HKN ECE 110 Final Exam Review Session

COREY SNYDER

Before we get started…

- There are special office hours for finals week

- HKN PHYS 212 Review Session 6-8pm in ECEB 1013 TODAY!
- HKN PHYS 213 Review Session 2-4pm in ECEB 1013 on Sunday (5/6)!

- Don’t forget about HKN!
  - Review Sessions
  - Tutoring
  - Tech Talks
  - Peer Mentoring
  - Resume Reviews
  - Mock Interviews

	Thursday (5/3)	Friday (5/4)	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday (5/10)
10:00 AM								Micki
11:00 AM		Prof. Schmitz						Micki
12:00 PM	Review Session			Steven				
1:00 PM	Review Session		Micki	Steven	Prof. Fang	Corey	Corey	Corey
2:00 PM	Review Session	Prof. Gruev	Micki			Corey	Corey	Corey
3:00 PM					Prof. Choi			
4:00 PM	Steven	Steven						
5:00 PM	Steven	Steven						
6:00 PM								
All office hours are held in ECEB 1005 (lab classroom)								

Legit Tips and Tricks to Show Off Your Wits

Concise Advice to Make Your Grade Look Nice

Wise Words to Make your Score Soar

Lessons to Lend to a Friend for the End (of the semester)

- Use your study sheet more like a study tool
- Spend your time showing what you know
- Pace yourself, you have three hours!
- Take some time to relax before the exam

Beyond ECE 110…

- You have a right to be skeptical
- "You're not as good as you think you are when you win, and you're not as bad as you think you are when you lose"
- Don't forget about HKN!

Topics

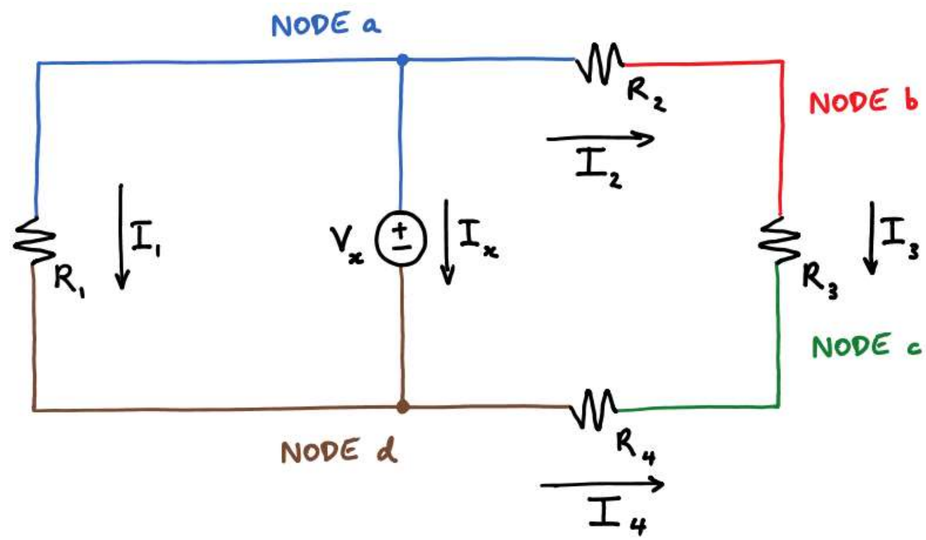
- Circuit Analysis Fundamentals
  - Ohm's Law, KVL, KCL, Resistor Combinations, CDR, VDR
- Power Conventions
  - Standard/Non-standard Current, V_{rms}/P_{avg}
- I-V Characteristics and Thevenin/Norton Equivalents
- Node Method
  - Supernode
- Diodes
  - Clippers, "How many are on?"
- BJTs
  - CE BJT Circuit, I-V Characteristic, BJT Amplifier
- MOSFETs
  - CS MOSFET Circuit, I-V Characteristic, CMOS
- Signal-to-Noise Ratio
- Sampling and Quantization
  - Nyquist, Data Conversion
- Information Theory and Compression
  - Entropy, Huffman Coding
- Solar Cells and Photodiodes

Ohm’s Law, Resistance, and Power

- Ohm’s Law describes the relationship between the voltage *across* and current *through* a resistive element
  - Ohm’s law only applies for linear components, i.e. resistors
  - More on linear components with Thevenin/Norton Equivalents (and in ECE 210!)
- $V = IR$
- Resistance of an element can found by: $R = \frac{\rho l}{A}$
- Power dissipated by an element can be found by: $P = IV, \mathrm{P} = I^2 R, P = \frac{V^2}{R}$
  - You can go between the three forms using Ohm’s Law!
- Resistors in series share the same current
- Resistors in parallel share the same two nodes and thus the same voltage

Nodes, KVL, and KCL

- A node is any part of a circuit that is at an *equipotential*
  - Wires are equipotentials
- Kirchhoff's Voltage Law
  - Conservation of Energy
  - Performed on a loop
- $\sum V_{rises} = \sum V_{drops}$
- Kirchhoff's Current Law
  - Conservation of Charge
  - Performed at a node
  - Bubble method
- $\sum I_{in} = \sum I_{out}$



Equivalent Resistance Example #1

For the following circuit:

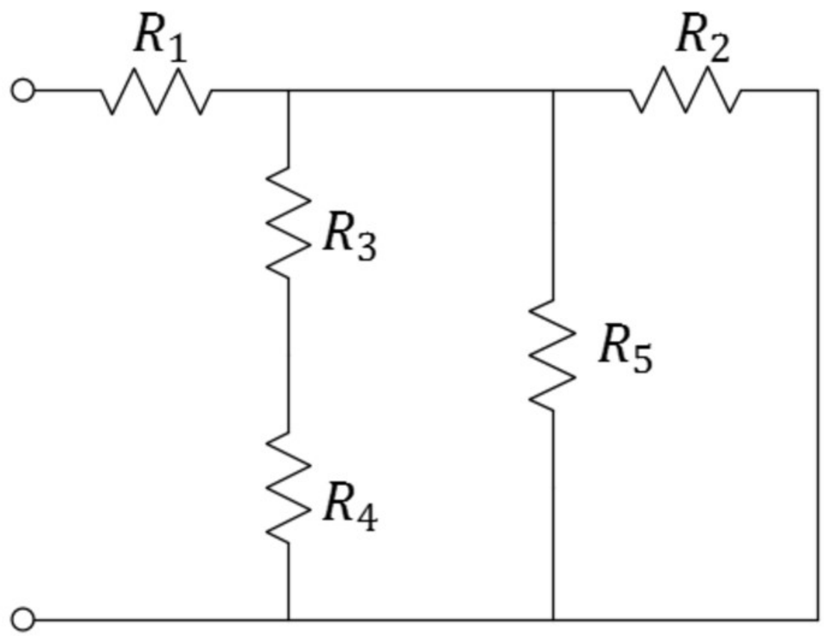
What is the relationship between R_3 and R_4?

A. **Series**

B. Parallel

C. Neither

Any current that passes through R_3 will exclusively pass through R_4, thus the two resistors share the same current and are in series.



Equivalent Resistance Example #1

For the following circuit:

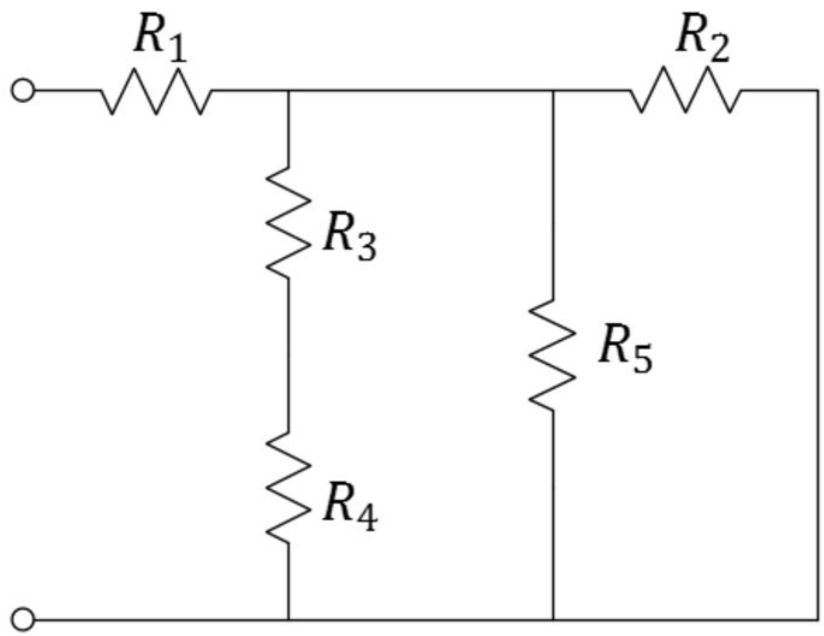
What is the relationship between R_1 and R_2?

A. Series

B. Parallel

C. **Neither**

R_1 and R_2 do not share the same two nodes. Additionally, any current that passes through R_1 can split to other branches including R_3 or R_5. Therefore, they are neither in series nor parallel.



Equivalent Resistance Example #1

For the following circuit:

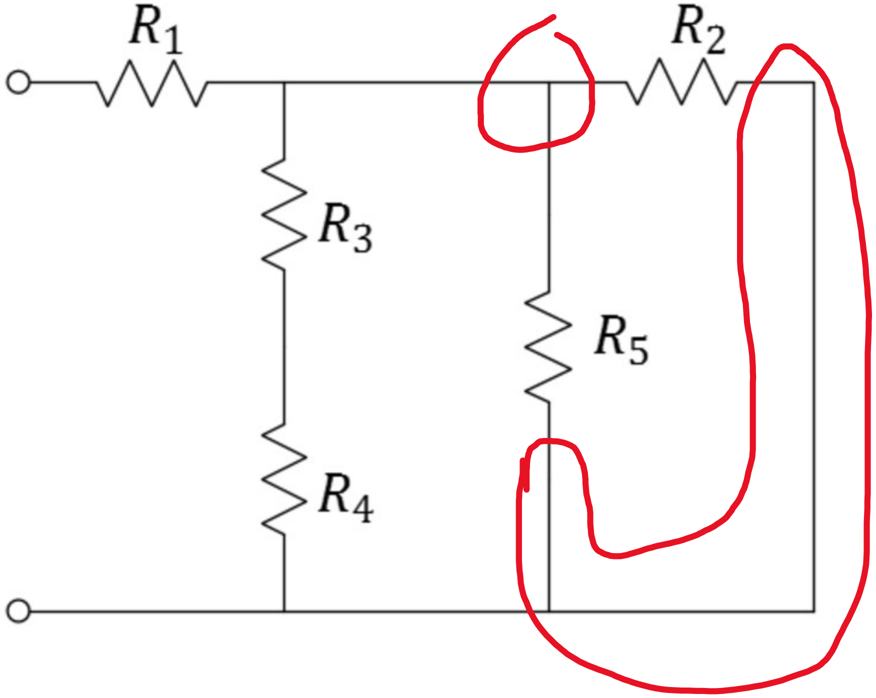
What is the relationship between R_2 and R_5?

A. Series

B. Parallel

C. Neither

R_2 and R_5 share the same two nodes, as indicated on the right. Therefore, they must be in parallel.



Equivalent Resistance Example #2

Find the value of R_1 that makes $R_{eq} = 27.5\Omega$

A. 4Ω

B. 8Ω

C. 11Ω

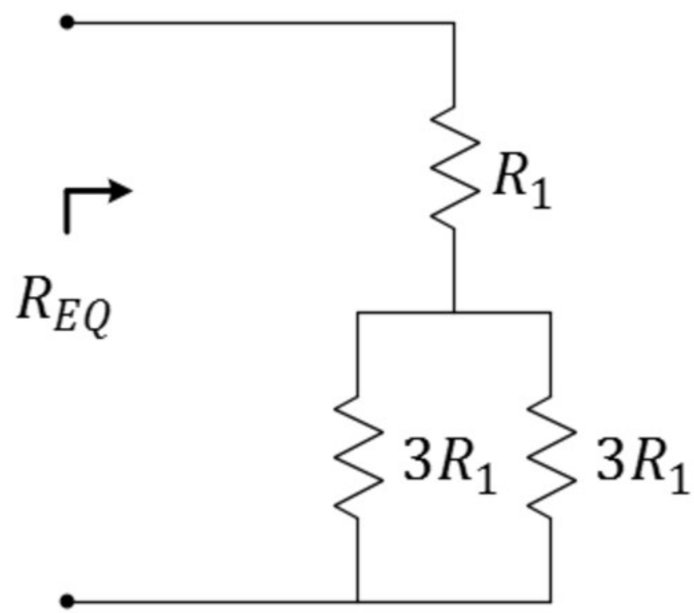
D. 12Ω

E. Unsure

$$R_{eq} = (3R_1||3R_1) + R_1$$
$$= \frac{3}{2}R_1 + R_1$$
$$R_{eq} = \frac{5}{2}R_1$$

$$\therefore R_1 = \frac{2}{5}(27.5)$$

$$R_1 = 11\Omega$$



KVL and KCL Example

Find the unknown voltages, V_2, V_3, V_5.

A. $V_2 = 0V, V_3 = -3V, V_5 = 0V$

B. $V_2 = 6V, V_3 = -3V, V_5 = 6V$

C. $V_2 = 6V, V_3 = 3V, V_5 = -6V$

D. $\boldsymbol{V_2 = 6V, V_3 = -3V, V_5 = -6V}$

E. Yeah, uh, no.

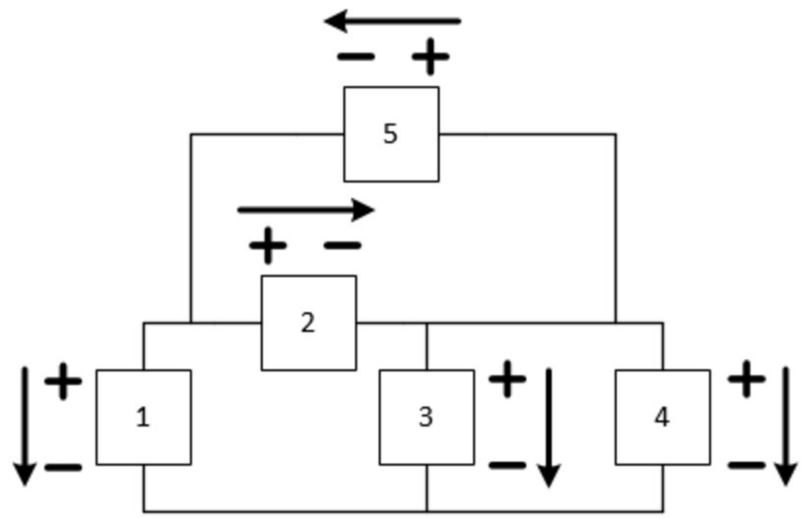
$V_1 = 3\,V$

$V_4 = -3\,V$

$I_2 = -2\,A$

$I_3 = -1\,A$

$I_5 = 3\,A$



Finding V_2:

$$V_1 = V_2 + V_4$$
$$V_2 = V_1 - V_4$$
$$V_2 = 6V$$

Finding V_3:

$$V_3 = V_4$$
$$V_3 = -3V$$

Finding V_5:

$$V_1 + V_5 = V_4$$
$$V_5 = V_4 - V_1$$
$$V_5 = -6V$$

$$\sum V_{rises} = \sum V_{drops}$$

KVL and KCL Example

Find the unknown currents, I_1, I_4.

A. $\boldsymbol{I_1 = 5A, I_4 = -4A}$

B. $I_1 = 1A, I_4 = -4A$

C. $I_1 = 1A, I_4 = -1A$

D. $I_1 = 5A, I_4 = -1A$

E. Please. Help me.

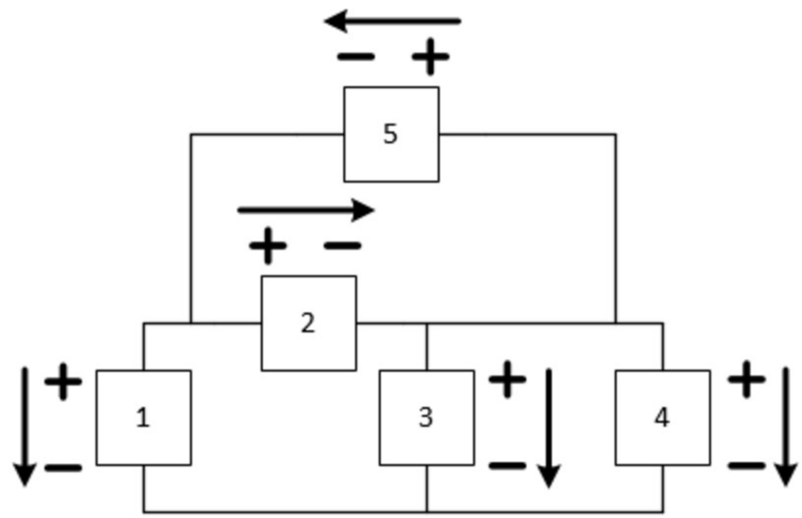
$V_1 = 3\,V$

$V_4 = -3\,V$

$I_2 = -2A$

$I_3 = -1A$

$I_5 = 3A$



Finding I_1:

$$I_5 = I_1 + I_2$$
$$I_1 = I_5 - I_2$$
$$I_1 = 5A$$

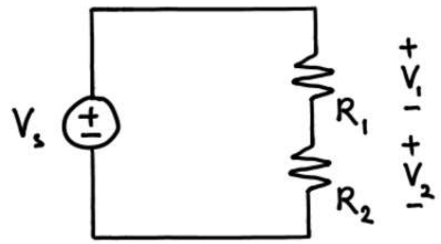
Finding I_4:

$$I_2 = I_3 + I_4 + I_5$$
$$I_4 = I_2 - I_3 - I_5$$
$$I_4 = -4A$$

$$\sum I_{in} = \sum I_{out}$$

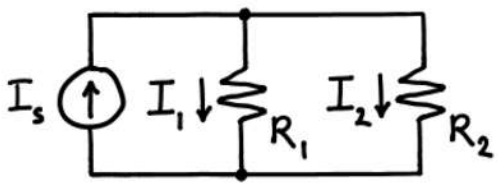
Voltage Divider and Current Divider

- We can use voltage divider rule (VDR) in order to find the voltage across individual resistors in series



$$V_1 = \frac{R_1}{R_1 + R_2} V_s \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$$

- We can use current divider rule (CDR) in order to find the current through individual resistors in parallel



$$I_1 = \frac{R_2}{R_1 + R_2} I_s \qquad I_2 = \frac{R_1}{R_1 + R_2} I_s$$

Root-mean-square Voltage (V_{rms})

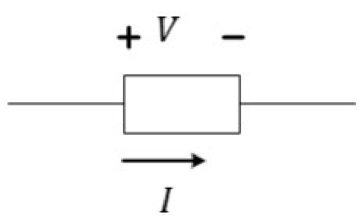
- The exact definition of V_{rms} is:

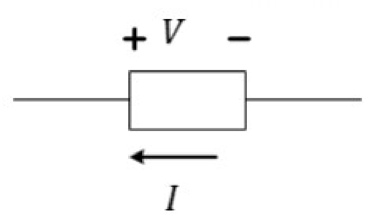
$$V_{rms} = \sqrt{\frac{\left(\int_0^T f^2(t)dt\right)}{T}}$$

- We will mainly ask you to use the following two formulas:
- $V_{rms}(sinusoid) = \frac{Amplitude}{\sqrt{2}}$
- $V_{rms}(square\ wave) = Amplitude\sqrt{\%DC}$
- We use V_{rms} to determine the power delivered to a load from a time-varying source
  - $P_{avg} = \frac{V_{rms}^2}{R}$

Power and Labeling

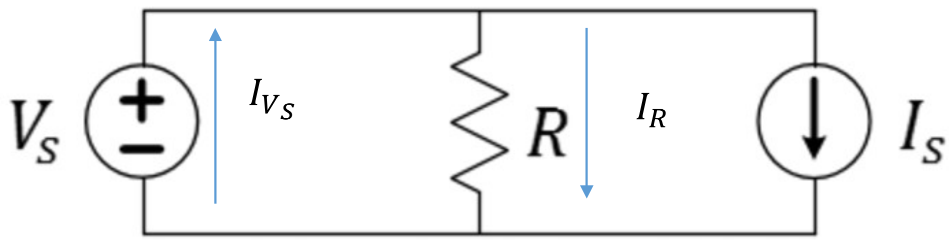
- We know that power can be expressed in three ways: $P = IV = I^2R = \frac{V^2}{R}$
- If the value of power is positive, the element is absorbing power
- If the value of power is negative, the element is supplying power
- Standard vs. Non-Standard Labeling
- Standard: $P = IV, V = IR$, Current goes + to -
- Non-Standard: $P = -IV, V = -IR$, Current goes – to +





Circuit Power Example

For the following circuit, determine the power of the resistor, current source, and voltage source.



All of the elements are in parallel. Thus, they must all share the same voltage. This voltage will be V_s since one branch is entirely composed of that voltage source. We can immediately find the power for the resistor and current source.

$$P_R = \frac{V^2}{R} = \frac{V_S^2}{R}$$
$$\boldsymbol{P_R = 57.6mW}$$

$$P_{I_S} = IV = I_S V_S$$
$$\boldsymbol{P_{I_S} = 36mW}$$

$$V_s = 2.4\ V$$
$$R = 100\ \Omega$$
$$I_s = 15\ mA$$

Two ways to compute P_{V_S}: sum of powers or directly by finding the current through V_S.

#1: $\sum P = 0$

$$P_R + P_{I_S} + P_{V_S} = 0$$
$$\boldsymbol{P_{V_S} = -93.6mW}$$

#2: $I_{V_S} = I_R + I_S$

$$= \frac{V_S}{R} + I_S$$

$$I_{V_S} = 39mA$$

$P_{V_S} = -I_{V_S} V_S$ (non-standard current!)

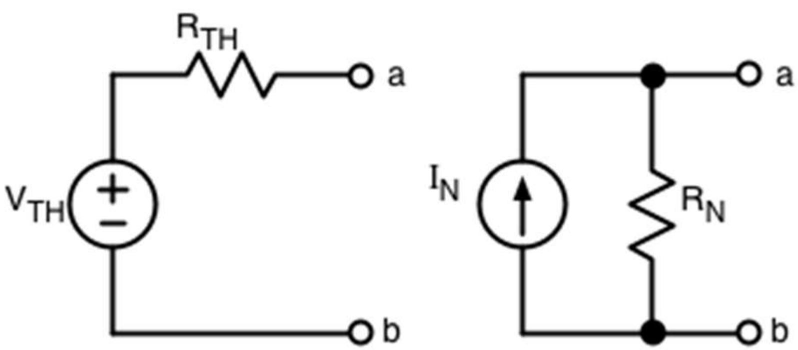
$$\boldsymbol{P_{V_S} = -93.6mW}$$

I-V Characteristics

- We can characterize circuits where the current is a function of the voltage
- For ECE 110, we typically want to characterize linear circuits, where the I-V Characteristic is of the form
  - $I = mV + b$
- In order to obtain this equation, we want to find two points:
  - V_{oc} and I_{sc}
- V_{oc} $is\ the\ x - intercept, I_{sc}\ is\ the\ y - intercept$

Thevenin and Norton Equivalents

- We can express any linear circuit as a simple circuit involving a source and a resistor (Wow)
  - More on this in ECE 210!
- We want to find V_{oc}, I_{sc}, and R_{eff}
- If we find two of these parameters, we can find the third through Ohm's Law!
- If current is defined as standard:
  - $I = \frac{-I_{SC}}{V_{OC}}V + I_{SC}$
- If current is defined as non-standard:
  - $I = \frac{I_{SC}}{V_{OC}}V - I_{SC}$



Thevenin and Norton Equivalents

- There is a lot of vocabulary with Thevenin and Norton Equivalents, and this can be confusing! Keep the following things in mind:

$$I_{sc} = I_N$$
$$V_{oc} = V_T$$
$$R_T = R_N = R_{eff} = R_{eq}$$

- Remember what each parameter represents in the IV characteristic and the corresponding graph.
- I_{sc} is the y-intercept
- V_{oc} is the x-intercept
- $\frac{1}{R_{eff}}$ is the **magnitude** of the slope. Remember the sign is determined by whether the current definition is standard or non-standard

Thevenin and Norton Example #1

The open-circuit voltage of the component C is $6V$, its short-circuit current is $400mA$, and $V_s = 3V$. Determine the power of the component C.

A. $P = 0.6W$

B. $\boldsymbol{P = -0.6W}$

C. $P = 2.4W$

D. $P = -2.4W$

E. I have a moral objection to Thevenin and Norton Equivalents.

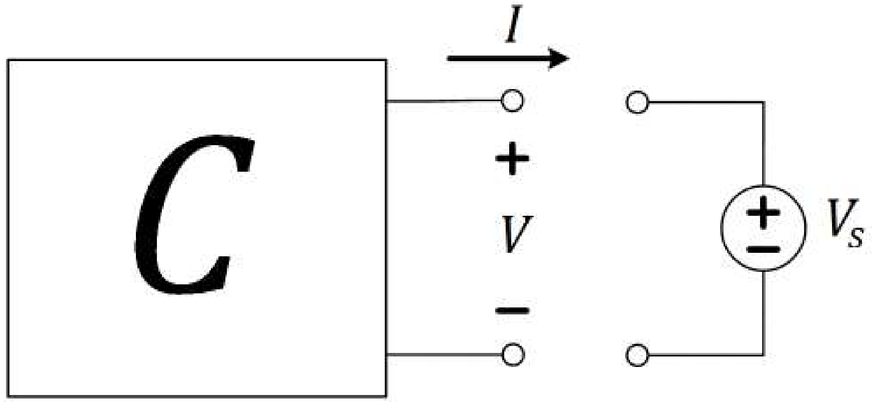
I-V Characteristic of Component C:

$$I = -\frac{I_{sc}}{V_{oc}}V + I_{sc}$$

$$I = -\frac{1}{15}V + \frac{2}{5}$$

I-V Characteristic of Source V_S:

$$V = V_S$$



Setting the two I-V Characteristics equal:

$$V = V_s = 3V$$

$$I = 200mA$$

$$P = -IV = -0.6W$$

The current I that enters into the component C will travel from "-" to "+", thus we have non-standard current

Thevenin and Norton Example #2

For the following circuit, determine the Thevenin Equivalent. That is, find $V_T, I_N, and\ R_T$.

What is R_T?

A. 0Ω

B. 3Ω

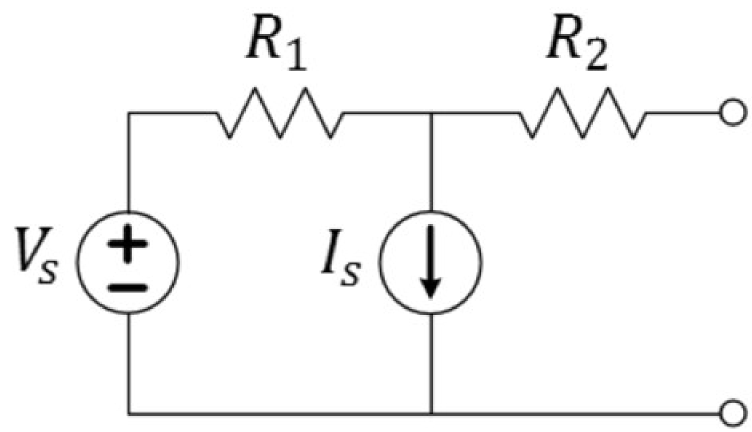
C. **8Ω**

D. 10Ω

Apply source suppression: voltage sources become wires and current sources become open circuits. This will leave R_1 and R_2 in series.

Thus,

$$R_T = R_1 + R_2$$
$$R_T = 8\Omega$$



$R_1 = 5\ \Omega$

$R_2 = 3\ \Omega$

$I_S = 1\ A$

$V_S = 7\ V$

Thevenin and Norton Example #2

For the following circuit, determine the Thevenin Equivalent. That is, find $V_T, I_N, and\ R_T$.

What is V_T?

A. $\frac{7}{5}$V

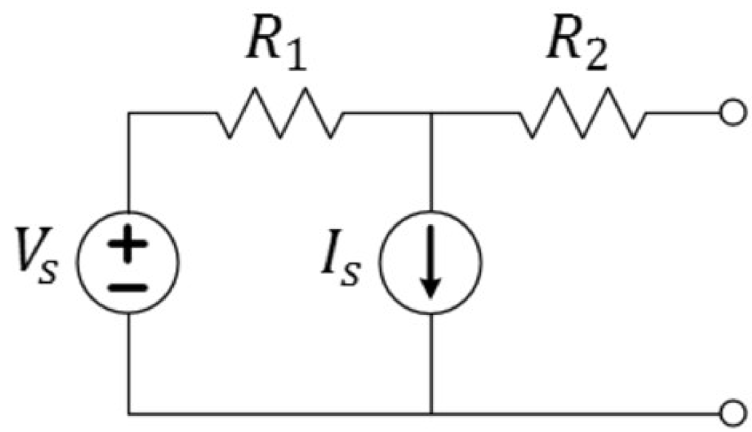
B. $\mathbf{2V}$

C. $5V$

D. $7V$

Leave terminals open and find voltage across them (V_{OC}). This is the same as the voltage across the current source since no voltage will be dropped across R_2.

$$V_T = V_{I_S}$$
$$V_T = V_S - I_S R_1$$
$$V_T = 2V$$



$R_1 = 5\ \Omega$

$R_2 = 3\ \Omega$

$I_S = 1\ A$

$V_S = 7\ V$

Thevenin and Norton Example #2

For the following circuit, determine the Thevenin Equivalent. That is, find $V_T, I_N, and\ R_T$.

What is I_N?

A. $\boldsymbol{\frac{1}{4}A}$

B. $1A$

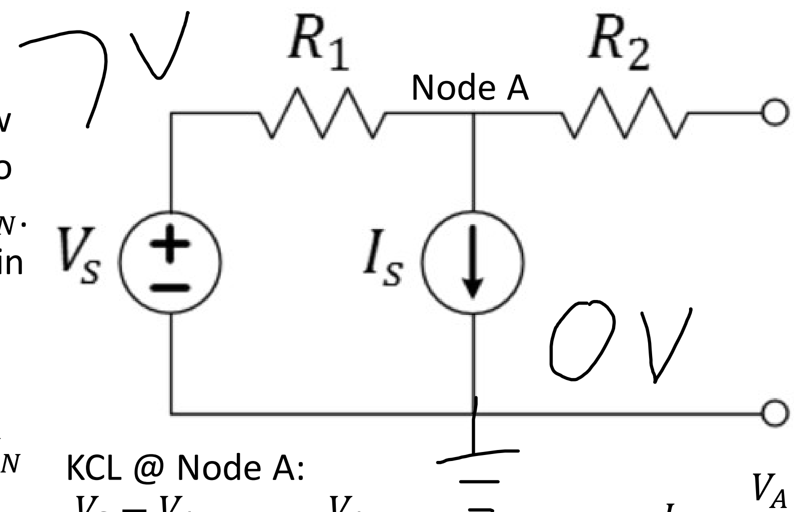
C. $\frac{7}{8}A$

D. $4A$

E. PLEASE MAKE IT STOP

Could use Ohm's Law and the previous two answers to deduce I_N. But where's the fun in that? Let's find it directly!

Remember we find I_N by determining the current between the terminals when they are shorted (I_{SC}).



$R_1 = 5\ \Omega$

$R_2 = 3\ \Omega$

$I_S = 1\ A$

$V_S = 7\ V$

KCL @ Node A:

$$\frac{V_S - V_A}{R_1} = I_S + \frac{V_A}{R_2}$$

$$V_A = \frac{3}{4}V$$

$$I_N = \frac{V_A}{R_2}$$

$$I_N = \frac{1}{4}A$$

Node Voltage Method

- We want to express the KCL in terms of node voltages using Ohm's Law. How do we do this?

1. Pick a ground node that touches as many sources as possible.
2. Label the nodes that touch ground through a voltage source.
3. Pick a node where you need to find the node voltage. Write KCL at that node.
4. Express each current using Ohm's Law and the node voltages. Be careful of your current directions. We express each current as the voltage "where you start" minus the voltage "where you end" over the resistance.

$$I_x = \frac{V_{start} - V_{end}}{R_x}$$

5. Solve! (May need a system of equations if there is more than one unknown node)

Supernode Example

Given $V_S = 5V$, find the value of the current I in amps.

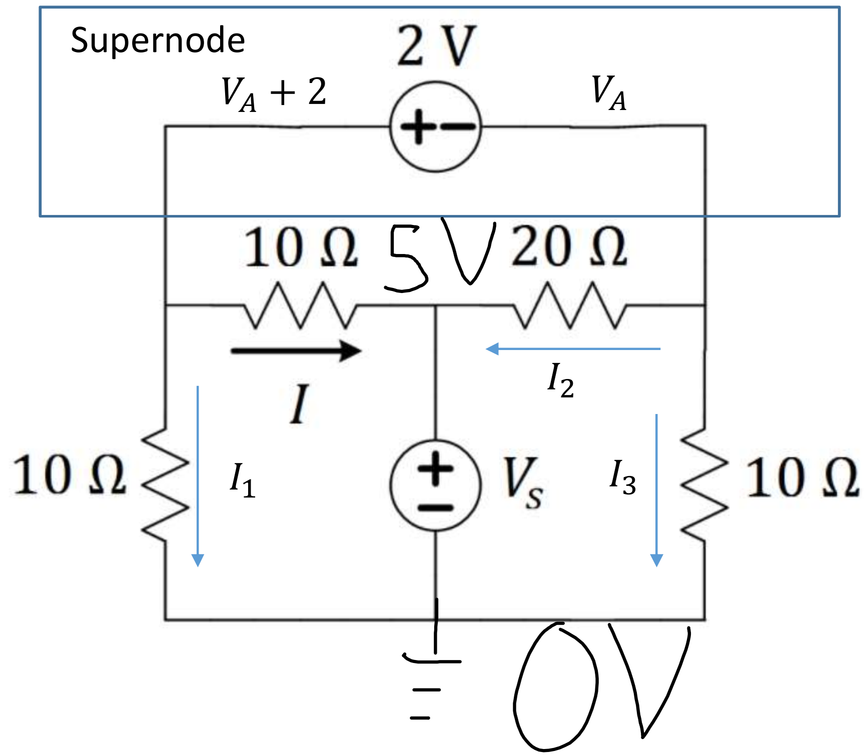
We perform KCL together at the two nodes that form our supernode. Thus, we have the following:

$$I_1 + I + I_2 + I_3 = 0$$

$$\frac{V_A + 2}{10} + \frac{(V_A + 2) - 5}{10} + \frac{V_A - 5}{20} + \frac{V_A}{10} = 0$$

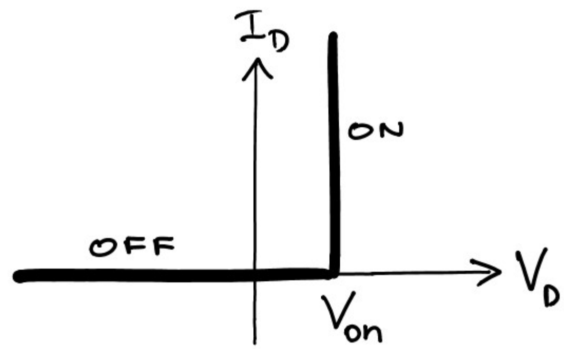
$$2V_A + 4 + 2V_A - 6 + V_A - 5 + 2V_A = 0$$
$$V_A = 1V$$

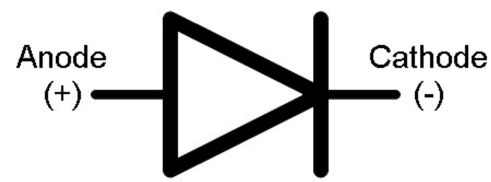
$$I = \frac{(V_A + 2) - 5}{10} \qquad \boldsymbol{I = -\frac{1}{5}A}$$



Diodes

- Diodes are non-linear devices that allow current to pass in only one direction
- Offset-Ideal Model
- How do we know if a diode is on? Guess and Check!
- If we guess off, we say the current is zero and $V_D < V_{on}$
- Guess on, positive current flows and V_{on} volts are across diode
- If our guess is wrong, one of these conditions will be violated
  - E.g. We guess on, but reverse current flows through the diode
- Clipper circuits





Diode Example – How many are on?

In the following circuit, $V_{ON} = 2V$. How many diodes are on?

A. 1

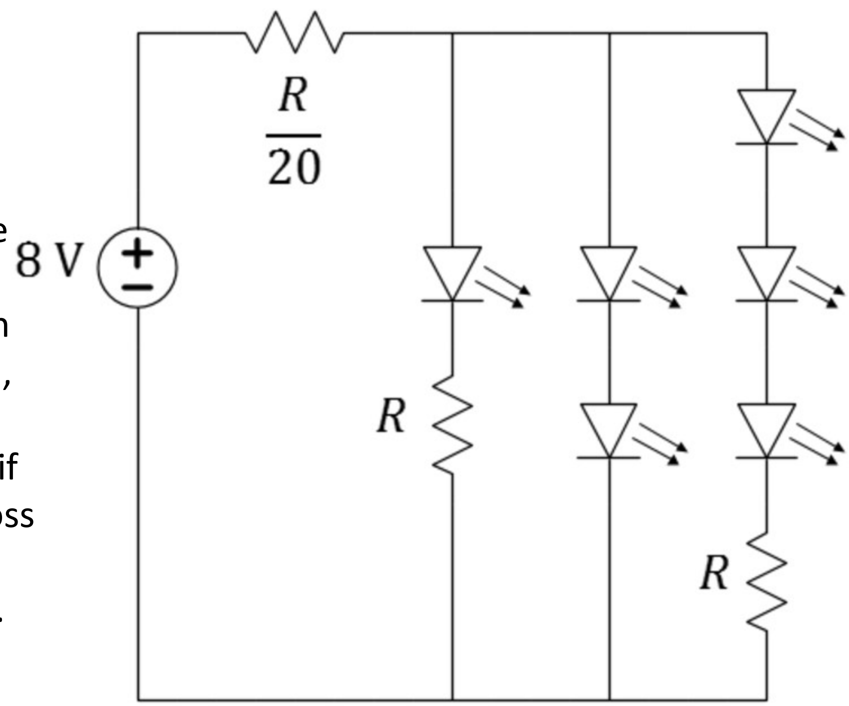
B. 2

C. 3

D. 6

E. Good Question

We start by assuming only the left branch is on since it will require the least voltage to turn on ($\geq 2V$). We see that the voltage across the left branch in this case will be greater than $4V$ since the series resistor R is 20 times larger than the top-left resistor. Next, we say the middle branch will also be on. Since all branches are in parallel, they will all share the $4V$ required by the two middle diodes. It is also important to note that if the middle branch is on, **exactly $4V$** will be across this branch. Therefore, the far right branch cannot be on and we will have just **3 diodes on**.



Diode Example – Clipper Circuit

For the following clipper circuit, determine the minimum and maximum output voltages. $V_S = 3.3\cos(\omega t)\,, V_{ON} = 0.7V, and\ V_1 = -0.5V$.

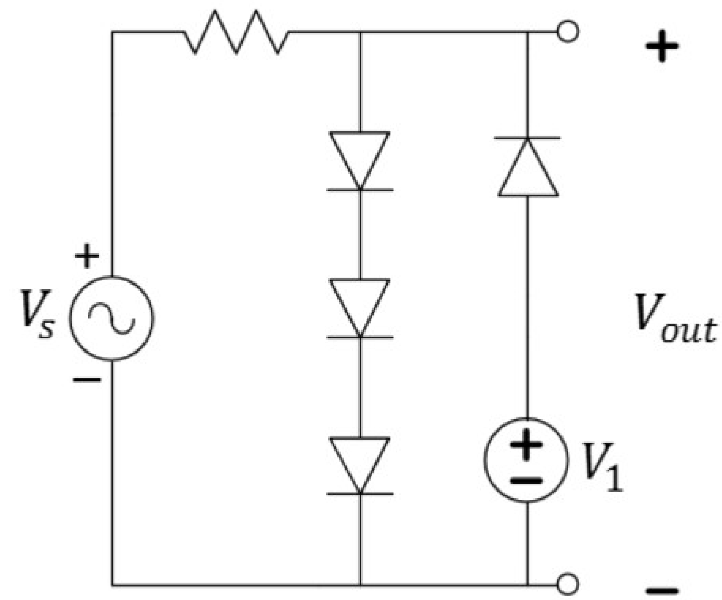
Left branch will have $2.1V$ across it if on.

Right branch will have $-1.2V$ across it if on.

The input is bounded by $V_S \in [-3.3,3.3]$

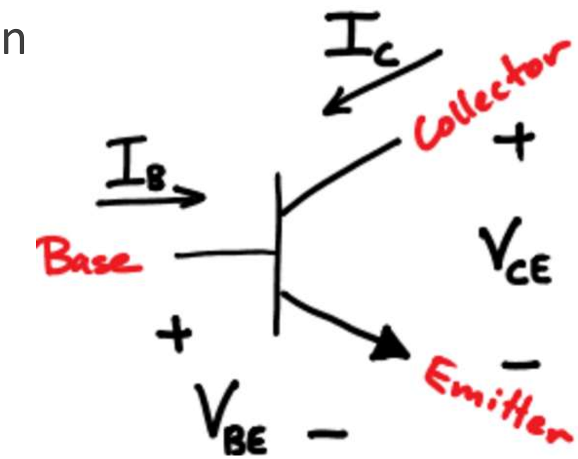
Therefore, whenever $V_S > 2.1V$, the left branch will turn on and clip the output. If $V_S < -1.2V$, the right branch will turn on and clip the output.

Thus, $V_{max} = 2.1V, V_{min} = -1.2V$



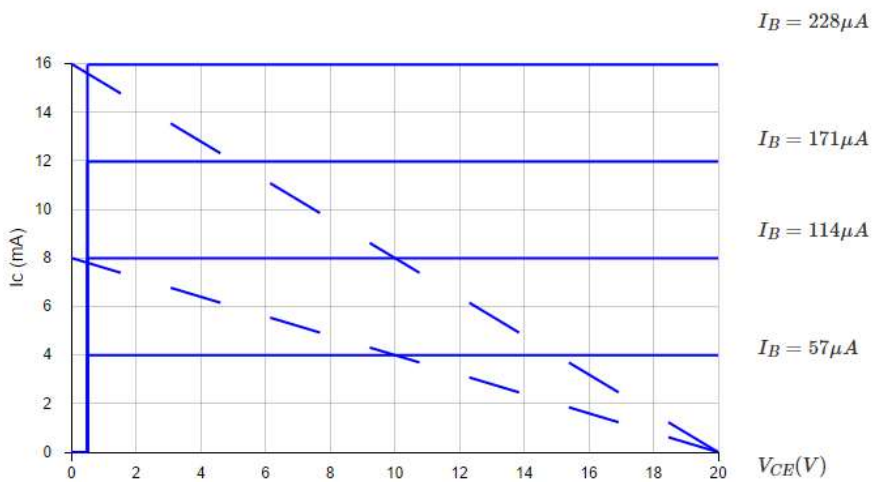
Bipolar Junction Transistor (BJT)

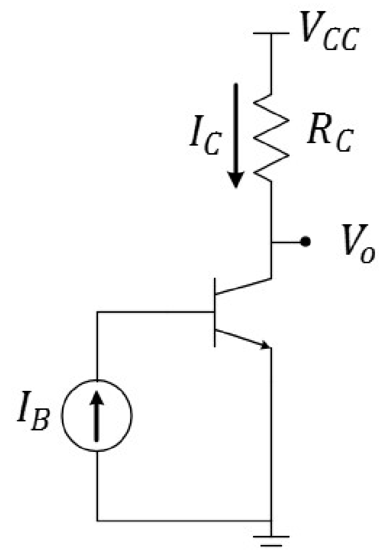
- Three terminal device: Base, collector, emitter
- $V_{BE,ON}$ and $V_{CE,SAT}$ are properties of the BJT
- In ECE 110 we consider the Common-Emitter (CE) configuration
- Three regions of operation: Off (Cutoff), Active, Saturation
- Off: $V_{BE} < V_{BE,ON}$, all currents are zero!
- Active: $V_{BE} > V_{BE,ON}$, $I_C = \beta I_B$
- Saturation: $V_{BE} > V_{BE,ON}$, $V_{CE} = V_{CE,SAT}$, $I_C \neq \beta I_B$!



BJT IV Characteristics

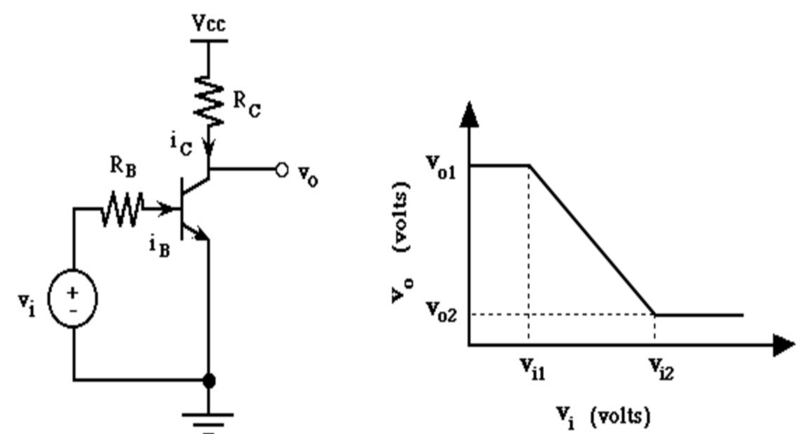
- The solid lines represent the IV characteristics of the BJT. Note that the IV characteristic depends on the bias conditions: the value of I_B
- The dotted line represents the IV characteristic of the right-hand loop: the collector side





BJT Transfer Characteristic

- The transfer characteristic of a BJT relates the output voltage, $V_{CE} = V_o$, to its input voltage, V_i.
- We can deduce the regions of operation and thus the important values from the graph
- $V_{o1} = V_{CC}$
- $V_{o2} = V_{CE,SAT}$
- $V_{i1} = V_{BE,ON}$
- $V_{i2} = V_i^* =$ minimum input to enter saturation
- Gain $= G = -\frac{\beta R_C}{R_B} =$ slope in active region



BJT Power

- In order to calculate the power of a BJT, we consider the power contributed by each junction we analyze: Base-Emitter and Collector-Emitter
- $P_{BJT} = V_{BE,ON} I_B + V_{CE} I_C$
- If the BJT is off, power is zero.
- We must check to see if the transistor is in the active or saturated region as usual
- Notice that the Base-Emitter (1st) term is typically much smaller than the Collector-Emitter (2nd) term since $I_C \gg I_B$ due to β

BJT Example – IV Characteristic

Let's examine the values of I_C and V_0 for different biasing currents, I_B. $V_{CE,SAT} = 0.2V$.

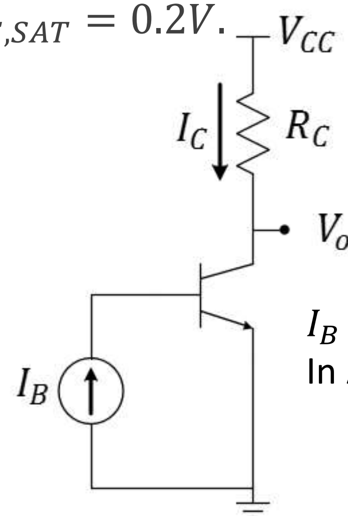
- $I_B = 57\mu A$
- $I_B = 114\mu A$
- $I_B = 228\mu A$
- What is β?

IV-Characteristic of Right Side:

$$V_{oc} = V_{CC} = 20V$$

$$I_{sc} = \frac{V_{CC}}{R_C} = 16mA$$

Thus, we use the top dashed line to represent the right side.



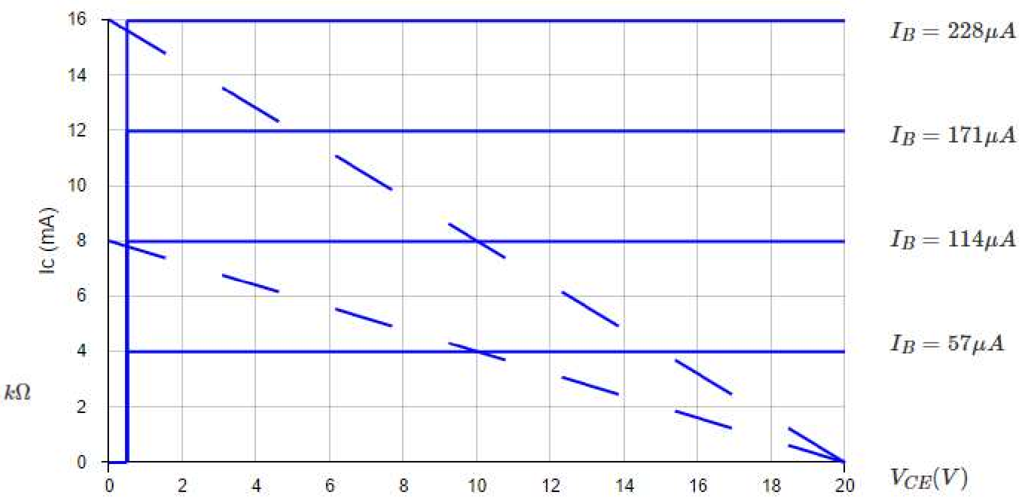
$V_{CC} = 20\ V$

$R_C = 1.2500\ k\Omega$

$I_B = 57\mu A$:
In Active region,

$$I_C = 4mA$$
$$V_{CE} = 15V$$

$$\beta = \frac{I_C}{I_B}$$
$$\beta \approx 70.1754$$



$I_B = 114\mu A$:
In Active region,

$$I_C = 8mA$$
$$V_{CE} = 10V$$

$I_B = 228\mu A$:
In Saturation region,

$$V_{CE} = V_{CE,SAT} = 0.2V$$
$$I_C = I_{C,SAT} = 15.84mA$$

BJT Example – Transfer Characteristic

Determine $V_{i1}, V_{i2}, V_{o1}, and\ V_{o2}$ and the Gain for the following BJT circuit.

What is V_{i1}?

A. $\mathbf{0.7V}$

B. $0.2V$

C. $9V$

V_{i1} = minimum input to turn on BJT.

$$V_{i1} = V_{BE,ON} = 0.7V$$

$V_{CC} = 9\ V$

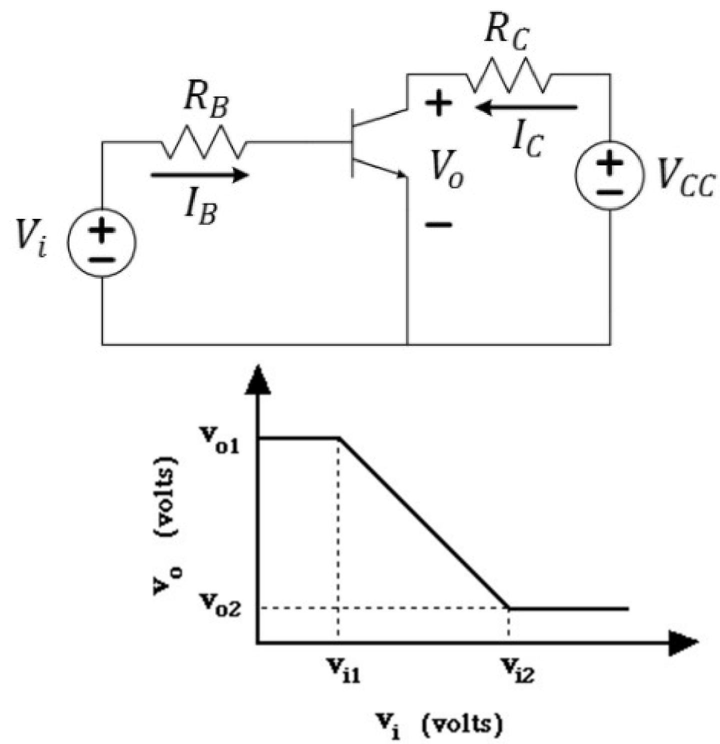
$V_{CE,sat} = 0.2\ V$

$V_{BEon} = 0.7\ V$

$R_C = 6\ k\Omega$

$R_B = 12\ k\Omega$

$\beta = 160$



BJT Example – Transfer Characteristic

Determine $V_{i1}, V_{i2}, V_{o1}, and\ V_{o2}$ and the Gain for the following BJT circuit.

What is V_{o1}?

A. $0.7V$

B. $0.2V$

C. $\boldsymbol{9V}$

V_{o1} = output in Off region

$$V_{o1} = V_{CC} = 9V$$

$V_{CC} = 9\ V$

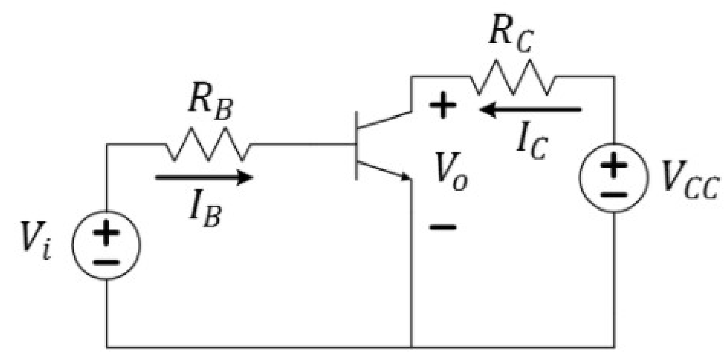
$V_{CE,sat} = 0.2\ V$

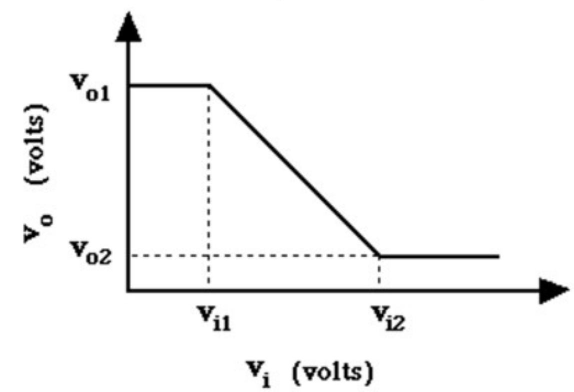
$V_{BEon} = 0.7\ V$

$R_C = 6\ k\Omega$

$R_B = 12\ k\Omega$

$\beta = 160$





BJT Example – Transfer Characteristic

Determine $V_{i1}, V_{i2}, V_{o1}, and\ V_{o2}$ and the Gain for the following BJT circuit.

What is V_{o2}?

A. $0.7V$

B. $\mathbf{0.2V}$

C. $9V$

V_{o2} = output in Saturation region

$$V_{o2} = V_{CE,SAT} = 0.2V$$

$V_{CC} = 9\ V$

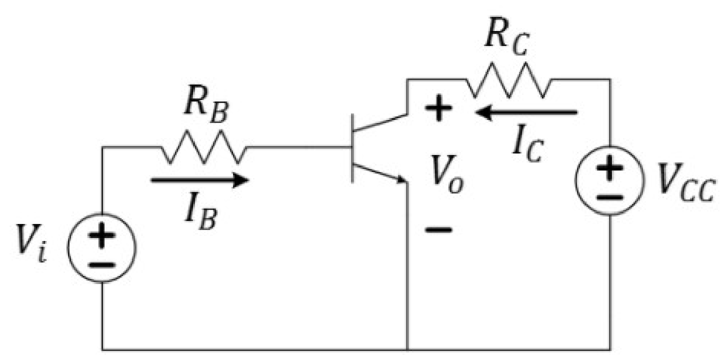
$V_{CE,sat} = 0.2\ V$

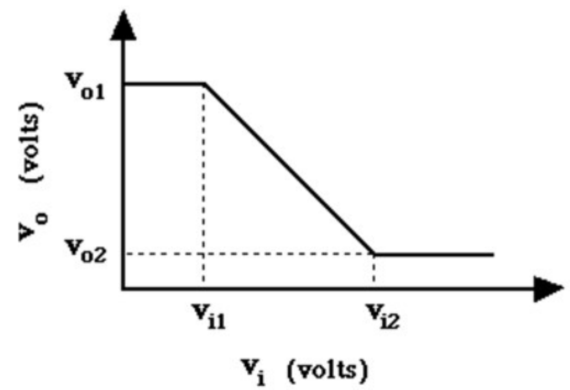
$V_{BEon} = 0.7\ V$

$R_C = 6\ k\Omega$

$R_B = 12\ k\Omega$

$\beta = 160$





BJT Example – Transfer Characteristic

Determine $V_{i1}, V_{i2}, V_{o1}, and\ V_{o2}$ and the Gain for the following BJT circuit.

What is V_{i2}?

A. $\mathbf{0.810}\boldsymbol{V}$

B. $1.111V$

C. $1.909V$

D. $2.000V$

E. True.

V_{i2} is the minimum possible input to reach saturation. Thus, we are in both the Active and Saturation regions. We will use both regions' assumptions.

$$I_C = I_{C,SAT} = \frac{V_{CC} - V_{CE,SAT}}{R_C}$$

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{CE,SAT}}{\beta R_C}$$

$$V_{i2} = V_{BE,ON} + I_B R_B$$

$$V_{i2} = V_{BE,ON} + \left(\frac{R_B}{\beta R_C}\right)(V_{CC} - V_{CE,SAT}) \qquad V_{i2} = 0.81V$$

$V_{CC} = 9\ V$

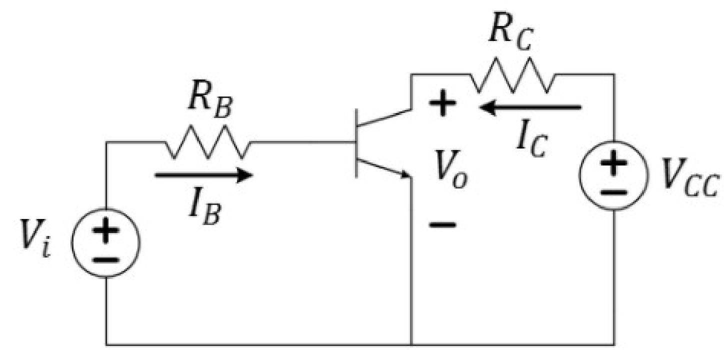
$V_{CE,sat} = 0.2\ V$

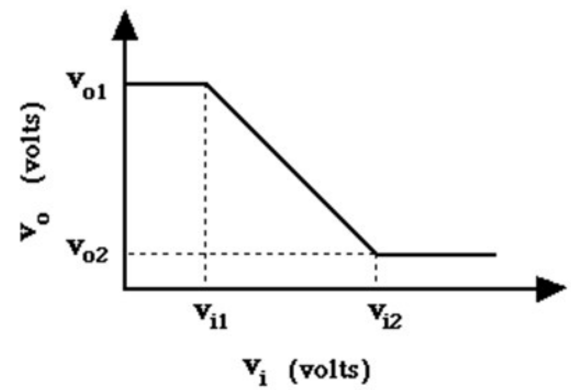
$V_{BEon} = 0.7\ V$

$R_C = 6\ k\Omega$

$R_B = 12\ k\Omega$

$\beta = 160$





BJT Example – Transfer Characteristic

Determine $V_{i1}, V_{i2}, V_{o1}, and\ V_{o2}$ and the Gain for the following BJT circuit.

What is the Gain?

A. $\mathbf{-80\ V/V}$

B. $80V/V$

C. $\frac{-1}{80}V/V$

D. $\frac{1}{80}V/V$

E. This joke is getting old.

$$Gain = \frac{V_{o1} - V_{o2}}{V_{i1} - V_{i2}}$$

$$Gain = -80V/V$$

$V_{CC} = 9\ V$

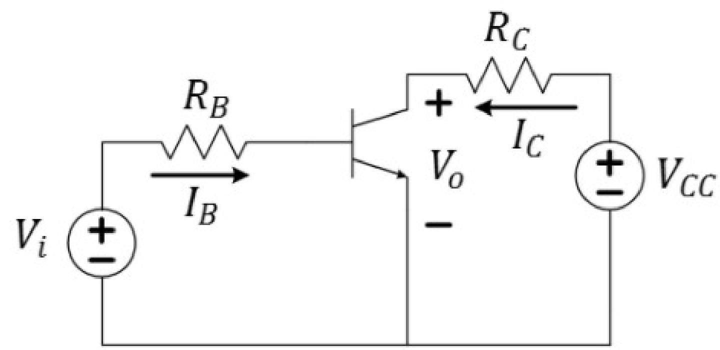
$V_{CE,sat} = 0.2\ V$

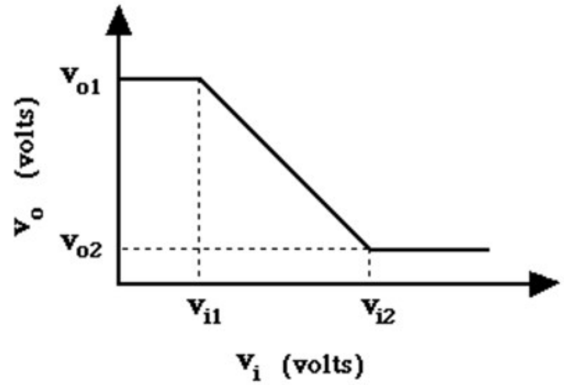
$V_{BEon} = 0.7\ V$

$R_C = 6\ k\Omega$

$R_B = 12\ k\Omega$

$\beta = 160$





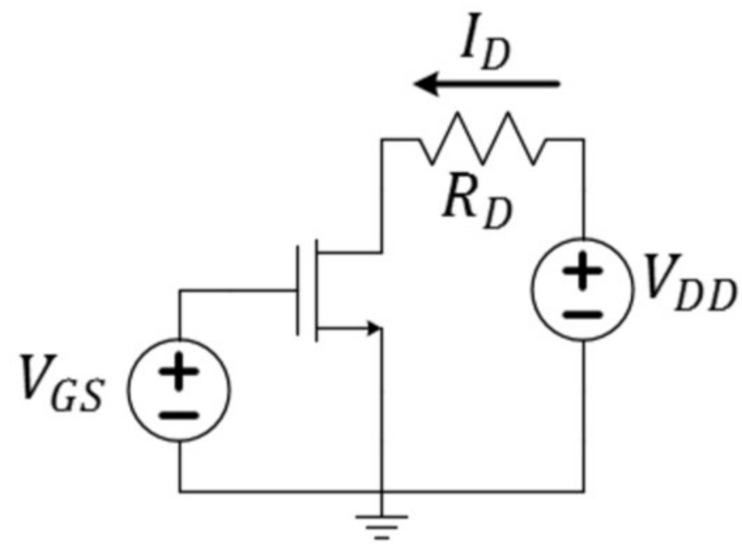
Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET)

- Three terminal device: gate, source, drain
- Comes in two flavors, NMOS and PMOS, more on this in the next slide!
- V_{TH} is a property of the specific MOSFET
- Be comfortable interpreting I-V Characteristic of MOSFET

Conditions	Mode	Behavior under Linear Model
$V_{GS} < V_{TH}$	OFF	$I_D = 0$
$V_{GS} > V_{TH}$ $V_{DS} > V_{GS} - V_{TH}$	ACTIVE	$I_D = k(V_{GS} - V_{TH})^2$
$V_{GS} > V_{TH}$ $V_{DS} < V_{GS} - V_{TH}$	OHMIC	$I_D = k(V_{GS} - V_{TH})V_{DS}$

Solving MOSFET IV Questions

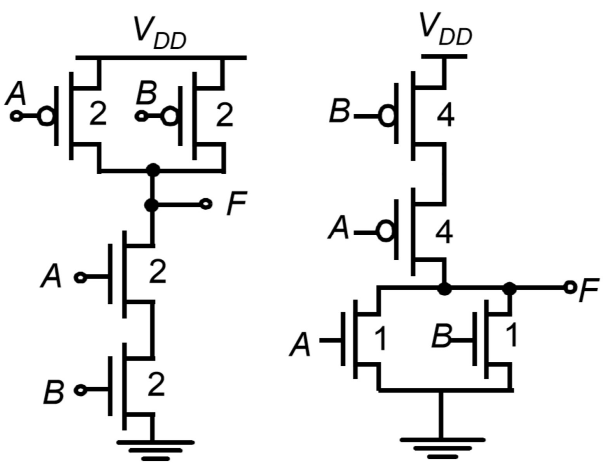
- You are frequently asked to solve for the operating point of a Common-Source MOSFET circuit. The best approach for solving is the graphical method involving the MOSFET IV characteristics.
- Notice that the right side of the circuit, including R_D and V_{DD}, represents a Thevenin equivalent!
- We can use the load-line method to plot the IV characteristics against each other to determine the operating point
- If the operating point is in the active region, the I_D should be easy to read and V_{DS} will follow by KVL.
- If the operating point is in the Ohmic region, we set the Ohmic region current equation equal to the current equation we can derive through KVL.

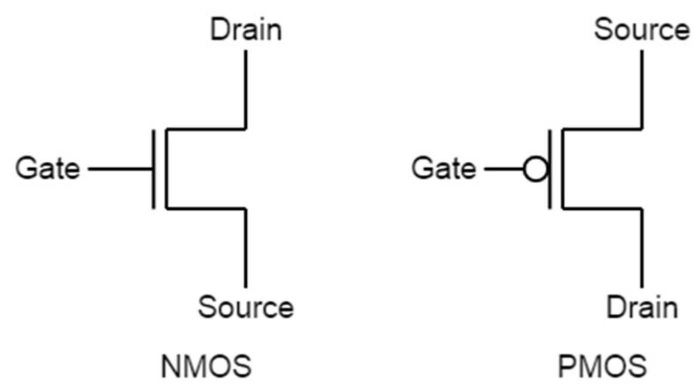


Complementary MOS Logic (cMOS)

- Combine NMOS and PMOS transistors in order to perform a logical operation
  - i.e. AND, NOR, NOT
- NMOS and PMOS are biased differently
  - NMOS, source connects to ground; PMOS, source connects to V_{DD}

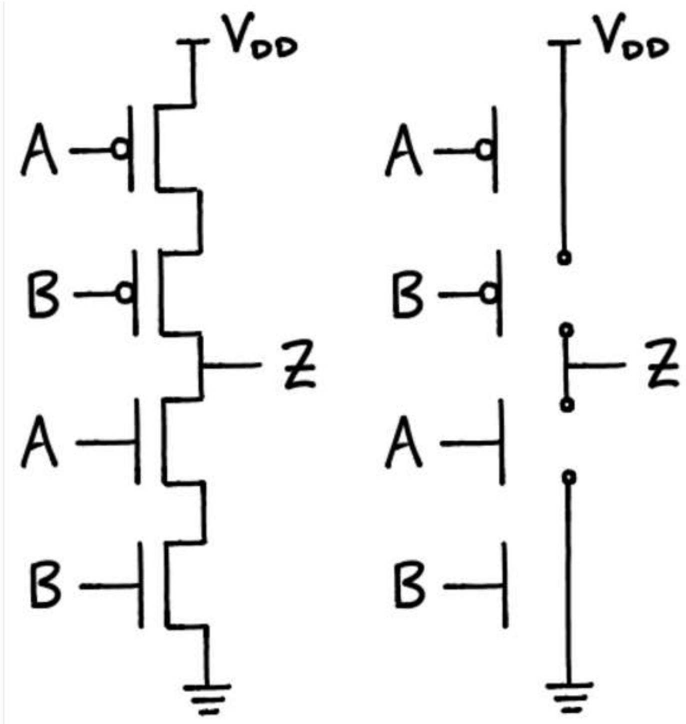
Input (at gate)	nMOS	pMOS
0	Non-conducting	Conducting
1	Conducting	Non-conducting





Improper CMOS

- A CMOS circuit is considered *proper* if:
  - The output can only be connected to one of V_{DD} **or** Ground at a time
- A CMOS circuit is considered *improper* if:
  - The output can be connected to both V_{DD} **and** Ground simultaneously by some input combination
  - The output can be connected to neither V_{DD} **nor** Ground simultaneously by some input combination



CMOS Power

$$P = nafCV_{DD}^2$$

- n = number of capacitors/transistors
- a = activity factor
- f = frequency
- C = capacitance
- V_{DD} = supplied voltage

MOSFET Example – IV Characteristic

The I-V Characteristic of a MOSFET is shown at right. $V_{TH} = 1V, R_D = 400\Omega, V_{DD} = 6V$, and $V_{DS} = 1.2V$.

$V_{GS} - V_{TH} = 3$ curve:

$$45mA = k(3)^2$$

$$\boldsymbol{k = 5mA/V^2}$$

Works for any curve!

I-V Characteristics for V_{DD} and R_D:

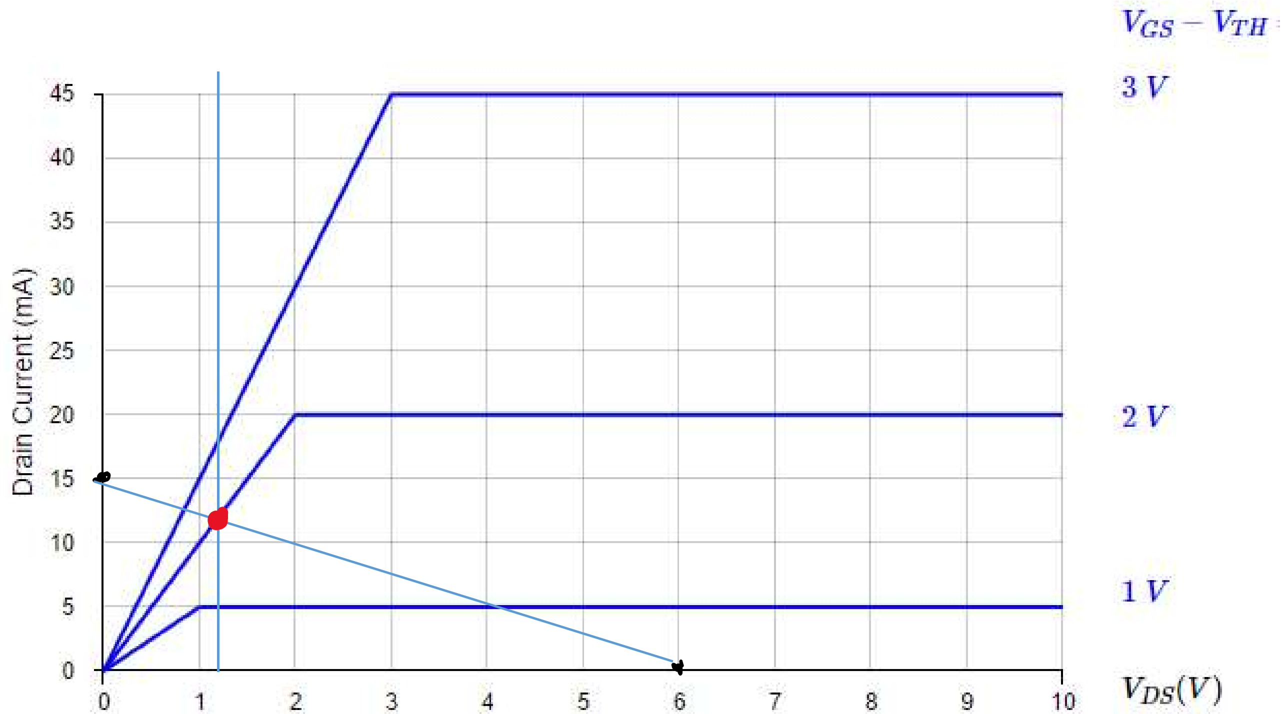
$$V_{oc} = V_{DD} = 6V$$

$$I_{sc} = \frac{V_{DD}}{R_D} = 15mA$$

- What is k?
- What is V_{GS}?

$V_{GS} - V_{TH} = 2\text{V}$

$\boldsymbol{V_{GS} = 3V}$

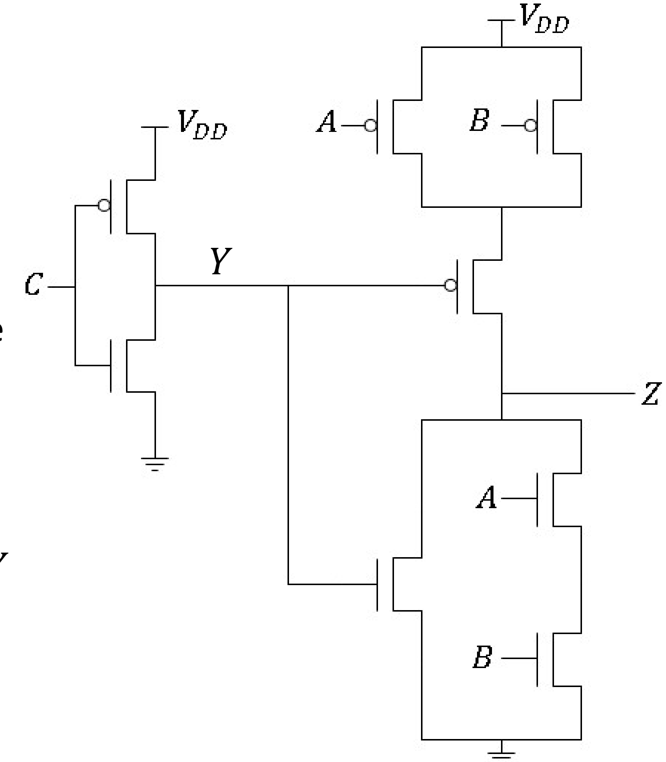


MOSFET Example – CMOS # 1

Fill in the following truth table for the provided cMOS circuit.

ABC	Z
000	0
001	1
010	0
011	1
100	0
101	1
110	0
111	0

This circuit is a two-stage cMOS circuit. We have the first stage on the left with the input C. When C is low, the output of the first stage Y is high; when C is high, Y is low. Y serves as an input to the second stage along with A and B. We obtain a low output Z when either Y is high or A and B are high. Conversely, we obtain a high output Z when Y is low and A or B are low.



MOSFET Example – CMOS #2

What input combination ABC will result in an improper output for Z?

A. 000

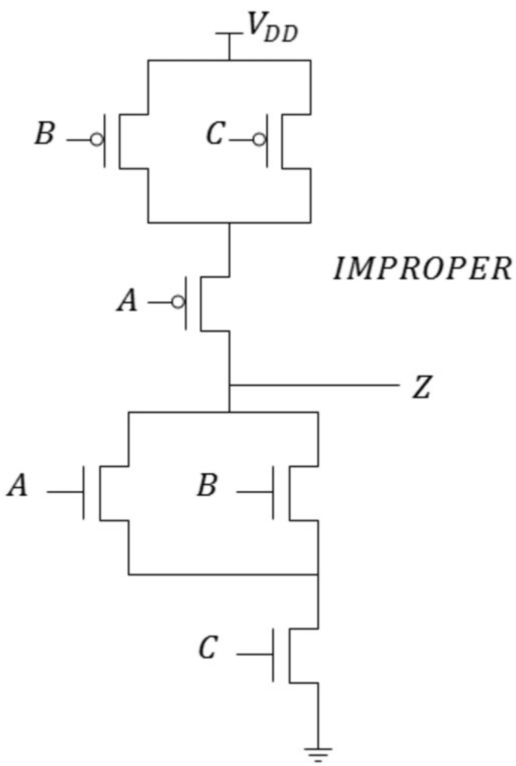
B. 111

C. 011

D. 110

E. 101

When $ABC = 110$, we neither have a path to V_{DD} nor ground.



Signal-to-Noise Ratio (SNR)

- Signal-to-Noise Ratio gives us an expression of the amount of noise distortion in a system
- $SNR = \frac{P_{signal}}{P_{noise}}$
- Remember that the average power for a time-varying signal requires V_{rms}
- $P_{avg} = \frac{V_{rms}^2}{R}$
- We can also express SNR as follows
- $SNR = \frac{P_{signal}}{P_{noise}} = \frac{\frac{V_{rms,signal}^2}{R}}{\frac{V_{rms,noise}^2}{R}} = \left(\frac{V_{rms,signal}}{V_{rms,noise}}\right)^2$

SNR Example

A sinusoidal signal is measured across a 50Ω resistor. If the sinusoidal signal component is given by $s(t) = 16\sin(\omega t)\,V$, what rms noise voltage $V_{rms,noise}$ must also be present to give a signal-to-noise ratio of 8?

A. 2V

B. 4V

C. 6V

D. 8V

E. I'm good. Thanks.

$$V_{rms,signal} = \frac{A}{\sqrt{2}} = 8\sqrt{2}$$

$$SNR = \left(\frac{V_{rms,signal}}{V_{rms,noise}}\right)^2$$

$$V_{rms,noise} = \frac{V_{rms,signal}}{\sqrt{SNR}} = \frac{8\sqrt{2}}{2\sqrt{2}}$$

$$V_{rms,noise} = 4V$$

Sampling and Quantization

- When we convert from a continuous-time analog signal (function of time) to a discrete-time digital signal (function of n), we are performing sampling, or Analog/Digital (A/D) conversion
- We relate the sampled (digital) signal to the original (analog) signal by:

$$x[n] = x(nT_s)$$

- n is the index for the "nth" sample, T_s is the sampling period
- In order to retain all the information in the original signal, we must sample above the Nyquist Rate

$$f_{Nyquist} = 2f_{max}$$

Sampling and Quantization

- We cannot store every single sampled value exactly since computers do not have infinite storage!
- Therefore, we must quantize our digital values to a finite set of representative values
- Each quantized level is represented by a bit pattern
  - $Number\ of\ levels = 2^n, n = number\ of\ bits$
- For example, if I know my digitized signal is bounded in the range $[-2,1.5]V$ and I want to quantize eight unique values, I will assign them as follows

$$\begin{array}{ccc} 1 & 1 & 1 = 1.5V \\ 1 & 1 & 0 = 1.0V \\ 1 & 0 & 1 = 0.5V \\ 1 & 0 & 0 = 0.0V \\ 0 & 1 & 1 = -0.5V \\ 0 & 1 & 0 = -1.0V \\ 0 & 0 & 1 = -1.5V \\ 0 & 0 & 0 = -2.0V \end{array}$$

Quantization Example

What is the minimum number of bits per sample needed to digitize a 0 to 8 volt signal if the quantization error (half of the level spacing) should be less than 0.2 volts?

A. 3

B. 4

C. 5

D. 6

E. A lot.

Remember that maximum quantization error is the half-level spacing because voltages halfway between levels will maximize the shortest distance to a level. The number of spaces is as follows:

$$\#spaces = \frac{8V}{\frac{0.4V}{space}} = 20$$

Remember that if we have L spaces, we have $L + 1$ levels. Thus, we have 21 levels.

If we have N bits, we can encode 2^N levels. So, we need to find the minimum number of bits to represent at least 21 levels. This would be 5 bits.

Entropy

- Entropy represents the amount of information or uncertainty in a system.
- “Bits/symbol”
- In information theory, we refer to this as the minimum amount of information that must be retained for the signal or system to be accurately represented.
  - In other words, if we compress a signal below entropy, we cannot completely recover it upon decompression

$$Entropy = H = \sum_{i \in symbols} p_i \log_2\left(\frac{1}{p_i}\right)$$

Compression and Huffman Encoding

- $Data\ Compression\ Ratio = \frac{original\ data\ rate}{compressed\ data\ rate}$
- $Savings = 1 - \frac{1}{DCR}$
- There are two types of compression, lossless and lossy
  - Example of lossless compression: PNG
  - Examples of lossy compression: JPEG, MPEG, mp3, mp4, H.264, HEVC/H.265,
- Huffman Encoding
  - Used for Lossless compression
  1. Pick two least frequent elements
  2. Pair these elements, pick a standard (bigger on the left or right)
  3. Eliminate paired elements, replace with new combined node
  4. Repeat 1-4 until you reach root node (sum is 1)
  5. Label left branches 1's, right branches 0's (technically the opposite works too, again be consistent!)

Entropy and Compression Example

Consider the set of symbols $X_1, X_2, \dots, X_5$ with probabilities $\frac{1}{16}, \frac{1}{8}, \frac{1}{8}, \frac{3}{16}, \frac{1}{2}$.

If we were to use the same number of bits per symbol for all symbols in this set, how many bits would we need for each symbol?

A. 1

B. 2

C. 3

D. 4

E. I don't want to talk about it.

We have 5 symbols. Remember that N bits can encode 2^N levels. Thus, 3 symbols will provide 8 symbols, which will be enough to encode this symbol set.

Entropy and Compression Example

Consider the set of symbols $X_1, X_2, \dots, X_5$ with probabilities $\frac{1}{16}, \frac{1}{8}, \frac{1}{8}, \frac{3}{16}, \frac{1}{2}$.

What is the entropy of this symbol set?

A. 1.000

B. 1.953

C. 2.163

D. 2.500

E. I plead the fifth.

$$H = \sum_{i=1}^{N} p_i \log_2 \frac{1}{p_i}$$

$$H = \left(\frac{1}{16}\right)\log_2 16 + 2\left(\frac{1}{8}\right)\log_2 8 + \left(\frac{3}{16}\right)\log_2 \frac{16}{3} + \left(\frac{1}{2}\right)\log_2 2$$

$$H = 1.9528\ bits$$

Entropy and Compression Example

Consider the set of symbols $X_1, X_2, \dots, X_5$ with probabilities $\frac{1}{16}, \frac{1}{8}, \frac{1}{8}, \frac{3}{16}, \frac{1}{2}$.

What is the maximum possible compression ratio for this system and the corresponding savings?

The maximum compression ratio will be the ratio between the worst-case and best-case bit rates. The worst-case is giving the same number of bits to each symbol, while the best case is entropy.

So…

$$DCR_{max} = \frac{3}{1.953}$$

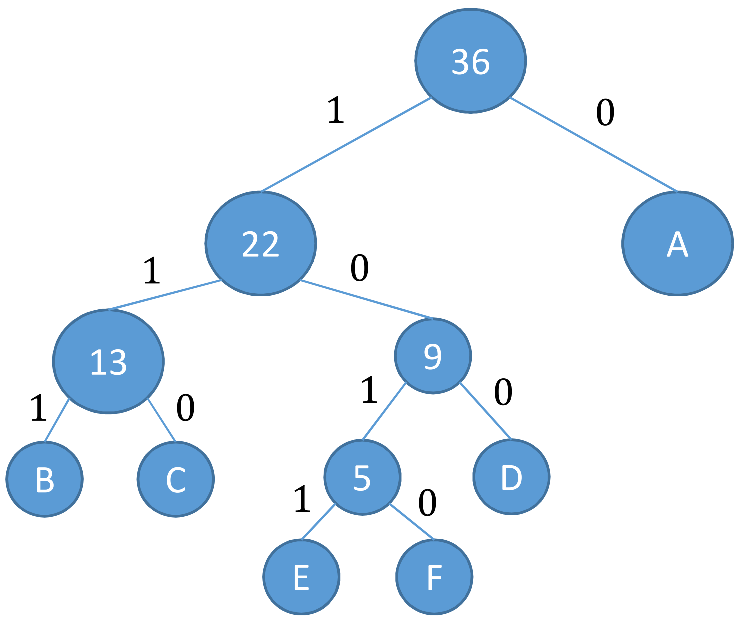
$$\boldsymbol{DCR_{max} = 1.536}$$

Huffman Coding Example

Suppose we have a collection of symbols with the following frequencies.

- A = 14
- B = 7
- C = 6
- D = 4
- E = 3
- F = 2

$$
\begin{aligned}
A &= 0\\
B &= 111\\
C &= 110\\
D &= 100\\
E &= 1011\\
F &= 1010
\end{aligned}
$$



Form the corresponding Huffman Tree and Huffman Code

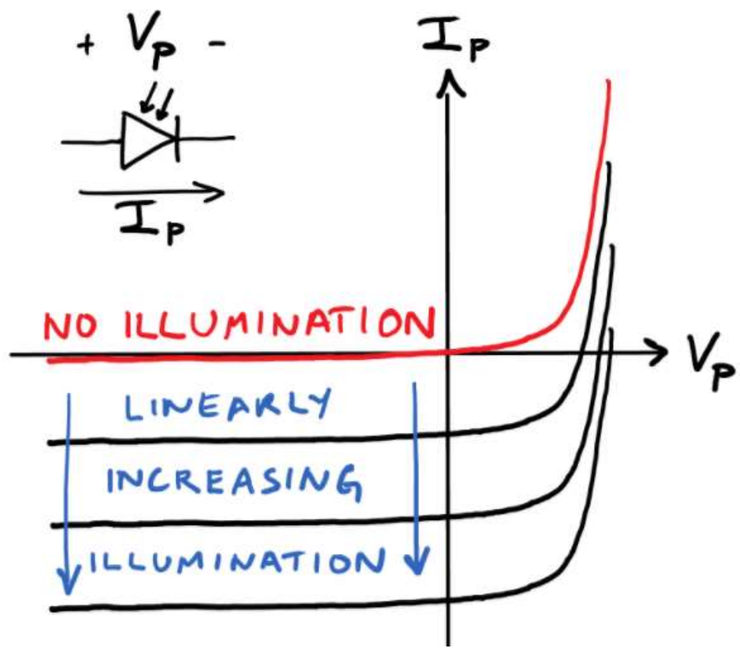
Solar Cells and Photodiodes

- Solar cells are used to turn solar energy into electrical energy
- You are not expected to understand the underlying device physics. Instead, understand the following relations

$$E_{photon}[eV] = \frac{1240}{\lambda\ [nm]}$$

$$Photon\ Emission\ Rate = \frac{Output\ Power}{E_{photon}}$$

- Photons are absorbed only if their energy is greater than or equal to the bandgap energy
- Photodiodes have different V_{oc} and I_{sc} for different levels of illumination
  - $P_{max} = FillFactor * I_{sc}V_{oc}$



Photodiode Example

We have three identical and equally illuminated photodiodes each with $V_{oc} = 4V$ and $I_{sc} = 1A$

If we were to connect these photodiodes together in series, what would be the new V_{oc} and I_{sc} of the resulting combination?

A. $V_{oc} = 4V, I_{sc} = 3A$

B. $V_{oc} = 4V, I_{sc} = 1A$

C. $V_{oc} = 12V, I_{sc} = 3A$

D. $\boldsymbol{V_{oc} = 12V, I_{sc} = 1A}$

E. Sorry, I was looking at my phone. What's up?

Current is shared in series and voltage adds in series.

So, we add the V_{oc}'s and use the same I_{sc}.

Photodiode Example

We have three identical and equally illuminated photodiodes each with $V_{oc} = 4V$ and $I_{sc} = 1A$

If we were to connect these photodiodes together in parallel, what would be the new V_{oc} and I_{sc} of the resulting combination?

A. $\boldsymbol{V_{oc} = 4V, I_{sc} = 3A}$
B. $V_{oc} = 4V, I_{sc} = 1A$
C. $V_{oc} = 12V, I_{sc} = 3A$
D. $V_{oc} = 12V, I_{sc} = 1A$
E. Are we almost done?

Voltages are shared in parallel and currents are added in parallel.

Thus, we will use the same V_{oc} and add the I_{sc}'s

Thank You and Good Luck!