b. Let A = {1, 2, 3, 4, 5} and R {(1, 1), (1, 3), (1,5), (2, 3), (2, 4), (3, 3), (3, 5), (4, 2), (4, 4), (5, 4)}. Find the transitive closure of R using Warshall's algorithm.

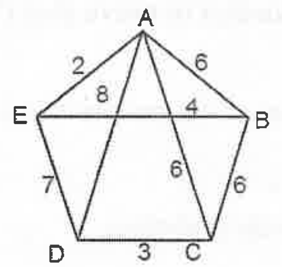
30. a. Solve the recurrence relation $a_{n+2}-6a_{n+1}+9a_n=3(2)^n+7(3)^n, n\geq 0$ given that $a_0=1, a_1=4$.

(OR)

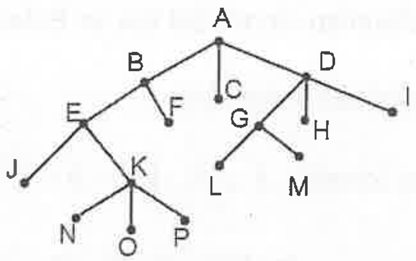
b.i. Prove that the intersection of two subgroups of a group G is also a subgroup of G.

ii. Prove the necessary and sufficient conditions for a nonempty subset H of a group $\{G, *\}$ to be a group.

31. a.i. Find the minimum spanning tree for the weighted graph using Kruskal's algorithm.



ii. In which order does (i) preorder (ii) post order traversal visit the vertices of the ordered rooted tree given in the diagram.



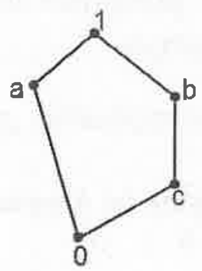
(OR)

b.i. Prove that the number of edges in a bipartite graph with 'n' vertices is atmost $\left(\frac{n}{2}\right)^2$.

ii. Find the value of prefix expression $+-\uparrow 32\uparrow 23/8-42$.

32. a.i. If (L, ≤) be a lattice, then for any a, b, c ∈ L the following properties hold. If b ≤ c, then (i) $a\vee b\leq a\vee c$ (ii) $a\wedge b\leq a\wedge c$.

ii. Verify whether the lattice given by Hasse diagram is distributive.



(OR)

b.i. Prove that $(a+b)'=a'\circ b'$ and $(a-b)'=a'+b'$, for all a, b ∈ B.

ii. In any Boolean algebra, show that $(x+y)(x'+z)=xz+x'y+yz=xz+x'y$.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2019

3rd to 8th Semester

15MA302 – DISCRETE MATHEMATICS

(For the candidates admitted during the academic year 2015 – 2016 to 2017 – 2018)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minutes

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. Which one is the contrapositive of $q\rightarrow p$?
(A) $p\rightarrow q$ (B) $\neg p\rightarrow \neg q$
(C) $\neg q\rightarrow \neg p$ (D) $p\wedge q$

2. What is the dual of $(p\rightarrow q)\rightarrow(\neg q\rightarrow \neg p)\equiv T$
(A) $\neg(\neg p\wedge q)\wedge(q\wedge \neg p)\equiv F$ (B) $\neg(p\wedge q)\wedge(q\wedge p)\equiv T$
(C) $(\neg p\wedge q)\wedge(q\wedge \neg p)\equiv F$ (D) $p\wedge q$

3. The negation of $(x)(R(x)\rightarrow D(x))$ is
(A) $(x)(\neg R(x)\rightarrow D(x))$ (B) $(x)R(x)\rightarrow D(x)$
(C) $\exists x(R(x)\wedge \neg D(x))$ (D) $R(x)\rightarrow \neg D(x)$

4. $P\rightarrow Q, Q\rightarrow R\Rightarrow$ ________.
(A) $P\rightarrow R$ (B) $R\rightarrow P$
(C) Q (D) P

5. An equivalence relation R on a set A is said to possess
(A) Reflexive, antisymmetric and transitive (B) Reflexive, symmetric and transitive
(C) Reflexive, nonsymmetric and antisymmetric (D) Reflexive, symmetric

6. A digraph representing the partial order relation is a
(A) Helmut hasse (B) Poset
(C) Graph relation (D) Hasse diagram

7. In a group of 100 people, several will have birthdays in the same month. Atleast how many must have birthdays in the same month
(A) 10 (B) 8
(C) 9 (D) 12

8. If $A=\{1, 2, 3, 4\}$, $B=\{x, y, z\}$ and $f=\{(1,x),(2,y),(3,z),(4,x)\}$, then the function 'f' is
   (A) Both 1-1 and onto (B) 1-1 but not onto
   (C) Onto but not 1-1 (D) Neither 1-1 nor onto

9. The solution of the recurrence relation $a_{n-2}a_{n-1}=0$ is
   (A) $C.3^n$ (B) $C.2^n$
   (C) $n.2^n$ (D) $C.2^n-1$

10. The generating function of the sequence 1, 1, 1.... is given by
   (A) $\frac{1}{1+x}$ (B) $\frac{1}{1-x}$
   (C) $\frac{1}{(1-x)^2}$ (D) $\frac{1}{x}$

11. The order of the identity element of a group of order 3 is
   (A) 1 (B) 0
   (C) 3 (D) 2

12. If a is an element of a group (G, *), with identity e such that $a^2=a,$ then
   (A) a = e (B) $a = a^{-1}$
   (C) a is a generator of G (D) Order of a is 3

13. A connected graph without any circuit is called ________.
   (A) Leaf (B) Flower
   (C) Tree (D) Loop

14. A graph in which loops and parallel edges are allowed is called ______ graph.
   (A) Pseudo (B) Multi
   (C) Simple (D) Null

15. A maximum height of a 11 vertex binary tree is
   (A) 4 (B) 5
   (C) 3 (D) 6

16. Every vertex which is reachable from a vertex V is called
   (A) Descendant (B) Leaf
   (C) Children (D) Root

17. State the value of aua
   (A) a (B) a^2
   (C) 0 (D) 1

18. If (L, ≤) is any lattice and if b is a complement of a, then $a\vee b$
   (A) 0 (B) 1
   (C) a (D) b

19. $1.a+0.a$ is
   (A) 1 (B) 2
   (C) a (D) b

20. $a.b.c+a.b$ is equal to
   (A) a (B) b
   (C) $a.b$ (D) $a+(b.c)$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Check whether the following proposition is tautology or not $(p\wedge q)\wedge\neg(p\vee q)$.

22. Use mathematical induction to prove that $1^2+3^2+5^2+....+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$.

23. Prove that in any group of six people atleast three must be mutual friends or atlest three must be mutual strangers.

24. Prove that a cyclic group is abelian.

25. Give an example of a group which contains
   (i) An Eulerian circuit that is also a Hamiltonian circuit
   (ii) A Hamiltonian circuit but not an Eulerian

26. State and prove modular inequality.

27. Simplify using set identify $\overline{A}\cup\overline{B}\cup(A\cap B\cap\overline{C})$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Construct the truthtable for each of the compound preposition $\neg(p\vee(q\wedge r))\leftrightarrow((p\vee q)\wedge(p\rightarrow r))$.

   ii. Without constructing the truthtable, prove that $p\rightarrow(q\rightarrow p)\equiv\neg p\rightarrow(p\rightarrow q)$.

(OR)

   b.i. Use indirect method to prove that the conclusion $\exists z\, Q(z)$ follows from the premises $\forall x(P(x)\rightarrow Q(x))$ and $\exists y P(y)$.

   ii. Give a direct proof for the implication $p\rightarrow(q\rightarrow s), (\neg r\vee p), q\Rightarrow(r\rightarrow s)$.

29. a.i. If R and S be relations on the set A represented by the matrices
$$M_R=\begin{bmatrix}0&1&0\\1&1&1\\1&0&0\end{bmatrix} M_S=\begin{bmatrix}0&1&0\\0&1&1\\1&1&1\end{bmatrix}.$$ Find (i) $R\cup S$ (ii) $R\cap S$ (iii) $R\circ S$

   ii. Draw the digraph representing the partial ordering $\{(a,b)/a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Reduce it to the Hasse diagram representing the given partial ordering.

(OR)