Phép biến đổi tích phân

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1 Phép Biến Đổi Tích Phân

1.1 Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$[-\pi, \pi]$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \quad (n = 1, 2, ...)$$

$[0, 2\pi] \to \int_0^{2\pi}$

1.1.1 Khoảng bất kỳ

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right)$$

$[-p, p]$

$$a_0 = \frac{1}{2p} \int_{-p}^{p} f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^{p} f(x) \cos \frac{n\pi}{p} x dx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{p} \int_{-p}^{p} f(x) \sin \frac{n\pi}{p} x dx \quad (n = 1, 2, ...)$$

1.1.2 Hàm chẳn lẻ

HÀM	TÍNH CHẤT	Điều kiện
Chẳn	$f(-x) = f(x)$	$\forall x$
Lẻ	$f(-x) = -f(x)$	$\forall x$

- Chẳn $\times$ Chẳn = Chẳn
- Chẳn $\times$ Lẻ = Lẻ
- Lẻ $\times$ Lẻ = Chẳn

Hàm chẳn

$$\int_{-p}^{p} f(x)dx = 2\int_{0}^{p} f(x)dx$$

Và

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x$$

Trong đó

$$a_0 = \frac{1}{p}\int_{0}^{p} f(x)dx$$

$$a_n = \frac{2}{p}\int_{0}^{p} f(x)\cos\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

Hàm lẻ

$$\int_{-p}^{p} f(x)dx = 0$$

Và

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

Trong đó

$$b_n = \frac{2}{p}\int_{0}^{p} f(x)\sin\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

1.2 Differentiation

$$(c)^{'} = 0$$

$$(kx)^{'} = k\ ,\ (ku)^{'} = ku^{'}$$

$$(x^n)^{'} = nx^{n-1}\ ,\ (u^n)^{'} = u^{'}nu^{n-1}$$

$$(uv)^{'} = u^{'}v + v^{'}u$$

$$\left(\frac{u}{v}\right)^{'} = \frac{u^{'}v - v^{'}u}{v^2}$$

$$\left(\frac{1}{x}\right)^{'} = -\frac{1}{x^2}\ ,\ \left(\frac{1}{u}\right)^{'} = \frac{u^{'}}{u^2}$$

$$(\sqrt{x})^{'} = \frac{1}{2\sqrt{x}}\ ,\ (\sqrt{u})^{'} = \frac{u^{'}}{2\sqrt{u}}$$

$$(\sin x)^{'} = \cos x\ ,\ (\sin u)^{'} = u^{'}\cos u$$

$$(\cos x)^{'} = -\sin x\ ,\ (\cos u)^{'} = -u^{'}\sin u$$

$$(e^x)^{'} = e^x\ ,\ (e^u)^{'} = u^{'}e^u$$

$$(a^x)^{'} = a^x \ln a\ ,\ (a^u)^{'} = u^{'}a^u \ln a$$

$$(\ln x)^{'} = \frac{1}{x}\ ,\ (\ln u)^{'} = \frac{u^{'}}{u}$$

$$(\log_a x)^{'} = \frac{1}{x \ln a}\ ,\ (\log_a u)^{'} = \frac{u^{'}}{u \ln a}$$

$$(u \pm v)^{'} = u^{'} \pm v^{'}$$

$$(u_1 \pm ... \pm u_n)^{'} = u_1^{'} \pm ... \pm u_n^{'}$$

$$y = y[u(x)] \Rightarrow y^{'}(x) = y^{'}(u)u^{'}(x)$$

$$\left(\frac{ax+b}{cx+d}\right)^{'} = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$$

1.3 Integral

$$\int dx = x \; , \; \int du = u$$
$$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} \; , \; \int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1}$$

$$\int a^x dx = \frac{a^x}{\ln a} \; , \; \int a^u dx = \frac{a^u}{\ln a}$$
$$\int \frac{dx}{x} = \ln |x| \; , \; \int \frac{du}{u} = \ln |u|$$

$$\int e^x dx = e^x \; , \; \int e^u du = e^u$$
$$\int e^{kx} dx = \frac{e^k x}{k}$$

$$\int \cos x dc = \sin x \; , \; \int \cos u du = \sin u$$
$$\int \sin x dx = -\cos x \; , \; \int \sin u du = -\cos u$$
$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$$
$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b)$$

$$\int (ax+b)^{\alpha} = \frac{1}{a} \frac{(ax+b)^{\alpha+1}}{\alpha+1}$$

1.4 Root of trigonometric

$$\sin u = 0 \Rightarrow u = k\pi$$
$$\sin u = 1 \Rightarrow u = \frac{\pi}{2} + k2\pi$$
$$\sin u = -1 \Rightarrow u = -\frac{\pi}{2} + k2\pi$$

$$\cos u = 0 \Rightarrow u = \frac{\pi}{2} + k\pi$$
$$\cos u = 1 \Rightarrow u = k2\pi$$
$$\cos u = -1 \Rightarrow u = \pi + k2\pi$$