Adaptive flexibility function in smart energy systems: A linearized price-demand mapping approach

Seyed Shahabaldin Tohidi, Henrik Madsen, Georgios Tsaousoglou and Tobias K. S. Ritschel

Abstract— The transactive control paradigm enables a flexible electricity-consuming asset to offer its flexibility upstream by simply adapting its consumption profile in response to pricing signals. A consumer's response to prices over time is modeled through a so-termed flexibility function. However, a consumer's flexibility function needs to be adaptive to account for changes to the consumer's internal dynamics over time. This paper proposes an adaptive mechanism for price signal generation using a piecewise linear approximation of a flexibility function with unknown parameters. In this adaptive approach, the price signal is parameterized and the parameters are changed adaptively such that the output of the flexibility function follows a reference demand signal. This is guaranteed using the Lyapunov stability theorem. The proposed method does not require an estimation algorithm for unknown parameters, which eliminates the need for persistency of excitation of signals, and consequently, simplifies practical adoption and deployment.

I. INTRODUCTION

Expanding renewable energy sources, like solar and wind power, decentralizes energy production. Consequently, it is the energy demand that needs to be adjusted to meet the available generation [1]. Thereby, the energy system is in a transition from a centralized system with relatively few power generation facilities to a decentralized system where the balance is ensured by the demand-side response and local intelligent systems [2]–[4].

Demand Side Management (DSM) consists of various control strategies for load shifting, peak shaving, or demand reduction [5]. This requires the demand side profile to be flexible, that is, it should be capable of managing its demand and generation based on user needs, grid balancing, and local climate conditions [6], [7]. For example, the flexibility potential of the thermal dynamics of a building is dependent on its inherent thermal mass and storage options such as water tanks, along with the Heating, Ventilation, and Air Conditioning (HVAC) system. Advanced control design has shown to have great potential for activating this flexibility potential [5], [8].

The flexibility function, a mapping between prices and energy consumption over a time horizon in a price-responsive system, is proposed as a minimum interoperability mechanism between the aggregator and the individual flexible assets (e.g., buildings) [9]. A generalized version of the flexibility function involving a nonlinear mapping between price and demand is provided in [10]. Specifically, the mapping describes the temporal evolution of the energy demand in response to changes in the energy price [11]. Therefore, it is the essential component of a transactive-control DSM framework [12], where a resource's controller receives a set of prices (communicated by an upstream controller) and adapts its consumption profile in response to these prices.

However, this relationship between prices and energy demand is not static. It changes in the course of days or weeks, due to numerous factors including changes in ambient conditions (e.g. humidity) and consumer behavior. Thus, the dynamics of the price-demand relationship evolve over time, which motivates the design of a mechanism that accounts for these variations. Such considerations can be modeled by introducing parametric uncertainty into the flexibility function. Passive methods are based on robust fixed-structure control systems considering bounded parametric uncertainty [13]–[16], whereas active methods are based on adaptive control methods that adjust the control law based on the changes in system parameters [17]–[21].

This paper proposes an adaptive flexibility controller capable of updating the control law, i.e. the price signal, based on the changes in the price-demand dynamics. To the best of our knowledge, existing methods based on the flexibility function do not account for parametric uncertainty. In this adaptive approach, the price signal is parameterized and the parameters are changed adaptively such that the demand is able to closely follow a reference demand signal. Another benefit of employing this approach is that it is not based on system identification methods. Hence, it does not require any persistency of excitation assumption on the input signals [22]. Moreover, a projection algorithm has been employed to confine the adaptive parameters within a prespecified compact set to guarantee the prices' boundedness [23], [24].

Finally, on the practical side, the controller's adaptation capability effectively eliminates the burden of conducting a manual, customized modeling-and-control study for each flexibility resource separately; rather, the adaptive controller can be deployed seamlessly across different assets, in a plug-and-play fashion, which allows for mass adoption.

This paper is organized as follows. Section II provides an overview of the flexibility function considered in this work. Section III presents a linearized version of the flexibility function. Section IV provides an optimal control signal assuming that all parameters are known. Considering unknown parameters, Section V proposes an adaptive flexibility function mechanism while ensuring the boundedness of the control signal. Section VI presents the simulation results, and a summary is provided in Section VII.

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II. FLEXIBILITY FUNCTION

Nonlinear dynamics of the price-demand relationship are proposed in [10]. In this study, we focus on the deterministic dynamics of the price-demand relationship, as given in the following differential equation form

$$dX_t/dt = \frac{1}{C}(D_t - B_t), \tag{1}$$

$$\delta_t = \ell(f(X_t) + g(u_t)), \tag{2}$$

$$D_t = B_t + \delta_t \Delta(\mathbb{1}_{\delta_t > 0}(1 - B_t) + \mathbb{1}_{\delta_t < 0}B_t), \tag{3}$$

where $X \in [0, 1] \subset \mathbb{R}$ is the state of charge, $B \in [0, 1] \subset \mathbb{R}$ is the baseline demand, $u \in [0, 1] \subset \mathbb{R}$ is the energy price, $\delta \in [0, 1] \subset \mathbb{R}$ is the demand change, $D \in [0, 1] \subset \mathbb{R}$ is the expected demand, C is the capacity of flexible energy, and $\Delta \in [0, 1]$ is the proportion of flexible demand. The above equations are constructed based on the normalized parameters between 0 and 1. Moreover, the function $\mathbb{1}_{\delta_t < 0}$ is equal to 1 when $\delta_t < 0$ and 0 otherwise, and the function $\mathbb{1}_{\delta_t > 0}$ is equal to 1 when $\delta_t > 0$ and 0 otherwise. The nonlinear functions involved in the flexibility function are given by

$$g(u) = \beta_1 Is_1(u) + \cdots + \beta_7 Is_7(u), \tag{4}$$

$$f(X) = (1 - 2X + \alpha_1(1 - (2X - 1)^2))(\alpha_2 + \alpha_3(2X - 1)^2 + \alpha_4(2X - 1)^6), \tag{5}$$

$$\ell(f(X) + g(u)) = -1 + \frac{2}{1 + e^{-k(f(X) + g(u))}}, \tag{6}$$

where $Is_1, \ldots, Is_7$ are I-spline functions [25], and the parameters $\beta_1, \ldots, \beta_7, \alpha_1, \ldots, \alpha_4$, and k are assumed to be unknown. They can be identified using different approaches, e.g., by maximizing the likelihood of observing the actual measurements [26]. By design, the functions $f(.)$ and $g(.)$ are monotonically decreasing and $\ell(.)$ is monotonically increasing (see Figure 1).

Remark 1: The relationship between energy price and energy demand is described by the nonlinear function $f(.)$, while the relationship between state of charge and energy demand is described by the nonlinear function $g(.)$. If these functions have positive values, it results in an increase in demand, and if they have negative values, it leads to a decrease in demand.

In the sequel, we first linearize the flexibility function in Section III. Then, two controllers are designed to generate price such that the demand follows its reference. In Section IV, the controller is designed assuming that the parameters are known. In Section V, we design an adaptive price generator when some parameters of the system are not known.

III. DETERMINISTIC LINEARIZED FLEXIBILITY FUNCTION

The nonlinear functions f and g are identified for a water tower and three buildings in [10]. However, it is shown in this paper that these functions are approximately linear in a wide range of energy prices and states of charge. A piecewise linear version of these functions has been used in [27] to describe demand response potential. Inspired by the above-mentioned approaches and considering an example of the nonlinear functions of the flexibility function, $f(X)$, $g(u)$, and $\ell(f(X) + g(u))$, in Figure 1, one can assume that $f(X)$, $g(u)$, and $\ell(f(X) + g(u))$ behave linearly in a range $[\epsilon_1, \epsilon_2]$ for some ϵ_1 and ϵ_2, $0 \le \epsilon_1, \epsilon_2 < 1$. One can find the slope of $f(X)$, $g(u)$, and $\ell(f(X) + g(u))$ around a point in $[\epsilon_1, \epsilon_2]$, as η_1, η_2, and η_3, respectively. Also, the biases for $f(X)$ and $g(u)$ can be considered as λ_1 and λ_2, respectively. Using the linearized version of the mentioned functions, (1)–(3) can be rewritten as

$$dX_t/dt = \frac{\Delta}{C}\eta_3(\eta_1 X_t + \lambda_1 + \eta_2 u_t + \lambda_2)$$
$$\times (\mathbb{1}_{\delta_t > 0}(1 - B_t) + \mathbb{1}_{\delta_t < 0}B_t), \tag{7}$$

$$\delta_t = \eta_3(\eta_1 X_t + \lambda_1 + \eta_2 u_t + \lambda_2), \tag{8}$$

$$D_t = B_t + \delta_t \Delta(\mathbb{1}_{\delta_t > 0}(1 - B_t) + \mathbb{1}_{\delta_t < 0}B_t). \tag{9}$$

The state equation (7) can be rewritten as the following piecewise-defined function

$$dX_t/dt = \tag{10}$$

$$\begin{cases} \frac{\Delta}{C}\eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3)(1 - B_t), & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) > 0, \\ \frac{\Delta}{C}\eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3)B_t, & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) < 0, \end{cases}$$

where $\lambda_3 = \lambda_1 + \lambda_2$. This implies that the state and output dynamics can be written in the form of a linear time-varying dynamical system as

$$dX_t/dt = a_t X_t + b_t u_t + d_t, \tag{11}$$

$$D_t = B_t + Ca_t X_t + Cb_t u_t + Cd_t, \tag{12}$$

where a_t and b_t are defined as

$$a_t = \begin{cases} \frac{\Delta}{C}\eta_3\eta_1(1 - B_t), & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) > 0, \\ \frac{\Delta}{C}\eta_3\eta_1 B_t, & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) < 0, \end{cases} \tag{13}$$

$$b_t = \begin{cases} \frac{\Delta}{C}\eta_3\eta_2(1 - B_t), & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) > 0, \\ \frac{\Delta}{C}\eta_3\eta_2 B_t, & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) < 0, \end{cases} \tag{14}$$

and

$$d_t = \begin{cases} \frac{\Delta}{C}\eta_3\lambda_3(1 - B_t), & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) > 0, \\ \frac{\Delta}{C}\eta_3\lambda_3 B_t, & \eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) < 0. \end{cases} \tag{15}$$

If $\eta_3(\eta_1 X_t + \eta_2 u_t + \lambda_3) = 0$, then $a_t = b_t = d_t = 0$.

Remark 2: By design, the functions f and g are monotonously decreasing and ℓ is monotonously increasing [10]. Therefore, $\eta_1 < 0$, $\eta_2 < 0$, and $\eta_3 > 0$ and a_t and b_t are negative for all $t \ge 0$. Furthermore, λ_1 and λ_2 are positive scalars. The negativity of a_t is important to the stability analysis of the linearized flexibility function. Also, having information on the sign of b_t is required for the adaptive flexibility function design and will be utilized in Section V.

IV. OPTIMAL PRICE GENERATOR

Suppose that the scalar C is known and that the values of the scalar functions a_t and b_t are known for all $t \ge 0$. Then, by setting $D_t = D_{ref_t}$ and isolating the input in (12), it can be shown that applying the price signal

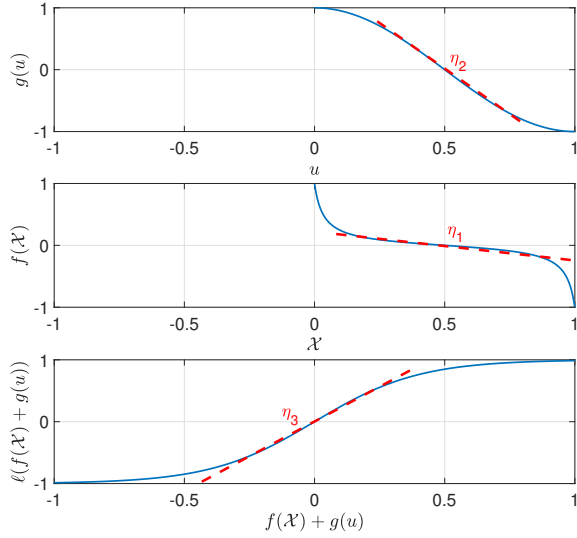$$u_t = \frac{1}{Cb_t}\left(-Ca_t X_t - Cd_t - B_t + D_{ref_t}\right) \tag{16}$$

Fig. 1. Schematic of nonlinear functions of the flexibility function and their linear approximations.

to (11) ensures that the output of the flexibility function, D_t, is equal to the reference demand signal, D_{ref_t}. It is noted that Cb_t has a nonzero value for all $t \geq 0$. However, (16) may not be feasible since it does not consider the restrictions of u_t, that is, $u_t \in [0, 1]$. To this end, an optimization problem needs to be solved, that minimizes $u_t - \frac{1}{Cb_t}(-Ca_tX_t - Cd_t - B_t + D_{ref_t})$.

Remark 3: Assume that the values of a_t, b_t, and C are known for all $t \geq 0$, and that the signals D_{ref_t} and B_t are provided for all $t \geq 0$. Then, the optimization problem

$$min_{u_t} \int_{t_k}^{t_{k+1}} (Cb_tu_t + Ca_tX_t + Cd_t + B_t - D_{ref_t})^2,$$
$$dX_t/dt = a_tX_t + b_tu_t + d_t,$$
$$u_t \in [0, 1], \qquad (17)$$

finds the optimal price signal for $t \in [t_k, t_{k+1})$. It is noted that u_t is considered constant throughout the interval $[t_k, t_{k+1})$.

Remark 4: Once a daily demand is purchased by an aggregator, and the daily baseline demand is provided, the bounds of the integral in the optimization problem (17) can be extended and an optimal price signal can be calculated for the whole day.

The procedure for implementing the proposed method of Section III is given in Implementation Procedure 1.

Implementation procedure 1 Price signal generation algorithm for linearized flexibility function

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-Given D_{ref_t}, B_t, C, η_1, η_2, η_3, λ_1 and λ_2
-Solve the optimization problem (17)

V. ADAPTIVE FLEXIBILITY FUNCTION

In this section, we describe an approach for computing a control signal, u_t, when a_t and b_t are unknown, such that the demand, D_t, converges to its reference value, D_{ref_t}. Assume that a_t and b_t are piecewise constant.

Rewrite the state dynamics of the linearized flexibility function (11) as

$$dX_t/dt = aX_t + b(u_t + \bar{d}), \qquad (18)$$

where $\bar{d} = \lambda_3/\eta_2$.

Let the reference dynamics be defined as

$$dY_t/dt = \lambda Y_t + \frac{1}{C}r_t, \qquad (19)$$

where Y_t is the state of the reference dynamics and $r_t = D_{ref_t} - B_t$. Notice that the dynamics (19) is selected such that Y_t mimics the behaviour of (1) when $D_t = D_{ref_t}$. This can be done by choosing a negative λ. The negativity of λ ensures that the reference dynamics are stable.

A controller has to be designed such that it captures the changes of r_t and X, and generates a stabilizing control signal. Thus, we employ the control law

$$u_t = \hat{\alpha}_tX_t + \hat{\beta}_tr_t + \hat{\zeta}_t, \qquad (20)$$

where $\hat{\alpha}_t$, $\hat{\beta}_t$ and $\hat{\zeta}_t$ are control gains. With the control law (20), the closed loop dynamics can be written as

$$dX_t/dt = (a + b\hat{\alpha}_t)X_t + b\hat{\beta}_tr_t + b(\hat{\zeta}_t + \bar{d}). \qquad (21)$$

If the flexibility function parameters were known, the ideal parameters could be calculated by comparing the closed-loop dynamics and the reference dynamics, i.e., $\alpha^* = \frac{\lambda-a}{b}$, $\beta^* = \frac{1}{bC}$ and $\zeta^* = -\bar{d}$.

By defining the error dynamics as $e_t = X_t - Y_t$, the error dynamics can be obtained as

$$de_t/dt = (a + b\hat{\alpha}_t)X_t + b\hat{\beta}_tr_t + b(\hat{\zeta}_t + \bar{d}) - \lambda Y_t - \frac{1}{C}r_t. \qquad (22)$$

Defining the parameter errors as $\tilde{\alpha}_t = \hat{\alpha}_t - \alpha^*$, $\tilde{\beta}_t = \hat{\beta}_t - \beta^*$ and $\tilde{\zeta}_t = \hat{\zeta}_t - \zeta^*$, the error dynamics (23) can be rewritten as

$$de_t/dt = \lambda e_t + b\tilde{\alpha}_tX_t + b\tilde{\beta}_tr_t + b\tilde{\zeta}_t. \qquad (23)$$

Definition 1: The projection operator, denoted as Proj, for two scalars θ and Y is defined as

$$\mathrm{Proj}(\theta, Y) \equiv \begin{cases} Y - Yh(\theta) & if \ h(\theta) > 0 \ \& \ Y\left(\frac{dh(\theta)}{d\theta}\right) > 0 \\ Y & \text{otherwise}, \end{cases} \qquad (24)$$

where $h(.): \mathbb{R} \to \mathbb{R}$ is a convex function defined as

$$h(\theta) = \frac{(\theta - \theta_{min} - \varepsilon_\theta)(\theta - \theta_{max} + \varepsilon_\theta)}{(\theta_{max} - \theta_{min} - \varepsilon_\theta)\varepsilon_\theta}, \qquad (25)$$

where ε_θ is the projection tolerance, and θ_{max} and θ_{min} are the upper and lower bound of θ. These bounds also form the projection boundary.

The following theorem provides the main results of this paper. It provides the projection based adaptive laws along with stability analysis and convergence results.

Theorem 1: Consider the flexibility function dynamics (11) and the reference model (19), and assume that a_t and b_t are piecewise constant unknown parameters, but the sign of b_t is considered to be known. Suppose that the price signal u_t, given in (20), is the control input of the flexibility function dynamics (11)–(12) with the adaptive parameters, $\hat{\alpha}_t$, $\hat{\beta}_t$ and $\hat{\zeta}_t$, that are updated using the following projection-based adaptive laws,

$$d\hat{\alpha}_t/dt = \gamma_\alpha \text{Proj}\left(\hat{\alpha}_t, -sgn(b_t)\mathcal{X}_t e_t\right), \quad (26)$$

$$d\hat{\beta}_t/dt = \gamma_\beta \text{Proj}\left(\hat{\beta}_t, -sgn(b_t)r_t e_t\right), \quad (27)$$

$$d\hat{\zeta}_t/dt = \gamma_\zeta \text{Proj}\left(\hat{\zeta}_t, -sgn(b_t)e_t\right), \quad (28)$$

where a projection operator "Proj" is employed to keep the adaptive parameters bounded, and γ_α, γ_β and γ_ζ are three positive adaptation gains. Then, given any initial condition $e_0 \in \mathbb{R}$, $\alpha_0 \in \Omega_\alpha$, $\beta_0 \in \Omega_\beta$, $\zeta_0 \in \Omega_\zeta$, and $\alpha^* \in [\alpha_{min} + \varepsilon_\alpha, \ \alpha_{max} - \varepsilon_\alpha]$, $\beta^* \in [\beta_{min} + \varepsilon_\beta, \ \beta_{max} - \varepsilon_\beta]$ and $\zeta^* \in [\zeta_{min} + \varepsilon_\beta, \ \zeta_{max} - \varepsilon_\beta]$, $\tilde{\alpha}_t$, $\tilde{\beta}_t$ and $\tilde{\zeta}_t$ remain uniformly bounded for all $t \geq 0$ and e_t converges to 0 as $t \to \infty$. Furthermore, u_t remains bounded and D_t converges to D_{ref_t}.

Proof: Consider the candidate Lyapunov function

$$V(e_t, \tilde{\alpha}_t, \tilde{\beta}_t, \tilde{\zeta}_t)$$
$$= \frac{1}{2}e_t^2 + \frac{1}{2\gamma_\alpha}|b_t|\tilde{\alpha}_t^2 + \frac{1}{2\gamma_\beta}|b_t|\tilde{\beta}_t^2 + \frac{1}{2\gamma_\zeta}|b_t|\tilde{\zeta}_t^2. \quad (29)$$

The time derivative of (29) along the trajectories of (23) and (26)–(28) can be calculated as

$$dV(e_t, \tilde{\alpha}_t, \tilde{\beta}_t, \tilde{\zeta}_t)/dt$$
$$= \lambda e_t^2 + |b_t|\left(sgn(b_t)\mathcal{X}_t e_t + \text{Proj}\left(\hat{\alpha}_t, -sgn(b_t)\mathcal{X}_t e_t\right)\right)\tilde{\alpha}_t$$
$$+ |b_t|\left(sgn(b_t)r_t e_t + \text{Proj}\left(\hat{\beta}_t, -sgn(b_t)r_t e_t\right)\right)\tilde{\beta}_t$$
$$+ |b_t|\left(sgn(b_t)e_t + \text{Proj}\left(\hat{\zeta}_t, -sgn(b_t)e_t\right)\right)\tilde{\zeta}_t. \quad (30)$$

Using the properties of projection algorithm [24], implies that

$$dV(e_t, \tilde{\alpha}_t, \tilde{\beta}_t, \tilde{\zeta}_t)/dt \leq \lambda e_t^2 \leq 0. \quad (31)$$

The negativity of dV/dt implies that e_t, $\tilde{\alpha}_t$, $\tilde{\beta}_t$ and $\tilde{\zeta}_t$ are bounded, which causes de_t/dt to be bounded as well. It also implies that

$$\int_0^t |\lambda|e_t^2 dt \leq -\int_0^t (dV/dt) \ dt = V_0 - V_t \leq V_0, \quad (32)$$

for all $t \geq 0$, which shows that $e_t \in \mathcal{L}_2$. Given that $e_t \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and $de_t/dt \in \mathcal{L}_\infty$, and using Barbalat's lemma, one can confirm that $\lim_{t\to\infty} e_t = 0$. Therefore, $\mathcal{X}_t$ converges to $\mathcal{Y}_t$, and since $\mathcal{Y}_t$, by design, follows the trajectories of (1) with $D_t = D_{ref_t}$, the same holds for $\mathcal{X}_t$. Therefore, the adaptive price signal u_t, with the adaptation laws (26) and (27), leads to the convergence of D_t to D_{ref_t}. Moreover, by knowing the range of change of $\mathcal{X}_t$ and r_t, and by the selection of the upper and lower bounds of the projection algorithm, one can guarantee that u_t is bounded and is kept in the range $[0, \ 1]$. ∎

Remark 5: The adaptive update laws (26)–(28) employ a projection algorithm to keep the parameters of the adaptive
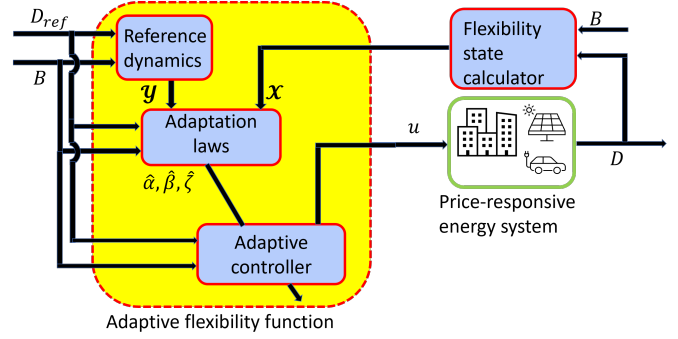

Fig. 2. Block diagram of the proposed adaptive flexibility function.

system bounded, regardless of any stability condition (see Definition 1). Detailed discussion about the projection algorithm, its properties, and its novel modified versions can be found in [20], [23], [24].

Remark 6: As described in Remark 2, the sign of b_t is always negative by design. Thus, the adaptation laws (26)–(28) are implementable.

Remark 7: In order to implement the adaptation laws (26)–(28), one requires to know the values of $\mathcal{X}_t$. Having the demand or its estimation, D_t, the baseline, B_t and the flexibility capacity, C, $\mathcal{X}_t$ can be calculated using (1) as $\mathcal{X}_t = \int_0^t \frac{1}{C}(D_t - B_t)dt$.

Remark 8: Every day, an aggregator purchases a specific amount of energy for each hour, $\bar{D}_{ref}$. If an estimate of the hourly demand, $\bar{D}$, of a price-responsive energy system is available along with the hourly baseline, $\bar{B}$, one can implement the proposed adaptive approach and find the hourly price signal throughout each day. This hourly price signal can then be used in a model predictive controller or an energy management system (EMS).

A block diagram of the proposed adaptive flexibility function is shown in Figure 2. In addition, the procedure for implementing the proposed method of Theorem 1 is given in Implementation procedure 2.

Implementation procedure 2 Adaptive price signal generation algorithm based on the linearized flexibility function

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-**Given** D_t, D_{ref_t}, B_t, C
-**Set** λ, γ_α, γ_β **and** γ_ζ
-**Set the projection algorithm parameters** ζ_α, ζ_β, α_{min}, α_{max}, β_{min}, β_{max}, ζ_{min} **and** ζ_{max}
-**Provide the reference dynamics** $\mathcal{Y}_t$ **using (19)**
-**Calculate** $\mathcal{X}_t$ **using Remark 7**
-**Calculate the error** $e_t = \mathcal{X}_t - \mathcal{Y}_t$
-**Implement the adaptation laws (26)–(28) and find** $\hat{\alpha}_t$, $\hat{\beta}_t$ **and** $\hat{\zeta}_t$
-**Employ the control (20)**

VI. SIMULATION RESULTS

The linearized flexibility function is utilized to demonstrate the effectiveness of the proposed approaches. The
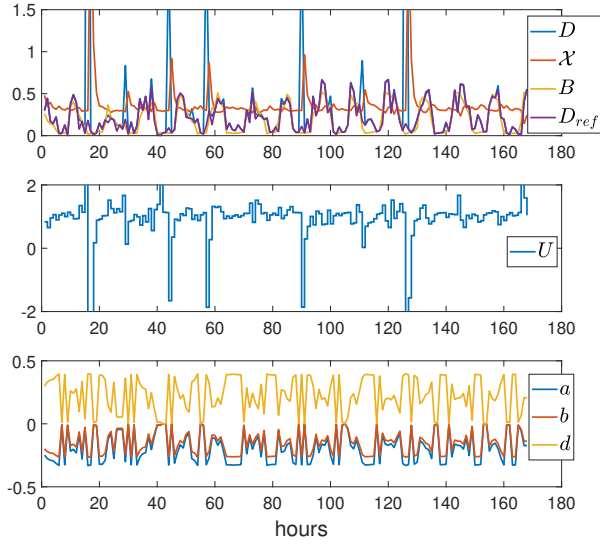
Fig. 3. Price signal generation using the controller (16) without bounds on the price signal.
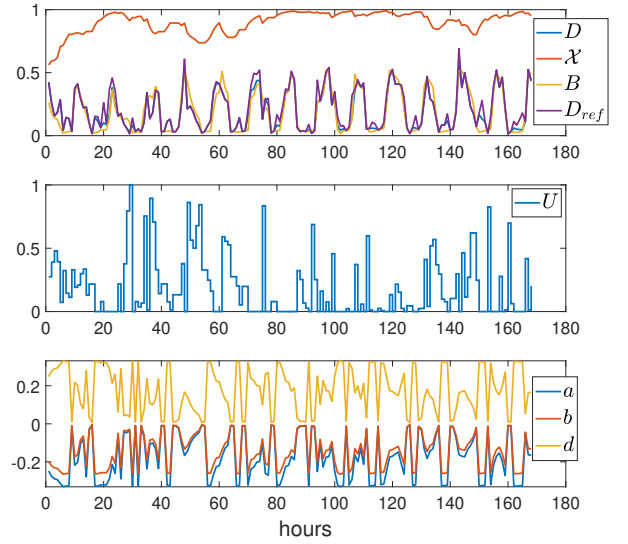

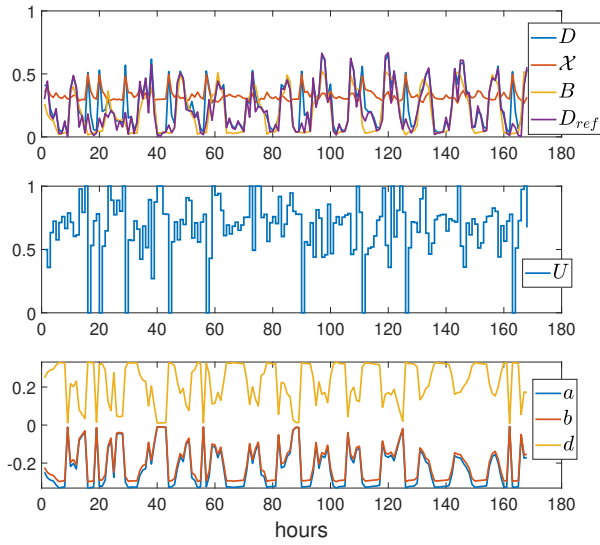Fig. 5. Price signal generation using Implementation procedure 1.

Fig. 4. Price signal generation using the controller (16) with bounds on the price signal.

Fig. 6. Price signal generation using Implementation procedure 2.

parameters of the linearized flexibility function are $\eta_1 = -1$, $\eta_2 = -0.9$, $\eta_3 = 1$, $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $C = 2.97$ and $\Delta = 1$.

The first scenario considers all of the parameters to be known. Thus, we follow Implementation procedure 1. First, we implement (16) without solving optimization or any bound on the price signal. Figure 3 demonstrates the results of the linearized flexibility function. The top panel shows the flexibility state, X, its output, D, baseline signal, B, and the reference demand, D_{ref}. It is seen that D follows D_{ref} conveniently. However, there are some mismatches between these two at the switching times, i.e. when the sign of δ_t changes. At the switching times, the control mechanism
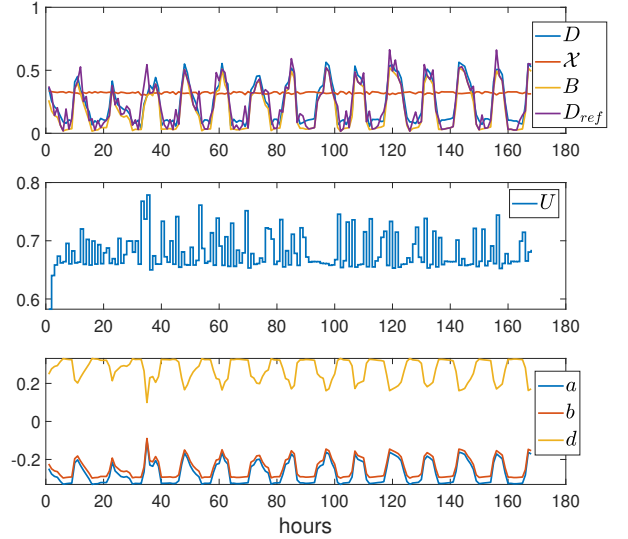
compensates such that D follows D_{ref} after a short period of time. The middle panel illustrates the generated price signal. It is seen that the generated price signal is not limited between 0 and 1. The third panel shows the time-varying parameters of the linearized flexibility function.

Figure 4 follows the first scenario without solving an optimization problem but with a software limitation. It is seen in the middle panel of this figure that the control signal, the price signal, is limited between 0 and 1, using a software limitation. It is observed that bounding the price signal does not cause instability. It even pushes the state and output of the linearized flexibility function to the prespecified limit of 0 and 1, as can be seen in the top panel. Also, the demand follows its reference. The third panel shows the time-varying

parameters of the linearized flexibility function.

The results of Implementation procedure 1, with optimization problem (17) is illustrated in Figure 5. The top panel of this figure shows the results of the flexibility state, its output, the baseline signal, and the reference demand. It is seen that the demand follows the reference demand. The middle panel shows the price signal. The optimization problem finds the optimal solution while considering the constraints. The time-varying parameters of the linearized system are demonstrated in the bottom panel.

The results of Implementation procedure 2, where the parameters of the flexibility function are not known, are illustrated in Figure 6. The top panel of this figure shows the results of the flexibility state, its output, the baseline signal, and the reference demand. It is seen that the demand follows the reference demand. The middle panel shows the price signal. The optimization problem finds the optimal solution while considering the constraints. The time-varying parameters of the linearized system are demonstrated in the bottom panel.

VII. CONCLUSIONS

An adaptive flexibility function based on an adaptive model reference controller structure is proposed in this paper. The method utilizes the linearized price-demand mapping and generates an adaptive price signal to diminish the difference between the demand and the reference demand in an energy system. The proposed method considers price signal constraints using a projection algorithm. Furthermore, the method needs neither uncertainty identification nor persistence of excitation assumption. This property along with the adaptation capability simplifies offering the flexibility services, e.g., in a plug-and-play manner and without the need to conduct a manual, customized modeling-and-control study for each resource separately. Simulation results show the effectiveness of the proposed method.

ACKNOWLEDGMENT

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