(The Best)

HKN ECE210 FA17 Review

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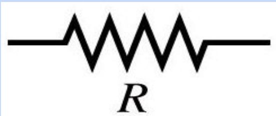
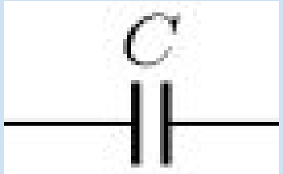
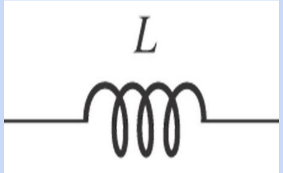
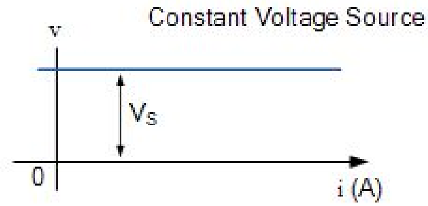
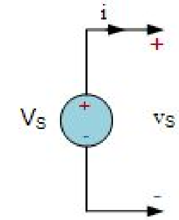
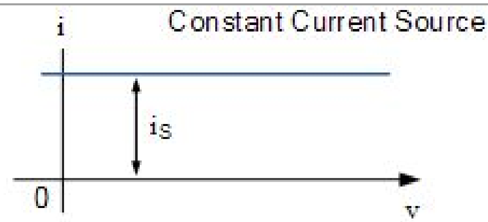
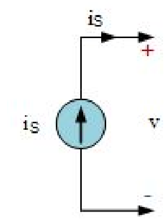
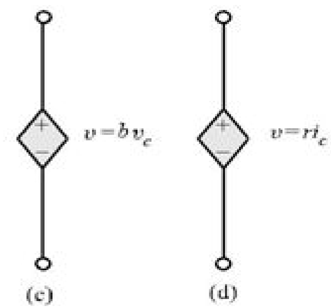
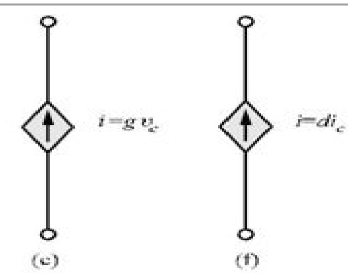
Today's Agenda

1) Touch on the concepts you will need to know for your exam
2) Go through the Fall 2016 Exam
3) Open up for questions and do extra example problems related to the needs of the students
4) RL and RC circuit problems!!!!!!!!!!!

The Basics: Electrical Loads

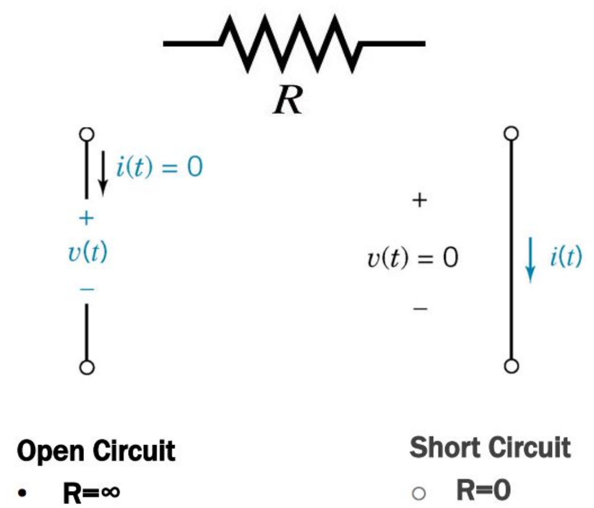
Circuit Element	Voltage-Current Law	Circuit Symbol
Resistor	$v = iR$	
Capacitor	$i = C\frac{dv}{dt}$	
Inductor	$v = L\frac{di}{dt}$	

The Basics: Electrical Sources

Circuit Element	Voltage-Current Law	Circuit Symbol
Independent Voltage Source		
Independent Current Source		
Dependent Voltage Source	c) Voltage controlled $V = kV_x$ d) Current Controlled $V = kI_x$	
Dependent Current Source	e) Voltage controlled $I = kV_x$ f) Current controlled $I = kI_x$	

Special Cases of Resistors

- We shall consider two special cases of the resistor:
  - A short-circuit/**short** is a resistor with **zero** resistance.
  - A open-circuit/**open** is a resistor with **infinite** resistance.
- As a consequence of ohm's law:
  - A short has zero voltage drop across it, independent of the current through the short.
  - A open carries zero current, independent of the voltage across its terminals
- Their special symbols are shown here:



Special Cases of Capacitors and Inductors

- Recall the I-V characteristic equation of capacitors and inductors:

$$i = C\frac{\mathrm{dv}}{\mathrm{dt}} \qquad v = L\frac{\mathrm{di}}{\mathrm{dt}}$$

- In non-time varying DC (Direct Current) circuits, the voltages and currents across all circuit elements are *constant*.
- Then the current through the capacitor is zero, independent of the voltage across it. Similarly, the voltage across the inductor is zero, independent of the current through it.
- Thus, <u>in non-time varying DC circuits, capacitors act as open-circuits and inductors act as short-circuits!</u>

Complex numbers

Rectangular Form: a + bj

- Good for adding and subtracting
- To convert: $|A| = \sqrt{a^2 + b^2}$
- Q1&Q4: $\omega = tan^{-1}(\frac{a}{b})$
- Q2: $\omega = \pi + tan^{-1}(\frac{a}{b})$
- Q3: $\omega = tan^{-1}(\frac{a}{b}) - \pi$

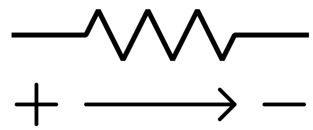
Polar Form: Ae^{jw}

- Good for multiplying and dividing
- Convert with Euler's identity $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- Other useful identities

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

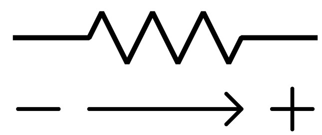
Sign Conventions

Standard Flow:



$V = I * R$ $\quad P = I * V$

Nonstandard Flow:



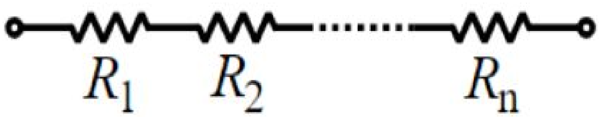
$V = -I * R$ $\quad P = -I * V$

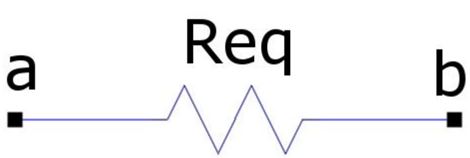
Absorbing power if: $P > 0$ Injecting power if: $P < 0$

Resistor Combinations

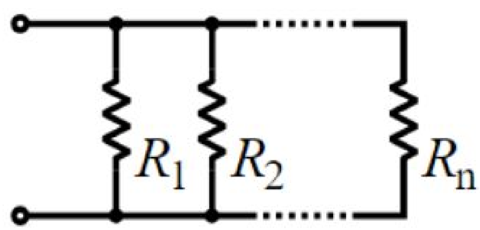
- When a set of resistors carry the same current through a single branch, they are in *series*.
- When a set of resistors support the same voltage between the same pair of nodes, they are in *parallel*.
- Simplify circuits by finding equivalent resistance

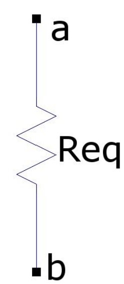
$$R_{eq} = R_1 + R_2 + \ \ldots + R_n$$



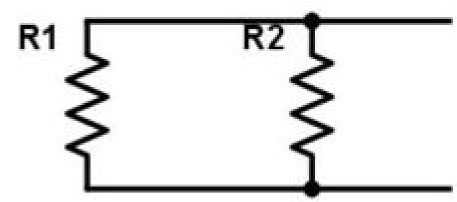


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$





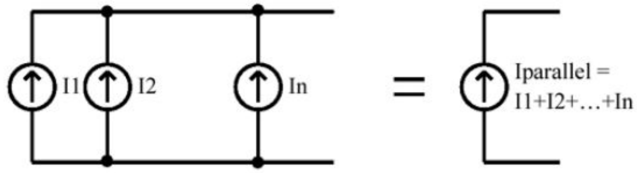
$$R_{eq} = \frac{R_1 * R_2}{R_1 + R_2}$$



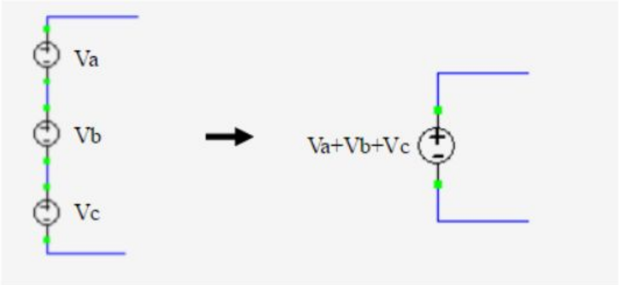
Source Combinations

- Voltage sources in series can be added to produce one equivalent voltage source.
- Current Sources in parallel can be added to produce one equivalent current source.

$$I_{out} = I_1 + I_2 + \cdots + I_n$$



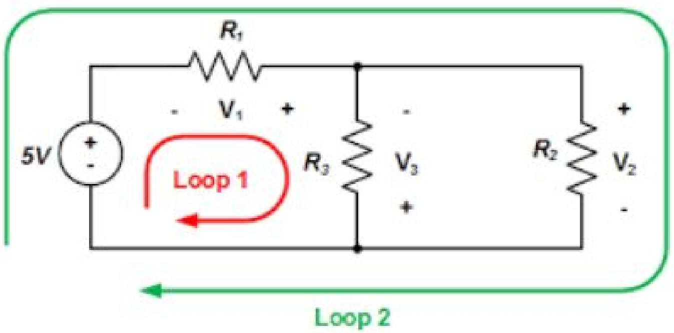
$$V_{out} = V_1 + V_2 + \cdots + V_n$$



Kirchhoff's Voltage Law

- Kirchhoff's Voltage Law (KVL): The sum of all voltage drops equals the sum of all voltage rises around a closed loop.

$$\sum V_{Rises} = \sum V_{Drops}$$



Loop 1: $5 + V_1 + V_3 = 0$

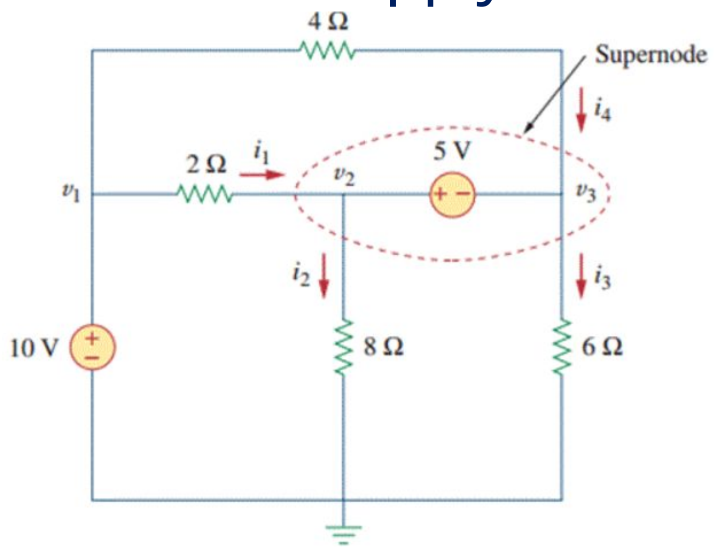
Loop 2: $5 + V_1 = V_2$

Kirchhoff's Current Law

- KCL: The sum of all currents entering a particular node is equal to the sum of all currents exiting that particular node.

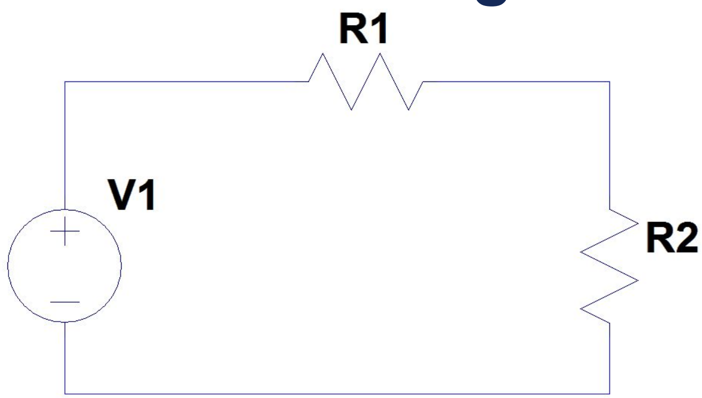
$$\sum_{Entering} I = \sum_{Exiting} I$$

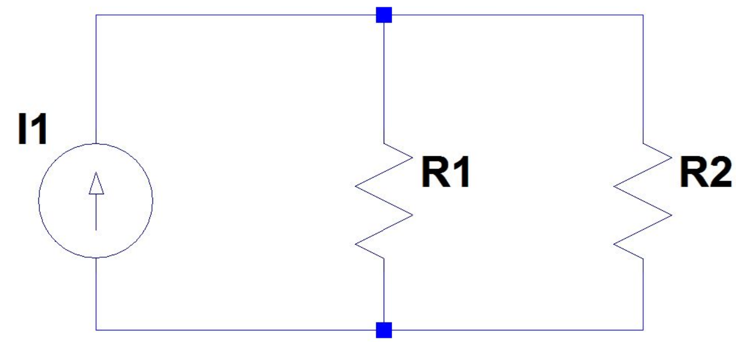
- We can also apply KCL at *super nodes:*



$$i_1 + i_4 = i_2 + i_3$$

Current & Voltage Divider





$$V_{R1} = V_1 * \frac{R1}{R1+R2}$$

$$I_{R1} = I_s * \frac{R2}{R1+R2}$$

In general...

$$V_{Rn} = V_s * \frac{Rn}{\sum R}$$

In general...

$$I_{Rn} = I_s * \frac{R_{eff}}{R_n+R_{eff}}$$

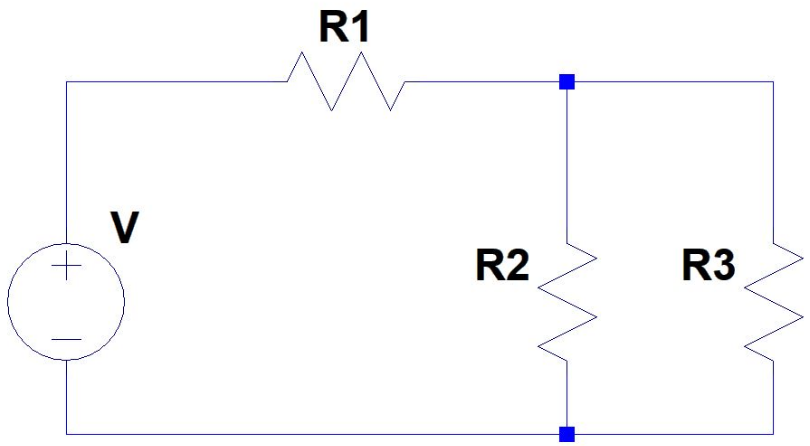
Where R_{eff} is the effective resistance of all the resistors except the one being inspected

Node-Voltage Method

Using the fact that voltage is the same in a node

1. Assign relative ground node
2. Assign variable names to unknown voltage points
   Ex) $V_1, V_2, V_3 \ldots$
3. Set up KCL : $I_{IN} = I_{OUT}$ at unknown voltage nodes
4. Solve systems of equations

$$I_n = \frac{V_{START} - V_{END}}{R_n}$$



Loop-Current Method/ Mesh Current Method

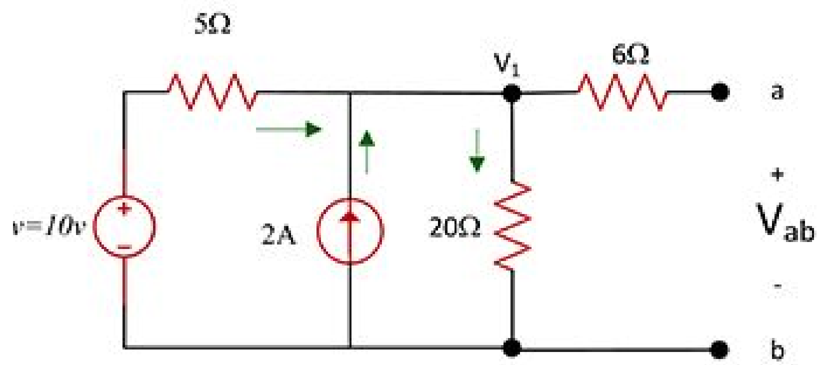
- What is it? Simultaneously using KVL and Ohm's law to solve for unknown currents and voltages.

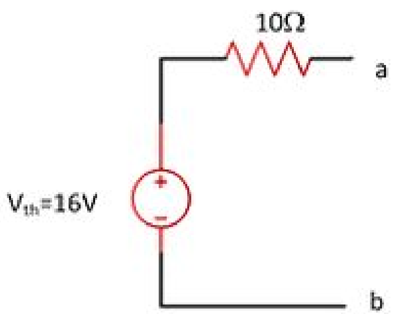
Principle of Superposition

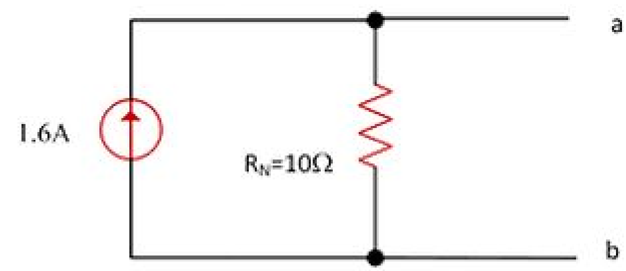
- How it works:
  - For a circuit with N independent voltage and current sources, redraw the circuit N times.
  - With each new circuit, choose a new source to be active while suppressing all other sources(a suppressed voltage source is a short and a suppressed current source is an open)
  - In each of the N circuits, calculate the voltage or current of interest
  - The current or voltage we were initially interested in is just the algebraic sum of each of the calculated current or voltage responses due to each individual source.

Thevenin and Norton Equivalents

- Thevenin Equivalent: Voltage source and resistor in series
- Norton Equivalent: Current source and resistor in parallel
- Every resistive network can be expressed as either its Thevenin *and/or* its Norton equivalents.







Equivalent circuit with <u>only independent sources</u>

1) **Thevenin Voltage**: Leave the output terminal open
2) **Norton Current**: Connect a short between the output terminals.
3) **Thevenin/Norton Resistance**: Suppress all sources. Calculate the equivalent resistance "looking in" from the output terminals.

$$V_{oc} = V_T \qquad I_{sc} = I_N \qquad R_N = R_T$$

Note: You only need to do two of the three steps above. One you know any of the two, the third can be calculated by rearranging the equation:

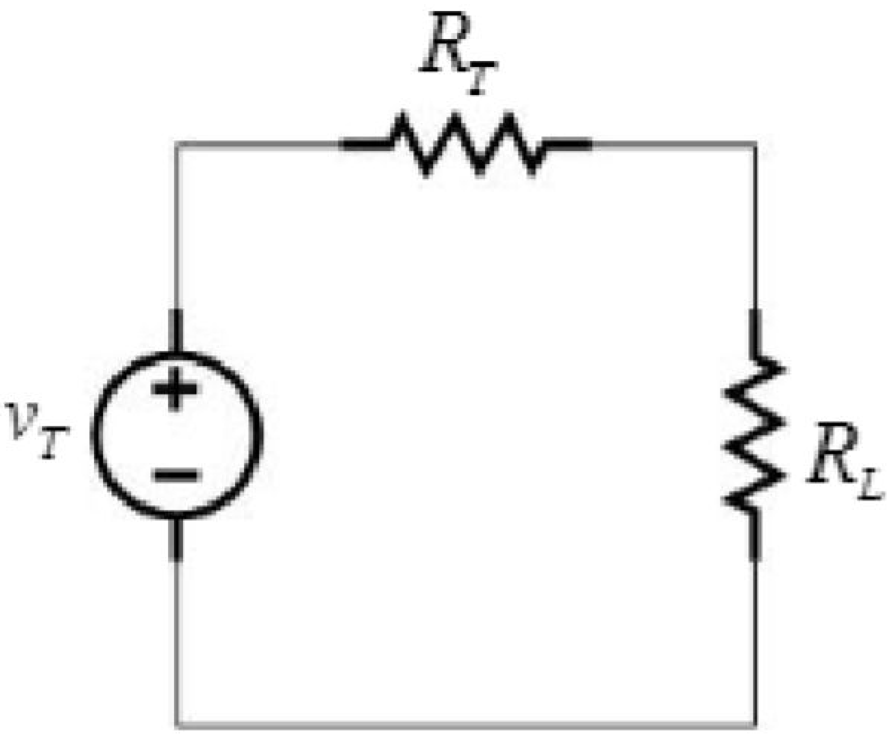
So pick the easier two of the three steps!

$$V_T = I_N R_T$$

Equivalent Circuits with Dependent Sources.

- The steps for calculating the thevenin voltage and norton current remain the same.
- However, source suppression no longer works for calculating the Thevenin/Norton Resistance.
- Instead, we must use the *test signal method.*
  - Connect a 1A independent current source to the output terminals of the circuit.
  - Suppress all independent sources.
  - Apply circuit analysis techniques to determine the voltage across this current source.
  - That voltage has the same value as the Thevenin/Norton resistance, just in ohms instead of volts, of course.
- The relation $V_T = I_N R_T$ still holds, so the test signal method is unnecessary if you already know the Thevenin voltage and Norton current. Again, you should choose to do the two steps that are easiest for the particular circuit you are given.

Available Power (Maximum Power)

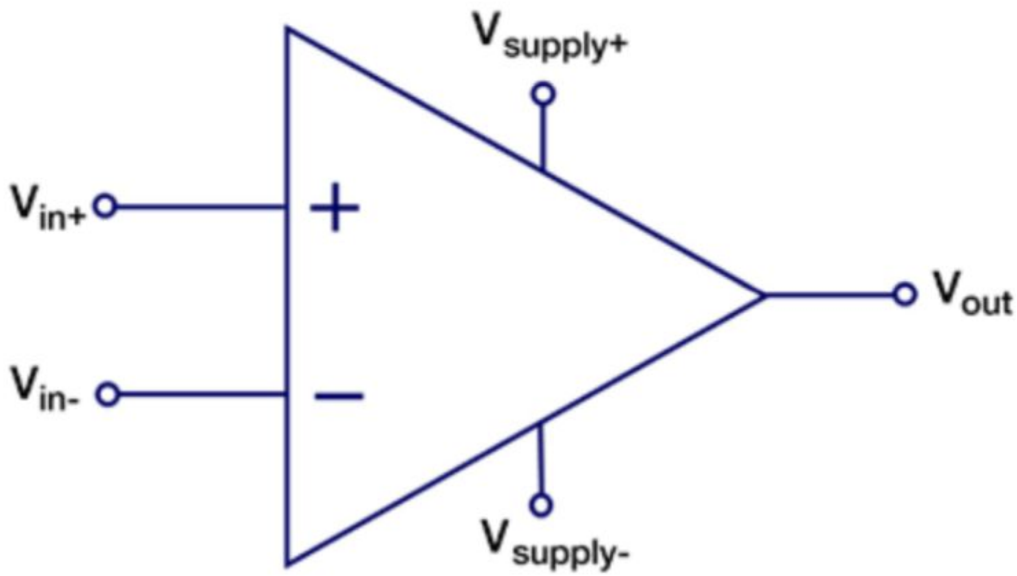


$$\boxed{\text{Set } R_L = R_T}$$

$$P_a = V_T^2/4R_T$$

Ideal Op-amp Model

$$v_- = v_+$$
$$i_+ = 0$$
$$i_- = 0$$



RL, RC, and RLC circuits

- So far, we have solved linear resistive networks. The math involved setting up a system of equations.
- RL and RC circuits require setting up and solving a first-order ODE
- RLC circuits require setting up and solving a second-order ODE. You will never need to actually solve it, but you simply need to know that the equation describing a circuit with both a capacitor and an inductor is a second order ODE.

First Order Differential Equations

Given equation, where α and β are constants

$$\frac{dy}{dt} + \alpha y(t) = \beta$$

Then, $y(t) = A + Be^{-\alpha t}$, where A and B are also constants

Get A by finding the limit as $t \rightarrow \infty$, and B from initial conditions

The limit $t \rightarrow \infty$ is the steady state

Time constant: $\tau = \dfrac{1}{\alpha}$

1. Homogeneous solution – the exponential term
2. Particular solution – the constant

Zero Input and Zero State

- Zero input is what would the voltage or current across an inductor or capacitor be if it was connected only to a resistor
  - Only depends on initial condition!

- Zero state is what would the voltage or current across an inductor or capacitor be if it's initial charge/current was 0
  - Only depends on source!

For an inductor

Zero Input: $i_{zi}(t) = i(0)e^{-\frac{t}{\tau}}$

Zero State: $i_{zs}(t) = i_s - i_s e^{-\frac{t}{\tau}}$

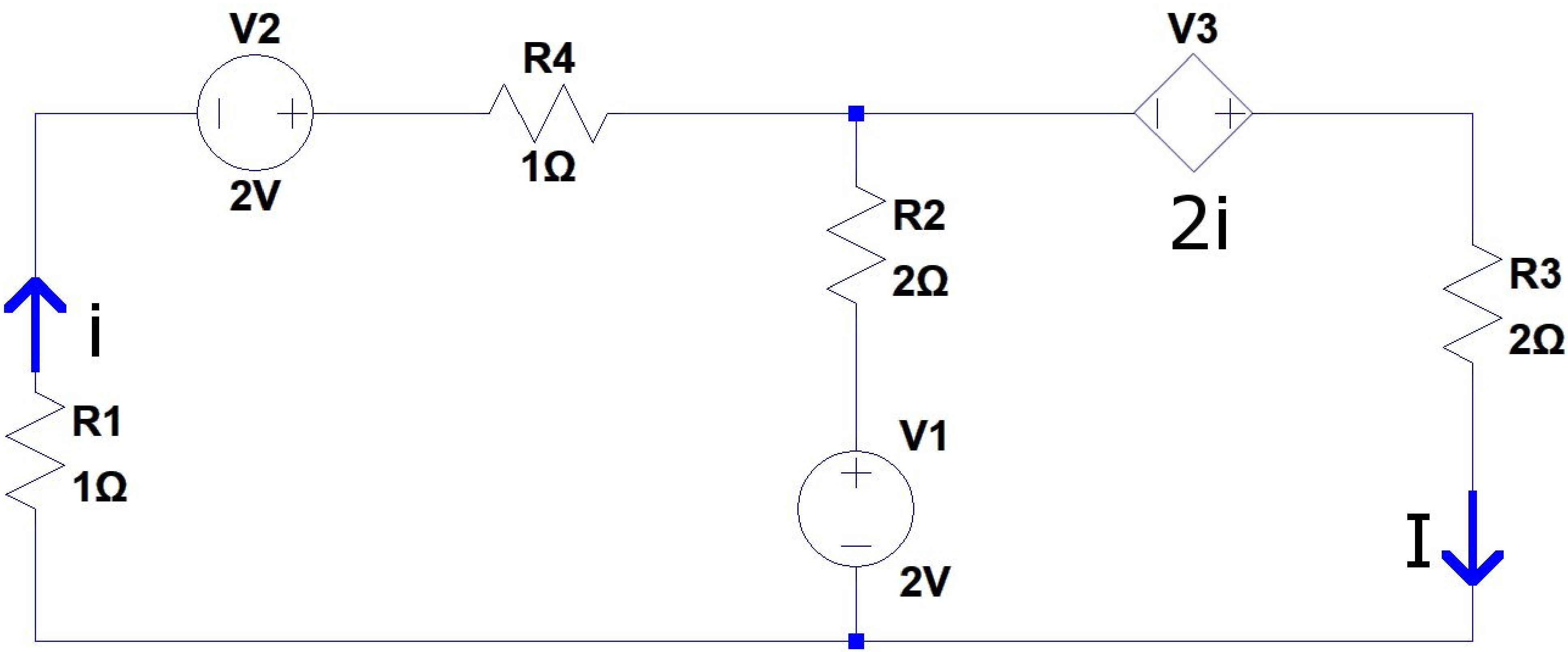
For a capacitor

Zero Input: $V_{zi}(t) = V(0)e^{-\frac{t}{\tau}}$

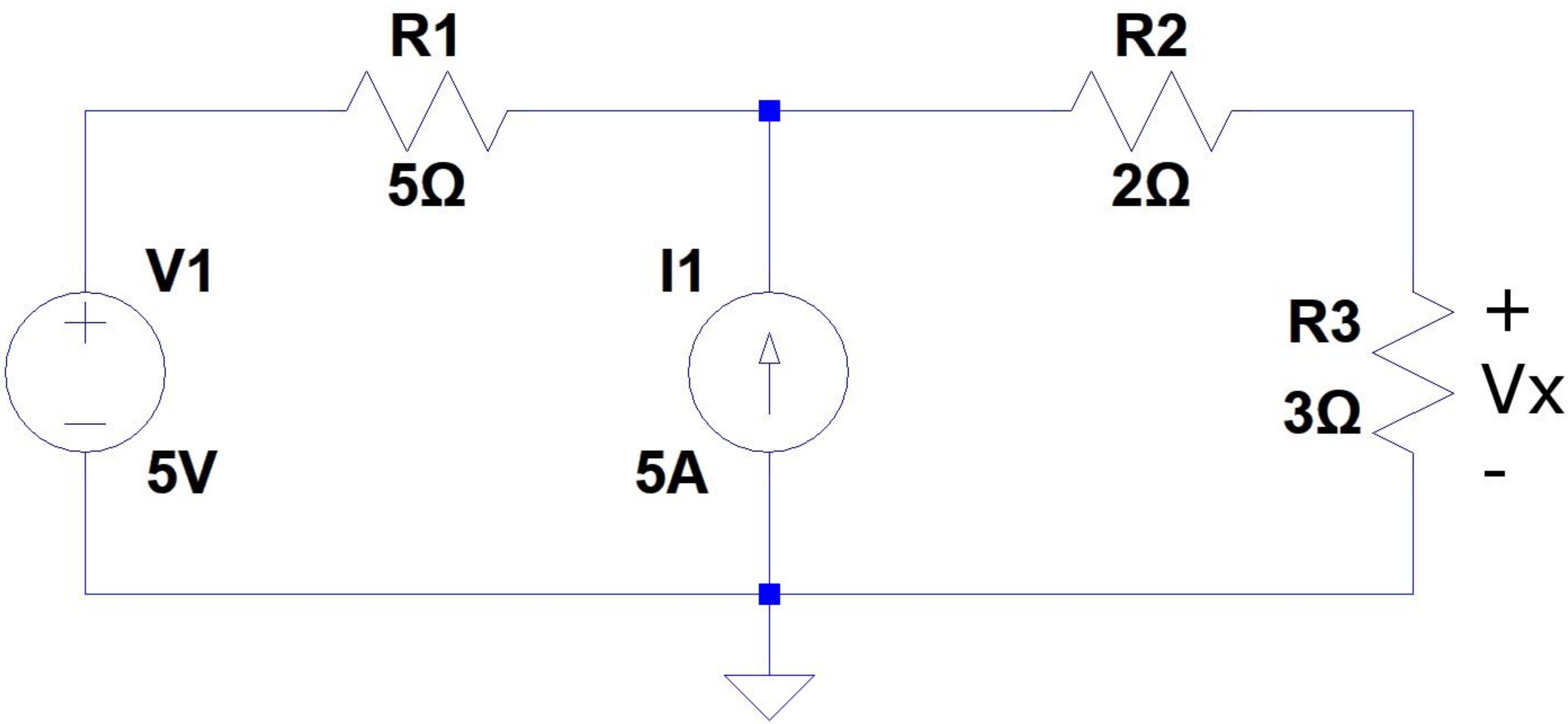
Zero State: $V_{zs}(t) = V_s - V_s e^{-\frac{t}{\tau}}$

Fall 2016 Exam 1

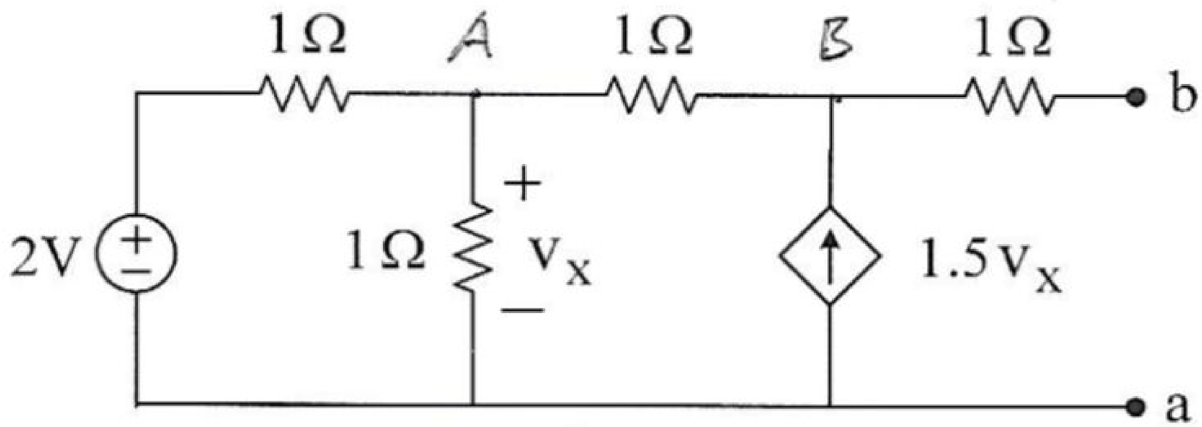
1 b) Find the value of I



1c) Find Vx and the power absourbed/injected by the 5V source



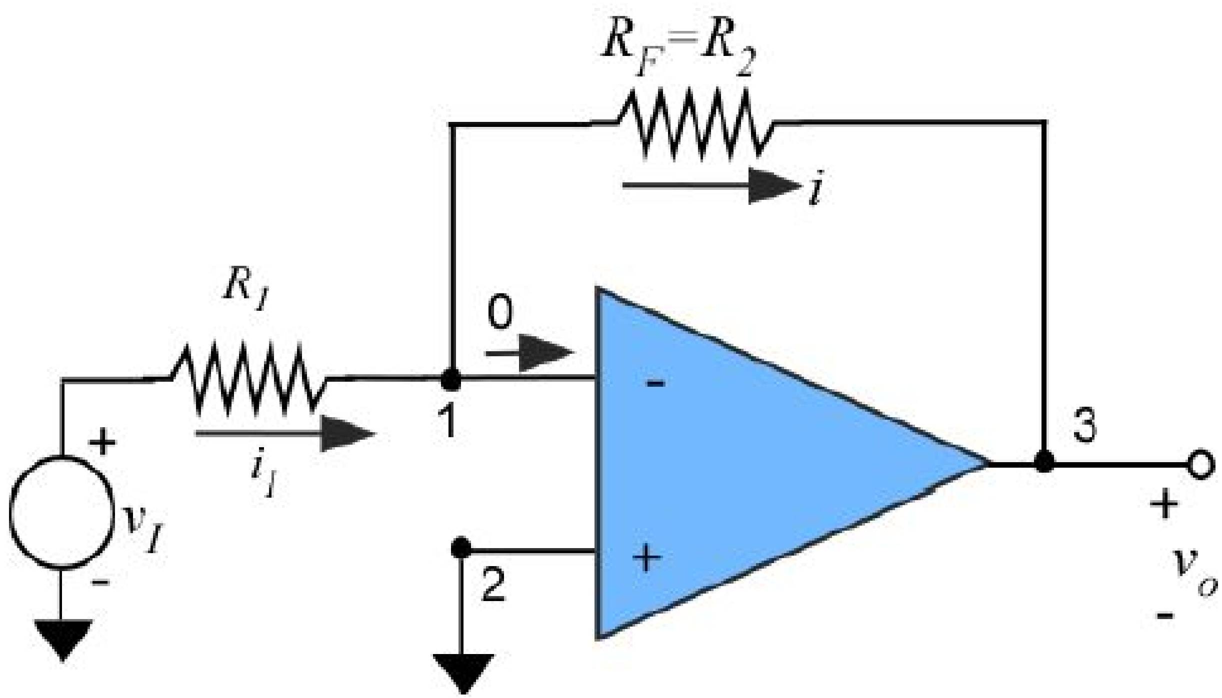
Thevenin Norton Problem 2



- Find Thevenin and Norton equivalents

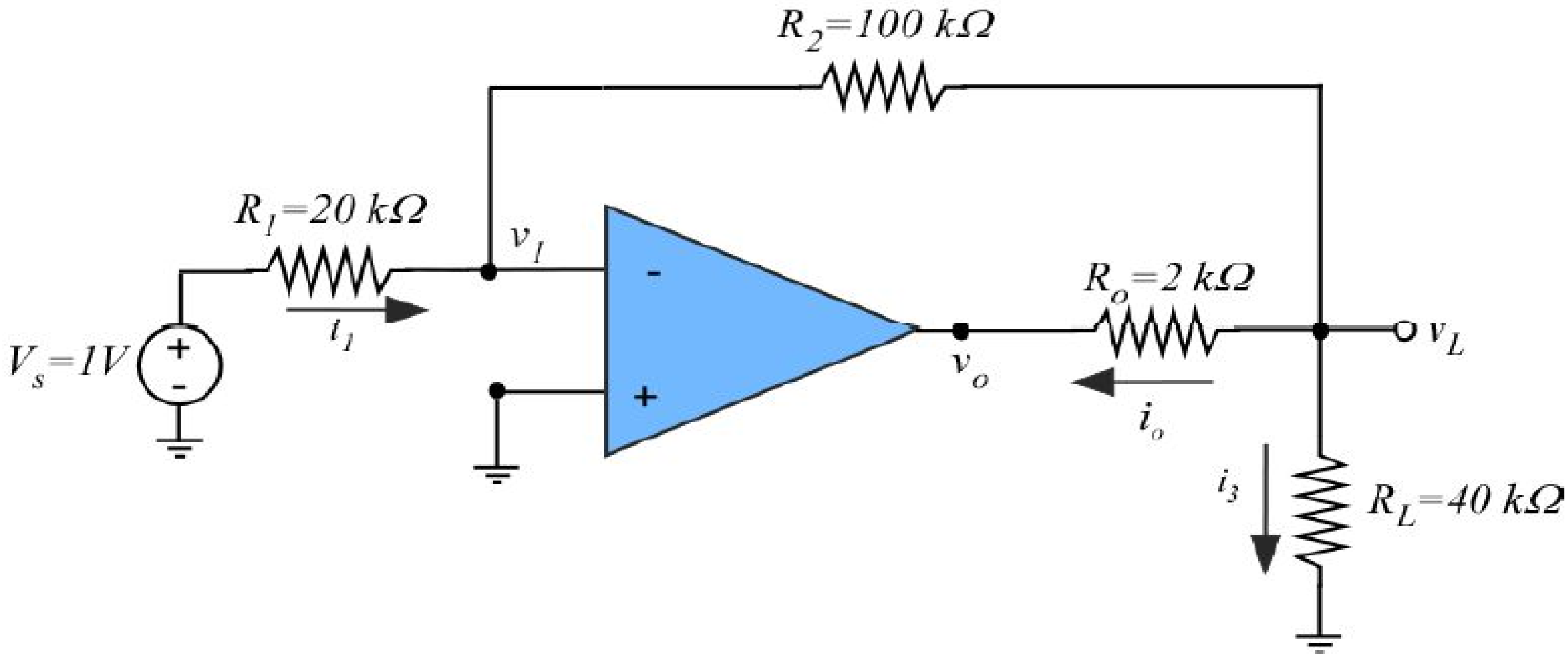
Fall 16 Problem 3 Part (a)

a) For the circuit shown, the voltage gain is -5 V/V and the total value of resistance used is 120 kΩ. Determine R_1 and R_2.



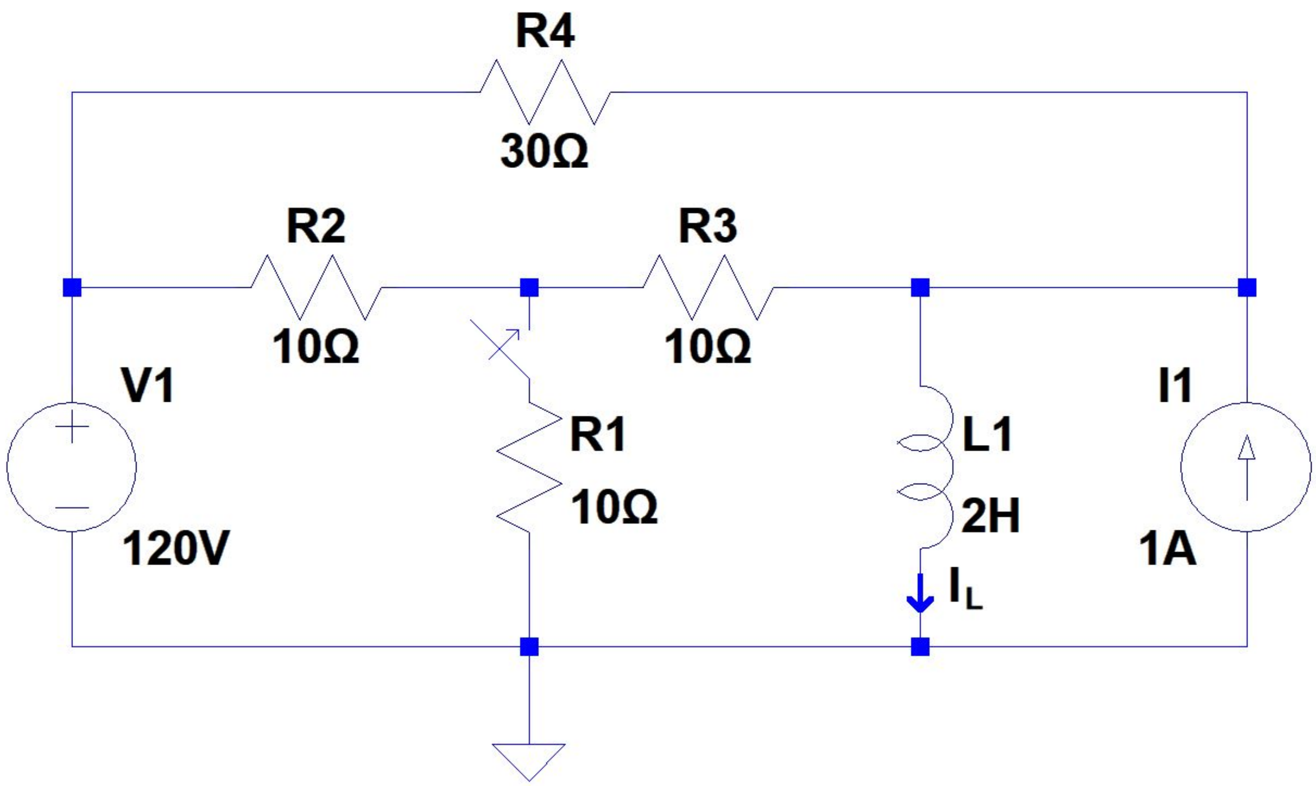
Fall 16 Problem 3 Part (b)

b) For the circuit shown below, assume that the op amp is ideal. Find the voltage at the load v_L.



LC circuit Fall 2016

The circuit is in DC steady state before the switch flips at t = 0.



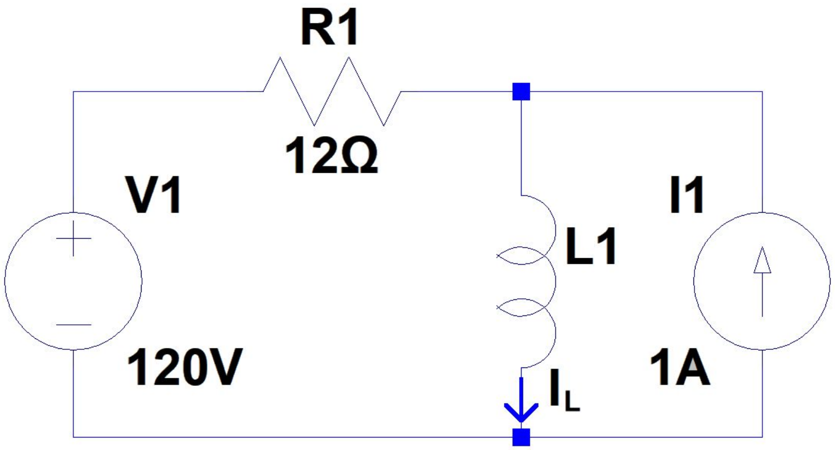
$$i_L(0^-) =$$

$$i_L(t \to \infty) =$$

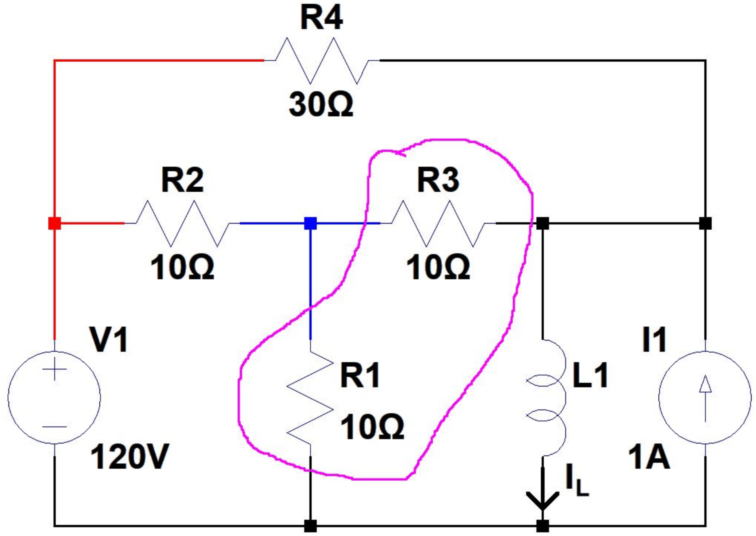
$$V_L(0^-) =$$

$$V_L(t \to \infty) =$$

LC circuit Fall 2016



$$i_L(0^-) = 11\text{A}$$



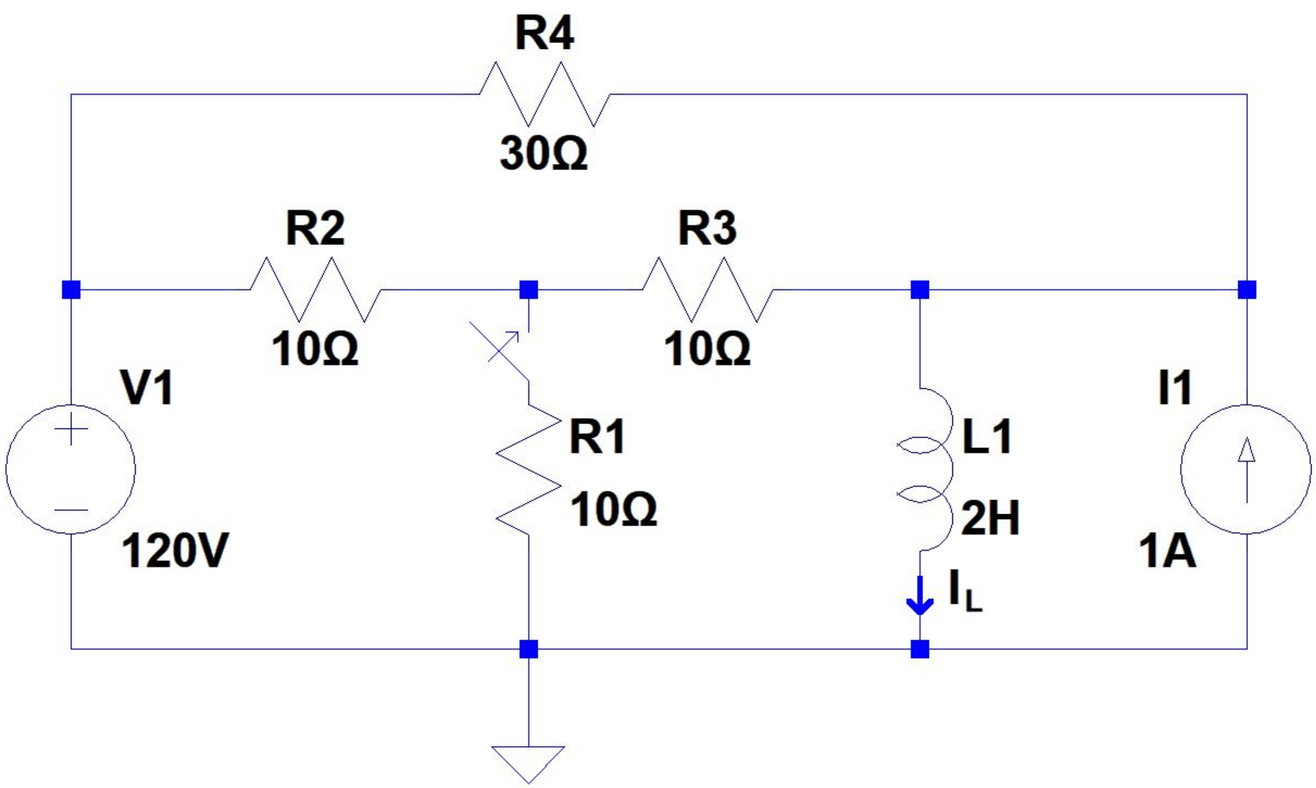
$$i_L(t \to \infty) = 9\text{A}$$

$$V_L(0^-) = 0$$

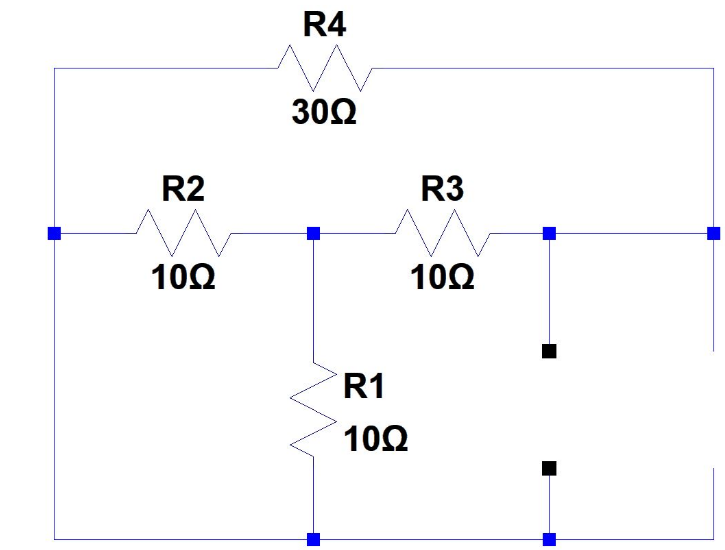
$$V_L(t \to \infty) = 0$$

LC circuit Fall 2016 part 2

Find the time constant, as well as current and voltage across the inductor as a function of time for t>0



LC circuit Fall 2016 part 2



$$i_L(t) = A + Be^{-\frac{t}{\tau}}$$

$$\text{A} = \text{final current (9A)}$$

$$\text{B} = \text{change in current (2A)}$$

$$i_L(t) = 9 + 2e^{-5t}$$

$$\tau = \frac{L}{R_T}$$

$$R_T = 10$$

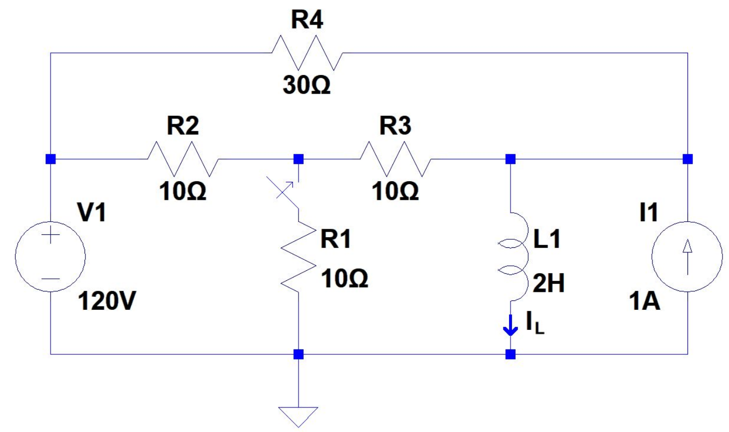
$$\tau = \frac{1}{5}$$

$$V_L = L * \frac{di}{dt}$$

$$V_L(t) = 2 * \frac{d}{dt}(9 + 2e^{-5t})$$

$$V_L(t) = 2 * -10e^{-5t}$$

$$V_L(t) = -20e^{-5t}$$



EXTRA PRACTICE

Practice Problems

Simplify and express your answer in the form you prefer:

1. $\frac{3+4j}{5e^{2j}}$ 2. $\frac{2+j}{3-j}$

Simplify and find phase and magnitude

3. $j(1+j)e^{j\frac{\pi}{4}}$ 4. $(\frac{2+2j}{3-3j})^2$

Solutions

1. Polar: $e^{j(\tan^{-1}(\frac{4}{3})-2)}$

   Rectangular: $\cos(\tan^{-1}(\frac{4}{3})-2)+j*\sin((\tan^{-1}(\frac{4}{3})-2)$

2. $\frac{2+j}{3-j}$

   $\frac{(2+j)(3+j)}{(3-j)(3+j)}$

   $\frac{6+2j+3j+j^2}{9+3j-3j-j^2}$

   $\frac{6+6j-1}{9+1}$

   $\frac{5+6j}{10}$

   $\frac{1}{2}+\frac{3}{5}j$

3. $j(1+j)e^{j\frac{\pi}{4}}$

   $(-1+j)e^{j\frac{\pi}{4}}$

   $\sqrt{2}e^{j\frac{3\pi}{4}}*e^{j\frac{\pi}{4}}$

   $\sqrt{2}e^{j\pi}$

4. $(\frac{2+2j}{3-3j})^2$

   $(\frac{2\sqrt{2}e^{j\frac{\pi}{4}}}{3\sqrt{2}e^{-j\frac{\pi}{4}}})^2$

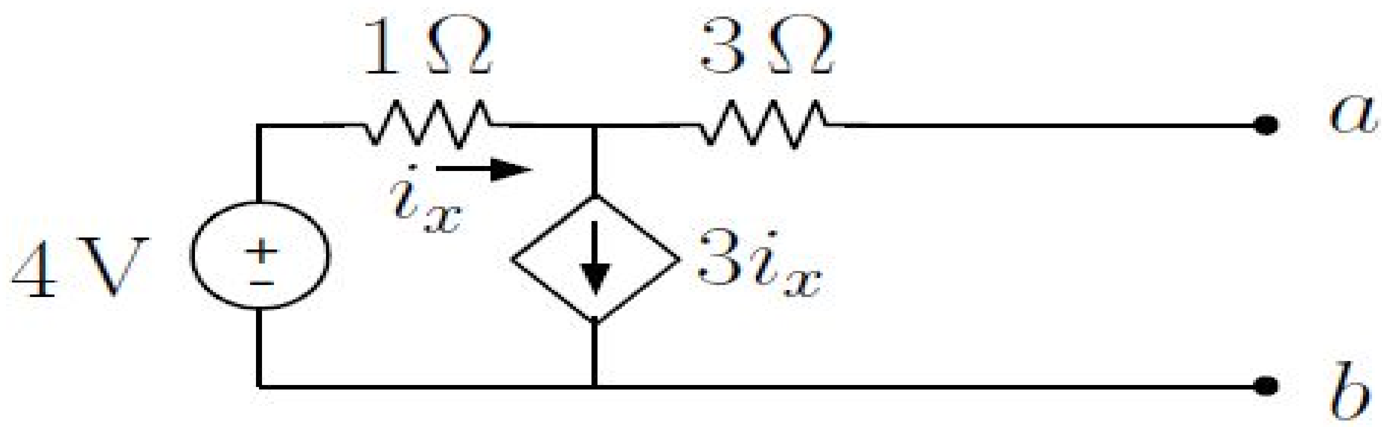
   $(\frac{2}{3}e^{(j\frac{\pi}{4}-(-j\frac{\pi}{4}))})^2$

   $(\frac{2}{3}e^{(j\frac{\pi}{4}-(-j\frac{\pi}{4}))})^2$

   $(\frac{2}{3}e^{j\frac{\pi}{2}})^2$

   $\frac{4}{9}e^{j\pi}$

Node Voltage Method Practice Problem

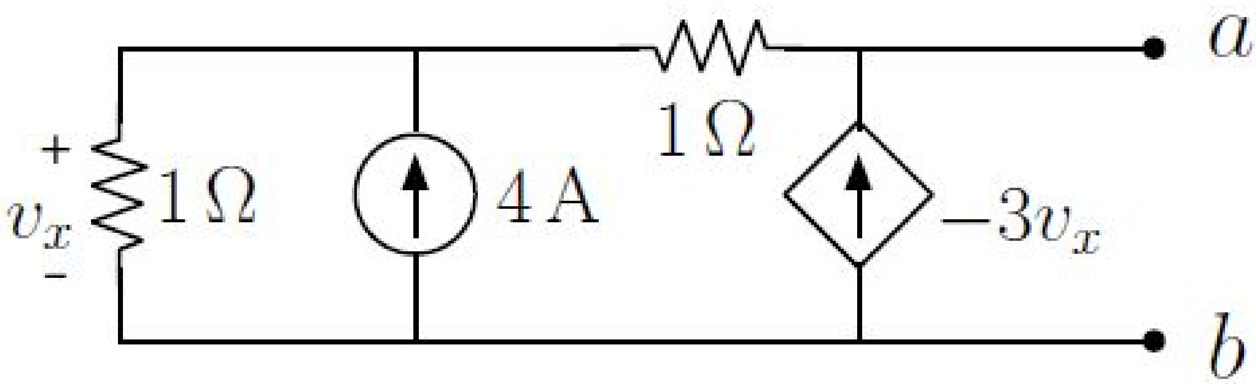


Find I_X and Open Circuit Voltage V_{AB}

1. Use node-voltage method
2. Set up systems of equations
3. Solve for ix and Vab

$$i_x = 0, V_{ab} = 4V$$

Node Voltage Method Practice Problem

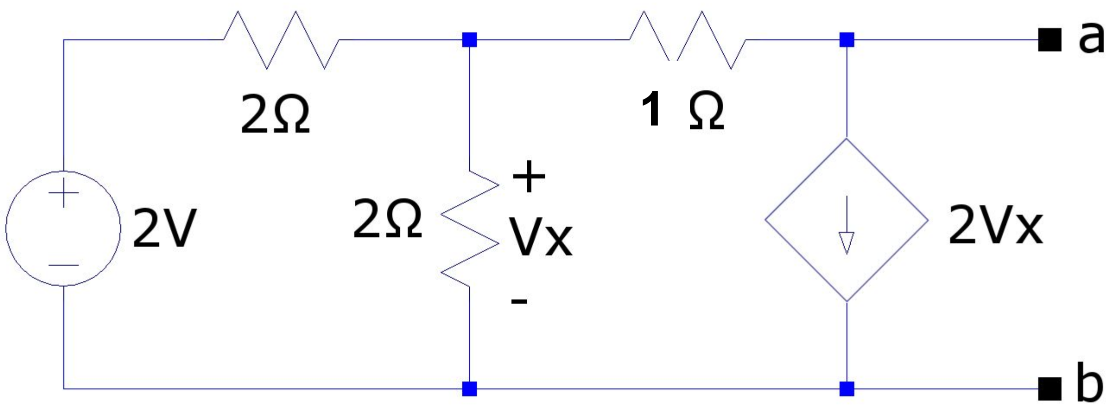


Find V_X and Thevinin Voltage V_T

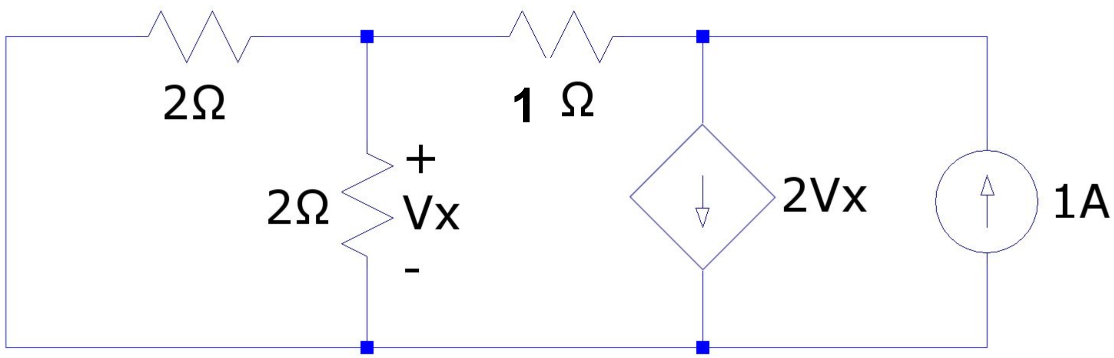
1. Use node voltage method
2. Set systems of equations
3. Solve for Vx and Vab or Vx and In

$$V_x = 1V, V_T = -2V$$

Test Signal Practice Problem

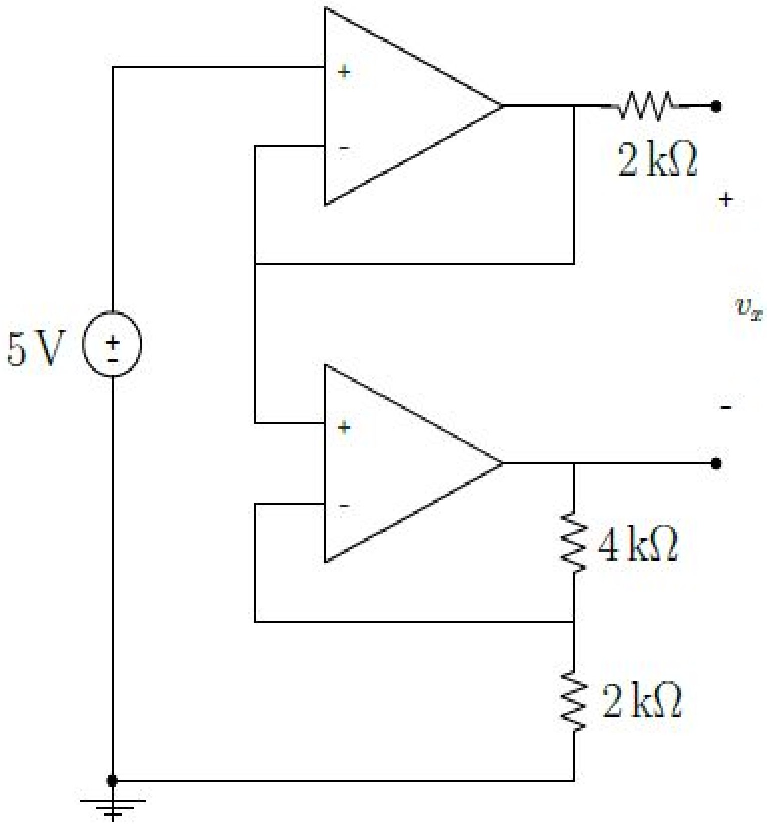


- Turns into the following to find Thevenin resistance using test signal method.



Rt=7, Vt=10, In=10/7

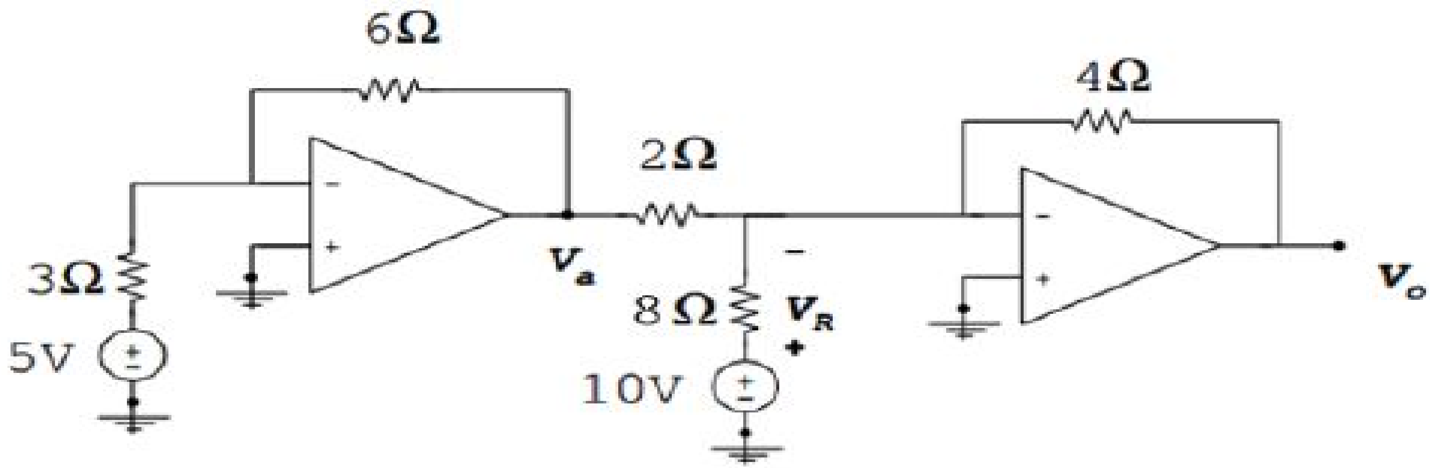
Op-Amps Practice Problems



Solve for V_x:

Answer: $-10V$

Op-Amps Practice Problems

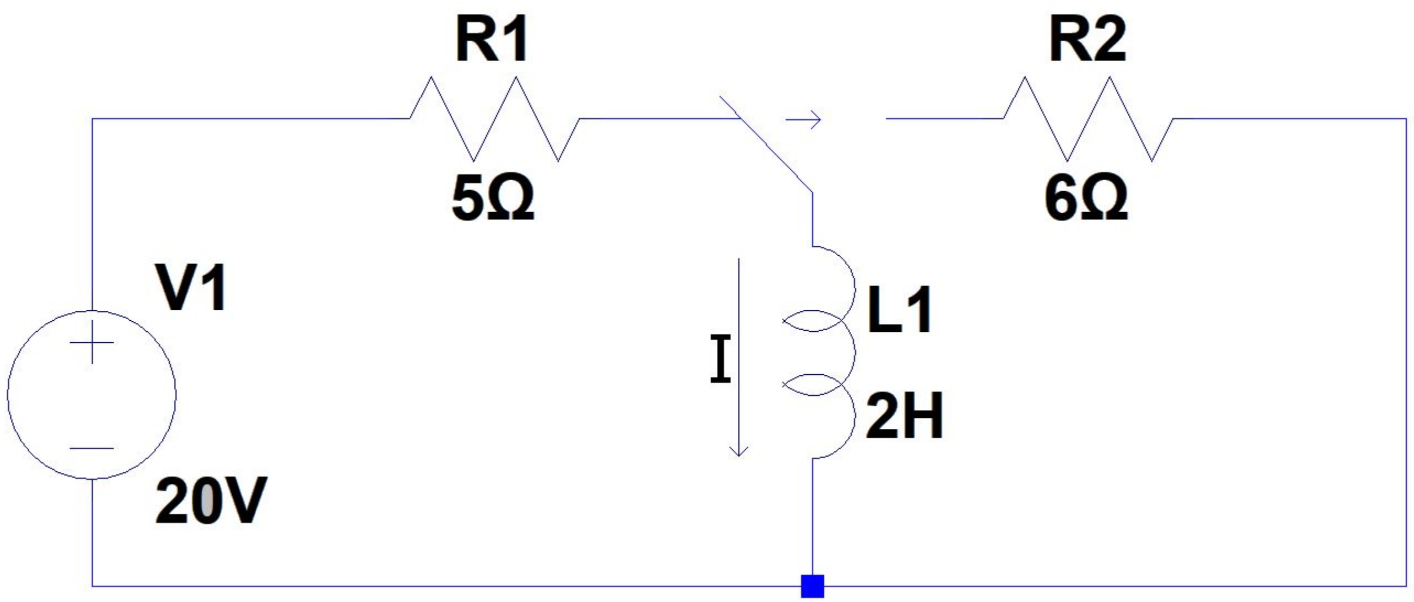


Solve for V_a, V_R, V_o:

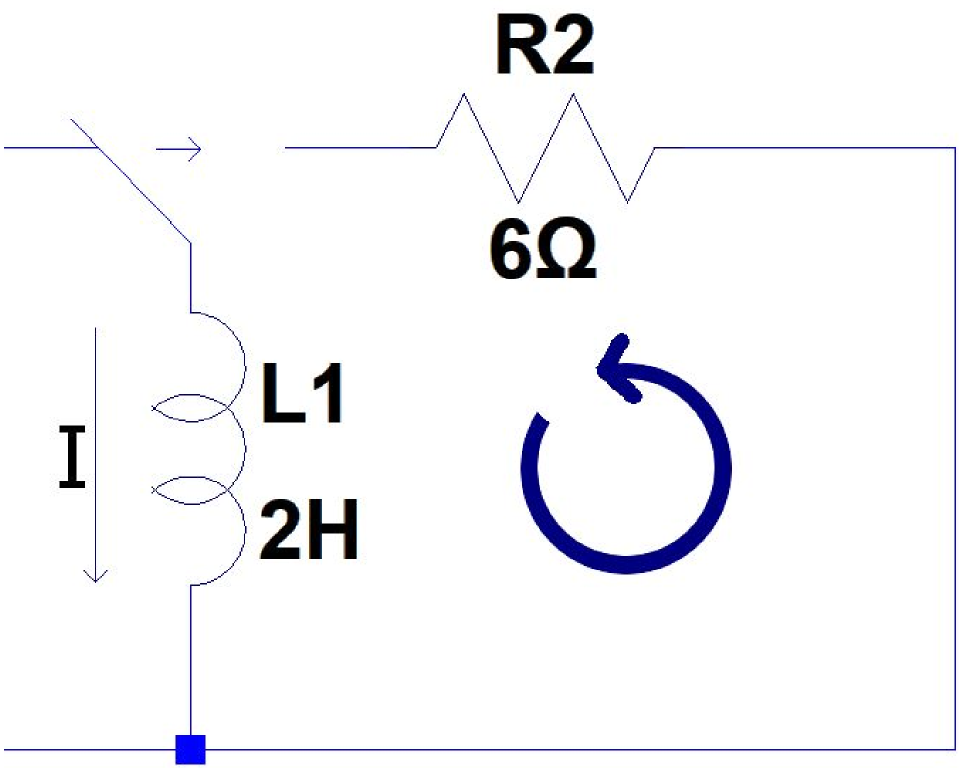
$$V_A = -10V, V_R = 10V, V_o = 15V$$

LC circuit

Assume that the circuit is in this layout for a long time. At time t = 0, the switch flips. Find the expression for current across the inductor



LC circuit solutions



$$V_{inductor} = L\frac{di}{dt} \text{ and } V_{resistor} = iR$$

$$V_{inductor} + V_{resistor} = 0$$

$$L\frac{di}{dt} = -iR$$

$$2 * \frac{di}{dt} = -i * 6$$

$$\frac{di}{dt} = -i * 3$$

$$\frac{1}{i}\frac{di}{dt} = -3$$

$$\int \frac{1}{i}\, di = \int -3\, dt$$

$$\ln(i) = -3t$$

$$i = Ce^{-3t}$$

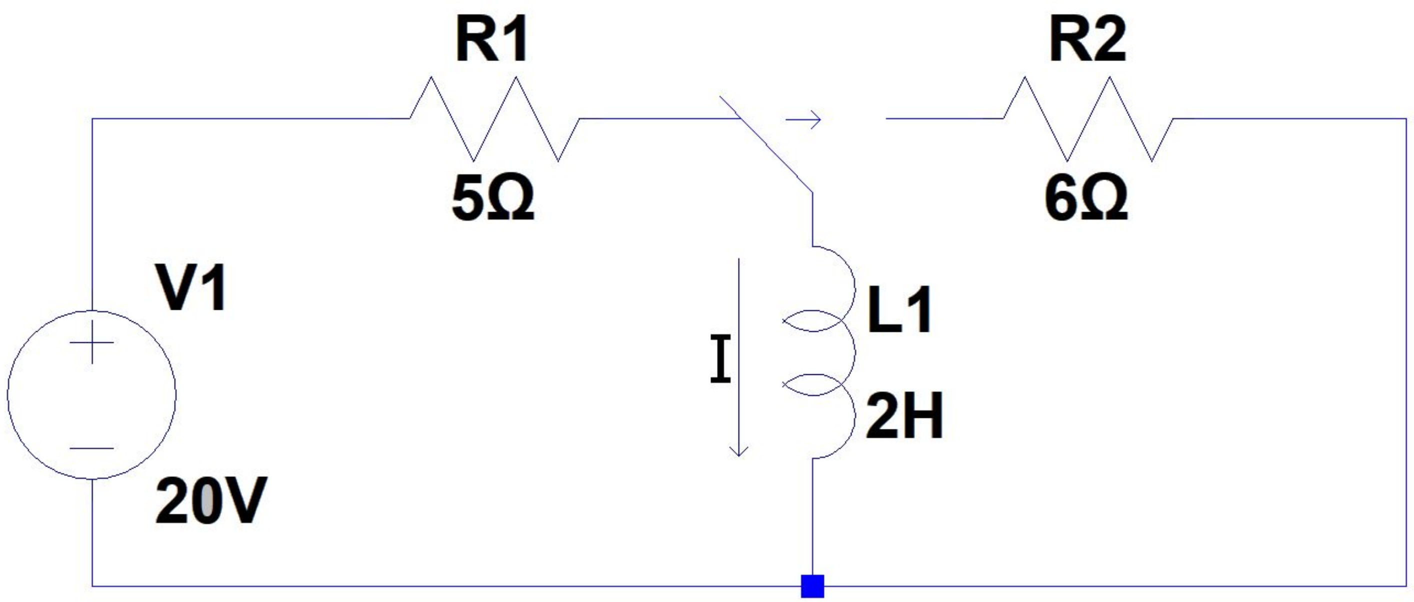
$$i(0) = 4 \text{ because of steady state conditions}$$

$$C = 4$$

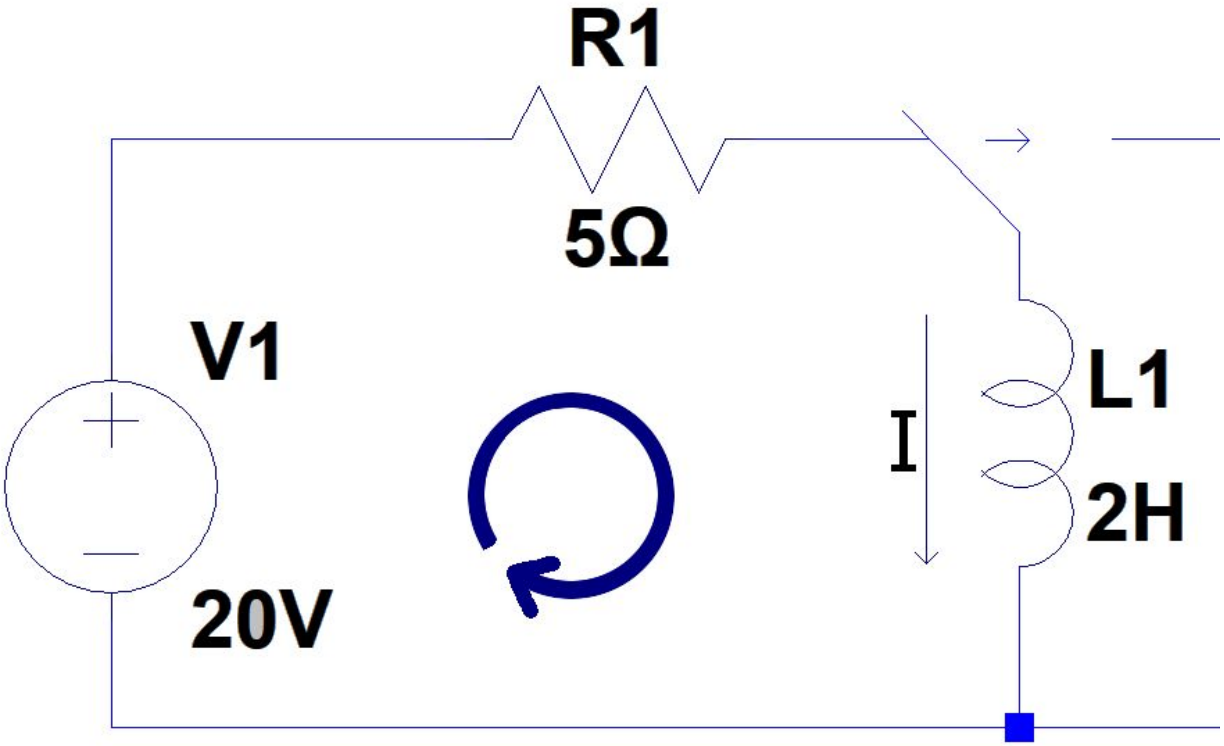
$$i(t) = 4e^{-3t}$$

LC circuit part 2

Assume the switch does not flip. Find the current as a function of time, zero input, and zero state. At time t = 0, there is -2A of current current through the inductor



LC circuit solution



$$L\frac{di}{dt} + iR = 20$$

$$2\frac{di}{dt} + i * 5 = 20$$

$$2\frac{di}{dt} = 20 - 5i$$

$$\frac{di}{dt} = \frac{20-5i}{2}$$

$$\int \frac{2}{20-5i}\, di = \int 1\, dt$$

$$-\frac{2}{5}\int \frac{1}{i-4}\, di = t + c$$

$$-\frac{2}{5}\ln(i-4) = t + c$$

$$\ln(i-4) = -\frac{5}{2} * t + c$$

$$i - 4 = Ce^{\frac{-5}{2}t}$$

$$i = 4 + Ce^{\frac{-5}{2}t}$$

$$i(0) = -2 \text{ so } C = -6$$

$$i(t) = 4 - 6e^{\frac{-5}{2}t}$$

$$i = 4 - Ce^{\frac{-5}{2}t}$$

$$i(0) = 0 \text{ so } C = 4 \text{ because zero state}$$

$$i(t)_{zs} = 4 - 4e^{\frac{-5}{2}t}$$

$$i(t) = i(t)_{zs} + i(t)_{zi}$$

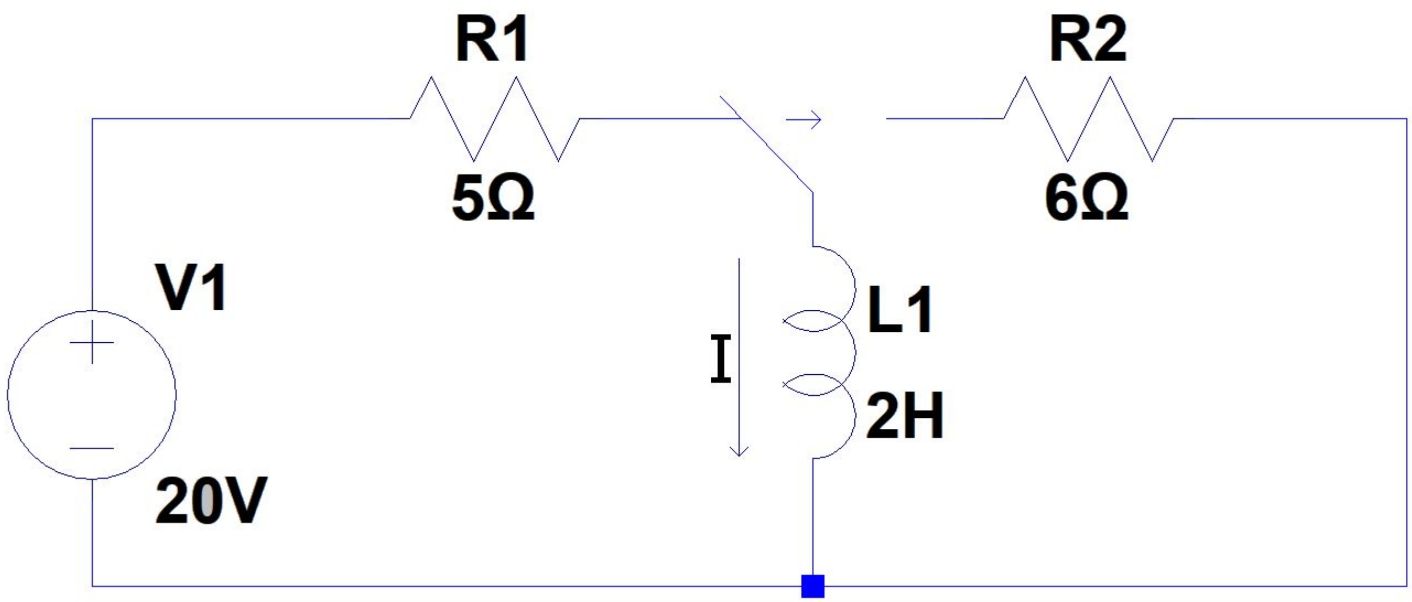
$$i(t)_{zi} = i(t) - i(t)_{zs}$$

$$i(t)_{zi} = 4 - 6e^{\frac{-5}{2}t} - (4 - 4e^{\frac{-5}{2}t})$$

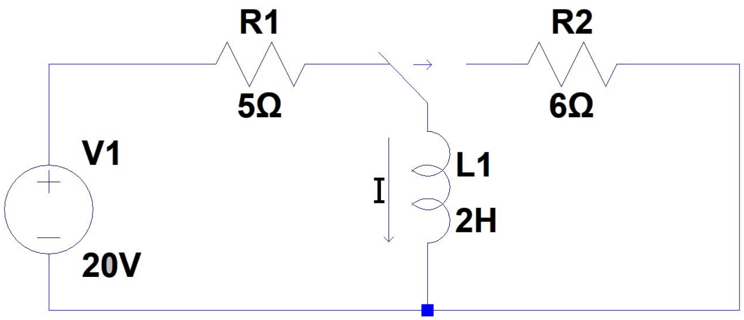
$$i(t)_{zi} = -2e^{\frac{-5}{2}t}$$

LC circuit part 3

At time t = 0 the inductor has no current. At time t = 2, the switch flips. Find the expression for the current across the inductor (hint, use your work from parts 1 and 2)



LC circuit part 3 solution



From part 2, the current of the inductor at time t when it is connected to the voltage source is

$$i(t) = 4 - 4e^{-\frac{5}{2}t}$$

so

$$i(2) = 4 - 4e^{-5} \text{ A}$$

From part one, the current of the inductor at time t when it is not connected to the voltage source is

$$i(t) = Ce^{-3t}$$

where C is the initial current.

If we plug in the initial current, we get

$$i(t) = (4 - 4e^{-5})e^{-3t}$$

However, because we have a 2 second offset

$$i(t) = (4 - 4e^{-5})e^{-3(t-2)}$$

Concepts

-Ohm's law and ohmic devices

-Sign conventions for ohm's law and power calculations

-euler's identity

-1st order ODE

  -zero-state

  -zero-input

  -particular solution/homogenous solution

  -steady-state term/transient term

-RLC circuits result in solving a second order ODE

-DC vs AC circuits

-Capacitors and Inductors in DC vs AC circuits

-Time constants

Formulas

1)Euler's identity
2)Time constant
3)Voltage Divider
4)Current Divider
5)Combining Resistors in series/parallel
6)voltage across capacitor as sum of final and initial voltages across capacitor.
7)ideal op-amp model

Formula Cheat Sheet - LC stuff

$V = L\frac{di}{dt}$

$I = C\frac{dv}{dt}$

$P_{max} = \frac{V_t^2}{4R_t}$

Maximum power at $R_L = R_T$

$F(t) = A + Be^{-\frac{t}{\tau}}$

$\tau = RC$

$\tau = \frac{L}{R}$

Where A is steady state and B is difference between initial and steady state

For an inductor

Zero Input: $i_{zi}(t) = i(0)e^{-\frac{t}{\tau}}$

Zero State: $i_{zs}(t) = i_s - i_s e^{-\frac{t}{\tau}}$

For a capacitor

Zero Input: $V_{zi}(t) = V(0)e^{-\frac{t}{\tau}}$

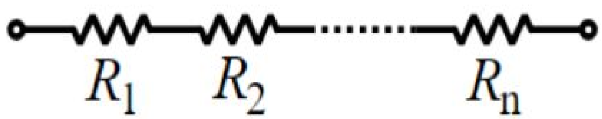
Zero State: $V_{zs}(t) = V_s - V_s e^{-\frac{t}{\tau}}$

$F(t) = A + Be^{-\frac{t}{\tau}}$

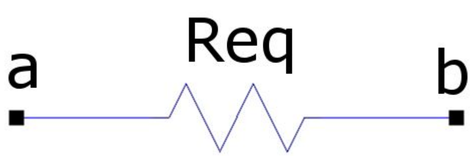
$\tau = RC$

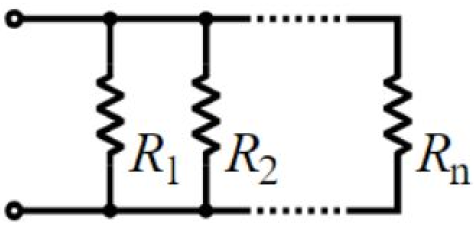
$\tau = \frac{L}{R}$

Formula Cheat Sheet - Resistors

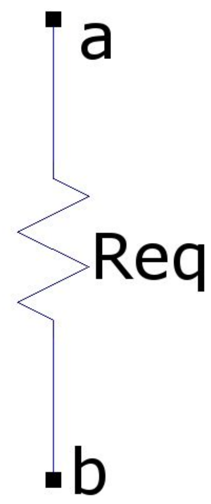


$$R_{eq} = R_1 + R_2 + \ \ldots + R_n$$





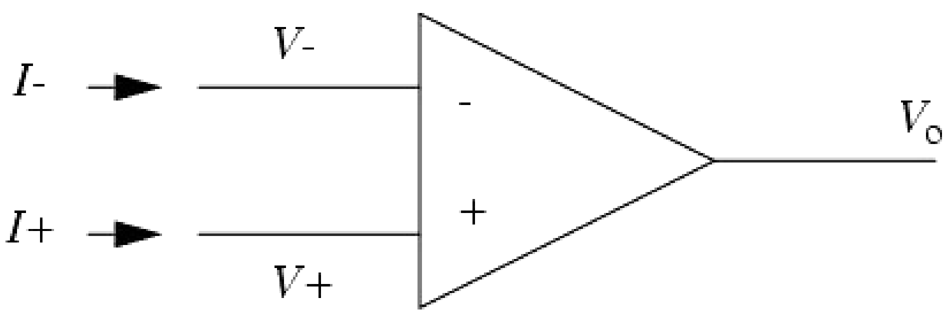
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



Formula cheat sheet - OpAmps

Ideal Op-Amps : $V+ = V-$, $I+ = I- = 0A$

Vout, Iout are the only unknown!



Note: for analysis use,

$I_- = I_+ = 0$

$V_- = V_+$

(The Best)

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