ECE 329 Midterm 3 - HKN Review Session

April 15, 2017

Topics Covered

Lectures 23 to 33

1. Wave equations in material media
2. Wave polarization
3. Reflection and transmission
4. Transmission lines

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f \mu\sigma}$	$\sim \sqrt{\pi f \mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	45°	$\sim \frac{2\pi}{\sqrt{\pi f \mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f \mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

Figure: Wave properties under different materials

Wave Equations in Material Media

- The propagation velocity of waves in different media is frequency dependent:

$$v_p = \frac{\omega}{\beta}$$

- From the wave equation, we can get a general solution for a x-polarized wave:

$$E_x = E_0 e^{\mp \alpha z} e^{\mp \beta z}$$

- Penetration depth, or skin depth:

$$\delta = \frac{1}{\alpha}$$

- Wavelength:

$$\lambda = \frac{2\pi}{\beta}$$

Wave Properties in Material Media

$$\bar{\gamma} = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)^{1/2}$$

attenuation

$$e^{-\alpha z}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)^{1/2}$$

velocity

$$v_p = \omega/\beta$$

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Wave Polarization

- Right-circular polarization: $cos(\omega t)\hat{x} + sin(\omega t)\hat{y} \Longleftrightarrow \hat{x} - j\hat{y}$. The x-component leads the y-component. This is the direction your right-hand fingers curl when your thumb is pointed in the z-direction.
- Left-circular polarization: $cos(\omega t)\hat{x} - sin(\omega t)\hat{y} \Longleftrightarrow \hat{x} + j\hat{y}$. The x-component lags the y-component. This is the direction your left-hand fingers curl when your thumb is pointed in the z-direction.

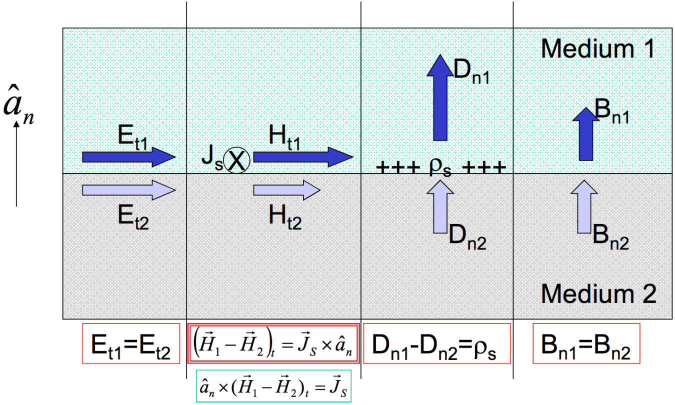
$$\mathbf{E_1} = \hat{x}\ \cos(\omega t \mp \beta z + \theta_1) \qquad \mathbf{E_2} = \hat{y}\ \cos(\omega t \mp \beta z + \theta_2)$$

$$\mathbf{E} = a\mathbf{E_1} + b\mathbf{E_2}$$

- If $\theta_1 - \theta_2 = 0$, E is linearily polarized. If $\theta_1 - \theta_2 = \pm\frac{\pi}{2}$, then E is circularly polarized. Else, it is elliptically polarized.

Reflection and Transmission

- As always, boundary conditions are your friend. The typical problem of normal incidence plane wave travelling from one region to the other has the following boundary conditions in place:



Reflection and Transmission

- If the wave encounters a PEC, it will be completely reflected. Total reflection of a normal planar boundary produces a standing wave which carries no net energy.
- If the wave encounters a region with matched impedance, it is completely transmitted.

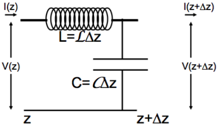
$$\Gamma = \frac{\overline{E}_1^-}{\overline{E}_1^+} = \frac{\overline{\eta}_2 - \overline{\eta}_1}{\overline{\eta}_1 + \overline{\eta}_2}$$

$$\tau = \frac{\overline{E}_2^+}{\overline{E}_1^+} = 1 + \Gamma = \frac{2\overline{\eta}_2}{\overline{\eta}_1 + \overline{\eta}_2}$$

Transmission Lines

- At microwave frequencies, lumped elements cannot be used. Transmission lines are a good alternative. If you find this material interesting, look into ECE 447, 451, and 457.
- The transmission line can be modelled as such:

One slice of the TL:



$$V(z+\Delta z)-V(z)=-L\frac{\partial I}{\partial t} \qquad I(z)-I(z+\Delta z)=C\frac{\partial V}{\partial t}$$

$$\therefore \frac{\partial V}{\partial z}=-\mathcal{L}\frac{\partial I}{\partial t} \qquad \therefore \frac{\partial I}{\partial z}=-\mathcal{C}\frac{\partial V}{\partial t}$$

$$V=V^{+}+V^{-}$$

$$I=\frac{1}{Z_0}(V^{+}-V^{-})=I^{+}+I^{-}$$

$$I^{+}=\frac{V^{+}}{Z_0} \qquad I^{-}=-\frac{V^{-}}{Z_0}$$

Transmission Lines

- The reflection coefficient for a transmission line terminated with a load is shown on the left. The current reflection coefficient is on the right (note the negative sign - this will be important in bounce diagrams).

$$\Gamma = \frac{V^-}{V^+} = \frac{R_L - Z_0}{R_L + Z_0} \qquad \frac{I^-}{I^+} = \frac{-(V^- / Z_0)}{(V^+ / Z_0)} = -\frac{V^-}{V^+} = -\Gamma$$

- The power relates to the incident, reflected, and transmitted voltages as:

$$P_{incident} = V^+ I^+$$

$$P_{reflected} = V^- I^-$$

$$P_{transmitted} = V^{++} I^{++}$$

Bounce Diagrams

- Bounce diagrams are a good way of representing voltage and current propagation across a transmission line. The following equation is used to model the voltage at each moment of time. Note that the period, T, is 2 times the length of the line divided by the propagation velocity.

$$
\begin{aligned}
V(z,t) &= \tau_g \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v}) \\
&+ \tau_g \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})
\end{aligned}
$$

$$
\begin{aligned}
I(z,t) &= \tfrac{\tau_g}{Z_o} \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v}) \\
&- \frac{\tau_g}{Z_o} \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})
\end{aligned}
$$

Shorted Stubs

- By terminating a transmission line with a short or open stub, we can get different input impedances depending on the frequency and the length of the line.
- The shorted line forms standing voltage waves on the line. At even integers of $\frac{\lambda}{4}$, the line is seen as a short. At odd integers, it is seen as an open.

$$Z_{in} = jZ_0 \tan \beta l$$

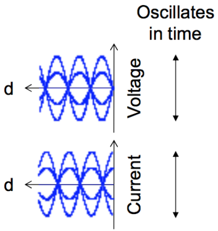
If tan(βl)>0, shorted TL is inductive

- If tan(βl)<0, shorted TL is capacitive

$$V(d,t) = -2|V^+|\sin(\beta d)\sin(\omega t + \theta)$$

$$I(d,t) = 2Y_0|V^+|\cos(\beta d)\cos(\omega t + \theta)$$

V(0,t)=0 always (voltage null)
I(0,t) varies (current maxima)



Open Stubs

- A similar response happens with an open stub.
- At even integers of $\frac{\lambda}{4}$, the line is seen as an open. At odd integers, it is seen as a short.

$$Y_{in} = jY_0 \tan \beta l$$

- If tan(βl)<0, shorted TL is inductive

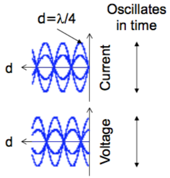
  If tan(βl)>0, shorted TL is capacitive

Same phasor algebra as before with current & voltage reversed!

$\beta = 2\pi/\lambda$

$$I(d,t) = -2Y_0 \left|V^+\right| \sin(\beta d)\sin(\omega t + \theta)$$

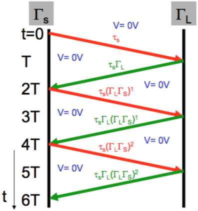
$$V(d,t) = 2\left|V^+\right| \cos(\beta d)\cos(\omega t + \theta)$$



I(0,t)=0 always (current null)
V(0,t) varies (voltage maxima)

More Notes on Bounce Diagrams

- At steady-state, a transmission line is simply a short.
- If the source has a time-shifted response, for example $f(t) = u(t-2)$, make sure to shift the bounce diagram up or down accordingly.
- When asked about the voltage or current from a bounce diagram at a certain time or position along the line, draw a vertical line (for position) or a horizontal line (for time) and add the values you 'see' at the point. An example will be discussed in the next.
- Fall 2012 and Spring 2013 exams cover transmission line reflections pretty well.



More notes on Power Conversion

- For problems with a plane-wave incident on an interface, the sum of the transmitted and reflected power equals that of the incident power.

NOTE! on Formula sheet:

$$\Gamma^2 + \frac{\eta_1}{\eta_2}\tau^2 = 1 \quad ^2 \quad \Rightarrow \quad \frac{\langle S_r \rangle}{\langle S_i \rangle} + \frac{\langle S_t \rangle}{\langle S_i \rangle} = \frac{\langle S_i \rangle}{\langle S_i \rangle}$$

- Go over microwave resonators from Lecture 33. Fall 2012 Problem 3 has a good problem on that topic.