1) C = 1/3 , Mean = 0 , Variance = 5/6

①

$$f(x) = \begin{cases} c(2+x) & -2 \le x \le -1 \\ c & -1 < x < 1 \\ c(2-x) & 1 \le x \le 2 \end{cases}$$

A> Integrate over the interval and equal to 1

$$\int_{-2}^{-1} c(2+x)\,dx + \int_{-1}^{1} c\,dx + \int_{1}^{2} c(2-x)\,dx = 1$$

$$c\left(2x + \frac{x^2}{2}\right)\Big]_{-2}^{-1} + cx\Big]_{-1}^{1} + c\left(2x - \frac{x^2}{2}\right)\Big]_{1}^{2} = 1$$

$$c\left(-2 + \frac{1}{2} + 4 - 2\right) + c(2) + c\left(4 - 2 - 2 + \frac{1}{2}\right) = 1$$

$$3c = 1 \qquad \boxed{c = \frac{1}{3}}$$

B> Mean and Variance

$$\text{Mean} = \int x f(x)\,dx = \int_{-2}^{-1} \frac{1}{3} \times x \times (2+x)\,dx + \int_{-1}^{1} \frac{1}{3} x\,dx + \int_{1}^{2} \frac{1}{3} \times x(2-x)\,dx$$

$$= \frac{1}{3}\left[x^2 + \frac{x^3}{3}\right]_{-2}^{-1} + \frac{1}{3}\frac{x^2}{2}\Big]_{-1}^{1} + \frac{1}{3}\left[x^2 - \frac{x^3}{3}\right]_{1}^{2}$$

$$= -\frac{1}{3}\left[4 - \frac{8}{3} - 1 + \frac{1}{3}\right] + \frac{1}{3}\left[4 - \frac{8}{3} - 1 + \frac{1}{3}\right]$$

$$\boxed{\text{Mean} = -\frac{2}{9} + \frac{2}{9} =} \qquad \boxed{\text{Mean} = 0}$$

$$\text{Variance} = \int (x-\mu)^2 f(x)\,dx$$

$$= \int x^2 f(x)\,dx - \mu^2$$

$$= \int_{-2}^{-1} \frac{1}{3} x^2 (2+x)\,dx + \int_{-1}^{1} \frac{1}{3} x^2\,dx + \int_{1}^{2} \frac{1}{3} x^2 (2-x)\,dx$$

$$= \frac{1}{3}\left[\frac{2x^3}{3} + \frac{x^4}{4}\right]_{-2}^{-1} + \frac{1}{3}\frac{x^3}{3}\Big]_{-1}^{1} + \frac{1}{3}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_{1}^{2}$$

$$= \frac{1}{3}\left[\frac{-2}{3} + \frac{1}{4} + \frac{16}{3} - \frac{16}{4}\right]$$

$$+ \frac{1}{3}\left[\frac{1}{3} + \frac{1}{3}\right]$$

$$+ \frac{1}{3}\left[\frac{16}{3} - \frac{16}{4} - \frac{2}{3} + \frac{1}{4}\right]$$

$$\boxed{\text{Variance} = \frac{5}{6}}$$

2) C = ¾ , Mean = 0 , Variance = 1/5

② $f(x) = c(1-x^2) \quad -1 < x < 1$

A) Integrate over the interval and equal to 1

$$\int f(x)\,dx = 1 \qquad \int_{-1}^{1} c(1-x^2)\,dx = 1$$

$$c\left(x - \frac{x^3}{3}\right)\Bigg]_{-1}^{1} = 1$$

$$c\left(1 - \frac{1}{3}\right) - c\left(-1 + \frac{1}{3}\right) = 1$$

$$\boxed{c = \frac{3}{4}}$$

$$f(x) = \frac{3}{4}(1-x^2) \quad -1<x<1$$

B) Mean

$$\bar{x} = \int x f(x)\,dx = \int_{-1}^{1} x \times \frac{3}{4}(1-x^2)\,dx$$

$$\frac{3}{4}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1} = \frac{3}{4}\left[\frac{1}{2} - \frac{1}{4} - \frac{1}{2} + \frac{1}{4}\right]$$

$$\boxed{\bar{x} = 0}$$

$$\text{Variance} = \int (x-\mu)^2 f(x)\,dx = \int x^2 f(x)\,dx - \mu^2$$

$$= \int_{-1}^{1} x^2 \frac{3}{4}(1-x^2)\,dx$$

$$= \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5}\right]$$

$$\boxed{\text{Variance} = \frac{1}{5}}$$

3) C = 2/3, Mean = 7/9, Variance = 37/162

3) A)

$$f(x) = \begin{cases} c & 0 \le x < 1 \\ c(2-x) & 1 \le x \le 2 \end{cases}$$

$$\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx = 1$$

$$cx\Big]_0^1 + c\left(2x - \frac{x^2}{2}\right)\Big]_1^2 = 1$$

$$c + c\left(4 - 2 - 2 + \frac{1}{2}\right) = 1$$

$$\boxed{c = \frac{2}{3}}$$

B) Mean

$$\bar{x} = \int x f(x)\,dx = \int_0^1 \frac{2}{3} x\,dx + \int_1^2 \frac{2}{3}(2-x)x\,dx$$

$$= \frac{2}{3}\frac{x^2}{2}\Big]_0^1 + \frac{2}{3}\left[x^2 - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \frac{2}{3}\left[4 - \frac{8}{3} - 1 + \frac{1}{3}\right]$$

$$\boxed{\bar{x} = \frac{7}{9}}$$

Variance

$$= \int (x-\mu)^2 f(x)\,dx = \int x^2 f(x)\,dx - \mu^2$$

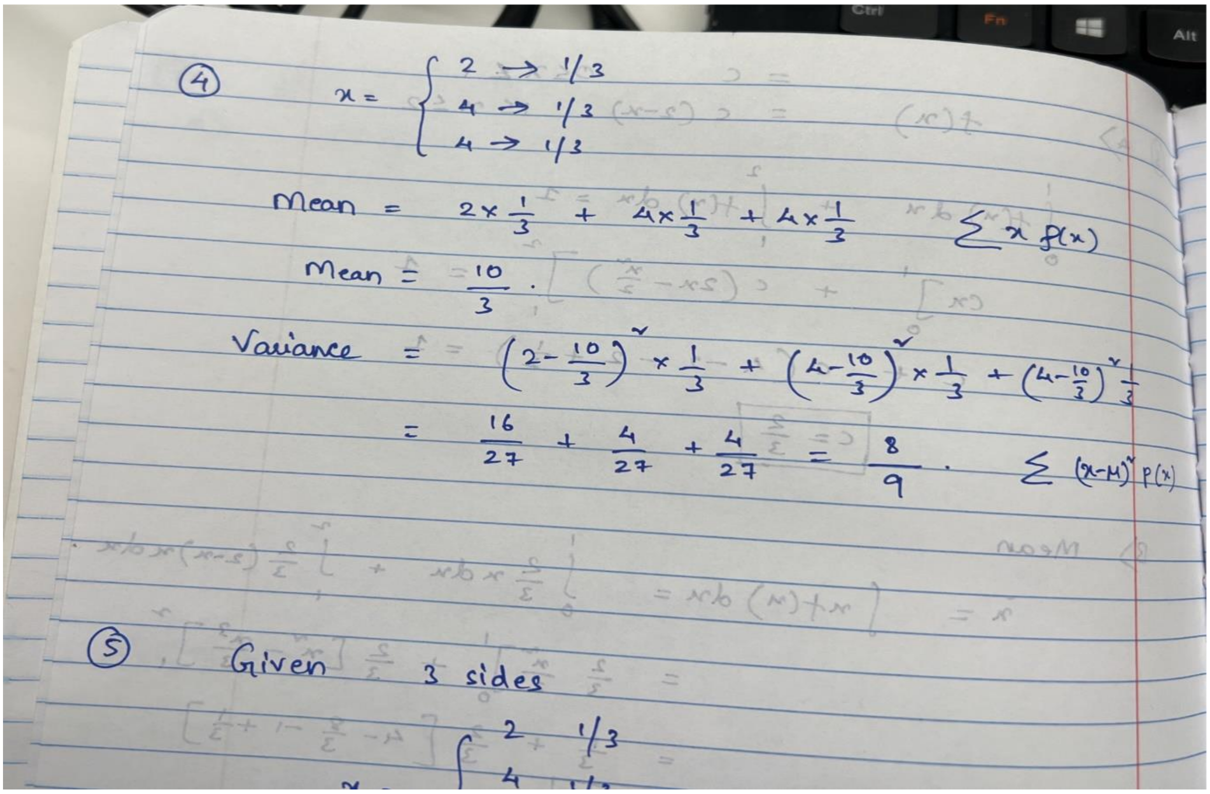
$$= \int_0^1 \frac{2}{3} x^2\,dx + \int_1^2 \frac{2}{3}(2-x)\,x^2\,dx - \left(\frac{7}{9}\right)^2$$

$$= \frac{2}{3}\left[\frac{x^3}{3}\right]_0^1 + \frac{2}{3}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2 - \left(\frac{7}{9}\right)^2$$

$$= \frac{2}{3} \times \frac{1}{3} + \frac{2}{3}\left[\frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4}\right] - \left(\frac{7}{9}\right)^2$$

$$\frac{5}{6} - \left(\frac{7}{9}\right)^2 = \boxed{\frac{37}{162} = \text{Variance}}$$

4) Mean = 10/3, Variance = 8/9



④ $x = \begin{cases} 2 \rightarrow 1/3 \\ 4 \rightarrow 1/3 \\ 4 \rightarrow 1/3 \end{cases}$

Mean $= 2 \times \frac{1}{3} + 4 \times \frac{1}{3} + 4 \times \frac{1}{3}$ $\quad \sum x f(x)$

Mean $= \frac{10}{3}$.

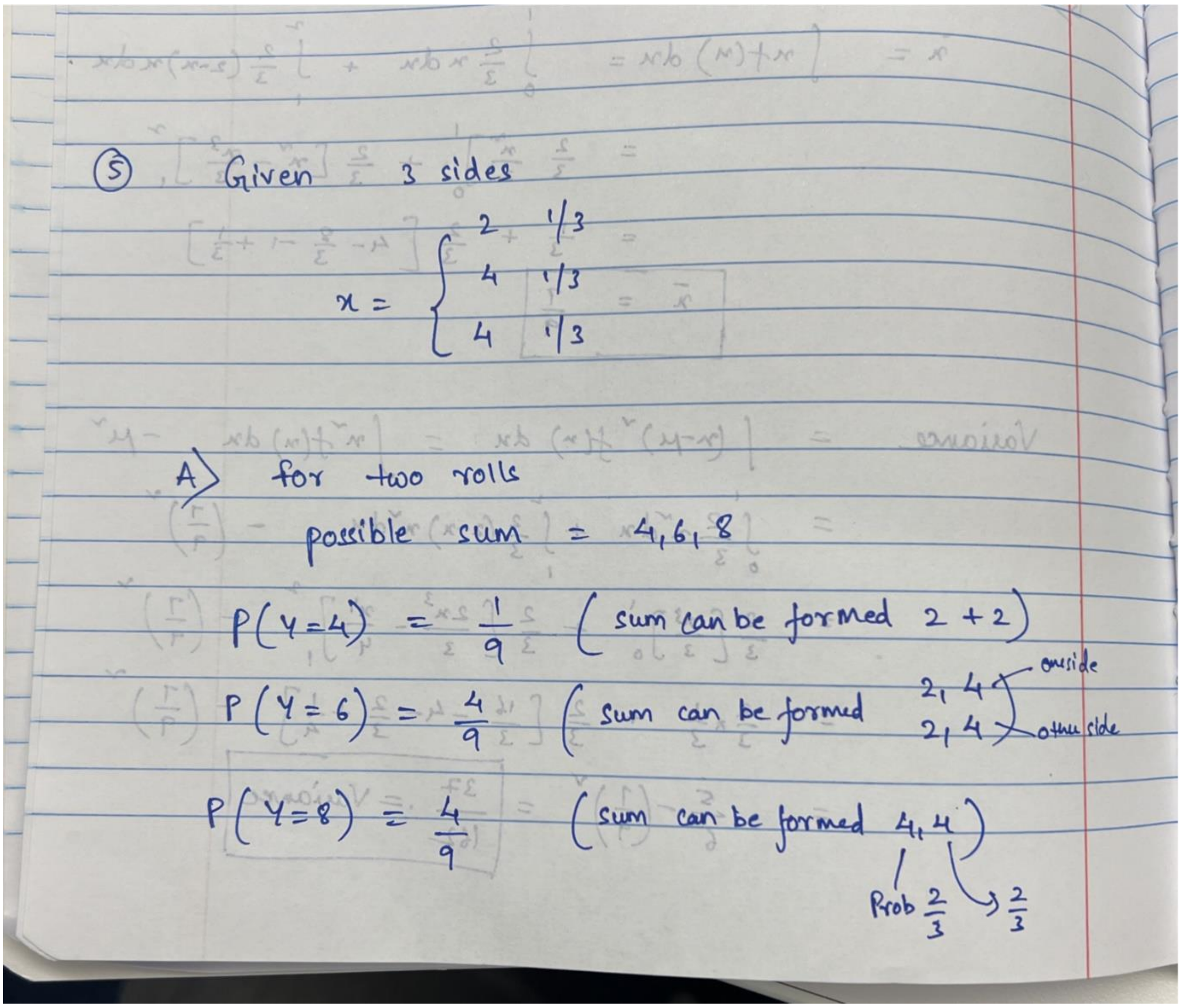
Variance $= \left(2 - \frac{10}{3}\right)^2 \times \frac{1}{3} + \left(4 - \frac{10}{3}\right)^2 \times \frac{1}{3} + \left(4 - \frac{10}{3}\right)^2 \frac{1}{3}$

$= \frac{16}{27} + \frac{4}{27} + \frac{4}{27} = \frac{8}{9}$. $\quad \sum (x-\mu)^2 p(x)$

⑤ Given 3 sides

$\begin{cases} 2 & 1/3 \\ 4 & 1/3 \end{cases}$

5) A) PMF -> {4: 1/9, 6:4/9, 8:4/9}



⑤ Given 3 sides

$x = \begin{cases} 2 & 1/3 \\ 4 & 1/3 \\ 4 & 1/3 \end{cases}$

A) for two rolls

possible sum $= 4, 6, 8$

$P(Y=4) = \frac{1}{9}$ (sum can be formed $2+2$)

$P(Y=6) = \frac{4}{9}$ (sum can be formed $2, 4$ one side, $2, 4$ other side)

$P(Y=8) = \frac{4}{9}$ (sum can be formed $4, 4$)

Prob $\frac{2}{3}$ $\quad \frac{2}{3}$

B) PMF {6:1/27, 8: 6/27, 10:12/27 , 12:8/27 }

Y	P(Y
4	1/9
6	4/9
8	4/9

B) find pmf of y for thue rolls of the die

possible sum = 6, 8, 10, 12

$P(Y=6) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

(2) (2) (2)

$P(Y=8) = \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \times 3 = \frac{2}{27} \times 3 = \frac{6}{27}$

[(2) (4) (2)] × 3 combinations

$P(Y=10) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \times 3 = \frac{4}{27} \times 3 = \frac{12}{27}$

[(4) (4) (2)] × 3 combinations

$P(Y=12) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

(4) (4) (4)

Y	P(Y)
6	1/27
8	6/27
10	12/27
12	8/27

6) C = 1/64, Mean = 16/5, Variance = 32/75

⑥ $f(x) = Cx^3 \quad 0 \le x \le 4$

A) Integrate over the range.

$$\int_0^4 Cx^3\,dx = 1$$

$$\left[\frac{Cx^4}{4}\right]_0^4 = 1$$

$$\boxed{C = \frac{1}{64}}$$

B) Mean

$$\bar{x} = \int x f(x)\,dx = \int_0^4 x \times Cx^3\,dx = \frac{1}{64}\left[\frac{x^5}{5}\right]_0^4$$

$$\boxed{\bar{x} = \frac{16}{5}}$$

$$\text{Variance} = \int (x-\mu)^2 f(x)\,dx$$

$$= \int_0^4 \left(x - \frac{16}{5}\right)^2 f(x)\,dx = \int_0^4 x^2 \times \frac{1}{64} \times x^3\,dx - \left(\frac{16}{5}\right)^2$$

$$= \frac{1}{64} \times \left.\frac{x^6}{6}\right]_0^4 - \left(\frac{16}{5}\right)^2$$

$$= \frac{1}{64} \times \frac{4^6}{6} - \left(\frac{16}{5}\right)^2$$

$$= \frac{32}{3} - \left(\frac{16}{5}\right)^2$$

$$= \frac{32}{3} - \frac{256}{25} = \frac{800-768}{75}$$

$$= 32/75$$

$$\boxed{\text{Variance} = \frac{32}{75}}$$

7) Part A : R (.rmd , knitted pdf) and Stata(.do and .log) files uploaded
   Part B : Calculation below, similarly calculated the same in R

```
```{r 7A}

sample = c(1,2,3,4,5)

n = length(sample)

prob = c(1/15, 2/15, 3/15, 4/15, 5/15)

mean_distribution = sum(sample*prob)

print('mean_distribution = ')
print(mean_distribution)

var_distribution = sum(((sample- mean_distribution)^2)*prob)

print('var_distribution = ')
print(var_distribution)



```

 [1] "mean_distribution = "
 [1] 3.666667
 [1] "var_distribution = "
 [1] 1.555556


```{r 7B}
#Question 7B
mean_error = mean_distribution - mean_dice_value
print('Error or difference in mean of distribution and mean of sample drawn    = ')
print(mean_error)

var_error = var_distribution - var_dice_value
print('Error or difference in variance of distribution and variance of sample drawn    = ')
print(var_error)

#Response to question 7B
#In the sample drawn in R and Stata, we have set the seed to 10072021 and we have made 100 observations
#that are randomly generated using the random integer generator.
#Hence the observed mean/variance is different from the calculate mean/variance.
#The observed mean/variance will again change if the number of observations are changed.

```
```

1. Sample = (1, 2, 3, 4, 5)
probabilities = (1/15, 2/15, 3/15, 4/15, 5/15)

Mean $= \sum x\, p(x)$

$= 1\times\frac{1}{15} + 2\times\frac{2}{15} + 3\times\frac{3}{15} + 4\times\frac{4}{15} + 5\times\frac{5}{15}$

$= \frac{1}{15} + \frac{4}{15} + \frac{9}{15} + \frac{16}{15} + \frac{25}{15}$

$\boxed{\text{Mean} = \frac{55}{15} = \frac{11}{3} = 3.67}$

Variance $= \sum (x-\mu)^2 p(x)$

$= \sum x^2 p(x) - \mu^2$

$= 1^2\times\frac{1}{15} + 2^2\times\frac{2}{15} + 3^2\times\frac{3}{15} + 4^2\times\frac{4}{15} + 5^2\times\frac{5}{15} - \left(\frac{11}{3}\right)^2$

$= \frac{1^3+2^3+3^3+4^3+5^3}{15} - \frac{121}{9}$

$\boxed{\text{Variance} = \frac{45}{3} - \frac{121}{9} = \frac{14}{9}}$

Mean, variance in R = 3.63, 1.73
Mean, variance in Stata = 3.59, 1.759

Error in R = Mean error = 0.0367
Variance error = −0.1748

Error in Stata = Mean error = 0.08
Variance error = −0.202