# Phép biến đổi tích phân

April 10, 2025

# 1 Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

 $[-\pi,\pi]$ 

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \ dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \ dx \quad (n = 1, 2, \ldots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \; dx \quad (n=1,2,\ldots) \label{eq:bn}$$

$$[0,2\pi]\to \int_0^{2\pi}$$

# 1.1 Khoảng bất kỳ

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right)$$

[-p,p]

$$a_0 = \frac{1}{2p} \int_{-p}^p f(x) \; dx$$

$$a_n = \frac{1}{p} \int_{-p}^p f(x) \cos \frac{n\pi}{p} x \, dx \quad (n = 1, 2, \ldots)$$

$$b_n = \frac{1}{p} \int_{-p}^p f(x) \sin \frac{n\pi}{p} x \, dx \quad (n = 1, 2, \ldots)$$

# 1.2 Hàm chẳn lẻ

HÀM	TÍNH CHẤT	Điều kiện
Chẳn	J ( - )	$\forall x$
Lẻ	f(-x) = -f(x)	$\forall x$

- $Ch\mathring{a}n \times Ch\mathring{a}n = Ch\mathring{a}n$
- $Ch\mathring{a}n \times L\mathring{e} = L\mathring{e}$
- Lẻ  $\times$  Lẻ = Chẳn
- $\cos \rightarrow \mathrm{Ch} \dot{\tilde{a}} \mathrm{n}$ .
- $\sin \rightarrow L\dot{e}$ .

### 1.2.1 Hàm chẳn

$$\int_{-p}^{p} f(x)dx = 2\int_{0}^{p} f(x) dx$$

Và

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x$$

Trong đó

$$a_0 = \frac{1}{p} \int_0^p f(x) \, dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi}{p} x \, dx \quad (n = 1, 2, ...)$$

### 1.2.2 Hàm lẻ

$$\int_{-p}^{p} f(x) \ dx = 0$$

Và

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{p} x$$

Trong đó

$$b_n = \frac{2}{p} \int_0^p f(x) \sin \frac{n\pi}{p} x \, dx \quad (n = 1, 2, \ldots)$$

### 1.3 Parseval

$$\frac{1}{2p} \int_{-p}^{p} f(x)^2 dx = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

# 2 Fourier Transform

### 2.1 Biểu diễn

$$f(x) = \int_0^\infty [A(\omega)\cos\omega x + B(\omega)\sin\omega x] d\omega \quad (-\infty < x < \infty), \ \forall \omega \ge 0$$

Trong đó

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt$$

$$B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(t) \sin \omega t \, dt$$

Không liên tục tại x

$$\frac{f(x+) + f(x-)}{2}$$

# 2.2 Phép biến đổi

$$\hat{f}(\omega) = \mathcal{F}(f(x))(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx \quad (-\infty < \omega < \infty)$$

$$\hat{f}(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \check{f}(x) \, dx$$

Ngược

$$\check{f}(x) = f(x) = \mathcal{F}^{-1}(\hat{f}(\omega))(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega x} \hat{f}(\omega) \, d\omega \quad (-\infty < x < \infty)$$

### 2.3 Operational properties

#### 2.3.1 AXTT

$$\mathcal{F}(\alpha f + \beta q) = \alpha \mathcal{F}(f) + \beta \mathcal{F}(q)$$

#### 2.3.2 Đạo hàm

$$\mathcal{F}(f^{(n)}) = (i\omega)^n \mathcal{F}(f)$$

$$\mathcal{F}(xf(x))(\omega) = i \bigg[ \hat{f} \bigg]'(\omega) = i \frac{d}{d\omega} \mathcal{F}(f)(\omega)$$

$$\mathcal{F}(x^n f(x))(\omega) = i^n \left[ \hat{f} \right]^{(n)}(\omega)$$

### 2.3.3 Tích chập

$$f*g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x-t)g(t) \ dt$$

$$g*f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x-t) f(t) \; dt$$

$$\mathcal{F}(f * g) = \mathcal{F}(f)\mathcal{F}(g)$$

$$\mathcal{F}(f*f)(\omega) = \hat{f}(\omega)^2$$

Tìm  $\mathcal{F}(g) \to \text{Chọn } f(x)$ sao cho

$$\mathcal{F}(f(x)) = g(\omega)$$
$$\mathcal{F}(\mathcal{F}(f)) = \mathcal{F}(g(\omega))$$
$$f(-x) = \mathcal{F}(g)$$

### 2.3.4 Gaussian Transform

a > 0

$$\mathcal{F}\left(e^{-\frac{ax^2}{2}}\right)(\omega) = \frac{1}{\sqrt{a}}e^{-\frac{\omega^2}{2a}}$$

$$a > 0 \quad (a = 2a)$$

$$\mathcal{F}\left(e^{-ax^2}\right)(\omega) = \frac{1}{\sqrt{2a}}e^{-\frac{\omega^2}{4a}}$$

a = 1

$$\mathcal{F}\left(e^{-\frac{x^2}{2}}\right)(\omega) = e^{-\frac{\omega^2}{2}}$$

### 2.4 Partial Derivatives Transform

$$\mathcal{F}\bigg(\frac{\partial^n}{\partial t^n}u(x,t)\bigg)(\omega)=\frac{d^n}{dt^n}\hat{u}(\omega,t),\quad n=\overline{1,n}$$

$$\mathcal{F}\bigg(\frac{\partial^n}{\partial x^n}u(x,t)\bigg)(\omega)=(i\omega)^n\hat{u}(\omega,t),\quad n=\overline{1,n}$$

### 2.5 Boundary Value Problem

- Step 1: FT the given BVP (Hint: Operational properties)  $\rightarrow$  ODE.
- Step 2: Sovle *ODE* (Hint: Integrated the given *Boundary value*)  $\rightarrow \hat{u}(\omega, t)$ .
- Step 3: Inverse FT  $\hat{u}(\omega,t) \rightarrow u(x,t)$ .

# 3 Laplace Transform

$$\mathcal{L}(f)(s) = \int_0^\infty f(t) e^{-st} dt$$

Với

$$|f(t)| \le Me^{at} \quad \forall t \ge 0$$

Ngược

$$\mathcal{L}(y(t)) = Y(s)$$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

Tính chất

$$\mathcal{L}^{-1}(\alpha F + \beta G) = \alpha \mathcal{L}^{-1}(F) + \beta \mathcal{L}^{-1}(G)$$
 
$$f(t) = \mathcal{L}^{-1}(F(s)) ,$$
 
$$g(t) = \mathcal{L}^{-1}(G(s))$$

# 3.1 Operational properties

### 3.1.1 AXTT

$$\mathcal{L}(\alpha f + \beta g) = \alpha \mathcal{L}(f) + \beta \mathcal{L}(g)$$

### 3.1.2 Đạo hàm

$$\begin{split} \mathcal{L}(f^{(n)}) &= s^n \mathcal{L}(f) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0) \\ \\ \mathcal{L}(f') &= s \mathcal{L}(f) - f(0) \\ \\ \mathcal{L}(f'') &= s^2 \mathcal{L}(f) - s f(0) - f'(0) \\ \\ \mathcal{L}(f''') &= s^3 \mathcal{L}(f) - s^2 f(0) - s f'(0) - f''(0) \end{split}$$

### 3.2 Partial Fractions

$$\begin{split} q(x) &= a(x-a_1)^{m_1}(x-a_2)^{m_2}\dots(x^2+b_1x+c_1)^{n_1}(x^2+b_2x+c_2)^{n_2}\dots\\ &\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_m}{(x-a)^m} \\ &\frac{B_1x+C_1}{(x^2+bx+c)} + \frac{B_2x+C_2}{(x^2+bx+c)^2} + \dots + \frac{B_nx+C_n}{(x^2+bx+c)^n} \end{split}$$

# 3.3 Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	$e^{at}$	
3.	$t^n, \ n=1,2,3,$	$\frac{s-a}{n!}$
4.	$t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\frac{\alpha}{s^{p+1}}}{\sqrt{\pi}}$
6.	$t^{n-\frac{1}{2}},\ n=1,2,3,$	$\frac{\overline{1\cdot 3\cdot 5\cdots (2n-1)\sqrt{\pi}}}{a}$
7.	$\sin(at)$	$\frac{a}{s^2 + a^2}$
8.	$\cos(at)$	$\frac{s}{s^2 + a^2}$ $\frac{2as}{2as}$
9.	$t\sin(at)$	
10.	$t\cos(at)$	$\frac{(s^2 + a^2)^2}{s^2 - a^2}$ $\frac{(s^2 + a^2)^2}{(s^2 + a^2)^2}$

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$f(t) = C^{-1}(E(t))$	E(z) = C(f(t))
$\begin{array}{lllll} & \sin(at) - at \cos(at) & \frac{(s^2 + a^2)^2}{2as^2} \\ & \frac{2as^2}{(s^2 + a^2)^2} \\ & \frac{2as^2}{(s^2 + a^2)^2} \\ & 13. & \cos(at) - at \sin(at) & \frac{s(s^2 - a^2)}{(s^2 + a^2)^2} \\ & 14. & \cos(at) + at \sin(at) & \frac{s(s^2 + a^2)^2}{(s^2 + a^2)^2} \\ & 15. & \sin(at + b) & \frac{s\sin(b) + a\cos(b)}{s^2 + a^2} \\ & 16. & \cos(at + b) & \frac{s\cos(b) - a\sin(b)}{s^2 + a^2} \\ & 17. & \sinh(at) & \frac{a}{s^2 - a^2} \\ & 18. & \cosh(at) & \frac{s^2 - a^2}{s^2 - a^2} \\ & 19. & e^{at}\sin(bt) & \frac{(s - a)^2 + b^2}{s - a} \\ & 20. & e^{at}\cos(bt) & \frac{(s - a)^2 + b^2}{s - a} \\ & 21. & e^{at}\sinh(bt) & \frac{b}{(s - a)^2 - b^2} \\ & 22. & e^{at}\cosh(bt) & \frac{1}{(s - a)^2 - b^2} \\ & 23. & t^n e^{at}, & n = 1, 2, 3, \dots & \frac{n!}{(s - a)^{n+1}} \\ & 24. & f(ct) & \frac{1}{c}F\begin{pmatrix} s \\ c \\ c \\ c \\ s \\ u_c(t) = u(t - c) & (Heaviside Function) \\ & 26. & \delta(t - c) & (Dirac Delta Function) \\ & 26. & \delta(t - c) & (Dirac Delta Function) \\ & 27. & u_c(t) f(t - c) & e^{-cs}F(s) \\ & 28. & u_c(t) g(t) & e^{-cs}\mathcal{E}\{g(t + c)\} \\ & 29. & e^{ct}f(t) & F(s - c) \\ & 30. & t^n f(t), & n = 1, 2, 3, \dots & (-1)^n F^{(n)}(s) \\ & 31. & \frac{1}{t}f(t) & \int_s^\infty F(u)  du \\ & \end{array}$		$f(t) = \mathcal{L}^{-1}\{F(s)\}$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	11.	$\sin(at) - at\cos(at)$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			$\frac{(s^2+a^2)^2}{2as^2}$
$\begin{array}{llllllllllllllllllllllllllllllllllll$	12.	$\sin(at) + at\cos(at)$	$\overline{(s^2+a^2)^2}$
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21. $e^{at} \sinh(bt)$ $\frac{b}{(s-a)^2 - b^2}$ 22. $e^{at} \cosh(bt)$ $\frac{n!}{(s-a)^{n+1}}$ 24. $f(ct)$ $\frac{1}{c} F\left(\frac{s}{c}\right)$ 25. $u_c(t) = u(t-c)$ (Heaviside Function) $\frac{e^{-cs}}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_c(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n = 1, 2, 3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$		,	$(s-a)^2 + b^2$
21. $e^{at} \sinh(bt)$ 22. $e^{at} \cosh(bt)$ 23. $t^n e^{at}$ , $n = 1, 2, 3,$ 24. $f(ct)$ 25. $u_c(t) = u(t-c)$ (Heaviside Function) 26. $\delta(t-c)$ (Dirac Delta Function) 27. $u_c(t)f(t-c)$ 28. $u_c(t)g(t)$ 29. $e^{ct}f(t)$ 30. $t^n f(t)$ , $n = 1, 2, 3,$ 31. $\frac{1}{t}f(t)$ $(s-a)^2-b^2$ $(s-a)^2-b^2$ $(s-a)^{n+1}$ $(s-a)^$	20.	$e^{av}\cos(bt)$	$\overline{(s-a)^2+b^2}$
22. $e^{-cc} \cosh(bt)$ $ \frac{(s-a)^2 - b^2}{n!} $ 23. $t^n e^{at}$ , $n = 1, 2, 3,$ $ \frac{n!}{(s-a)^{n+1}} $ 24. $f(ct)$ $ \frac{1}{c} F\left(\frac{s}{c}\right) $ 25. $u_c(t) = u(t-c)$ (Heaviside Function) $ \frac{e^{-cs}}{s} $ 26. $\delta(t-c)$ (Dirac Delta Function) $ e^{-cs} F(s) $ 27. $u_c(t) f(t-c)$ $ e^{-cs} F(s) $ 28. $u_c(t) g(t)$ $ e^{-cs} \mathcal{L}\{g(t+c)\} $ 29. $e^{ct} f(t)$ $ F(s-c) $ 30. $t^n f(t)$ , $n = 1, 2, 3,$ $ (-1)^n F^{(n)}(s) $ 31. $\frac{1}{t} f(t)$ $ \int_s^{\infty} F(u)  du $	21.	$e^{at}\sinh(bt)$	*
23. $t^n e^{at}$ , $n = 1, 2, 3,$ $\frac{n!}{(s-a)^{n+1}}$ 24. $f(ct)$ $\frac{1}{c} F\left(\frac{s}{c}\right)$ 25. $u_c(t) = u(t-c)$ (Heaviside Function) $\frac{e^{-cs}}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_c(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n = 1, 2, 3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$	22	$e^{at}\cosh(bt)$	$\frac{(s-a)^2-0^2}{s-a}$
23. $t^{n}e^{us}$ , $n = 1, 2, 3,$ $\frac{1}{(s-a)^{n+1}}$ 24. $f(ct)$ $\frac{1}{c}F\left(\frac{s}{c}\right)$ 25. $u_{c}(t) = u(t-c)$ (Heaviside Function) $\frac{e^{-cs}}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_{c}(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_{c}(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^{n}f(t)$ , $n = 1, 2, 3,$ $(-1)^{n}F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_{s}^{\infty}F(u) du$	22.	. ,	$(s-a)^2 - b^2$
25. $u_c(t) = u(t-c)$ (Readstate Function) $\frac{s}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_c(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n=1,2,3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_s^{\infty} F(u)  du$	23.	$t^n e^{at},  n = 1, 2, 3, \dots$	$\frac{n!}{(s-a)^{n+1}}$
25. $u_c(t) = u(t-c)$ (Readstate Function) $\frac{s}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_c(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n=1,2,3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_s^{\infty} F(u)  du$	24.	f(ct)	$\frac{1}{r}F\left(\frac{s}{r}\right)$
25. $u_c(t) = u(t-c)$ (Readstate Function) $\frac{s}{s}$ 26. $\delta(t-c)$ (Dirac Delta Function) $e^{-cs}$ 27. $u_c(t)f(t-c)$ $e^{-cs}F(s)$ 28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n=1,2,3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_s^{\infty} F(u)  du$			$\stackrel{c}{e^{-cs}} \stackrel{c}{\langle c \rangle}$
$ \begin{array}{lll} 27. & u_c(t)f(t-c) & e^{-cs}F(s) \\ 28. & u_c(t)g(t) & e^{-cs}\mathcal{L}\{g(t+c)\} \\ 29. & e^{ct}f(t) & F(s-c) \\ 30. & t^nf(t), & n=1,2,3,\dots & (-1)^nF^{(n)}(s) \\ 31. & \frac{1}{t}f(t) & \int_s^\infty F(u)du \end{array} $			
28. $u_c(t)g(t)$ $e^{-cs}\mathcal{L}\{g(t+c)\}$ 29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t),  n=1,2,3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_s^{\infty} F(u)  du$			
29. $e^{ct}f(t)$ $F(s-c)$ 30. $t^n f(t)$ , $n = 1, 2, 3,$ $(-1)^n F^{(n)}(s)$ 31. $\frac{1}{t}f(t)$ $\int_s^{\infty} F(u) du$			
31. $\frac{1}{t}f(t)$ $\int_{s}^{\infty} F(u) du$	29.	$e^{ct}f(t)$	F(s-c)
	31.	$\frac{1}{t}f(t)$	$\int_{s}^{\infty} F(u) du$
J(0, 0, 1)			$\underline{F(s)}$
33. $\int_0^t f(t-\tau)g(\tau) d\tau$ $F(s)G(s)$		0	S = F(s)G(s)
$\int_{0}^{T} e^{-st} f(t) dt$	55.	O .	
34. $f(t+T) = f(t)$ $\frac{\int_0^{\infty} e^{-t} f(t) dt}{1 - e^{-sT}}$	34.	f(t+T) = f(t)	$\frac{J_0 - J(v) \cdot w}{1 - e^{-sT}}$

# 4 ODE

# 4.1 PTVP CẤP 1

# 4.1.1 Tách biến

$$f(x)dx = g(y)dy$$

phuong phap

$$\int f(x)dx = \int g(y)dy$$
$$F(x) = G(y) + C$$

# 4.1.2 Tuyến tính cấp 1

# Thuần nhất

$$y' + p(x)y = 0$$

phuong phap

$$\begin{split} \frac{dy}{dx} &= -p(x)y \\ \frac{1}{y}dy &= -p(x)dx \\ \int \frac{1}{y}dy &= -\int p(x)dx \\ \ln|y| &= -\int p(x)dx + \ln|c| \\ \ln|y| - \ln|c| &= -\int p(x)dx \\ \ln|\frac{y}{c}| &= -\int p(x)dx \\ \frac{y}{c} &= e^{-\int p(x)dx} \\ y &= c.e^{-\int p(x)dx} \end{split}$$

# Không thuần nhất

$$y' + p(x)y = q(x)$$

phuong phap (biến thiên hằng số)

$$y' + p(x)y = 0 \implies y = C.e^{-\int p(x)dx}$$

$$C = C(x)$$

$$\begin{cases} y = C(x).e^{-\int p(x)dx} \\ y' = C'(x).e^{-\int p(x)dx} - p(x)C(x).e^{-\int p(x)dx} \end{cases}$$

$$\Rightarrow y' + p(x)y = q(x)$$

$$C'(x) \cdot e^{-\int p(x)dx} - p(x)C(x) \cdot e^{-\int p(x)dx} + p(x)C(x) \cdot e^{-\int p(x)dx} = q(x)$$

$$C'(x) \cdot e^{-\int p(x)dx} = q(x)$$

$$C'(x) = q(x) \cdot e^{\int p(x)dx}$$

$$C(x) = \int q(x) \cdot e^{\int p(x)dx} dx + K$$

$$\implies y = C(x).e^{-\int p(x)dx}$$
 
$$y = \left(\int q(x).e^{\int p(x)dx}dx + K\right)e^{-\int p(x)dx}$$

### 4.1.3 Bernoulli

$$y' + p(x)y = q(x).y^{\alpha}$$

phuong phap

$$y^{-\alpha}.y' + p(x)y^{1-\alpha} = q(x)$$

$$z = y^{1-\alpha}$$

$$z' = (1-\alpha).y^{-\alpha}.y'$$

$$y^{-\alpha}.y' = \frac{1}{1-\alpha}z'$$

$$\implies y^{-\alpha}.y' + p(x)y^{1-\alpha} = q(x)$$
 
$$\frac{1}{1-\alpha}z' + p(x).z = q(x)$$
 
$$z' + (1-\alpha)p(x).z = q(x)(1-\alpha)$$

### 4.1.4 Toàn phần

$$P(x,y)dx + Q(x,y)dy = 0$$

$$P'_{u} = Q'_{x}$$

$$u(x,y) = \int_{x_0}^x P(x,y_0)dx + \int_{y_0}^y Q(x,y)dy + C$$
 or 
$$u(x,y) = \int_x^x P(x,y)dx + \int_y^y Q(x_0,y)dy + C$$

# 4.2 PTVP CÁP 2

### 4.2.1 Thuần nhất

$$y'' + py' + qy = 0$$

phuong phap

$$\lambda^2 + p\lambda + q = 0 \implies \text{giai ptrinh} \implies \lambda_1, \lambda_2$$

- $\lambda_1 \neq \lambda_2 \implies y = C_1 \cdot e^{\lambda_1 x} + C_2 \cdot e^{\lambda_2 x}$
- $\lambda_1 = \lambda_2 = \lambda \implies y = (C_1 + x.C_2)e^{\lambda x}$
- $\lambda_{1,2} = \alpha \pm i\beta \implies y = e^{\alpha x}(C_1 \cdot \cos \beta x + C_2 \cdot \sin \beta x)$

### 4.2.2 Không thuần nhất

$$y'' + py' + qy = f(x)$$

1)  $f(x) = e^{ax}Q_n(x)$  phuong phap

$$y'' + py' + qy = 0 \implies \lambda_1, \lambda_2 \implies y_{tn}$$

$$(P_n(x)$$
 cung bac voi  $Q_n(x),$  vi du:  $P_n(x)=Ax+B)$ 

- $a \neq \lambda_1 \land a \neq \lambda_2 \implies y_r = e^{ax} P_n(x)$
- $\begin{array}{ll} \bullet & a = \lambda_1 \vee a = \lambda_2 \implies y_r = x e^{ax} P_n(x) \\ \bullet & a = \lambda_1 = \lambda_2 \implies y_r = x^2 e^{ax} P_n(x) \end{array}$

$$\Rightarrow y'' + py' + qy = f(x)$$

$$y''_r + py'_r + qy_r = f(x)$$

$$\Rightarrow \text{He so } A, B, \dots \Rightarrow y_r \Rightarrow y = y_{tn} + y_r$$

2)  $f(x) = P_n(x) \sin \beta x + Q_m(x) \cos \beta x$  phuong phap

$$y'' + py' + qy = 0 \implies \lambda_1, \lambda_2 \implies y_{tn}$$

$$l = \max\{m,n\}R_l(x) = Ax + B \quad H_l(x) = Cx + D$$

- $\pm i\beta \neq \lambda_{1,2} \implies y_r = R_l(x)\sin\beta x + H_l(x)\cos\beta x$
- $\pm i\beta = \lambda_{1,2}^{1,2} \implies y_r = x(R_l(x)\sin\beta x + H_l(x)\cos\beta x)$

$$\implies y'' + py' + qy = f(x)$$
 
$$y''_r + py'_r + qy_r = f(x)$$
 
$$\implies \text{He so } A, B, C, D, \dots \implies y_r \implies y = y_{tn} + y_r$$

# 3) Biến thiên hằng số

$$y''+py'+qy=0 \implies \lambda_2, \lambda_2 \implies y_{tn}=C_1.y_1(x)+C_2.y_2(x)$$
 
$$C_1=C_1(x)C_2=C_2(x)$$
 
$$\mathrm{HPT} \implies \begin{cases} C_1'y_1+C_2'y_2=0\\ C_1'y_1'+C_2'y_2'=f(x) \end{cases} \implies C_1',C_2' \implies C_1,C_2$$
 
$$\implies y=C_1.y_1(x)+C_2.y_2(x)$$

# 5 Basics

### 5.1 Euler formula

$$\begin{split} e^{i\omega x} &= \cos(\omega x) + i\sin(\omega x) \\ e^{-i\omega x} &= \cos(\omega x) - i\sin(\omega x) \\ |e^{-i\omega x}| &= |\cos(\omega x) - i\sin(\omega x)| = \sqrt{\cos^2(\omega x) + \sin^2(\omega x)} = 1 \\ \cos u &= \frac{1}{2}(e^{iu} + e^{-iu}) \\ \sin u &= \frac{1}{2i}(e^{iu} - e^{-iu}) \end{split}$$

#### 5.2 Differentiation

$$(c)' = 0$$

$$(kx)' = k , (ku)' = ku'$$

$$(x^n)' = nx^{n-1} , (u^n)' = u'nu^{n-1}$$

$$(uv)' = u'v + v'u$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2} , \left(\frac{1}{u}\right)' = \frac{u'}{u^2}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} , (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$(\sin x)' = \cos x , (\sin u)' = u'\cos u$$

$$(\cos x)' = -\sin x , (\cos u)' = -u'\sin u$$

$$(e^x)' = e^x , (e^u)' = u'e^u$$

$$(a^x)' = a^x \ln a , (a^u)' = u'a^u \ln a$$

$$(\ln x)' = \frac{1}{x} , (\ln u)' = \frac{u'}{u}$$

$$(\log_a x)' = \frac{1}{x \ln a} , (\log_a u)' = \frac{u'}{u \ln a}$$

$$(u \pm v)' = u' \pm v'$$

$$(u_1 \pm \dots \pm u_n)' = u'_1 \pm \dots \pm u'_n$$

$$y = y[u(x)] \Rightarrow y'(x) = y'(u)u'(x)$$

$$\left(\frac{ax + b}{cx + d}\right)' = \frac{ab}{(cx + d)^2}$$

#### 5.3 Integral

$$\int dx = x \,, \int du = u$$

$$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} \,, \int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1}$$

$$\int a^x dx = \frac{a^x}{\ln a} \,, \int a^u dx = \frac{a^u}{\ln a}$$

$$\int \frac{dx}{x} = \ln |x| \,, \int \frac{du}{u} = \ln |u|$$

$$\int e^x dx = e^x \,, \int e^u du = e^u$$

$$\int e^{kx} dx = \frac{e^k x}{k} \,, \int e^{ax+b} dx = \frac{1}{a} e^{ax+b}$$

$$\int \cos x dc = \sin x \,, \int \cos u du = \sin u$$

$$\int \sin x dx = -\cos x \,, \int \sin u du = -\cos u$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b)$$

$$\int (ax+b)^{\alpha} = \frac{1}{a} \frac{(ax+b)^{\alpha+1}}{\alpha+1}$$

# 5.3.1 Từng phần

$$\int_{a}^{b} u \, dv = uv|_{a}^{b} - \int_{a}^{b} v \, du$$

$$I = \int_{a}^{b} P(x)Q(x) \, dx$$

$$u = P(x)$$

$$dv = Q(x)dx$$

Chọn P(x) đặt u: Nhất log - Nhì đa - Tam lượng - Tứ mũ. Múa cột

VD1: Tính nguyên hàm :  $I = \int (2x^2 - 3) e^x dx$ 

	(đạo hàm )	dấu	(nguyên hàm)		
	$u=2x^2-3$		$dv = e^x dx$		
	4 <i>x</i>	+ 🗼	$e^x$		
	4	,	$e^x$		
	0	+	e <sup>x</sup>		
		_			

$$\Rightarrow I = e^{x}(2x^{2} - 3) - 4x \cdot e^{x} + 4e^{x} + C$$
$$= e^{x}(2x^{2} - 4x + 1) + C$$

### 5.4 Trigonometry

$$\sin^2 a + \cos^2 a = 1$$

$$\tan a = \frac{\sin a}{\cos a}$$

$$\cot a = \frac{\cos a}{\sin a}$$

$$1 + \tan^2 a = \frac{1}{\cos^2 a}$$

$$1 + \cot^2 a = \frac{1}{\sin^2 a}$$

 $\tan a \cot a = 1$ 

### 5.4.1 Cộng - Trừ

$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

### 5.4.2 Nhân

$$\sin 2a = 2\sin a\cos a$$

$$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

$$\cot 2a = \frac{\cot^2 a - 1}{2 \cot a}$$

$$\sin 3a = 3\sin a - 4\sin^3 a$$

$$\cos 3a = 4\cos^3 a - 3\cos a$$

$$\tan 3a = \frac{3\tan a - \tan^3 a}{1 - 3\tan^2 a}$$

$$\cot 3a = \frac{3\cot^2 a - 1}{\cot^3 a - 3\cot a}$$

### 5.4.3 Hạ bậc

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\tan^2 a = \frac{1 - \cos 2a}{1 + \cos 2a}$$

$$\sin^2 a \cos^2 a = \frac{1 - \cos 4a}{8}$$

$$\cos^3 a = \frac{3\cos a + \cos 3a}{4}$$

$$\sin^3 a = \frac{3\sin a - \sin 3a}{4}$$

$$\sin^4 a = \frac{\cos 4a - 4\cos 2a + 3}{8}$$

$$\cos^4 a = \frac{\cos 4a + 4\cos 2a + 3}{8}$$

# 5.4.4 Tích tổng

$$\cos a \cos b = \frac{1}{2} \left[ \cos(a – b) + \cos(a + b) \right]$$

$$\sin a \sin b = \frac{1}{2} \left[ \cos(a – b) – \cos(a + b) \right]$$

$$\sin a \cos b = \frac{1}{2} \left[ \sin(a+b) + \sin(a\!-\!b) \right]$$

# 5.4.5 Tổng tích

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\cos a + \sin a = \sqrt{2}\cos\left(\frac{\pi}{4} - a\right) = \sqrt{2}\sin\left(\frac{\pi}{4} + a\right)$$

$$\cos a - \sin a = \sqrt{2}\cos\left(\frac{\pi}{4} + a\right) = \sqrt{2}\sin\left(\frac{\pi}{4} - a\right)$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cos b}$$

$$\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cos b}$$

$$\cot a + \cot b = \frac{\sin(a+b)}{\sin a \sin b}$$

$$\cot a - \cot b = \frac{\sin(b-a)}{\sin a \sin b}$$

$$\cot a + \cot b = \frac{2\sin(b-a)}{\sin a \sin b}$$

$$\cot a + \cot b = \frac{2\sin(b-a)}{\sin a \sin b}$$

$$\cot a + \cot b = \frac{2\sin(b-a)}{\sin a \sin b}$$

# 5.5 Cấp số nhân

$$S_n=u_1+u_2+\ldots+u_n=\frac{u_1(1-q^n)}{1-q}$$
 
$$u_n=u_{n-1}.q$$
 
$$u_n=u_1.q^{n-1}$$

 $\cot a - \tan a = 2 \cot 2a$