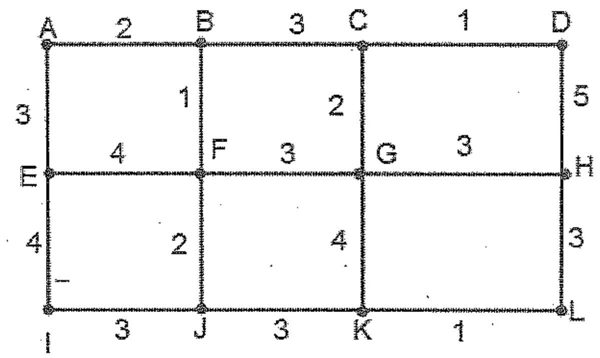
b.i. Using generating function, solve the recurrence relation $a_n - a_{n-1} - 6a_{n-2} = 0,\ a_0 = 2,\ a_1 = 1$. (8 Marks)

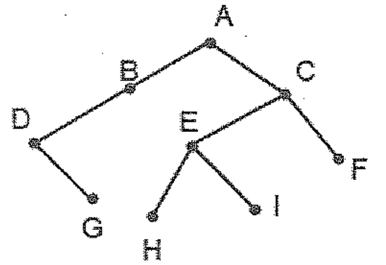
ii. If $f: G \to G'$ is a group homomorphism from $(G,*) \to (G',\Delta)$, show that kernel of f is a subgroup of G. (4 Marks)

31. a. Find the minimum spanning tree for the weighted graph given below.



(OR)

b.i. List the order in which the vertices of the binary tree given below are processed using preoder, inorder and post order traversals. (8 Marks)



ii. Give an example of a graph which is
(A) Eulerian and Hamiltonian
(B) Eulerian but not Hamiltonian
(C) Hamiltonian but not Eulerian
(D) Neither Eulerian nor Hamiltonian

32. a.i. Let $(L, \le)$ be a lattice. For any $a, b, c \in L$ prove that if $b \le c$, then $a \vee b \le a \vee c$ and $a \wedge b \le a \wedge c$.

ii. Given $(D_{42}, \le)$ where $D_{42} = \{Divisors\ of\ 42\}$ and $a \vee b = LCM\{a,b\}$, $a \wedge b = GCD\{a,b\}$ Find the complements of all elements of D_{42}.

(OR)

b.i. Simplify the Boolean expression $a + a'bc' + (b+c)'$.

ii. In any Boolean algebra, simplify $(x+y)(x'+z)$..

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Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third to Seventh Semester

15MA302 – DISCRETE MATHEMATICS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. $(p \to q) \wedge (r \to q)$ is equivalent to
(A) $(p \vee r) \to q$ (B) $(p \wedge r) \to q$
(C) A tautology (D) A contradiction

2. The negation of the statement "2 is even and –3 is negative" is
(A) 2 is even and –3 is not negative (B) 2 is odd and –3 is negative
(C) 2 is even or –3 is negative (D) 2 is odd or –3 is not negative

3. The contra positive of $q \to p$ is
(A) $p \to q$ (B) $\neg p \to \neg q$
(C) $\neg q \to \neg p$ (D) $\neg p \to q$

4. The statement $(p \wedge q) \Rightarrow p$ is a
(A) Tautology (B) A contradiction
(C) A conjunction (D) A negation

5. A relation R on a set A is an equivalence relation if R is
(A) Reflexive and symmetric (B) Reflexive, symmetric and transitive
(C) Symmetric and transitive (D) Reflexive, anti-symmetric and transitive

6. If $A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$ and $f = \{(1,x), (2,y), (3,z), (4,x)\}$ then f is
(A) Both one-one and onto (B) One-one but not onto
(C) Onto but not one-one (D) Neither one-one nor onto

7. If A is any non-empty set and B_1, B_2, B_3 are non-empty subsets of A, then the number of minisets generates by B_1, B_2, B_3 is
(A) 3 (B) 6
(C) 8 (D) 7

8. In a group of 100 people, the least number of people having birthdays in the same month is
(A) 10 (B) 8
(C) 9 (D) 12

9. If $D(k)=5.2^k$, $k\geq 0$, the corresponding recurrence relation is

(A) $D(k)-2D(k-1)=0,\ k\geq 1$ (B) $D(k)+2D(k-1)=0,\ k\geq 1$

(C) $D(k)-D(k-1)=0,\ k\geq 1$ (D) $2D(k)-D(k-1)=0,\ k\geq 1$

10. The general solution of $D(k)-8D(k-1)+16D(k-2)=0$ is

(A) $(c_0+c_1k)4^k$ (B) $c_0 4^k$

(C) $c_0 k 4^k$ (D) $c_0 4^{k+1}$

11. The order of the identity element of a group of order 3 is

(A) 3 (B) 1

(C) 0 (D) 2

12. If G is a group of order 15 and H is a subgroup of order 3, then the index of H in G is

(A) 3 (B) 1

(C) 5 (D) 15

13. A vertex which is not adjacent to every other vertex is called

(A) An isolated vertex (B) A pendant vertex

(C) An incident vertex (D) A simple vertex

14. The number of edges in a complete graph with 5 vertices is

(A) 10 (B) 15

(C) 20 (D) 5

15. A path of graph G which includes each vertex of G exactly once is called

(A) An Eulerian path (B) A Hamiltonian path

(C) A circuit (D) A tree

16. The number of vertices of a full binary tree is 13. The number of pendant vertices is

(A) 13 (B) 5

(C) 6 (D) 7

17. If $(L,\leq)$ is a lattice and if $a\leq b$, then $a\vee b=$

(A) b (B) a

(C) $a\wedge b$ (D) $a\vee b$

18. If B is a Boolean algebra, then for any a, b $\in$ B $(a+b)'=$

(A) $a'\cdot b'$ (B) $a'+b'$

(C) $a'+b$ (D) $a+b'$

19. In any Boolean algebra, if a = b, then $ab'+a'b'=$

(A) 1 (B) a'

(C) 0 (D) b

20. $a\cdot(b.c)+a\cdot b$ is equal to

(A) a (B) b

(C) $a\cdot b$ (D) $a+b\cdot c$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Construct the truth table for the following compound proposition $(\neg q\wedge(p\to q))\to \neg p$.

22. If $A=\{1,3,5\}$, R and S are relations on A defined by $R=\{(1,3),(3,5)\}$, $S=\{(1,1),(1,3),(1,5)\}$, find $M_R, M_S, M_{R\cup S}, M_{R\cap S}, M_{\bar{R}}$ and $M_{\bar{S}}$.

23. Let (G, *) be a group. For any a, b, $\in$ G, show that $(a*b)^{-1}=b^{-1}*a^{-1}$.

24. Let G(V, E) be an undirected graph with 'e' number of edges. Show that $\sum_{v\in V}\deg(v)=2e$ when $\deg(v)=$ degree of the vertex v.

25. In any Boolean algebra, show that (i) if $a'+b=1$, then $a\cdot b'=0$ (ii) if $a+b=b$, then $a\cdot b=a$.

26. Without constructing truthtable prove the following equivalence $\neg(p\to q)\equiv p\wedge \neg q$.

27. Draw the Hasse diagram representing the partial order relation on $A=\{1, 2, 3, 4, 6, 8, 12\}$ define by $R=\{(a,b)/a\mid b\}$ starting from the digraph.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Using indirect method of proof show that $r\to\neg q, r\vee s, s\to\neg q, p\to q\Rightarrow\neg p$.

ii. Use mathematical induction to prove $1.2+2.3+3.4+.....+n(n+1)=n^2$.

(OR)

b. Show that the premises 'one student in this class knows how to write programs in JAVA' and 'every one who knows how to write programs in JAVA get high paying job'. Imply the conclusion 'someone in this class can get a high paying job'. Let universe U = {students}.

29. a. Let $A=\{1, 2, 3, 4\}$. R is a relation on A defined by $R=\{(1, 2),(2, 1),(2, 3), (3, 4),(4, 3)\}$. Find the transitive closure of R, using Warshall's algorithm.

(OR)

b.i. If $f(x)=x+2$, $g(x)=x-2$ and $h(x)=3x$ where f, g, h are functions from $\mathbb{R}$ (set o real numbers) to $\mathbb{R}$. Prove that (A) $f\circ g=g\circ f$ (B) $g\circ h\neq h\circ g$ (C) $(f\circ g)\circ h=f\circ(g\circ h)$.

ii. Of any five points chosen within an equilateral triangle of side 1 cm each, show that there are atleast 2 points whose distance apart is 1/2 cm.

30. a. Solve the recurrence relation $a_n-7a_{n-1}+10a_{n-2}=8n+6,\ a_0=1,\ a_1=2$.

(OR)