DS 598 DEEP LEARNING

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Problem Set 2

1. $L(\phi) = \sum_{i=1}^{I}(\phi_0 + \phi_1 x_i - y_i)^2$
   with respect to ϕ_0

$$\frac{\partial L(\phi)}{\partial \phi_0} = 2\sum_{i=1}^{I}(\phi_0 + \phi_1 x_i - y_i)$$
$$= 2I\phi_0 + 2\phi_1 \sum_{i=1}^{I} x_i - 2\sum_{i=1}^{I} y_i$$

   with respect to ϕ_1

$$\frac{\partial L(\phi)}{\partial \phi_1} = 2\sum_{i=1}^{I} x_i(\phi_0 + \phi_1 x_i - y_i)$$
$$= 2\phi_0 \sum_{i=1}^{I} x_i + 2\phi_1 \sum_{i=1}^{I} x_i^2 - 2\sum_{i=1}^{I} x_i y_i$$

2. First Order Condition
   with respect to ϕ_0

$$\frac{\partial L(\phi)}{\partial \phi_0} = 2I\phi_0 + 2\phi_1 \sum_{i=1}^{I} x_i - 2\sum_{i=1}^{I} y_i$$
$$= 0$$

$$I\phi_0 + \phi_1 \sum_{i=1}^{I} x_i - \sum_{i=1}^{I} y_i = 0$$

$$\phi_0^* = \frac{1}{I}\sum_{i=1}^{I} y_i - \frac{\phi_1}{I}\sum_{i=1}^{I} x_i$$
$$= \bar{y} - \phi_1 \bar{x}$$

with respect to ϕ_1

$$\frac{\partial L(\phi)}{\partial \phi_1} = 2\sum_{i=1}^{I} x_i(\phi_0 + \phi_1 x_i - y_i)$$

Since $\phi_0^* = \bar{y} - \phi_1\bar{x}$,

$$\begin{aligned}
&= 2\sum_{i=1}^{I} x_i(\bar{y} - \phi_1\bar{x} + \phi_1 x_i - y_i) \\
&= 2\sum_{i=1}^{I} x_i(\phi_1(x_i - \bar{x}) - (y_i - \bar{y})) \\
&= 0
\end{aligned}$$

$$\phi_1^* = \frac{\sum_{i=1}^{I} x_i(y_i - \bar{y})}{\sum_{i=1}^{I} x_i(x_i - \bar{x})}$$

$$\phi_0^* = \bar{y} - \frac{\sum_{i=1}^{I} x_i(y_i - \bar{y})}{\sum_{i=1}^{I} x_i(x_i - \bar{x})}\bar{x}$$

Second Order Condition
with respect to ϕ_0

$$\frac{\partial^2 L(\phi)}{\partial \phi_0^2} = 2 > 0$$

$\therefore$ ϕ_0* attains the global minimum.
with respect to ϕ_1 $\because$ $\forall x_i,\ x_i \in \mathbb{R}^1$

$$\frac{\partial^2 L(\phi)}{\partial \phi_1^2} = 2\sum_{i=1}^{I} x_i^2 \geq 0$$

$\therefore$ ϕ_1^* attains the global minimum.