Question 1

1a.

P.S 9 (1226 5092)

Question 1

$$Consumption_{(it)} = \beta_0 + \beta_1 HeadRetired_{(it)} + \beta_2 FamilySize_{(it)} + \beta_3 HeadEducation_{(i)} + u_{(it)}$$

$HeadRetired \in \{0, 1\}$

(a) Some families will have higher consumption because typical income is higher, maybe correlated with family size.

F.E (fixed Effects) regression. The intercept will vary across families to control the income attributes across families.

Change in β_2 :- β_2 is the change of family size's effect on the change in consumption with the average family size held constant. Thus, after using F.E, β_2 should still be same as before (i.e) no effect on estimate of β_2.

Effect on β_3 :- As HeadEducation doesn't vary when we use F.E model, β_3 estimate will be zero. Effect on β_3 is that the estimate will become zero.

1a continued

Assumption on correlation:- Over time the error terms in the estimated consumption will be uncorrelated over time. The only correlation is the unobservable income which is part of η_{it}) error term is correlated with family size$_{(it)}$, however no other component which is part of η_{it}) will be uncorrelated with the variables HeadEducation (or) HeadRetired.

1b, 1c

(b) If it is infact the case that people will retire when their savings have had a higher return than expected, then the coefficeint β_1 would be affected. The resulted bias will be positive because the dependent variable consumption will either increase or stay the same upon retirement. β_1 is coefficient of HeadRetired

(c) Suppose many people are forced to retire before they have saved sufficiently because of poor health; then the bias in regression estimate β_1 will be negative as we expect the consumption to reduce upon retirement.

1d.

(d) FamilyHead's age as instrument for Head Retired$_{(it)}$

Assumptions:-

Exogeneity condition:- For HeadAge$_{(it)}$ to be an instrument variable, it should not be correlated with the error term $U_{(it)}$.

This property may not be satisfied due to the numerous unobservable terms in the $U_{(it)}$ such as health, wealth etc that might be correlated with the variable Head Age$_{(it)}$.

Correlation / Relevance condition:- For this property to be satisfied, HeadAge$_{(it)}$ should be in correlation with Head Retired$_{(it)}$. As we know, increase in age means increase in the chance of retirement. Thus this property will be satisfied.

As both the conditions are not satisfied at a time, HeadAge cannot be an instrument for HeadRetired$_{(it)}$.

Question 2

2a.

Question 2 Logged Inflation.

Estimate equation

$$\text{Interest Rate}_{(t)} = \beta_0 + \beta_1 \text{Inflation}_{(t)} + \beta_2 \text{Inflation}_{(t-1)} + u_{(t)}$$

(a) Suppose Inflation rate ↑ 2-3 % and stays same for several quarters.

$$① - IR_t = \beta_0 + \beta_1 If_{(t)} + \beta_2 If_{(t-1)} + u_{(t)}$$

$$② - IR_{t+1} = \beta_0 + \beta_1 If_{(t+1)} + \beta_2 If_{(t)} + u_{(t+1)}$$

$$③ - IR_{t+2} = \beta_0 + \beta_1 If_{(t+2)} + \beta_2 If_{(t+1)} + u_{(t+2)}.$$

$$\vdots$$

$$ⓝ - IR_{t+n} = \beta_0 + \beta_1 If_{(t+n)} + \beta_2 If_{(t+n-1)} + u_{(t+n)}.$$

* Assume Inflation rate changed in $If_{t-1} = 2\%$. to $If_t = 3\%$.

Then Eqn. $ⓝ - \cancel{(n-1)} + \cancel{(n-1)} - \cancel{(n-2)} + \cdots \cancel{②} - ①$

$ⓝ - ①$

$$IR_{t+n} - IR_t = \beta_0 - \beta_0 + \beta_1 (If_{t+n} - If_t) + \beta_2 (If_{t+n-1} - If_{t-1}) + (u_{t+n} - u_t)$$

$$= 0 + \beta_1 (0) + \beta_2 (3\% - 2\%) + 0$$

$$\boxed{\Delta I_{(t+n)\leftrightarrow t} = \beta_2 \%}$$

2a. continued in next page..

Assume inflation rate changed in between t, $t+n$

Then Eqn. (n) $-$ $\cancel{(n-1)}$ $+$ $\cancel{(n-1)}$ $-$ $\cancel{(n-2)}$ $+ \cdots +$ $\cancel{(2)}$ $-$ (1)

$$IR_{(t+n)} - IR_t = \beta_0 - \beta_0 + \beta_1 (IR_{t+n} - IR_t) + \beta_2 (IR_{t+n-1} - IR_{t-1}) + u_{t+n} - u_t$$

$$= 0 + \beta_1 (3\% - 2\%) + \beta_2 (3\% - 2\%) + 0$$

$$= (\beta_1 + \beta_2)(3\% - 2\%)$$

$$\boxed{\Delta IR_{(t+n) \leftrightarrow t} = (\beta_1 + \beta_2)\%}$$

Change in interest rate can be either $\beta_2\%$ or $(\beta_1 + \beta_2)\%$ depends on where the change is observed in Inflation rate

2b in next page…

2b.

(b) New model

$$IR_t = \beta_0 + \beta_1 If_t + \beta_2 If_{(t-1)} + IR_{(t-1)} + u_t$$

Here we are ~~using~~ seeing that the outcome variable (IR_t) depends on its own previous value (IR_{t-1}). This is a time series model called the Autoregressive model. This is used for forcasting when there is some correlation between values in a time series and the values that preceed or succeed them. As the model includes a imperfectly predictable stochastic term, this model is called a Autoregressive Stochastic process.