B.Tech. DEGREE EXAMINATION, MAY 2022
Fifth & Sixth Semester

18MAB302T – DISCRETE MATHEMATICS FOR ENGINEERS
(For the candidates admitted from the academic year 2018-2019 to 2019-2020)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours   Max. Marks: 75

PART – A (25 × 1 = 25 Marks)
Answer **ALL** Questions   Marks BL CO PO

1. If the cardinality of the sets A and B are 5 and 3 respectively then the cardinality of A×B is   1 2 1 2
(A) 32
(B) 8
(C) 25
(D) 15

2. Let A={0,1,2,3,4}, B={0,1,2,3} and aRb iff a+b=4, R={(1,3),(2,2),(3,1),(4,0)}. Then image of R is equal to   1 2 1 2
(A) {1,2,3,0}
(B) {1,2,3}
(C) {1,2,4}
(D) {1,3,4}

3. If A and B are non-empty sets then $A \cup (A-B)$ is   1 2 1 1
(A) ϕ
(B) A
(C) B
(D) A–B

4. The function $f : z^+ \to z^+$ defined by $f(x) = x^2 + 2$ is   1 2 1 2
(A) Bijective
(B) Injective, not surjective
(C) Surjective, not injective
(D) Neither injective nor surjective

5. Let $f(x) = x^3$ and $g(x) = 4x + 3$ be functions on R. Then fog is   1 2 1 2
(A) $4x^3 + 3$
(B) $4x^3$
(C) $(4x+3)^3$
(D) $4x+3$

6. Number of ways in which 'n' persons can be seated round a table is   1 2 2 2
(A) n!
(B) (n−1)!
(C) (n+1)!
(D) (n+2)!

7. If a and b are any two integers and non-zero then gcd (a,b) is   1 2 2 1
(A) *mn+ab*
(B) *ab divides mn*
(C) *ma−nb*
(D) *ma+nb*

8. Generalization of pigeonhole principle for n>m is   1 2 2 1
(A) $\left\lfloor \dfrac{n-1}{m} \right\rfloor + 1$
(B) $\left\lceil \dfrac{n}{m} \right\rceil + 1$
(C) $\left\lceil \dfrac{n-1}{m} \right\rceil - 1$
(D) $\left\lfloor \dfrac{n+1}{m} \right\rfloor + 1$

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b. Use the Euclidean algorithm to find gcd(12345, 54321) and express the gcd as a linear combination m,n of the given numbers and also find m and n.   10 3 2 2

28. a.i. Determine whether the following implication is a tautology, by using truth table technique.   6 3 3 2
$$p \to (p \to r) \Rightarrow (p \to q) \to (p \to r).$$

ii. Prove the equivalence without using truth table.   4 3 3 2
$$p \to (q \to p) \equiv \neg p \to (p \to q).$$

(OR)

b. Using rules of inference construct an argument to show that the following premises imply the conclusion "It rained". "If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on"; "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded".   10 3 3 2

29. a. State and prove the necessary and sufficient condition for a subset of a group G to be a subgroup of G.   10 3 4 1
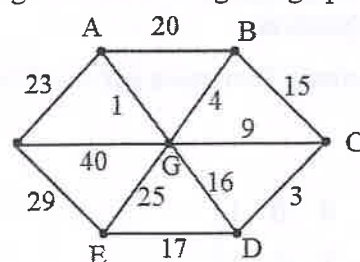
(OR)

b. Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding to the encoding function $e : B^3 \to B^6$, find the corresponding parity check matrix and use it to decode the following received words and hence, to find the original message. Are all the words decoded uniquely?   10 3 4 2
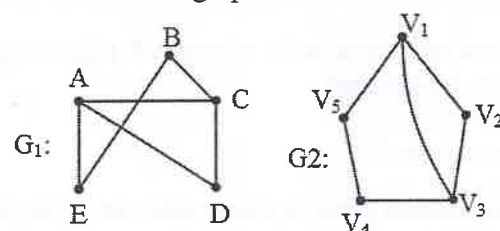(i) 110101
(ii) 001111
(iii) 111111

30. a. Find the minimum spanning tree for the weighted graph using Kruskal's algorithm.   10 3 5 2

(OR)

b.i. Give an example of a graph which contains   5 3 5 2
(1) An Eulerian circuit and a Hamiltonian circuit that are distinct
(2) Neither an Eulerian circuit nor a Hamiltonian circuit

ii. By using circuits, prove that the two graphs G₁ and G₂ are isomorphic.   5 3 5 2

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9. How many bit strings of length 10 contain atleast four 1's   1   2   2   2
   (A) 151200   (B) 5040
   (C) 848   (D) 720

10. The prime factorization of 6647 is   1   2   2   2
    (A) $(17^2)(23)$ (B) $(17)(23)$
    (C) $(17^3)(23)$ (D) $(23^2)(17)$

11. The dual of $\left(p \wedge (p \vee q)\right) \to q \equiv T$ is   1   2   3   2
    (A) $\neg\left(p \wedge (p \vee q)\right) \vee q \equiv T$ (B) $\neg\left(p \vee (p \wedge q)\right) \wedge q \equiv F$
    (C) $\neg\left(p \wedge (p \vee \dot{q})\right) \wedge q \equiv F$ (D) $\neg\left(p \wedge (p \vee q)\right) \vee q \equiv F$

12. The statement if a figure is a square then it is a quadrilateral is represented by p →q   1   2   3   2
    then it inverse is
    (A) If a figure is not a square then it is   (B) If a figure is not a quadrilateral then
    not a quadrilateral   it is not a square
    (C) If a figure is quadrilateral then it is   (D) If a figure is a square then it is not a
    a square   quadrilateral

13. Which of the following proposition is a tautology?   1   2   3   2
    (i) $(p \vee q) \to p$
    (ii) $p \vee (p \to q)$
    (iii) $p \vee (p \to \neg r)$
    (iv) $p \to (p \to q)$
    (A) (i)   (B) (ii)
    (C) (iii)   (D) (iv)

14. A set of formulae $H_1, H_2 \ldots H_m$ is said to be consistent if their conjunction implies a   1   2   3   1
    (A) Contradiction   (B) Tautology
    (C) Conclusion   (D) Implication

15. Select the correct option: 1   2   3   1
    (I)   The proposition p→q is F when p is T and q is F
    (II)   The proposition p↔q is F when p is T and q is F
    (A) (I) T (II) T   (B) (I) F (II) F
    (C) (I) T (II) F   (D) (I) F (II) T

16. The order of an element 'i' of the group G={1,−1,i,−i} under the usual multiplication   1   2   4   2
    is
    (A) 1   (B) 2
    (C) 4   (D) 3

17. The number of identity elements in a group is   1   2   4   2
    (A) Unique   (B) Distinct
    (C) More than one   (D) Not exist

18. The parity check matrix of the generator matrix $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ is   1   2   4   2
    (A) $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$
    (C) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

19. A code can detect a set of at most 5 errors iff the minimum distance between any two   1   2   4   2
    code words is
    (A) Atleast 6   (B) Atmost 6
    (C) Atleast 11   (D) Atmost 11

20. The weight of the word 1101001 is   1   2   4   2
    (A) 1   (B) 3
    (C) 4   (D) 7

21. If G=(V,E) is an undirected graph with 5 edges then $\sum_i \deg(Vi) =$ 1   2   5   2
    (A) 10   (B) 5
    (C) 11   (D) 12

22. The maximum number of edges in a simple disconnected graph G with n vertices and   1   2   5   2
    k components is
    (A) $\dfrac{(n+k)(n-k+1)}{2}$ (B) $\dfrac{(n-k)(n-k+1)}{2}$
    (C) $\dfrac{n(n-k+1)}{2}$ (D) $\dfrac{(n-k)^2}{2}$

23. A connected graph contains an Euler circuit iff each of its vertices is of   1   2   5   1
    (A) Even degree   (B) Odd degree
    (C) Prime degree   (D) Neither odd nor even

24. The chromatic number of a planar graph is not greater than   1   2   5   1
    (A) 3   (B) 4
    (C) 5   (D) 6

25. A tree with 100 vertices has   1   1   5   2
    (A) 98 edges   (B) 96 edges
    (C) 100 edges   (D) 99 edges

PART – B (5 × 10 = 50 Marks)
Answer ALL Questions

                                                  Marks   BL   CO   PO

26. a. If f:A→B and g:B→C are invertible functions, then prove gof:A→C is also invertible   10   3   1   1
    and prove that $(gof)^{-1}=f^{-1}\, og^{-1}$.

(OR)

b.i. Find the transitive closure of $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.   6   3   1   2

ii. Draw the Hasse diagram representing the partial ordering $P = \{(a,b \,/\, a\ divides\ b\}$ 4   3   1   2
    on {1,2,3,4,6,8,12} starting from the diagraph of P.

27. a.i. There are 3 piles of identical red, blue and green balls, where each pile contains atleast   6   3   2   2
    10 balls. In how many ways 10 balls be selected
    (1)   If there is no restriction?
    (2)   If atmost one red ball is selected?

ii. Of any 5 points chosen within an equilateral triangle whose sides are of length 1, show   4   3   2   2
    that atleast two points are within a distance of 1/2 of each other.

(OR)