Problem set 4

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Question 1

Question 1

(a)

$$t\,statistic = \frac{0.2153 - (-0.22)}{0.0586668} = 7.42$$

Hence the Ho: Coef (females) = −0.22 can be rejected. However the alternate hypothesis that coeff. not −0.22 cannot be rejected.

(b)

$$\frac{d\,\widehat{colgpa}}{d(hsize)}$$

$$colgpa = 2.495 + 0.0915\,hsize - 0.0149\,hsize^2 + 0.2153\,female - 0.0634\,hsize \times female + 0.0096\,hsize^2 \times female.$$

$$\frac{d\,\widehat{colgpa}}{d(hsize)} = 0.0915 - 0.03\,hsize - 0.0634\,female + 0.0192\,hsize \times female.$$

1c

(c) females = 1

$$\frac{\partial \widehat{colgpa}}{\partial (hsize)} = 0.0915 - 0.03\,hsize - 0.0634 + 0.0192\,hsize$$

$$= 0.0281 - 0.0108\,hsize \Rightarrow 0$$

Sign change $\frac{\partial \widehat{colgpa}}{\partial\, hsize} = 0$

$\boxed{hsize = 2.601}$ measured in 100s $\boxed{hsize = 260}$

for male students females = 0

$$\frac{\partial \widehat{colgpa}}{\partial\, hsize} = 0.0915 - 0.03\,hsize = 0$$

$\boxed{hsize = 3.05}$ measured in 100s.

$\boxed{hsize = 305}$

Level is higher for males. The level at which the sign of the effect of class size is higher for males at ~305 and for females at ~260.

1d

(d) Regression equation

$$\text{colgpa} = 2.495 + 0.0915\,\text{hsize} - 0.015\,\text{hsize}^2 + 0.2153\,\text{female} - 0.0634\,\text{hsize}\cdot\text{female} + 0.0096\,\text{hsize}^2\,\text{female}$$

females $= 0$ for males

$$E[\widehat{\text{colgpa}} \mid X_{\text{males}}]_{50 \to 60} = 2.495 + 0.0915 \times \frac{60}{100} - 0.015 \times 60^2 - 2.495 - 0.0915 \times \frac{50}{100} - (-0.015 \times 50^2)$$

$$= 0 + \frac{0.915}{100} - \frac{0.015 \times 1100}{100 \times 100} = \frac{0.75}{100}$$

$$\boxed{= 0.0075}$$

$E[\widehat{\text{colgpa}} \mid X_{\text{female}}]_{50-60}$ females $= 1$ for females

$$= \cancel{2.495} + \underbrace{0.0915 \times \frac{60}{100}}_{\text{hsize}} + \underbrace{\cancel{0.2153}}_{\text{female}} - \underbrace{0.0634 \times \frac{60}{100}}_{\text{hsize.female}}$$

$$- \cancel{2.495} - 0.0915 \times \frac{50}{100} - \cancel{0.2153} + 0.0634 \times \frac{50}{100}$$

$$- 0.015 \times \frac{60^2}{100^2} + 0.0096 \times \frac{60^2}{100^2}$$

$$+ \underbrace{0.015 \times \frac{50^2}{100^2}}_{\text{hsize}^2} - \underbrace{0.0096 \times \frac{50^2}{100^2}}_{\text{hsize}^2\,\text{female}}$$

$$= \frac{0.915}{100} - \frac{0.634}{100} - \frac{16.5}{100^2} + \frac{10.56}{100^2} = \boxed{0.002216}$$

→ Therefore predicted change is larger for males

Numerically the predicted change observed is larger for males, by a difference of 0.005. But looking at the magnitude, we can say that the values are almost same. The predicted change is same for both males and females.

1e

(e) Two restrictions given

$\beta_{hsize \cdot female}$ and $\beta_{hsize^2 \cdot female}$ are zero.

We use an f test on regression restricted (i.e) by keeping out the two terms ($hsize \cdot female$) and ($hsize^2 \cdot female$). Now we have both SSR_r and SSR_{ur}.

Using SSR's we calucute the f statistic and the p-value to test the null hypothesis. This can be interpreted by saying that dependence of student's GPA after fall semester (on) the size of graduating class is same in both females and males.

Question 3

3a ~~$H_0 : \beta = 0$~~ $H_0 : \beta = 0$

We reject if absolute value of z statistic is ≥ 1.96 (two tailed)

So for $\boxed{\hat{\beta} \text{ values} \geq 0.98 \text{ and } \leq -0.98}$

We know $\boxed{Pr(Z > -1.645) = 0.95}$

β value associated with 0.98

$$\frac{0.98 - \beta}{1/2} = -1.645 \quad \boxed{\beta = 1.8025} \quad \beta > 0 \checkmark$$

$\beta < 0$ ✗

$$\frac{-0.98 - \beta}{1/2} = -1.645 \quad \boxed{\beta = -0.16}$$

Reject this possibility

Hence for $\beta > 1.8$, probability that we can reject the null hypothesis is greater than 0.95.

Similarly for values of true change in mortality (β) for a value < -1.8 the probability the we can reject the null hypothesis is greater than 0.95.

3b

(b) Critical value at 1% significance level, two tailed is 2.576.

$H_0 : \beta = 0$

so $\hat{\beta} > 1.29$ $\hat{\beta} < -1.29$

Similarly to part (a)

$$\frac{1.29 - \beta}{1/2} = -1.645$$

$$\boxed{\beta = 2.11}$$

→ Hence for the true change in mortality values of $\beta > 2.11$ and $\beta < -2.11$, we can probably reject the null hypo this is greater than 0.95

→ from comparing (a) and (b) we can say that probability of rejecting the null hypothesis (H_0) decreases with decreasing in the test size or significance level.

3c

(c) Given sample size doubles

$$n \longrightarrow 2n$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}.$$

So . standard error error reduces by $\sqrt{2}$

$$n \longrightarrow 2n \qquad \hat{\beta}\ (0.98) \longrightarrow \frac{0.98}{\sqrt{2}} = 0.69$$

$$\sqrt{n} \longrightarrow \sqrt{2n}$$

New standard error $= \frac{0.5}{\sqrt{2}} = 0.3535$

$$\frac{0.69 - \beta}{0.3535} = -1.645$$

$$\beta = 1.2715$$

Hence when the sample size doubles, i.e $n \to 2n$, for values of true change in mortality (β) $\beta > 1.2715$ and $\beta < -1.2715$, we can say that probability with which we can reject the null hypothesis $H_0: \beta = 0$ will be greater than 0.95.