```
name: <unnamed>
      log: C:\Users\saiomkark\OneDrive - The University of Chicago\AdvStats\
> PS5\Question3.log
 log type: text
opened on: 5 Nov 2021, 23:18:58
. * clear memory in stata
. clear
. *Use Stata in this exercise. Suppose that x is drawn from the following "mix > ing distribution." Let y be a binary random variable with Pr(y = 1) = 0.9. I
> f y = 1, then x is drawn from a standard normal distribution. If y = 0, then
> x is drawn from a normal distribution with \mu = 100 and standard deviation \sigma
  = 20.
. * a) Find the mean of x.
. * b) For this distribution, use 10,000 draws from each of the following samp
> le sizes: n = 36, n = 64, n = 100, n = 225, n = 2500, and n = 12100. Discuss
> how well the normal approximation fits your simulated estimates of the mean
> at the critical values of 0.025 and 0.975.
. set seed 10072021
. set obs 100000
number of observations ( N) was 0, now 100,000
. * y is binary with Pr(y1=1) = 0.9
. gen y = runiform() < 0.9
. sum y
   Variable | Obs Mean Std. Dev.
                                                      Min
_________
         y | 100,000
                             .90021 .2997213
. * If y = 1, then x is drawn from a standard normal distribution. If y = 0, t
> hen x is drawn from a normal distribution with \mu = 100 and standard deviatio
> n \sigma = 20
. gen x = rnormal() if (y == 1)
(9,979 missing values generated)
. replace x = rnormal(100, 20) if (y == 0)(9,979 \text{ real changes made})
. sum x
   Variable | Obs Mean Std. Dev. Min Max
         x | 100,000 10.0146 30.76765 -4.810123 171.9138
. clear
```

```
. * In case the program "clt" is resident in memory drop it because we will re > define it in this program. Capture simply tells stata to ignore this comman
> d if the "clt" is not in memory
. capture program drop clt
. \star define program. This is the program that we will 10,000 times for each sa
> mple size; rclass tells us that we will be using results stored in the class
> r()
. program clt, rclass
  1. version 15.0
  2. *
. * Simulations for central limit theorem (clt)
. * declare argument of program; only one in this program the sample size
. args N
  3. *
. * clear any variables in memory
. clear
  4. *
. \star set observations to `N'. "quietly" tells stata not to put the results on y
> our screen
. quietly set obs `N'
  5. *
. \star Set temporary variables. Temporary variables are dropped whenever we star
> t a new "loop" through the data
. tempvar y x
  6. <sup>*</sup>
. * y is binary with Pr(y1=1) = 0.9
. gen `y' = uniform() < 0.9 7. *
. * If y = 1, then x is drawn from a standard normal distribution. If y = 0, t
> hen x is drawn from a normal distribution with \mu = 100 and standard deviatio
. gen x' = rnormal() if (y' == 1)
  8. replace x' = rnormal(100, 20) if (y' == 0)
. * calculate means and standard deviation; "quietly" command suppresses outpu
> t
. quietly sum `y'
. * return scalar mu"y" recovers the mean sd the standard deviation
. return scalar muy = r(mean)
11. return scalar sdy = r(sd)
. * calculate means and standard deviation; "quietly" command suppresses outpu
```

```
. quietly sum `x'
. * return scalar mu"x" recovers the mean sd the standard deviation . *  
. return scalar mux = r(mean)
14. return scalar sdx = r(sd)
15. *
. * end the program definition . *
. end
. * Set out seed
. set seed 24031997
. *
. * run simulation for 36 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 36
      command: clt 36
         muy: r(muy)
          mux: r(mux)
sdy: r(sdy)
sdx: r(sdx)
. * save this data
. save clt36, replace
file clt36.dta saved
. * Sample using 36 observations
. use clt36
(simulate: clt)
. gen z1 = (muy-0.9)
                         /(sdy/36^(1/2))
(221 missing values generated)
                         /(sdx/36^(1/2))
. gen z2 = (mux-10)
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
```

```
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 36 observations
                          Mean Std. Dev. Min Max
   Variable |
                 Obs
     rrej1 | 10,000 .0876 .2827264 0 1
lrej1 | 10,000 .0081 .0896392 0 1
rrej2 | 10,000 .006 .0772308 0 1
lrej2 | 10,000 .1031 .3041046 0 1
. /*
> 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
. bitest lrej1 = 0.025
Variable | N Observed k Expected k Assumed p Observed p
    lrej1 | 10000
                       81
                                 250 0.02500 0.00810
 Pr(k \ge 81) = 1.000000 (one-sided test)

Pr(k \le 81) = 0.000000 (one-sided test)
 Pr(k \le 81 \text{ or } k \ge 469) = 0.000000 \text{ (two-sided test)}
. bitest rrej1 = 0.025
Variable | N Observed k Expected k Assumed p Observed p
  rrej1 | 10000 876 250 0.02500 0.08760
 note: lower tail of two-sided p-value is empty
. bitest lrej2 = 0.025
  Variable |
              N Observed k Expected k Assumed p Observed p
250 0.02500 0.10310
    lrej2 | 10000 1031
 note: lower tail of two-sided p-value is empty
. bitest rrej2 = 0.025
 Variable | N Observed k Expected k Assumed p Observed p
```

rrej2 | 10000 60 250 0.02500 0.00600

```
. replace rrej1 = z1 > 0.975 \& z1 < .
(1,778 real changes made)
. replace lrej1 = z1 < -0.975
(1,393 real changes made)
. replace rrej2 = z2 > 0.975
(1,147 real changes made)
. replace lrej2 = z2 < -0.975
(1,345 real changes made)
. * Should be around 0.175 given that \sim 35\% of sample means remain outliers at
> z = 1 (~ 17.5% on each side) for a normal distribution
. sum rrej1-lrej2
  Variable | Obs Mean Std. Dev. Min Max
    rrej1 | 10,000 .2654 .441568 0 1
lrej1 | 10,000 .1474 .3545219 0 1
rrej2 | 10,000 .1207 .3257946 0 1
lrej2 | 10,000 .2376 .4256341 0 1
. bitest lrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
  lrej1 | 10000 1474 1750 0.17500 0.14740
 . bitest rrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
   rrej1 | 10000 2654 1750 0.17500 0.26540
 . bitest lrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
                           1750 0.17500 0.23760
  lrej2 | 10000 2376
 . bitest rrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
  rrej2 | 10000
                    1207 1750 0.17500 0.12070
```

```
. replace rrej1 = z1 > 0.025 \& z1 < .
(2,266 real changes made)
. replace lrej1 = z1 < -0.025
(3,385 real changes made)
. replace rrej2 = z2 > 0.025
(3,333 real changes made)
. replace lrej2 = z2 < -0.025
(2,913 real changes made)
. * Should be 0.50
. sum rrej1-lrej2
 Variable | Obs Mean Std. Dev. Min Max
   . * Should be around 0.50 on each side given that value of z is low at 0.025.
> Most sample means should fall as outliers for this range.
. bitest lrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
  lrej1 | 10000 4859 5000 0.50000 0.48590
 . bitest rrej1 = 0.50
  Variable | N Observed k Expected k Assumed p Observed p
rreil | 10000 4920 5000 0.50000 0.49200
 . bitest lrej2 = 0.50
 Variable | N Observed k Expected k Assumed p Observed p
   lrej2 | 10000
                  5289 5000 0.50000 0.52890
```

```
. bitest rrej2 = 0.50
```

```
Variable | N Observed k Expected k Assumed p Observed p
                                          5000
     rrej2 | 10000
                            4540
                                                    0.50000
                                                                 0.45400
 . clear
. * run simulation for 64 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 6\overline{4}
     command: clt 64
         muy: r(muy)
         mux: r(mux)
sdy: r(sdy)
sdx: r(sdx)
. * save this data
. save clt64, replace
file clt64.dta saved
. * Sample using 64 observations
. use clt64
(simulate: clt)
. gen z1 = (muy-0.9) /(sdy/64^{(1/2)})
(11 missing values generated)
. gen z2 = (mux-10) / (sdx/64^{(1/2)})
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 64 observations
                               Mean Std. Dev. Min Max
                  Obs
   Variable |
      rrej1 | 10,000 .1048 .3063109 0 1
lrej1 | 10,000 .0096 .097513 0 1
rrej2 | 10,000 .0082 .0901863 0 1
lrej2 | 10,000 .076 .2650114 0 1
```

```
> 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
. bitest lrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
-----
    lrej1 | 10000
                        96
                                250 0.02500 0.00960
 Pr(k \ge 96) = 1.000000 (one-sided test)

Pr(k \le 96) = 0.000000 (one-sided test)
 Pr(k \le 96) = 0.000000 (one-sided test)

Pr(k \le 96 \text{ or } k \ge 444) = 0.000000 (two-sided test)
. bitest rrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
  rrej1 | 10000 1048 250 0.02500 0.10480
 note: lower tail of two-sided p-value is empty
. bitest lrej2 = 0.025
Variable | N Observed k Expected k Assumed p Observed p
  lrej2 | 10000 760 250 0.02500 0.07600
 note: lower tail of two-sided p-value is empty
. bitest rrej2 = 0.025
  Variable |
              N Observed k Expected k Assumed p Observed p
                     82 250 0.02500 0.00820
    rrej2 | 10000
 . replace rrej1 = z1 > 0.975 \& z1 < .
(1,131 real changes made)
. replace lrej1 = z1 < -0.975
(970 real changes made)
. replace rrej2 = z2 > 0.975
(1,208 real changes made)
```

```
. replace lrej2 = z2 < -0.975
(1,360 real changes made)
^{\star} Should be around 0.175 given that \sim 35\% of sample means remain outliers at
> z = 1 (~ 17.5% on each side) for a normal distribution
. sum rrej1-lrej2
                     Mean Std. Dev.
  Variable |
             Obs
                                     Min
______
   rrej1 | 10,000 .2179 .4128397 0 1
lrej1 | 10,000 .1066 .3086194 0 1
rrej2 | 10,000 .129 .3352167 0 1
lrej2 | 10,000 .212 .4087453 0 1
. bitest lrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
   lrej1 | 10000 1066 1750 0.17500 0.10660
 . bitest rrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
 rrej1 | 10000 2179 1750 0.17500 0.21790
 . bitest lrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
   lrej2 | 10000 2120 1750 0.17500 0.21200
 . bitest rrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
                         1750 0.17500 0.12900
 rrej2 | 10000 1290
```

.

```
. replace rrej1 = z1 > 0.025 \& z1 < .
(3,223 real changes made)
replace lrej1 = z1 < -0.025
(3,521 real changes made)
. replace rrej2 = z2 > 0.025
(3,292 real changes made)
. replace lrej2 = z2 < -0.025
(3,087 real changes made)
. * Should be 0.50
. sum rrej1-lrej2
           Obs
                   Mean Std. Dev. Min Max
 Variable |
   1
. * Should be around 0.50 on each side given that value of z is low at 0.025.
> Most sample means should fall as outliers for this range.
. bitest lrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
  lrej1 | 10000 4587 5000 0.50000 0.45870
 . bitest rrej1 = 0.50
  Variable | N Observed k Expected k Assumed p Observed p
rreil | 10000 5402 5000 0.50000 0.54020
 . bitest lrej2 = 0.50
 Variable | N Observed k Expected k Assumed p Observed p
   lrej2 | 10000
                  5207 5000 0.50000 0.52070
```

```
. bitest rrej2 = 0.50
```

```
Variable | N Observed k Expected k Assumed p Observed p
                                          5000
     rrej2 | 10000
                            4582
                                                    0.50000
                                                                 0.45820
 . clear
. * run simulation for 100 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 100
     command: clt 100
  muy: r(muy)
  mux: r(mux)
         sdy: r(sdy)
sdx: r(sdx)
. * save this data
save clt100, replace
file clt100.dta saved
. * Sample using 100 observations
. use clt100
(simulate: clt)
. gen z1 = (muy-0.9)
                       /(sdy/100^(1/2))
(1 missing value generated)
. gen z2 = (mux-10) / (sdx/100^{(1/2)})
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 100 observations
                 Obs
                               Mean Std. Dev. Min
   Variable |
      rrej1 | 10,000 .0536 .2252379 0 1
lrej1 | 10,000 .0101 .0999949 0 1
rrej2 | 10,000 .0096 .097513 0 1
lrej2 | 10,000 .0582 .234133 0 1
                                                                   1
1
```

```
. /*
> 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
. bitest lrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
-----
    lrej1 | 10000
                        101
                                 250 0.02500 0.01010
 . bitest rrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
  rrej1 | 10000 536 250 0.02500 0.05360
 . bitest lrej2 = 0.025
Variable | N Observed k Expected k Assumed p Observed p
  lrej2 | 10000 582 250 0.02500 0.05820
                    = 0.000000 (one-sided test)
 Pr(k >= 582)
 Pr(k \le 582) = 1.000000 (one-sided test)

Pr(k \le 26 \text{ or } k \ge 582) = 0.000000 (two-sided test)
. bitest rrej2 = 0.025
Variable | N Observed k Expected k Assumed p Observed p rrej2 | 10000 96 250 0.02500 0.00960
 . replace rrej1 = z1 > 0.975 \& z1 < .
(1,469 real changes made)
. replace lrej1 = z1 < -0.975
(1,189 real changes made)
. replace rrej2 = z2 > 0.975
(1,283 real changes made)
. replace lrej2 = z2 < -0.975
(1,378 real changes made)
```

. \* . \* Should be around 0.175 given that ~35% of sample means remain outliers at > z = 1 (~ 17.5% on each side) for a normal distribution . \* . sum rrej1-lrej2 

Variable | Obs Mean Std. Dev. Min Max

Variable	Obs	Mean	Std. Dev.	Min	Max
rrej1	10,000	.2005	.4003945	0	1
lrej1	10,000	.129	.3352167	0	1
rrej2	10,000	.1379	.3448122	0	1
lrej2	10,000	.196	.3969884	0	1

. bitest lrej1 = 0.175

Variable	N	Observed k	Expected k	Assumed p	Observed p
lrej1	10000	1290	1750	0.17500	0.12900
Pr(k >= 1290) Pr(k <= 1290) Pr(k <= 1290 or	k >= 224	= 0.000000	(one-sided (one-sided (two-sided	test)	

. bitest rrej1 = 0.175

Variable	N O	bserved k	Expected k	Assumed p	Observed p
rrej1	10000	2005	1750	0.17500	0.20050
Pr(k >= 2005) Pr(k <= 2005) Pr(k <= 1503 or	k >= 2005	= 1.000000		test)	

. bitest lrej2 = 0.175

Variable	N	Observed k	Expected k	Assumed p	Observed p
lrej2	10000	1960	1750	0.17500	0.19600
Pr(k >= 1960)			O (one-sideo	,	

 $Pr(k \ge 1960)$  = 0.000000 (one-sided test)  $Pr(k \le 1960)$  = 1.000000 (one-sided test)  $Pr(k \le 1545 \text{ or } k \ge 1960)$  = 0.000000 (two-sided test)

. bitest rrej2 = 0.175

Variable	N	Observed	k Expected	k Assumed p	Observed p
rrej2	10000	1379	1750	0.17500	0.13790

\*

\*

<sup>. \*</sup> Should be around 0.50 on each side given that value of z is low at 0.025. > Most sample means should fall as outliers for this range.

```
. replace rrej1 = z1 > 0.025 \& z1 < .
(2,488 real changes made)
. replace lrej1 = z1 < -0.025
(2,846 real changes made)
. replace rrej2 = z2 > 0.025
(3,306 real changes made)
. replace lrej2 = z2 < -0.025
(3,130 real changes made)
. * Should be 0.50
. sum rrej1-lrej2
             Obs
                      Mean Std. Dev. Min
  Variable |
    1
. bitest lrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
  lrej1 | 10000 4136 5000 0.50000 0.41360
 . bitest rrej1 = 0.50
  Variable | N Observed k Expected k Assumed p Observed p
  rrejl | 10000 4493 5000 0.50000 0.44930
 . bitest lrej2 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
    lrej2 | 10000
                      5090
                                5000 0.50000 0.50900
 Pr(k >= 5090) = 0.036724 (one-sided test)

Pr(k <= 5090) = 0.964855 (one-sided test)

Pr(k <= 4910 \text{ or } k >= 5090) = 0.073449 (two-sided test)
. bitest rrej2 = 0.50
Variable | N Observed k Expected k Assumed p Observed p rrej2 | 10000 4685 5000 0.50000 0.46850
```

```
. clear
. * run simulation for 225 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 225
      command: clt 225
         muy: r(muy)
          mux: r(mux)
sdy: r(sdy)
sdx: r(sdx)
. * save this data
. save clt225, replace
file clt225.dta saved
. * Sample using 225 observations
. use clt225
(simulate: clt)
. gen z1 = (muy-0.9)
                          /(sdy/225^(1/2))
. gen z2 = (mux-10)
                         /(sdx/225^(1/2))
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 225 observations
   Variable |
                     Obs
                                 Mean Std. Dev.
                                                           Min
                                                                      Max
______
      rrej1 | 10,000 .0515 .2210263 0 1
lrej1 | 10,000 .0179 .1325947 0 1
rrej2 | 10,000 .0142 .1183206 0 1
lrej2 | 10,000 .0439 .2048829 0 1
> 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
```

> \*/

```
. bitest lrej1 = 0.025
```

```
Variable | N Observed k Expected k Assumed p Observed p
                  00 179 250 0.02500 0.01790
    lreil | 10000
 Pr(k >= 179) = 0.999999 (one-sided test)

Pr(k <= 179) = 0.000001 (one-sided test)

Pr(k <= 179 \text{ or } k >= 328) = 0.000002 (two-sided test)
. bitest rrej1 = 0.025
   Variable | N Observed k Expected k Assumed p Observed p
    rrej1 | 10000 515 250 0.02500 0.05150
 . bitest lrej2 = 0.025
Variable | N Observed k Expected k Assumed p Observed p
                                    250
    lrej2 | 10000
                                           0.02500 0.04390
                         439
 . bitest rrej2 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
  rrej2 | 10000 142 250 0.02500 0.01420
 . replace rrej1 = z1 > 0.975 \& z1 < .
(1,345 real changes made)
. replace lrej1 = z1 < -0.975
(1,107 real changes made)
. replace rrej2 = z2 > 0.975
(1,208 real changes made)
. replace lrej2 = z2 < -0.975
(1,459 real changes made)
. * Should be around 0.175 given that \sim 35\% of sample means remain outliers at
> z = 1 (~ 17.5% on each side) for a normal distribution
. sum rrej1-lrej2
Variable | Obs Mean Std. Dev. Min Max
     rrej1 | 10,000 .186 .3891261 0 1
lrej1 | 10,000 .1286 .3347734 0 1
rrej2 | 10,000 .135 .3417407 0 1
lrej2 | 10,000 .1898 .3921624 0 1
                                                          1
```

```
. bitest lrej1 = 0.175
 Variable | N Observed k Expected k Assumed p Observed p
  lrej1 | 10000 1286 1750 0.17500 0.12860
 . bitest rrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
   rrej1 | 10000
                   1860
                           1750 0.17500 0.18600
 . bitest lrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
  lrej2 | 10000 1898 1750 0.17500 0.18980
 . bitest rrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
  rrej2 | 10000 1350 1750 0.17500 0.13500
 . replace rrej1 = z1 > 0.025 \& z1 < .
(3,259 real changes made)
. replace lrej1 = z1 < -0.025
(3,595 real changes made)
. replace rrej2 = z2 > 0.025
(3,415 real changes made)
. replace lrej2 = z2 < -0.025
(3,145 real changes made)
. * Should be around 0.50 on each side given that value of z is low at 0.025.
> Most sample means should fall as outliers for this range.
```

. sum rrej1-lrej2

```
Obs
                                                          Min
   Variable |
                                 Mean Std. Dev.
                                                                       Max
       rrej1 | 10,000 .5119 .4998834
lrej1 | 10,000 .4881 .4998834
rrej2 | 10,000 .4765 .4994724
lrej2 | 10,000 .5043 .5000065
                                                        0
0
0
0
                                                                          1
                                                                            1
. bitest lrej1 = 0.50
    Variable | N Observed k Expected k Assumed p Observed p
     lrej1 | 10000 4881 5000 0.50000 0.48810
 Pr(k \ge 4881) = 0.991578 (one-sided test)

Pr(k \le 4881) = 0.008892 (one-sided test)

Pr(k \le 4881) or k \ge 5119) = 0.017784 (two-sided test)
. bitest rrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
      rrej1 | 10000
                                             5000
                                                         0.50000
                              5119
 Pr(k \ge 5119) = 0.008892 (one-sided test)

Pr(k \le 5119) = 0.991578 (one-sided test)

Pr(k \le 4881 \text{ or } k \ge 5119) = 0.017784 (two-sided test)
. bitest lrej2 = 0.50
   Variable | N Observed k Expected k Assumed p Observed p
   lrej2 | 10000 5043 5000 0.50000 0.50430
 Pr(k \ge 5043) = 0.197663 (one-sided test)

Pr(k \le 5043) = 0.807849 (one-sided test)

Pr(k \le 4957 \text{ or } k \ge 5043) = 0.395326 (two-sided test)
. bitest rrej2 = 0.50
                  N Observed k Expected k Assumed p Observed p
   Variable |
              ·<del>+</del>-----
     rrej2 | 10000 4765 5000 0.50000 0.47650
 . clear
. * run simulation for 2500 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 2500
      command: clt 2500
  muy: r(muy)
  mux: r(mux)
          sdy: r(sdy)
sdx: r(sdx)
```

```
. * save this data
. save clt2500, replace
file clt2500.dta saved
. * Sample using 2500 observations
. use clt2500
(simulate: clt)
. gen z1 = (muy-0.9)
                    /(sdy/2500^(1/2))
. gen z2 = (mux-10) / (sdx/2500^{(1/2)})
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 2500 observations
Variable | Obs Mean Std. Dev. Min Max
     . /* > 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
. bitest lrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
------
    lrej1 | 10000
                         229
                                  250 0.02500 0.02290
 Pr(k >= 229) = 0.917242 (one-sided test)

Pr(k <= 229) = 0.093273 (one-sided test)

Pr(k <= 229 \text{ or } k >= 271) = 0.189024 (two-sided test)
. bitest rrej1 = 0.025
  Variable |
               N Observed k Expected k Assumed p Observed p
rrej1 | 10000 309 250 0.02500 0.03090
```

```
. bitest lrej2 = 0.025
```

rrej1 | 10000

Pr(k >= 1673) = 0.979763 (one-sided test) Pr(k <= 1673) = 0.021576 (one-sided test) Pr(k <= 1673 or k >= 1828) = 0.042711 (two-sided test)

```
Variable | N Observed k Expected k Assumed p Observed p
                000 301 250 0.02500 0.03010
    lrej2 | 10000
 Pr(k \ge 301) = 0.000826 (one-sided test)

Pr(k \le 301) = 0.999329 (one-sided test)

Pr(k \le 201 \text{ or } k \ge 301) = 0.001507 (two-sided test)
. bitest rrej2 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
    rrej2 | 10000 210 250 0.02500 0.02100
 . replace rrej1 = z1 > 0.975 \& z1 < .
(1,364 real changes made)
. replace lrej1 = z1 < -0.975
(1,284 real changes made)
. replace rrej2 = z2 > 0.975
(1,390 real changes made)
. replace lrej2 = z2 < -0.975
(1,434 real changes made)
. * Should be around 0.175 given that \sim 35\% of sample means remain outliers at
> z = 1 (~ 17.5% on each side) for a normal distribution
. sum rrej1-lrej2
Variable | Obs Mean Std. Dev. Min Max
    . bitest lrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
                               1750 0.17500 0.15130
  lrej1 | 10000 1513
 . bitest rrej1 = 0.175
  Variable | N Observed k Expected k Assumed p Observed p
```

1673 1750 0.17500 0.16730

```
. bitest lrej2 = 0.175
```

```
Variable | N Observed k Expected k Assumed p Observed p
   lrej2 | 10000 1735 1750 0.17500 0.17350
 . bitest rrej2 = 0.175
  Variable | N Observed k Expected k Assumed p Observed p
rrej2 | 10000 1600 1750 0.17500 0.16000
 . replace rrej1 = z1 > 0.025 \& z1 < .
(3,259 real changes made)
. replace lrej1 = z1 < -0.025
(3,308 real changes made)
. replace rrej2 = z2 > 0.025
(3,257 real changes made)
. replace lrej2 = z2 < -0.025
(3,203 real changes made)
. \star Should be around 0.50 on each side given that value of z is low at 0.025.
> Most sample means should fall as outliers for this range.
. sum rrej1-lrej2
Variable | Obs Mean Std. Dev. Min Max
    . bitest lrej1 = 0.50
 Variable | N Observed k Expected k Assumed p Observed p
lrej1 | 10000 4821 5000 0.50000 0.48210
 . bitest rrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
   rrej1 | 10000 4932 5000 0.50000 0.49320
```

```
. bitest lrej2 = 0.50
  Variable | N Observed k Expected k Assumed p Observed p
     lrej2 | 10000 4938 5000 0.5
                                                   0.50000
                                                                0.49380
 Pr(k \ge 4938) = 0.894351 (one-sided test)

Pr(k \le 4938) = 0.109348 (one-sided test)

Pr(k \le 4938 \text{ or } k \ge 5062) = 0.218695 (two-sided test)
. bitest rrej2 = 0.50
   Variable | N Observed k Expected k Assumed p Observed p
     rrej2 | 10000 4857 5000 0.50000 0.48570
 . clear
. * run simulation for 12100 observations
. simulate muy = r(muy) mux = r(mux) sdy = r(sdy) sdx = r(sdx), reps(10000) no
> dots: clt 12100
     command: clt 12100
  muy: r(muy)
        mux: r(mux)
sdy: r(sdy)
sdx: r(sdx)
. * save this data
. save clt12100, replace
file clt12100.dta saved
. * Sample using 12100 observations
. use clt12100
(simulate: clt)
. gen z1 = (muy-0.9)
                       /(sdy/12100^(1/2))
. gen z2 = (mux-10)
                       /(sdx/12100^(1/2))
. * construct rejection level 5 percent rate; left and right tails
. gen rrej1 = z1 > 1.96 \& z1 < .
```

```
. gen lrej1 = z1 < -1.96
. gen rrej2 = z2 > 1.96
. gen lrej2 = z2 < -1.96
. * Should be 0.05, 1 binary 2 binary
. sum rrej1-lrej2 // 12100 observations
  Variable |
                 Obs
                          Mean Std. Dev.
                                              Min
    rrej1 | 10,000 .0265 .1606248 0 1
lrej1 | 10,000 .0236 .1518069 0 1
rrej2 | 10,000 .0236 .1518069 0 1
lrej2 | 10,000 .0278 .1644076 0 1
. /*
> 99% (-1%) 2.33
> 95% (-5%) 1.65
> 90% (-10%) 1.29
> 75% (-25%) 0.58
. bitest lrej1 = 0.025
. bitest rrej1 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
rrej1 | 10000 265 250 0.02500 0.026
 . bitest lrej2 = 0.025
  Variable | N Observed k Expected k Assumed p Observed p
                                250 0.02500 0.02780
    lrej2 | 10000
                        278
 Pr(k \ge 278) = 0.040818 (one-sided test)

Pr(k \le 278) = 0.964317 (one-sided test)

Pr(k \le 222 \text{ or } k \ge 278) = 0.078024 (two-sided test)
. bitest rrej2 = 0.025
  Variable |
              N Observed k Expected k Assumed p Observed p
rrei2 | 10000 236 250 0.02500 0.02360
```

```
. replace rrej1 = z1 > 0.975 \& z1 < .
(1,416 real changes made)
. replace lrej1 = z1 < -0.975
(1,416 real changes made)
. replace rrej2 = z2 > 0.975
(1,419 real changes made)
. replace lrej2 = z2 < -0.975
(1,358 real changes made)
. * Should be around 0.175 given that \sim 35\% of sample means remain outliers at
> z = 1 (~ 17.5% on each side) for a normal distribution
. sum rrej1-lrej2
  Variable | Obs Mean Std. Dev. Min Max
   . bitest lrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
 lrej1 | 10000 1652 1750 0.17500 0.16520
 . bitest rrej1 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
   rrej1 | 10000 1681 1750 0.17500 0.16810
 . bitest lrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
                        1750 0.17500 0.16360
 lrej2 | 10000 1636
 . bitest rrej2 = 0.175
Variable | N Observed k Expected k Assumed p Observed p
 rrej2 | 10000
                  1655 1750 0.17500 0.16550
```

```
. replace rrej1 = z1 > 0.025 \& z1 < .
(3,210 real changes made)
. replace lrej1 = z1 < -0.025
(3,337 real changes made)
. replace rrej2 = z2 > 0.025
(3,294 real changes made)
. replace lrej2 = z2 < -0.025
(3,218 real changes made)
. * Should be around 0.50 on each side given that value of z is low at 0.025.
> Most sample means should fall as outliers for this range.
. sum rrej1-lrej2
  Variable | Obs Mean Std. Dev. Min Max
. bitest lrej1 = 0.50
            N Observed k Expected k Assumed p Observed p
  Variable |
lrej1 | 10000 4989 5000 0.50000 0.49890
 . bitest rrej1 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
   rrej1 | 10000
                   4891
                             5000
                                    0.50000
                                             0.48910
 . bitest lrej2 = 0.50
  Variable | N Observed k Expected k Assumed p Observed p
lrej2 | 10000 4854 5000 0.50000 0.48540
                = 0.998306 (one-sided test)
 Pr(k >= 4854)
 Pr(k \le 4854) = 0.001806 (one-sided test)

Pr(k \le 4854) or k \ge 5146 = 0.003612 (two-sided test)
. bitest rrej2 = 0.50
Variable | N Observed k Expected k Assumed p Observed p
   rrej2 | 10000 4949 5000 0.50000 0.49490
```

```
. clear
. *Observations
. *Central Limit theorem is interpreted here.
. *a) Mean of the random variable 'x' following the 'Mixed distribution' = 10
. *1. As we increase N, the percentage of sample means that have a z-score bel > ow -0.025 and above 0.025 is ~99%.
. *2. For critical point z = 0.975: As we increase N, the percentage of sample
> means that have a z-score below -0.975 and above 0.975 is ~34%, which means
  66% of the sample means are between z score of 0.975.
. *These simulation results are in accordance with a typical normal distributi
> on where almost 68% of sample means lie within a z-score of 1 and where many
> sample means fall outside the z-score of 0.025 as the interval defined by \hat{t}
> he same is very very small.
. * close log file
. log close
     name: <unnamed>
       log: C:\Users\saiomkark\OneDrive - The University of Chicago\AdvStats\
> PS5\Question3.log
 log type: text
 closed on: 5 Nov 2021, 23:20:02
```