

General linear parameter-varying (LPV) system with input saturation, uncertainties, and matched disturbance

$$(\text{P}) \left\{ \begin{array}{lcl} \dot{\mathbf{x}}_p(t) & = & [\mathbf{A}_p(\boldsymbol{\rho}_t) + \boldsymbol{\Delta}\mathbf{A}_p(t)]\mathbf{x}_p(t) + [\mathbf{B}_p(\boldsymbol{\rho}_t) + \boldsymbol{\Delta}\mathbf{B}_p(t)]sat(\mathbf{u}(t - \theta(\boldsymbol{\rho}_t))) + \mathbf{d}_i(t)) + [\mathbf{D}_p(\boldsymbol{\rho}_t) + \boldsymbol{\Delta}\mathbf{D}_p(t)]\mathbf{d}(t), \\ \mathbf{z}(t) & = & \mathbf{C}_z(\boldsymbol{\rho}_t)\mathbf{x}_p(t) + \mathbf{D}_{zu}(\boldsymbol{\rho}_t)sat(\mathbf{u}(t - \theta(\boldsymbol{\rho}_t))) + \mathbf{d}_i(t)) + \mathbf{D}_{zd}(\boldsymbol{\rho}_t)\mathbf{d}(t), \\ \mathbf{y}(t) & = & \mathbf{C}_y\mathbf{x}_p(t), \\ \mathbf{x}_p(s) & = & \phi(s), \quad \forall s \in [-\bar{h}, 0] \end{array} \right.$$

Uncertainties:

$$\begin{bmatrix} \boldsymbol{\Delta}\mathbf{A}_p(t) \\ \boldsymbol{\Delta}\mathbf{B}_p(t) \\ \boldsymbol{\Delta}\mathbf{D}_p(t) \end{bmatrix} = \mathbf{E}\mathbf{F}(t) \begin{bmatrix} \mathbf{G}_A \\ \mathbf{G}_B \\ \mathbf{G}_D \end{bmatrix}$$

Matched disturbance dynamics with unknown but bounded energy input, $\boldsymbol{\nu}(t)$,

$$(\text{D}) \left\{ \begin{array}{lcl} \dot{\boldsymbol{\omega}}(t) & = & \mathbf{W}\boldsymbol{\omega}(t) + \mathbf{H}\boldsymbol{\nu}(t), \\ \mathbf{d}_i(t) & = & \mathbf{V}\boldsymbol{\omega}(t) + \mathbf{J}\boldsymbol{\nu}(t), \end{array} \right.$$

Proposed output-feedback controller

$$(\text{C}) \left\{ \begin{array}{lcl} \dot{\mathbf{x}}_K(t) & = & \mathbf{A}_K(\boldsymbol{\rho}_t)\mathbf{x}_K(t) + \mathbf{A}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{B}_K(\boldsymbol{\rho}_t)\mathbf{y}(t) + \mathbf{B}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{y}(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{E}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t) \\ & = & \mathbf{A}_K(\boldsymbol{\rho}_t)\mathbf{x}_K(t) + \mathbf{A}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{B}_K(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t) + \mathbf{B}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{E}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t) \\ & = & \begin{bmatrix} \mathbf{B}_K(\boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{A}_K(\boldsymbol{\rho}_t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t) \\ \mathbf{x}_k(t) \\ \mathbf{e}_\omega(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{A}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{x}_k(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{e}_\omega(t - \theta(\boldsymbol{\rho}_t)) \end{bmatrix} + \mathbf{E}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t), \\ \mathbf{u}(t - \theta(\boldsymbol{\rho}_t)) & = & \mathbf{C}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{y}(t - \theta(\boldsymbol{\rho}_t)) - \hat{\mathbf{d}}_i(t) \\ & = & \mathbf{C}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) - \mathbf{V}\hat{\boldsymbol{\omega}}(t) \\ & = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t) \\ \mathbf{x}_k(t) \\ \mathbf{e}_\omega(t) \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{C}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{x}_k(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{e}_\omega(t - \theta(\boldsymbol{\rho}_t)) \end{bmatrix} - \mathbf{V}\hat{\boldsymbol{\omega}}(t), \end{array} \right.$$

Proposed input disturbance observer

$$(\text{IDO}) \left\{ \begin{array}{lcl} \dot{\hat{\mathbf{x}}}_d(t) & = & \mathbf{W}\hat{\boldsymbol{\omega}}(t) + \mathbf{L}_K(\boldsymbol{\rho}_t)\mathbf{x}_K(t) + \mathbf{L}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{L}_y(\boldsymbol{\rho}_t)\mathbf{y}(t) + \mathbf{L}_{y\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{y}(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{F}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t) \\ & = & \mathbf{W}\hat{\boldsymbol{\omega}}(t) + \mathbf{L}_K(\boldsymbol{\rho}_t)\mathbf{x}_K(t) + \mathbf{L}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{L}_y(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t) + \mathbf{L}_{y\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{F}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t) \\ & = & \mathbf{W}\hat{\boldsymbol{\omega}}(t) + \mathbf{L}_y(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t) + \mathbf{L}_K(\boldsymbol{\rho}_t)\mathbf{x}_K(t) + \mathbf{L}_{y\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{L}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{x}_K(t - \theta(\boldsymbol{\rho}_t)) + \mathbf{F}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t) \\ & = & \mathbf{W}\hat{\boldsymbol{\omega}}(t) + \begin{bmatrix} \mathbf{L}_y(\boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{L}_K(\boldsymbol{\rho}_t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t) \\ \mathbf{x}_k(t) \\ \mathbf{e}_\omega(t) \end{bmatrix} + \begin{bmatrix} \mathbf{L}_{y\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{L}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{x}_k(t - \theta(\boldsymbol{\rho}_t)) \\ \mathbf{e}_\omega(t - \theta(\boldsymbol{\rho}_t)) \end{bmatrix} + \mathbf{F}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t)\bar{\boldsymbol{\Psi}}(t), \\ \hat{\boldsymbol{\omega}}(t) & = & \mathbf{x}_d(t) - \mathbf{L}_d(\boldsymbol{\rho}_t)\mathbf{y}(t), \\ \hat{\mathbf{d}}_i(t) & = & \mathbf{V}\hat{\boldsymbol{\omega}}(t) \end{array} \right.$$

Design constraints

$$\left[\begin{array}{cccccccccccccccc} \hat{\mathbf{A}}(\boldsymbol{\rho}_t) + \hat{\mathbf{A}}^T(\boldsymbol{\rho}_t) + \bar{\mathbf{S}} - \bar{\mathbf{R}} + \bar{\mathbf{Q}}(\boldsymbol{\rho}_t) + \left(\sum_{i=1}^{n_s} \dot{\rho}_i(t) \frac{\partial \bar{\mathbf{P}}(\boldsymbol{\rho}_t)}{\partial \rho_i(t)} \right) & \bar{\mathbf{P}}(\boldsymbol{\rho}_t) - \bar{\mathbf{P}}_1 + \kappa \hat{\mathbf{A}}^T(\boldsymbol{\rho}_t) & \bar{\mathbf{R}} - \bar{\mathbf{S}}_1 + \hat{\mathbf{A}}_d(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \bar{\mathbf{S}}_1 & \hat{\mathbf{B}}_\Psi(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) + \hat{\mathbf{G}}^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) - \hat{\mathbb{K}}^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) + 2\bar{\mathbf{V}}^T & \hat{\mathbf{B}}_w(\boldsymbol{\rho}_t) & \hat{\mathbf{C}}^T(\boldsymbol{\rho}_t) & \hat{\mathbf{E}}(\boldsymbol{\rho}_t) & \varepsilon \begin{bmatrix} \mathbf{G}_A \mathbf{Y} & \mathbf{G}_A & \mathbf{G}_B \mathbf{V} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\ * & -\kappa(\bar{\mathbf{P}}_1^T + \bar{\mathbf{P}}_1) + \bar{h}^2 \bar{\mathbf{R}} & \kappa \hat{\mathbf{A}}_d(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} & \kappa \hat{\mathbf{B}}_\Psi(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \kappa \hat{\mathbf{B}}_w(\boldsymbol{\rho}_t) & \mathbf{0} & \kappa \hat{\mathbf{E}}(\boldsymbol{\rho}_t) & \varepsilon \begin{bmatrix} \mathbf{G}_B \tilde{\mathbf{C}}_\theta(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{G}_B \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\ * & * & -2\bar{\mathbf{R}} + \bar{\mathbf{S}}_1 + \bar{\mathbf{S}}_1^T - (1 - \sum_{i=1}^{n_s} \dot{\rho}_i(t) \frac{\partial \theta}{\partial \rho_i(t)}) \bar{\mathbf{Q}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}) & \bar{\mathbf{R}} - \bar{\mathbf{S}}_1^T & \hat{\mathbf{G}}_1^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) + \hat{\mathbb{K}}^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} & \hat{\mathbf{C}}_d^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{0} & \varepsilon \begin{bmatrix} \mathbf{G}_B \tilde{\mathbf{C}}_\theta(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{G}_B \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\ * & * & * & -\bar{\mathbf{S}} - \bar{\mathbf{R}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & 2\bar{\mathbf{J}} & \bar{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \begin{bmatrix} -\mathbf{D}_{zu}(\boldsymbol{\rho}_t) \\ \mathbf{0} \end{bmatrix}^T & \mathbf{0} & \varepsilon \bar{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \begin{bmatrix} -\mathbf{G}_B \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}^T \\ * & * & * & * & * & -\mathbf{I} & \begin{bmatrix} \mathbf{D}_{zd}(\boldsymbol{\rho}_t) & \mathbf{D}_{zu}(\boldsymbol{\rho}_t) \mathbf{J} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T & \mathbf{0} & \varepsilon \begin{bmatrix} \mathbf{G}_D & \mathbf{G}_B \mathbf{J} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}^T \\ * & * & * & * & * & * & -\gamma^2 \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -\varepsilon \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & * & -\varepsilon \mathbf{I} & \mathbf{0} & \mathbf{0} \end{array} \right] \preceq \mathbf{0}$$

$$\begin{bmatrix} \bar{\mathbf{R}} & \bar{\mathbf{S}}_1 \\ * & \bar{\mathbf{R}} \end{bmatrix} \succeq \mathbf{0}$$

$$\begin{bmatrix} \beta & \mathbb{K}_{(i,:)}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) - \mathbf{G}_{1(i,:)}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \\ * & \bar{u}_i^2 \mathbf{P}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}) \end{bmatrix} \succeq \mathbf{0}, \quad i = 1, \dots, n_u$$

$$\begin{bmatrix} \beta & \begin{bmatrix} \tilde{\mathbf{C}}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \end{bmatrix}_{(i,:)} - \hat{G}_{1(i,:)}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \\ * & \bar{u}_i^2 \bar{\mathbf{P}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}) \end{bmatrix} \succeq \mathbf{0}, \quad i = 1, \dots, n_u$$

With matrices given by

$$\hat{\mathcal{A}}(\boldsymbol{\rho}_t) = \begin{bmatrix} \mathbf{A}_p(\boldsymbol{\rho}_t) \mathbf{Y} & \mathbf{A}_p(\boldsymbol{\rho}_t) & \mathbf{B}_p(\boldsymbol{\rho}_t) \mathbf{V} \\ \hat{\mathbf{A}}_K(\boldsymbol{\rho}_t) & \mathbf{X} \mathbf{A}_p(\boldsymbol{\rho}_t) + \mathbf{B}_K(\boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{X} \mathbf{B}_p(\boldsymbol{\rho}_t) \mathbf{V} \\ \tilde{\mathbf{L}}_K(\boldsymbol{\rho}_t) & \tilde{\mathbf{L}}_d(\boldsymbol{\rho}_t) \mathbf{C}_y \mathbf{A}_p(\boldsymbol{\rho}_t) + (\tilde{\mathbf{L}}_d(\boldsymbol{\rho}_t) - \tilde{\mathbf{L}}_y(\boldsymbol{\rho}_t)) \mathbf{C}_y & \mathbf{Z} \mathbf{W} + \tilde{\mathbf{L}}_d(\boldsymbol{\rho}_t) \mathbf{C}_y \mathbf{B}_p(\boldsymbol{\rho}_t) \mathbf{V} \end{bmatrix}$$

$$\bar{\mathbf{P}}_1 = \begin{bmatrix} \mathbf{Y} & \mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z} \end{bmatrix}$$

$$\hat{\mathcal{A}}_d(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) = \begin{bmatrix} \mathbf{B}_p(\boldsymbol{\rho}_t) \tilde{\mathbf{C}}_\theta(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{B}_p(\boldsymbol{\rho}_t) \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \\ \hat{\mathbf{A}}_\theta(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \tilde{\mathbf{B}}_\theta(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \\ \tilde{\mathbf{L}}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \tilde{\mathbf{L}}_{y\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \end{bmatrix}$$

$$\hat{\mathcal{B}}_\Psi(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) = \begin{bmatrix} -\mathbf{B}_p(\boldsymbol{\rho}_t) \bar{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \\ \hat{\mathbf{E}}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \\ \hat{\mathbf{F}}_K(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \end{bmatrix}$$

$$\hat{\mathcal{K}}^T(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) = \begin{bmatrix} \tilde{\mathbf{C}}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) & \mathbf{D}_{K\theta}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)}, \boldsymbol{\rho}_t) \mathbf{C}_y & \mathbf{0} \end{bmatrix}^T$$

$$\widehat{E}_w(\boldsymbol{\rho}_t) = \left[\begin{array}{cc} \mathbf{D}_p(\boldsymbol{\rho}_t) & \mathbf{B}_p(\boldsymbol{\rho}_t)\mathbf{J} \\ \mathbf{X}\mathbf{D}_p(\boldsymbol{\rho}_t) & \mathbf{X}\mathbf{B}_p(\boldsymbol{\rho}_t)\mathbf{J} \\ \widetilde{\mathbf{L}}_d(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{D}_p(\boldsymbol{\rho}_t) & \mathbf{Z}\mathbf{H} + \widetilde{\mathbf{L}}_d(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{B}_p(\boldsymbol{\rho}_t)\mathbf{J} \end{array} \right]$$

$$\widehat{C}^{\mathrm{T}}(\boldsymbol{\rho}_t) = \left[\begin{array}{ccc} \mathbf{C}_z(\boldsymbol{\rho}_t)\mathbf{Y} & \mathbf{C}_z(\boldsymbol{\rho}_t) & \mathbf{D}_{zu}(\boldsymbol{\rho}_t)\mathbf{V} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{e_{\omega}}(\boldsymbol{\rho}_t) \end{array} \right]^{\mathrm{T}}$$

$$\widehat{C}_d^{\mathrm{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t) = \left[\begin{array}{cc} \mathbf{D}_{zu}(\boldsymbol{\rho}_t)\widetilde{\mathbf{C}}_{K_{\theta}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t) & \mathbf{D}_{zu}(\boldsymbol{\rho}_t)\mathbf{D}_{K_{\theta}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t)\mathbf{C}_y \\ \mathbf{0} & \mathbf{0} \end{array} \right]^{\mathrm{T}}$$

$$\overline{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t)\mathbf{D}_{\Psi}^{\mathrm{T}}(\boldsymbol{\rho}_t) = \overline{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t) \left[\begin{array}{c} -\mathbf{D}_{zu}(\boldsymbol{\rho}_t) \\ \mathbf{0} \end{array} \right]^{\mathrm{T}}$$

$$\widehat{\mathcal{E}}(\boldsymbol{\rho}_t) = \mathbf{U}_1\mathbf{P}_1^{\mathrm{T}}\mathbf{E} = \mathbf{U}_2\mathbf{E} = \left[\begin{array}{ccc} \mathbf{E}_p & \mathbf{0} & \mathbf{0} \\ \mathbf{X}\mathbf{E}_p & \mathbf{0} & \mathbf{0} \\ \widetilde{\mathbf{L}}_d(\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{E}_p & \mathbf{0} & \mathbf{0} \end{array} \right]_{(2n_p+n_{\omega}) \times 3n_p}$$

$$\widehat{G}_A^{\mathrm{T}}(\boldsymbol{\rho}_t) = \left(\left[\begin{array}{ccc} \mathbf{G}_{A_p}\mathbf{Y} & \mathbf{G}_{A_p} & \mathbf{G}_{B_p}\mathbf{V} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right]_{3n_p \times (2n_p+n_{\omega})} \right)^{\mathrm{T}}$$

$$\widehat{G}_{A_d}^{\mathrm{T}}(\boldsymbol{\rho}_t) = \left(\left[\begin{array}{ccc} \mathbf{G}_{B_p}\mathbf{D}_{K_{\theta}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t)\mathbf{C}_y\mathbf{Y} + \mathbf{G}_B\mathbf{C}_{K_{\theta}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t)\mathbf{N} & \mathbf{G}_B\mathbf{D}_{K_{\theta}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t)\mathbf{C}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right]_{3n_p \times (2n_p+n_{\omega})} \right)^{\mathrm{T}}$$

$$\widehat{G}_{B_{\Psi}}^{\mathrm{T}}(\boldsymbol{\rho}_t) = \overline{\mathbf{T}}(\boldsymbol{\rho}_{t-\theta(\boldsymbol{\rho}_t)},\boldsymbol{\rho}_t) \left[\begin{array}{c} -\mathbf{G}_{B_p} \\ \mathbf{0} \\ \mathbf{0} \end{array} \right]^{\mathrm{T}}$$