

b.i. If the functions $f, g, h: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2 + 1}$ and $h(x) = x^4$ then find $\{(f \circ g) \circ h\}(x)$ and $\{f \circ (g \circ h)\}(x)$ and check if they are equal.

ii. A man liked for 10 hours and covered at total distance of 45 km. it is known that he liked 6 km in the first hour and only 3 km in the last hour. Show that he must have liked atleast 9 km within a certain period of 2 consecutive hours.

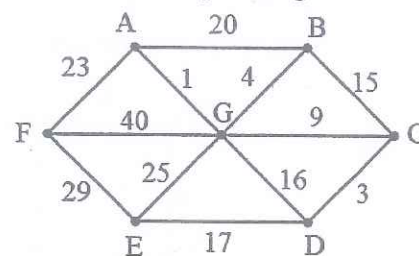
30.a. Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 2^n + n, (n \geq 2)$ given that $a_0=1$ and $a_1=4$.

(OR)

b.i. Use the method of generating functions to solve the recurrence relation $a_n = 3a_{n-1} + 1, n \geq 1$ given that $a_0=1$. (8 Marks)

ii. Prove that the identify element of a group $(G, *)$ is unique. (4 Marks)

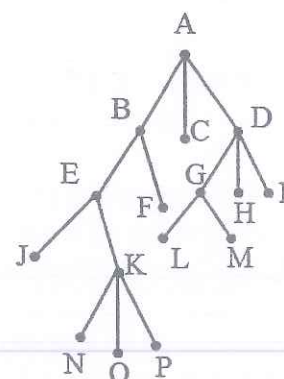
31.a. Use Kruskal's algorithm to find a minimum spanning tree of the weighted graph.



(OR)

b.i. Prove that in a bipartite graph with n vertices, the number of edges is atmost $\left(\frac{n}{2}\right)^2$.

ii. List the order in which the vertices of the following tree are processed using pre order, in order and post order.



32.a.i. If any Boolean algebra, show that $x[y + z(xy + xz)]' = xy$.

ii. Simplify the Boolean expression $a + a'bc' + (a+b)'$.

(OR)

b.i. Prove that $D_{42} = \{S_{42}, D\}$ is a complemented lattice by finding the complements of all the elements.

ii. State and prove dominance law in Boolean algebra.

Reg. No.

B.Tech. DEGREE EXAMINATION, DECEMBER 2019

First to Eighth Semester

15MA302 – DISCRETE MATHEMATICS

(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:

(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

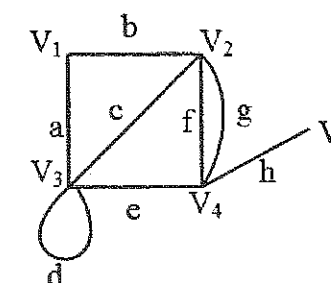
- Which of the following statement is the contra positive of the statement "if 4 is even and then 5 is negative"?
(A) If 5 is not negative and then 4 is not even (B) If 4 is even then 5 is not negative
(C) 4 is odd or 5 is not negative (D) 4 is even and 5 is not negative
- What is the negation of "every city in Canada is clean"?
(A) Every city in Canada is not clean (B) Some city in Canada is not clean
(C) Some city in Canada is clean (D) None of the above
- The dual of $\neg(p \wedge q) \rightarrow [\neg p \vee (\neg p \vee q)]$ is
(A) $(p \wedge q) \wedge [\neg p \wedge (\neg p \wedge q)]$ (B) $\neg p \wedge q$
(C) $(p \vee q) \wedge [\neg p \wedge q]$ (D) $(p \wedge q) \vee [p \vee (\neg p \vee q)]$
- A premise may be introduced at any point in the derivation is called _____.
(A) Rule P (B) Rule P and Rule T
(C) Rule T (D) Rule of CP
- A digraph representing the partial order relation is _____.
(A) Helmut Hasse (B) Poset
(C) Graph relation (D) Hasse diagram
- If set 'A' has m elements and 'B' has n elements then the number of functions defined from A to B is _____.
(A) m^n (B) n^m
(C) mn (D) n^2
- If $A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$ and $f = \{(1, x)(2, y)(3, z)(4, x)\}$ then the function f is _____.
(A) Both one to one and onto (B) One to one but not onto
(C) Onto but not one to one (D) Neither one to one nor onto

8. If the relation R is reflexive, symmetric and transitive then the relation R is called _____.
 (A) Equivalence class (B) Partial order relation
 (C) Partially order set (D) Equivalence relation
9. The solution of the recurrence relation $a_n - 2a_{n-1} = 0$ is _____.
 (A) $C.3^n$ (B) $C.2^n$
 (C) $n.2^n$ (D) $C2^n - 1$
10. The recurrence relation of the sequence 1, 3, 9, ... is _____.
 (A) $a_n = 3a_{n-1}$ (B) $a_n = 3a_{n+1}$
 (C) $a_n = 3a_n$ (D) $a_n = a_{n-1}$
11. The generator of a cyclic group $G = \{1, -1, i, -i\}$ is _____.
 (A) $i, -i$ (B) $1, -i$
 (C) $-1, -i$ (D) $1, -1$
12. If 'a' is an element of a group $(G, *)$ with identity 'e' such that $a^2 = a$ then
 (A) $a = a^{-1}$ (B) 'a' is generator of G
 (C) $a = e$ (D) Order of 'a' is 3
13. A connected graph without any circuit is called _____.
 (A) Leaf (B) Flower
 (C) Tree (D) Loop
14. A vertex which has degree one is called _____.
 (A) Isolated vertex (B) Pendent vertex
 (C) Incident vertex (D) Simple vertex
15. A tree with 'n' vertices has _____ edges.
 (A) n (B) n/2
 (C) n-1 (D) N+1
16. In a graph G all vertices are adjacent to each other then G is called _____.
 (A) Bipartite graph (B) Pseudo graph
 (C) Complete graph (D) Regular graph
17. Complement of any element in a Boolean algebra is _____.
 (A) Unique (B) 2
 (C) More than 2 (D) 0
18. All Boolean algebras of order 2^n are _____ to each other.
 (A) Homomorphism (B) Isomorphic
 (C) Subsets (D) Non-isomorphic
19. Every finite lattice is _____.
 (A) Bounded lattice (B) Unbounded lattice
 (C) Un countable lattice (D) Infinite lattice

20. A partial ordered set $\{<, \leq\}$ in which every pair of elements has a least upper bound and greatest lower bound is _____.
 (A) Boolean (B) Join
 (C) Lattice (D) Complement

PART - B (5 × 4 = 20 Marks)
 Answer ANY FIVE Questions

21. Construct the truth table for $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg p$.
22. Without using truth table, prove that the following $[p \wedge (p \leftrightarrow q)] \equiv T$.
23. If R is the relation of $A = \{1, 2, 3\}$ such that $(a, b) \in R$ if and only if $a + b = \text{even}$, find the relation matrix M_R , $M_{R^{-1}}$ and M_{R^2} .
24. Prove that the intersection of two subgroups is again a subgroup of G.
25. Define an incident matrix and write the incident matrix of the following graph.



26. In a Boolean algebra, show that $(a + b)' = a' \cdot b'$.
27. Draw the Hasse diagram representing the partial ordering divisibility on the set $\{1, 3, 5, 9, 15, 45, 60, 75\}$.

PART - C (5 × 12 = 60 Marks)
 Answer ALL Questions

- 28.a. Show that the following set of premises are inconsistent. "If Rama gets his degree, he will go for a job". "If he goes for a job, he will get married soon". "If he goes for higher studies, he will not get married". "Rama gets his degree and he goes for higher studies".
- (OR)
- b.i. Using indirect method of proof, derive $p \rightarrow \neg s$ from the premises
 $P \rightarrow (Q \vee R), Q \rightarrow \neg P, S \rightarrow \neg R$ and P . (8 Marks)
- ii. Using mathematical induction prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9, $n \geq 1$. (4 Marks)
- 29.a. Let $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 1), (1, 3), (1, 5), (2, 3), (2, 4), (3, 3), (3, 5), (4, 2), (4, 4), (5, 4)\}$ be the relation on A. find the transitive closure of R using Warshall's algorithm.

(OR)