b.i. If $f : A \to B$ and $g : B \to A$ are mappings such that $f \circ g = I_B$ and $g \circ f = I_A$. Prove that f and g are both invertible and also prove that $f^{-1} = g$ and $g^{-1} = f$. (6 Marks)

ii. Let $f : R \to R$ and $g : R \to R$ where R is the set of real numbers be given by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g$ and $g \circ f$. State whether these functions are injective, surjective and bijective. (6 Marks)

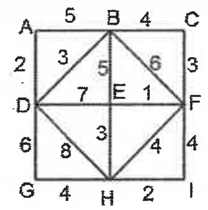
30. a. Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n),\ n \geq 0$ given that $a_0 = 1$ and $a_1 = 4$.

(OR)

b.i. State and prove Lagrange's theorem. (8 Marks)

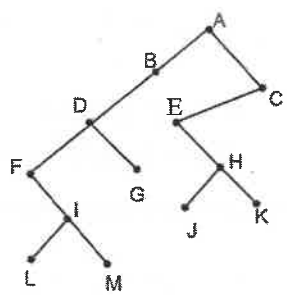
ii. Find the generating function for the recurrence relation $a_n = 4a_{n-1} + 3n.2^n,\ n \geq 1,\ a_0 = 4$. (4 Marks)

31.a. Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph.



(OR)

b. List the order in which the vertices of tree given in are processed using preorder, inorder and postorder traversal.



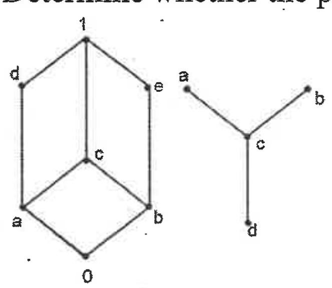
32.a.i. State and prove De Morgan's law. (8 Marks)

ii. In an Boolean algebra, prove that $ab + abc + a'b + ab'c = b + ac$. (4 Marks)

(OR)

b.i. Prove that $D_{42} \equiv \{S_{42}, D\}$ is a complemented lattice by finding the complements of all the elements. (8 Marks)

ii. Determine whether the posets represented by the Hasse diagram given are lattices.



(4 Marks)

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Reg. No.														

B.Tech. DEGREE EXAMINATION, JANUARY 2023

First to Eighth Semester

15MA302 – DISCRETE MATHEMATICS

(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. Which of the following is the negation of the statement "All the students of this class are intelligent"?
   (A) Atleast one student of this class is intelligent
   (B) Atleast one student of this class is not intelligent
   (C) Some student of the class are intelligent
   (D) Not all the students of this class are not intelligent

2. The statement $(p \wedge q) \Rightarrow p$ is a
   (A) Consistent
   (B) Contradiction
   (C) Tautology
   (D) Inverse

3. What is the dual of $(p \to q) \to (\sim q \to \sim p) \equiv T$
   (A) $\sim(\sim p \wedge q) \wedge (q \wedge \sim p) \equiv F$
   (B) $\sim(p \wedge q) \wedge (q \wedge p) \equiv T$
   (C) $(\sim p \wedge q) \wedge (q \wedge \sim p) \equiv F$
   (D) $(p \wedge q) \wedge (q \wedge \sim p) \equiv T$

4. If $A(x) : x$ is an animal, $B(x) : x$ is black and $C(x) : x$ is a cat, translate the following in words $(\exists x)(C(x) \wedge B(x))$
   (A) If x is a cat then it is black
   (B) All cats are black
   (C) Some animal are cats and are black
   (D) Some cats are black

5. Relative complement of S with respect to R is defined as
   (A) $\{x / x \in R \text{ and } x \notin S\}$
   (B) $\{x / x \in R \text{ and } x \in S\}$
   (C) $\{x / x \notin R \text{ and } x \in S\}$
   (D) $\{x / x \notin R \text{ and } x \notin S\}$

6. In a group of 100 people, several will have birthdays in the same month. atleast how many must have birth days in the same month.
   (A) 10
   (B) 8
   (C) 12
   (D) 9

7. How many possible functions we get $f : A \to B$, if $|A| = m$ and $|B| = n$
   (A) n^m
   (B) 2^n
   (C) 2^m
   (D) m^n

8. $R \subseteq Z^{+} \times Z^{+}$, where aRb if a divides b; determine the relation R.
(A) Equivalence relation (B) Totally ordered relation
(C) Partial order relation (D) Equivalence class of relations

9. Every group of order 4 is
(A) Abelian (B) Cyclic
(C) Normal group (D) Not abelian

10. The generating function of the sequence 1, 1, 1, ….. is given by
(A) $\frac{1}{1-x}$ (B) $\frac{1}{(1-x)^2}$
(C) $\frac{1}{1+x}$ (D) $\frac{1}{(1+x)^2}$

11. The characteristics equation of the recurrence relation $a_{n+2} - 6a_{n+1} - 9a_n = 0$ is
(A) $nC_1 3^n$ (B) $n(C_1 + nC_2)3^n$
(C) $(C_1 + nC_2)3^n$ (D) $n^2(C_1 + nC_2)3^n$

12. Find the formula for the general term F_n of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13….
(A) $F_{n+1} = F_n + F_{n-1}, n \geq 0$ (B) $F_{n+2} = F_{n+1} + F_n, n \geq 0$
(C) $F_n = F_{n+1} + F_{n+2}, n \geq 0$ (D) $F_{n-1} = F_n + F_{n+1}, n \geq 0$

13. Any ______ graph with n vertices and (n–1) edges is a tree.
(A) Connected (B) Circuitless
(C) Eulerian (D) Closed

14. If every internal vertex of a rooted tree has exactly 2 children the tree is called
(A) Full binary (B) Binary tree
(C) Tree (D) circuit

15. Every vertex which is reachable from a vertex V through a single edge are called ____ of V.
(A) Descendant (B) Leaf
(C) Children (D) Root

16. The number of edges in a complete graph with n vertices is
(A) $\frac{n(n-1)}{2}$ (B) $\frac{n(n+1)}{2}$
(C) $\frac{n!}{2}$ (D) $\frac{n(n+1)(2n+1)}{2}$

17. A _______ is lattice which contains a least element and a greatest element and which is both complemented and distributive.
(A) Boolean algebra (B) Algebra
(C) Lattice (D) Modular

18. Find the Boolean identity of $(a+b).(a'+b) =$
(A) $a+b$ (B) b
(C) a (D) $a.b$

19. If $\{L, \leq\}$ is a lattice, then for any $a, b, c \in L$ and $b \leq c$ then $a \vee b \leq a \vee c$ is
(A) Isotonic property (B) Modular inequality
(C) Distributive inequality (D) Absorption property

20. Find the complement of the Boolean expression $ab' + ac + b'c$
(A) $(a+b')+(a+c)+(b'+c)$ (B) $(a'+b).(a'+c').(b+c')$
(C) $(a \wedge b')+(a \wedge c)+(b' \wedge c)$ (D) $(a' \cdot b)+(a' \cdot c')+(b \cdot c')$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Show by indirect method of proof b can be derived from the premises $a \to b, c \to b, d \to (a \vee c), d$.

22. Nine points are in a unit square. Prove that there is a triangle formed by 3 of the points has an area not more than 1/8.

23. Prove that $(A-C) \cap (C-B) = \phi$ analytically.

24. If R is a relation on the set $\{1, 2, 3, 4, 5\}$, list the ordered pairs in R when aRb *if* $a+b=6$.

25. Show that $\{1, 3, 5, 7\}$ is an abelian group under multiplication modulo 8.

26. Prove that a tree with n vertices has $(n-1)$ edges.

27. In Boolean algebra, if $a+b=1$ and $a \cdot b = 0$, show that the complement of every element is unique.

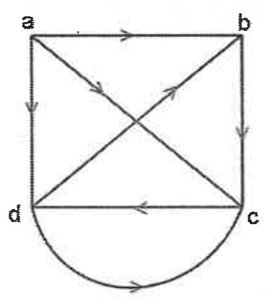
PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a.i. Show that the following premises are inconsistent: $p \to q, q \to r, s \to \urcorner r, p \wedge s$.
ii. Prove by mathematical induction that $n^3 - n$ is divisible by 6 for $n \in z^+$.

(OR)

b. Show that the premises "one student in this class knows how to write program in JAVA" and "everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "someone in this class can get a high paying job".

29. a. Find the relation matrix of a graph. Find the transitive closure of the path matrix of a given graph by using Warshall's algorithm.



(OR)