Phép biến đổi tích phân

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1 Phép Biến Đổi Tích Phân

1.1 Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$[-\pi, \pi]$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx \quad (n = 1, 2, ...)$$

$[0, 2\pi] \to \int_0^{2\pi}$

1.1.1 Khoảng bất kỳ

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{p}x + b_n \sin\frac{n\pi}{p}x\right)$$

$[-p, p]$

$$a_0 = \frac{1}{2p}\int_{-p}^{p} f(x)dx$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

1.1.2 Hàm chẵn lẻ

HÀM	TÍNH CHẤT	Điều kiện
Chẵn	$f(-x) = f(x)$	$\forall x$
Lẻ	$f(-x) = -f(x)$	$\forall x$

- Chẵn $\times$ Chẵn = Chẵn
- Chẵn $\times$ Lẻ = Lẻ
- Lẻ $\times$ Lẻ = Chẵn

Hàm chẵn

$$\int_{-p}^{p} f(x)dx = 2\int_{0}^{p} f(x)dx$$

Và

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x$$

Trong đó

$$a_0 = \frac{1}{p}\int_{0}^{p} f(x)dx$$

$$a_n = \frac{2}{p}\int_{0}^{p} f(x)\cos\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

Hàm lẻ

$$\int_{-p}^{p} f(x)dx = 0$$

Và

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

Trong đó

$$b_n = \frac{2}{p}\int_{0}^{p} f(x)\sin\frac{n\pi}{p}xdx \quad (n = 1, 2, ...)$$

1.2 Differentiation

$$(c)' = 0$$

$$(kx)' = k\,,\ (ku)' = ku'$$

$$(x^n)' = nx^{n-1}\,,\ (u^n)' = u'nu^{n-1}$$

$$(uv)' = u'v + v'u$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}\,,\ \left(\frac{1}{u}\right)' = \frac{u'}{u^2}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}\,,\ (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$(\sin x)' = \cos x\,,\ (\sin u)' = u'\cos u$$

$$(\cos x)' = -\sin x\,,\ (\cos u)' = -u'\sin u$$

$$(e^x)' = e^x\,,\ (e^u)' = u'e^u$$

$$(a^x)' = a^x \ln a\,,\ (a^u)' = u'a^u \ln a$$

$$(\ln x)' = \frac{1}{x}\,,\ (\ln u)' = \frac{u'}{u}$$

$$(\log_a x)' = \frac{1}{x \ln a}\,,\ (\log_a u)' = \frac{u'}{u \ln a}$$

$$(u \pm v)' = u' \pm v'$$

$$(u_1 \pm ... \pm u_n)' = u_1' \pm ... \pm u_n'$$

$$y = y[u(x)] \Rightarrow y'(x) = y'(u)u'(x)$$

$$\left(\frac{ax+b}{cx+d}\right)' = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$$

1.3 Integral

$$\int dx = x \, , \int du = u$$

$$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} \, , \int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1}$$

$$\int a^{x} dx = \frac{a^{x}}{\ln a} \, , \int a^{u} dx = \frac{a^{u}}{\ln a}$$

$$\int \frac{dx}{x} = \ln |x| \, , \int \frac{du}{u} = \ln |u|$$

$$\int e^{x} dx = e^{x} \, , \int e^{u} du = e^{u}$$

$$\int e^{kx} dx = \frac{e^{k} x}{k}$$

$$\int \cos x dc = \sin x \, , \int \cos u du = \sin u$$

$$\int \sin x dx = -\cos x \, , \int \sin u du = -\cos u$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b)$$

$$\int (ax+b)^{\alpha} = \frac{1}{a} \frac{(ax+b)^{\alpha+1}}{\alpha+1}$$

1.4 Trigonometry

$$\sin^2 a + \cos^2 a = 1$$

$$\tan a = \frac{\sin a}{\cos a}$$

$$\cot a = \frac{\cos a}{\sin a}$$

$$1 + \tan^2 a = \frac{1}{\cos^2 a}$$

$$1 + \cot^2 a = \frac{1}{\sin^2 a}$$

$$\tan a \cot a = 1$$

1.4.1 Cộng

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

1.4.2 Nhân

$$\sin 2a = 2\sin a\cos a$$

$$\cos 2a = \cos^2 a – \sin^2 a = 2\cos^2 a – 1 = 1 – 2\sin^2 a$$

$$\tan 2a = \frac{2\tan a}{1 – \tan^2 a}$$

$$\cot 2a = \frac{\cot^2 a – 1}{2\cot a}$$

$$\sin 3a = 3\sin a – 4\sin^3 a$$

$$\cos 3a = 4\cos^3 a – 3\cos a$$

$$\tan 3a = \frac{3\tan a – \tan^3 a}{1 – 3\tan^2 a}$$

$$\cot 3a = \frac{3\cot^2 a – 1}{\cot^3 a – 3\cot a}$$

1.4.3 Hạ bậc

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\tan^2 a = \frac{1 - \cos 2a}{1 + \cos 2a}$$

$$\sin^2 a \cos^2 a = \frac{1 - \cos 4a}{8}$$

$$\cos^3 a = \frac{3\cos a + \cos 3a}{4}$$

$$\sin^3 a = \frac{3\sin a - \sin 3a}{4}$$

$$\sin^4 a = \frac{\cos 4a - 4\cos 2a + 3}{8}$$

$$\cos^4 a = \frac{\cos 4a + 4\cos 2a + 3}{8}$$

1.4.4 Tích tổng

$$\cos a \cos b = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin a \sin b = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right]$$

$$\sin a \cos b = \frac{1}{2}\left[\sin(a + b) + \sin(a - b)\right]$$

1.4.5 Tổng tích

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\cos a + \sin a = \sqrt{2}\cos\left(\frac{\pi}{4}-a\right) = \sqrt{2}\sin\left(\frac{\pi}{4}+a\right)$$

$$\cos a - \sin a = \sqrt{2}\cos\left(\frac{\pi}{4}+a\right) = \sqrt{2}\sin\left(\frac{\pi}{4}-a\right)$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a\cos b}$$

$$\tan a - \tan b = \frac{\sin(a-b)}{\cos a\cos b}$$

$$\cot a + \cot b = \frac{\sin(a+b)}{\sin a\sin b}$$

$$\cot a - \cot b = \frac{\sin(b-a)}{\sin a\sin b}$$

$$\cot a + \tan a = \frac{2}{\sin 2a}$$

$$\cot a - \tan a = 2\cot 2a$$