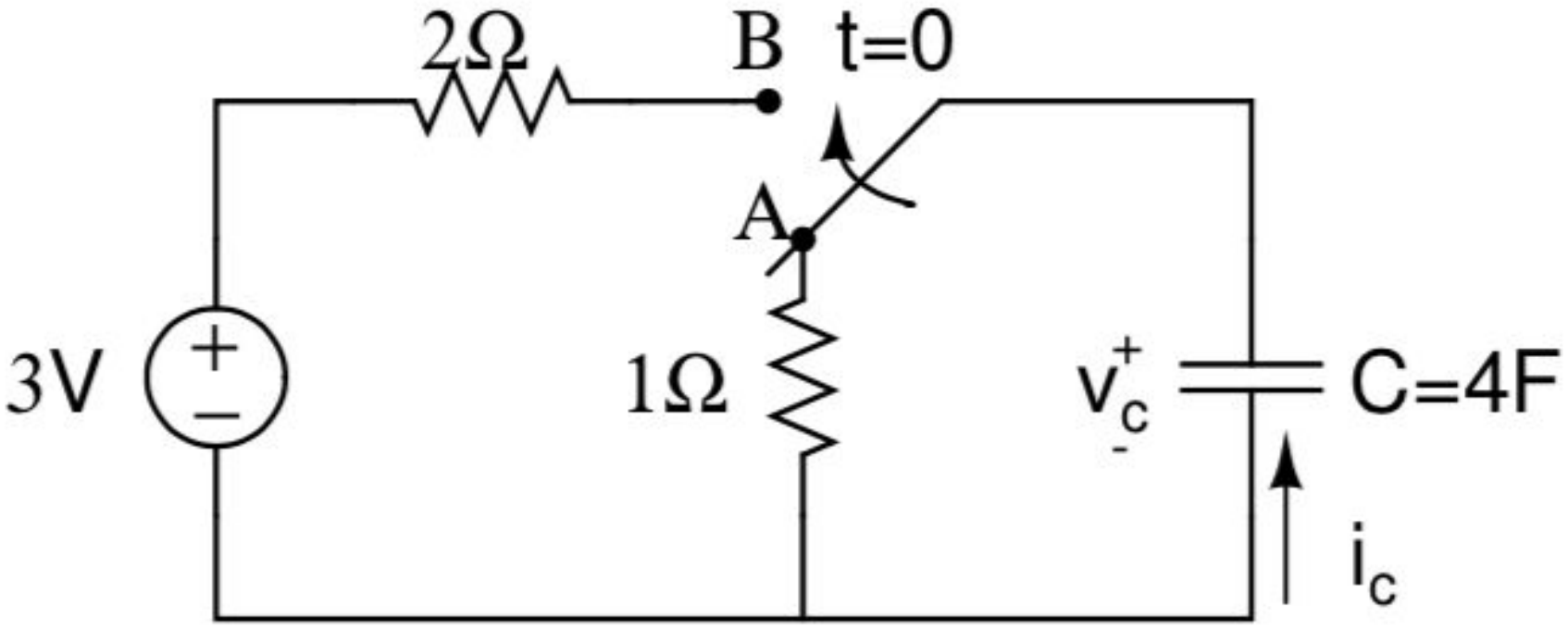


Switch in position A for a long time. T=0, move switch to position B.

Find $v_C(0^-)$ and $v_C(0^+)$



Switch in position A for a long time. T=0, move switch to position B.

For $t > 0$, $v_C(t)$ can be expressed as $v_C(t) = K_1 e^{-at} + K_2$.

What are K_1, K_2, and a?