Phép biến đổi tích phân

February 25, 2025

1 Phép Biến Đổi Tích Phân

1.1 Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$[-\pi, \pi]$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \quad (n = 1, 2, ...)$$

$[0, 2\pi] \to \int_0^{2\pi}$

1.1.1 Khoảng bất kỳ

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{p}x + b_n \sin\frac{n\pi}{p}x\right)$$

$[-p, p]$

$$a_0 = \frac{1}{2p}\int_{-p}^{p} f(x)dx$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi}{p}x dx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}x dx \quad (n = 1, 2, ...)$$

1.1.2 Hàm chẵn lẻ

HÀM	TÍNH CHẤT	Điều kiện
Chẵn	$f(-x) = f(x)$	$\forall x$
Lẻ	$f(-x) = -f(x)$	$\forall x$

- Chẵn $\times$ Chẵn $=$ Chẵn
- Chẵn $\times$ Lẻ $=$ Lẻ
- Lẻ $\times$ Lẻ $=$ Chẵn

Hàm chẵn

$$\int_{-p}^{p} f(x)dx = 2\int_{0}^{p} f(x)dx$$

Và

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x$$

Trong đó

$$a_0 = \frac{1}{p}\int_{0}^{p} f(x)dx$$

$$a_n = \frac{2}{p}\int_{0}^{p} f(x)\cos\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

Hàm lẻ

$$\int_{-p}^{p} f(x)dx = 0$$

Và

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

Trong đó

$$b_n = \frac{2}{p}\int_{0}^{p} f(x)\sin\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

1.1.3 Parseval

$$\frac{1}{2p}\int_{-p}^{p} f(x)^2 dx = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

1.2 Basics

1.2.1 Differentiation

$$(c)^{'} = 0$$

$$(kx)^{'} = k\ ,\ (ku)^{'} = ku^{'}$$

$$(x^n)^{'} = nx^{n-1}\ ,\ (u^n)^{'} = u^{'}nu^{n-1}$$

$$(uv)^{'} = u^{'}v + v^{'}u$$

$$\left(\frac{u}{v}\right)^{'} = \frac{u^{'}v - v^{'}u}{v^2}$$

$$\left(\frac{1}{x}\right)^{'} = -\frac{1}{x^2}\ ,\ \left(\frac{1}{u}\right)^{'} = \frac{u^{'}}{u^2}$$

$$(\sqrt{x})^{'} = \frac{1}{2\sqrt{x}}\ ,\ (\sqrt{u})^{'} = \frac{u^{'}}{2\sqrt{u}}$$

$$(\sin x)^{'} = \cos x\ ,\ (\sin u)^{'} = u^{'}\cos u$$

$$(\cos x)^{'} = -\sin x\ ,\ (\cos u)^{'} = -u^{'}\sin u$$

$$(e^x)^{'} = e^x\ ,\ (e^u)^{'} = u^{'}e^u$$

$$(a^x)^{'} = a^x \ln a\ ,\ (a^u)^{'} = u^{'}a^u \ln a$$

$$(\ln x)^{'} = \frac{1}{x}\ ,\ (\ln u)^{'} = \frac{u^{'}}{u}$$

$$(\log_a x)^{'} = \frac{1}{x\ln a}\ ,\ (\log_a u)^{'} = \frac{u^{'}}{u\ln a}$$

$$(u \pm v)^{'} = u^{'} \pm v^{'}$$

$$(u_1 \pm ... \pm u_n)^{'} = u_1^{'} \pm ... \pm u_n^{'}$$

$$y = y[u(x)] \Rightarrow y^{'}(x) = y^{'}(u)u^{'}(x)$$

$$\left(\frac{ax+b}{cx+d}\right)^{'} = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$$

1.2.2 Integral

$$\int dx = x \; , \; \int du = u$$

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} \; , \; \int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1}$$

$$\int a^x dx = \frac{a^x}{\ln a} \; , \; \int a^u dx = \frac{a^u}{\ln a}$$

$$\int \frac{dx}{x} = \ln|x| \; , \; \int \frac{du}{u} = \ln|u|$$

$$\int e^x dx = e^x \; , \; \int e^u du = e^u$$

$$\int e^{kx} dx = \frac{e^k x}{k}$$

$$\int \cos x dc = \sin x \; , \; \int \cos u du = \sin u$$

$$\int \sin x dx = -\cos x \; , \; \int \sin u du = -\cos u$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)$$

$$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)$$

$$\int (ax+b)^\alpha = \frac{1}{a}\frac{(ax+b)^{\alpha+1}}{\alpha+1}$$

1.2.3 Trigonometry

$$\sin^2 a + \cos^2 a = 1$$

$$\tan a = \frac{\sin a}{\cos a}$$

$$\cot a = \frac{\cos a}{\sin a}$$

$$1 + \tan^2 a = \frac{1}{\cos^2 a}$$

$$1 + \cot^2 a = \frac{1}{\sin^2 a}$$

$$\tan a \cot a = 1$$

Cộng

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Nhân

$$\sin 2a = 2 \sin a \cos a$$

$$\cos 2a = \cos^2 a – \sin^2 a = 2\cos^2 a – 1 = 1 – 2\sin^2 a$$

$$\tan 2a = \frac{2 \tan a}{1 – \tan^2 a}$$

$$\cot 2a = \frac{\cot^2 a – 1}{2 \cot a}$$

$$\sin 3a = 3 \sin a – 4\sin^3 a$$

$$\cos 3a = 4\cos^3 a – 3 \cos a$$

$$\tan 3a = \frac{3 \tan a – \tan^3 a}{1 – 3\tan^2 a}$$

$$\cot 3a = \frac{3\cot^2 a – 1}{\cot^3 a – 3 \cot a}$$

Hạ bậc

$$\cos^2 a = \frac{1+\cos 2a}{2}$$

$$\sin^2 a = \frac{1-\cos 2a}{2}$$

$$\tan^2 a = \frac{1-\cos 2a}{1+\cos 2a}$$

$$\sin^2 a \cos^2 a = \frac{1-\cos 4a}{8}$$

$$\cos^3 a = \frac{3\cos a + \cos 3a}{4}$$

$$\sin^3 a = \frac{3\sin a - \sin 3a}{4}$$

$$\sin^4 a = \frac{\cos 4a - 4\cos 2a + 3}{8}$$

$$\cos^4 a = \frac{\cos 4a + 4\cos 2a + 3}{8}$$

Tích tổng

$$\cos a \cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

Tổng tích

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a–b}{2}$$

$$\cos a – \cos b = –2\sin\frac{a+b}{2}\sin\frac{a–b}{2}$$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a–b}{2}$$

$$\sin a – \sin b = 2\cos\frac{a+b}{2}\sin\frac{a–b}{2}$$

$$\cos a + \sin a = \sqrt{2}\cos\left(\frac{\pi}{4}–a\right) = \sqrt{2}\sin\left(\frac{\pi}{4}+a\right)$$

$$\cos a – \sin a = \sqrt{2}\cos\left(\frac{\pi}{4}+a\right) = \sqrt{2}\sin\left(\frac{\pi}{4}–a\right)$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a\cos b}$$

$$\tan a – \tan b = \frac{\sin(a–b)}{\cos a\cos b}$$

$$\cot a + \cot b = \frac{\sin(a+b)}{\sin a\sin b}$$

$$\cot a – \cot b = \frac{\sin(b–a)}{\sin a\sin b}$$

$$\cot a + \tan a = \frac{2}{\sin 2a}$$

$$\cot a – \tan a = 2\cot 2a$$