ii. If $f: Z \to \mathbb{N} \cup \{0\}$ defined by $f(x)=\begin{cases} 2x-1, & x>0 \\ -2x & x<0 \end{cases}$ prove f is 1–1 and onto.

30. a.i. Solve the recurrence relation $a_{n+2}-6a_{n+1}+9a_n=7(3)^n, n\geq 0$ given that $a_0=1, a_1=4$.

ii. Prove that sub group of a cyclic group is cyclic.

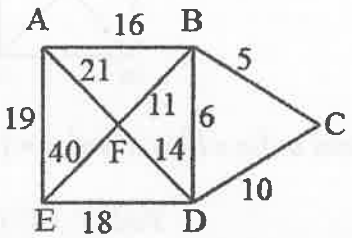
(OR)

b.i. Using generating functions, solve $a_n-3a_{n-1}=1, n\geq 1$ with $a_0=1$.

ii. Show that the set Q^+ of all +ve rational numbers forms an abelian group under the operation '*' defined by $a*b=\frac{ab}{2}, a,b\in Q^+$.

31. a.i. Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n}{2}\right)^2$. (4 Marks)

ii. Find the minimum spanning tree for the weighted graph.



(8 Marks)

(OR)

b.i. Prove that the number of vertices in a full binary tree is odd and the number of pendant vertices of a tree is equal to $\frac{n+1}{2}$.

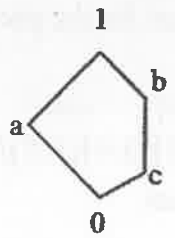
ii. Construct the binary tree whose inorder and pre order traversals are EACIFHDBG and FAEICDHGB.

32. a.i. Let (L, ≤) be any lattice. Then prove the following statements are equivalent. $(b\leq c)\Leftrightarrow(b\vee c=c)\Leftrightarrow(b\wedge c\Leftrightarrow b)$.

ii. Simplify the Boolean expression $x[y+z(xy+xz)']$.

(OR)

b.i. Verify whether the lattice given by the Hasse diagram is distributive or not.



ii. In any Boolean algebra, show that $(a+b')(b+c')(c+a')=(a'+b)(b'+c)(c'+a)$.

* * * * *

(3)

Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2019

1st to 7th Semester

15MA302 – DISCRETE MATHEMATICS

(For the candidates admitted during the academic year 2015 – 2016 to 2017 – 2018)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minutes

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Which of the following proposition is a tautology?
(A) $(p\vee q)\to p$ (B) $p\vee(q\to p)$
(C) $p\vee(p\to q)$ (D) $p\to(p\to q)$

2. Which one is the inverse of $q\to p$?
(A) $p\to q$ (B) $\sim p\to\sim q$
(C) $\sim q\to\sim p$ (D) $p\vee q$

3. Let P: he is tall and q: he is handsome be 2 statements then the statement "it is not true that he is short or handsome" is
(A) $p\vee q$ (B) $\sim(\sim p\vee q)$
(C) $p\vee(\sim q)$ (D) $\sim p\vee q$

4. Which one is the dual of $\neg(p\vee q)\vee T$
(A) $\neg(p\wedge q)\wedge T$ (B) $\neg(p\wedge q)\wedge F$
(C) $\neg(p\vee q)\vee F$ (D) $\neg(p\wedge q)\vee T$

5. If f:A→B be a function with |A|=m and |B|=n then how many functions can be defined from A to B?
(A) 2^n (B) 2^m
(C) n^m (D) m^n

6. In a group of 100 people several will have birthdays in the same month. At least how many must have birthdays in the same month
(A) 10 (B) 8
(C) 9 (D) 12

7. Determine which one of the following relations on the set {1,2,3,4} is a function.
(A) $R_1=\{(1,1)(2,1)(3,1)(4,1)(3,3)\}$ (B) $R_2=\{(1,2)(2,3)(4,2)\}$
(C) $R_3=\{(4,4)(3,1)(1,2)(4,2)\}$ (D) $R_4=\{(1,1)(2,1)(3,2)(3,4)\}$

8. Determine which of the following relations has the property of symmetric and antisymmetric
(A) $R_1=\{(1,1)(2,2)(3,3)\}$ (B) $R_2=\{(1,1)(2,3)(3,2)(2,2)\}$
(C) $R_3=\{(2,2)(1,3)(3,2)(3,1)\}$ (D) $R_4=\{(1,1)(1,2)(2,1)(2,2)(3,3)(3,1)(1,3)\}$

9. The order of the recurrence relation $a_n - 4a_{n-1} - 11a_{n-2} + 30a_{n-3} = 4^n$ is
(A) 1 (B) 0
(C) 3 (D) 4

10. The generating function of the sequence 1,1,1,1…. is given by
(A) $\frac{1}{1+x}$ (B) $\frac{1}{1-x}$
(C) $\frac{1}{(1-x)^2}$ (D) $\frac{1}{x}$

11. A cyclic of order 8 has how many generators
(A) 1 (B) 4
(C) 2 (D) 8

12. A set (S,*) is called a monoid if the following axioms are satisfied
(A) Closure (B) Closure, associative, identity
(C) Closure, associative (D) Closure, associative, inverse, identity

13. A simple graph in which there is exactly one edge between each pair of distinct vertices is called
(A) Regular graph (B) Bipartite graph
(C) Complete graph (D) Tree

14. The number of vertices of odd degree in an undirected graph is
(A) ODD (B) EVEN
(C) ZERO (D) ONE

15. A connected graph contains an Euler circuit if and only if each of its vertices is of
(A) ODD degree (B) EVEN degree
(C) PRIME degree (D) No degree

16. If every vertex of a simple graph has the same degree then the graph is called
(A) Complete graph (B) Regular graph
(C) Bipartite graph (D) Sub graph

17. If (L,∨,∧) is a Boolean algebra then the complement of any element a ∈L is
(A) 2 (B) 3
(C) 0 (D) 1

18. Every totally ordered set is a
(A) Set with equivalence relation defined in it (B) Set with uncomparable elements in it
(C) Set with is complemented (D) Set which is complemented and distributive

19. In a lattice L if $a \le c$ then $a \vee (b \wedge c) \le (a \vee b) \wedge c$ is called
(A) Demorgan's law (B) Modular property
(C) Distributive property (D) Complement property

20. If $a \le c$ in L then
(A) $a \vee c = a$ (B) $a \vee c = c$
(C) $a \wedge c = c$ (D) $a = 1$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Construct truth table for $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$.

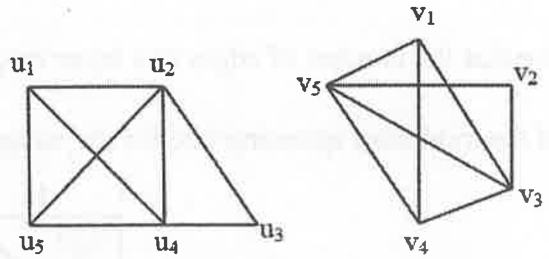
22. If R is the relation on the set of positive integers such that (a,b)∈R if and only if a^2+b is even. Prove that R is an equivalence relation.

23. If $f,g,h{:}R \to R$ are defined by $f(x) = x^2 - 4x, g(x) = \frac{1}{x^2+1}$ and $h(x) = x^4$. Find $(f_o(goh))(x)$ and $((fog)oh)(x)$.

24. Prove every cyclic group is abelian.

25. A man hiked for 10 hours and covered a total distance of 45kms. It is known that he hiked 6km in the first hour and only 3km in the last hour. Show that he must have hiked atleast 9km within a certain period of 2 consecutive hours.

26. Determine whether the following graphs are isomorphic.



27. Prove the laws $a \cdot (a+b) = a$ and $a + (a \cdot b) = a$ in a Boolean algebra.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Without using truth tables, prove $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$. (4 Marks)

ii. Show that the following premises "it is not sunny this afternoon and it is colder than yesterday. It is sunny if we will go to the playground. If we do not go to the ground then we will go to a movie. If we go to a movie then we will return home by sunset" lead to the conclusion "we will return home by sunset". (8 Marks)

(OR)

b.i. Use mathematical induction, to show that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \ge 2$.

ii. Establish the validity of the following argument:
Everyone who takes some fruit daily is healthy. Ram is not healthy. Therefore, Ram does not take fruit daily.

29. a.i. Draw Hasse diagram for the partial ordering "$x \le y \Leftrightarrow x$ divides y" on $S = \{1,2,3,4,6,8,12\}$. (4 Marks)

ii. If A={1,2,3,4,5} and R is the relation on A defined by $R = \{(1,2)(2,1)(2,3)(3,4)(4,2)(4,4)(5,1)(5,5)\}$. Find the transitive closure of R by using Warshall's algorithm. (8 Marks)

(OR)

b.i. If f:A→B and g:B→C are invertible functions then prove that $g \circ f : A \to C$ is also invertible.