Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
3rd to 7th Semester

15MA302 – DISCRETE MATHEMATICS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. Which of the following is tautology?
   (A) $(P \vee Q) \rightarrow P$ (B) $P \vee (Q \rightarrow P)$
   (C) $P \vee (P \rightarrow Q)$ (D) $P \rightarrow (P \rightarrow Q)$

2. A premise may be introduced at any point in the derivation is called
   (A) Rule P (B) Rule P and Rule T
   (C) Rule T (D) Rule C

3. If C(x):x is a cat, B(x) : x is black, then $(\exists x)(C(x) \wedge B(x))$ symbolizes the statement
   (A) Some cats are not black (B) All cats are black
   (C) Some cats are black (D) All cats are not black

4. Which of the following statement is the contrapositive of the statement, ' if 4 is even, then –5 is negative?
   (A) If 4 is even then –5 is not negative (B) 4 is even and –5 is not negative
   (C) If –5 is not negative, then 4 is not even (D) 4 is odd or –5 is not negative

5. In a group of 100 people, several will have birth days in the same month. Atleast how many must have birthdays in the same month.
   (A) 10 (B) 9
   (C) 8 (D) 12

6. If A= {1, 2, 3, 4}, B= {x, y, z} and $f\{(1,x),(2,y),(3,z),(4,x)\}$, then the function f is
   (A) Both 1–1 and onto (B) Neither 1–1 nor onto
   (C) 1–1 but not onto (D) Onto but not 1–1

7. A simplified form of digraph representing the partial order relation is
   (A) Helmut hasse (B) Poset
   (C) Graph relation (D) Hasse diagram

8. Which one of the following relations on the set {1, 2, 3, 4} is an equivalent relation
   (A) $\{(2,4),(4,2)\}$ (B) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
   (C) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$ (D) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

9. The order of the recurrence relation $S(K)-4S(K-1)-11S(K-2)+30S(K-3)=4^K$ is
   (A) 1 (B) 0
   (C) 3 (D) 4

10. The generating function for the recurrence relation $S(K)=2S(K-1), K \geq 1, S(0)=1$ is
   (A) $\frac{1}{1-z}$ (B) $\frac{1}{1+z}$
   (C) $\frac{1}{1+2z}$ (D) $\frac{1}{1-2z}$

11. The order of the identity element of a group of order 3 is
   (A) 1 (B) 0
   (C) 3 (D) 2

12. If 'a' is an element of a group (G, *) with identity e such that $a^2 = a$, then
   (A) $a = e$ (B) $a = a^{-1}$
   (C) 'a' is generator of G (D) Order of 'a' is 3

13. A circuit of a graph G is called _____ circuit if it includes each edge of G exactly once.
   (A) Hamiltonian (B) Konisberg
   (C) Closed (D) Eulerian

14. A connected graph without any circuit is called
   (A) Leaf (B) Flower
   (C) Tree (D) Loop

15. Every vertex which is reachable from a vertex v through a single edge are called ____ of v.
   (A) Descendant (B) Leaf
   (C) Children (D) Root

16. A vertex with zero in degree is called
   (A) Sink (B) Source
   (C) Terminal (D) Out degree

17. If (L, ≤) is any lattice and if 'b' is a complement of 'a', then $a \vee b$
   (A) 0 (B) 1
   (C) a (D) b

18. $X+(Y+Z)=$
   (A) X (B) (Y+Z)
   (C) (X+Y) + (X+Z) (D) XYZ

19. $a.b+a.b'$
   (A) b (B) a
   (C) ab (D) a′

20. State the value of $a \vee 1$
   (A) a (B) 0
   (C) a × 1 (D) 1

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

21. Show that the conclusion C follows from the premises H_1, H_2, H_3 in the following case using truthtable technique $H_1 : P \to (Q \to R), H_2 : P \wedge Q, C : R$.

22. Symbolize the following statements:
    (i) All roses are red (ii) Some men are clever

23. Draw the Hasse diagram representing the partial ordering $\{(A,B)/(A \subseteq B)\}$ on the power set P(S) where S = {a, b, c}.

24. If S = {1, 2, 3, 4, 5} and if the functions $f, g : S \to S$ are given by f = {(1, 2), (2, 1), (3, 4), (4, 5), (5, 3)}, g = {(1, 3), (2, 5), (3, 1), (4, 2), (5, 4)}. Show that $(f \bullet g)^{-1} = g^{-1} \bullet f^{-1} \neq f^{-1} \bullet g^{-1}$.

25. Prove that a cyclic group is abelian.

26. Give an example of a graph which contains:
    (i) An Eulerian circuit that is also a Hamiltonian circuit.
    (ii) An Eulerian circuit, but not an Hamiltonian circuit
    (iii) A Hamiltonian circuit, but not an Eulerian circuit
    (iv) Neither an Eulerian circuit nor a Hamiltonian circuit

27. In any Boolean algebra, show that $a = b$ *if* $ab' + a'b = 0$.

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

28. a.i. 'If you help me, then I will do my homework', 'If you do not help me, then I will go to sleep early'. 'If I go to bed early, the teacher will punish me'. Show that the above hypothesis leads to the conclusion "If I do not do my homework, then the teacher will punish me".

ii. Use mathematical induction , to prove $1.2 + 2.3 + 3.4 + ... + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

(OR)

b. Show that the premises, "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "someone in this class can get a high paying job".

29. a. Using Warshall's algorithm find the transitive closure of the relation whose matrix is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(OR)

b.i. A man hiked for 10 hours and covered a total distance of 45km. I is known that he hiked 6 km in the first hour and only 3km in the last hour. Show that he must have hiked alteast 9 km within a certain period of 2 consecutive hours.

ii. If $f: Z \to N$ is defined by

$$f(x) = \begin{cases} 2x-1, & if\ x > 0 \\ -2x\ , & if\ x \le 0 \end{cases}$$

(i) Prove that f is one to one and onto.

(ii) Determine f^{-1}.

30. a.i. Prove the necessary and sufficient condition for a non-empty subset H of a group {G, *} to be a subgroup is a, b ∈ H implies $a * b^{-1} \in H$.

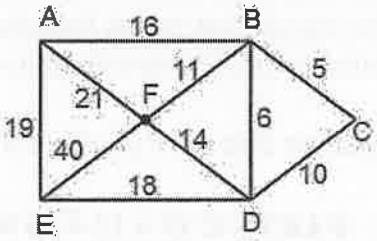
ii. Find the generating function of the recurrence relation $a_n = 3a_{n-1} + 1; n \ge 1$ given that $a_0 = 1$.

(OR)

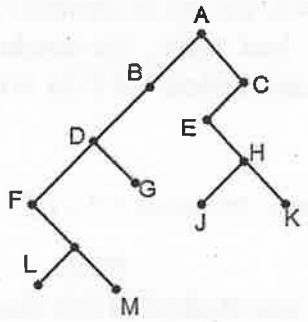
b.i. Solve the recurrence relation $a_n - 2a_{n-1} = 3^n; a_1 = 5$.

ii. Prove that union of two subgroups of a group {G, *} need not be a subgroup.

31. a.i. Find the minimum spanning tree for the weighted graph using Kruskal's algorithm.



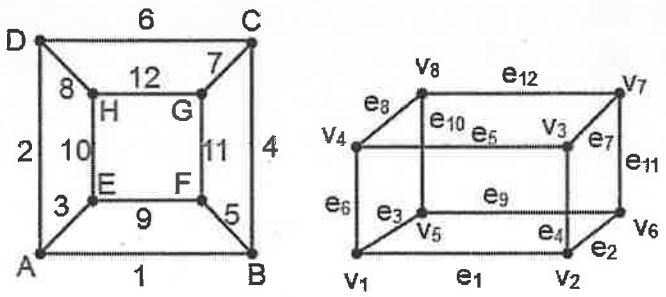
ii. Write the order in which the vertices of tree given in the figure are processed using preorder, inorder traversal.



(OR)

b.i. Prove that the number of edges in a bipartite graph with n vertices is atmost $\left(\frac{n}{2}\right)^2$.

ii. Verify whether the graphs are isomorphic or not:



32. a.i. Prove that $D_{42} = \{S_{42},\ D\}$ is a complemented lattice by finding the complements of all the elements $D_{42} = \{1,2,3,6,7,14,21,42\}$.

ii. Simplify the Boolean expression $a'.b'.c + a.b'.c + a'.b'.c'$ using Boolean algebra identities.

(OR)

b.i. If $\{L, \leq\}$ is a lattice, then for any a, b, c ∈ L, the following properties hold good: If $b \leq c$, then $(a \vee b) \leq (a \vee c)$ *and* $(a \wedge b) \leq (a \wedge c)$.

ii. In any Boolean algebra, show that $(xy'z' + xy'z + xyz + xyz')(x + y) = x$.

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