Adaptive digital twin for time-varying energy flexibility dynamics

Seyed Shahabaldin Tohidi[1], Nariman Mahdavi[2], Tobias K. S. Ritschel[1], Henrik Madsen[1]

Abstract— **This paper introduces an adaptive digital twin framework that represents the time-varying energy flexibility dynamics within energy systems. The proposed digital twin captures the real-time dynamic variations in demand response, price sensitivity, and energy consumption patterns by adapting to evolving system behaviors and external influences. By utilizing recursive modeling techniques, the digital twin dynamically calibrates its parameters to reflect current system conditions, enabling accurate and responsive representation of price-demand interactions. This tool can then be utilized by aggregators and flexibility management systems to effectively and accurately predict the demand and manage demand-response in Smart Energy Operating Systems. Simulation results demonstrate the ability to adaptively maintain accuracy under varying demand profiles and price fluctuations, highlighting its potential as a valuable asset for modern, responsive energy systems.**

I. INTRODUCTION

Recent climate change reports highlight a substantial increase in the Earth's surface temperature, commonly referred to as global warming. According to the State of the Climate in Europe report by the World Meteorological Organization (WMO) [1], Europe is warming faster than any of the other five regions defined by the WMO. The expansion of renewable energy sources, such as solar and wind power, has led to a more decentralized model of energy production. As a result, energy demand must increasingly adapt to align with the fluctuating availability of these renewable resources [2]. Consequently, the energy system is shifting from a centralized structure with a limited number of large power generation plants to a decentralized model, where demand-side management plays a crucial role in maintaining balance [3], [4].

Demand Side Management (DSM) is a strategy in energy systems aimed at optimizing energy use by shaping consumer demand rather than solely expanding supply. Load shifting, peak shaving, and demand reduction are common techniques for demand shaping [5]. Achieving this goal requires a high level of system flexibility. In this context, flexibility is the capability of consumers or systems to adjust their electricity consumption in response to direct or indirect requests from the grid operator, see for example [6], [7] for direct load control or [8] for indirect price-based control.

To actively support demand-side management, the aggregator, Distribution System Operator (DSO), and flexibility management system must be able to predict demand, assess flexibility potential, and accurately characterize flexibility. This capability enables more informed decision-making, improving both the efficiency of energy systems and the activation of flexible resources [9], [10].

Numerous studies have focused on methods for capturing demand behavior and modeling energy flexibility. Accurate prediction of energy flexibility requires dynamic, detailed modeling of energy systems that consider technical constraints, occupancy patterns, and boundary conditions, as highlighted in [11], [12]. For instance, [13] introduces time series analysis tools to quantify building flexibility in response to variable electricity prices. Additionally, [14] applies an inverse optimization method to estimate flexibility, showing that penalty adjustments can effectively shift demand from peak to off-peak hours. A linear, time-invariant dynamic model representing the price-demand relationship and characterizing energy flexibility for buildings and districts is presented in [15], where comparisons across buildings with varying flexibility characteristics are discussed. This model, which captures the temporal evolution of energy demand relative to price changes, is further employed in [16]. A nonlinear, and more realistic model of energy flexibility is developed in [17], termed the "flexibility function," which represents the price-demand relationship using a time-invariant nonlinear stochastic differential equation. The stability of the dynamics of the flexibility function is investigated in [18]. This function is utilized in an optimization algorithm to generate optimal price signals for incentive-based control and demand-side energy management [19]. In another study [20], the flexibility function is integrated into a holistic electricity market including transmission and distribution system operators.

In practice, the relationship between prices and energy demand is dynamic, varying across days or weeks due to factors such as shifts in ambient conditions (e.g., humidity), consumer behavior, and sudden price changes. These fluctuations drive the need for a mechanism that adapts to evolving price-demand dynamics. To address this, an adaptive flexibility function with a linearized price-demand mapping is proposed in [21]. This approach is based on the adaptive model reference controller structure [22], [23], and its stability is ensured through the Lyapunov stability theorem.

This paper introduces an adaptive digital twin of the time-varying price-demand dynamics in energy systems. The digital twin continuously updates its parameters in response to evolving price-demand relationships. Unlike the linear assumption in [21], our proposed adaptive approach models a nonlinear price-demand relationship and includes a stochastic

[1]Seyed Shahabaldin Tohidi, Tobias K. S. Ritschel, and Henrik Madsen are with Department of Applied Mathematics and Computer Science, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark {sshto, tobk, hmad}@dtu.dk.

[2]Nariman Mahdavi is with CSIRO Energy, Newcastle, NSW, Australia nariman.mahdavimazdeh@csiro.au

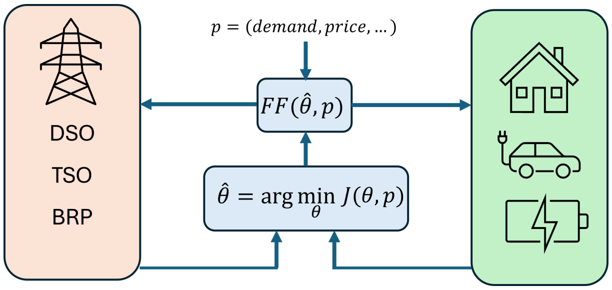

Fig. 1. Schematic of adaptive flexibility function. An optimization algorithm handles updating the parameters of the flexibility function (FF) using the data from the physical level (building, residents, etc.) and the grid level (DSO, TSO, etc.).

term, enhancing its realism. The adaptive digital twin's ability to self-adjust eliminates the need for manual, customized modeling for each flexibility resource, and hence, offers a practical approach. Additionally, it can be deployed across different assets in a plug-and-play fashion, facilitating broad and scalable adoption.

The developed digital twin models the nonlinear dynamic relationship between price and demand in price-responsive energy systems. Unlike traditional static models or non-adaptive dynamic flexibility functions, the proposed approach continuously updates its parameters using real-time data, including price signals, demand, and baseline consumption. This adaptability provides a more accurate energy system representation, ultimately enhancing its operational performance. As a result, the adaptive digital twin becomes a valuable decision support tool for energy operators, such as DSOs, TSOs, and aggregators, and enables more efficient and data-driven decision making (see Figure 1).

This paper is organized as follows. Section II provides an overview of the flexibility, demand side management, and the flexibility function considered in this work. The importance of an adaptive method is also discussed in this section. Section III introduces the adaptive digital twin for flexible energy systems. Section IV presents the simulation results, and a summary is provided in Section V.

II. PROBLEM STATEMENT

A. Flexibility

Demand Side Management (DSM) is a strategy in energy systems that seeks to manage electricity use by influencing consumer demand instead of solely focusing on increasing supply. This requires the energy system to be flexible. Flexibility refers to the ability of consumers or systems to adjust their electricity usage in response to direct or indirect requests from the grid operator. For example, a high degree of flexibility in a process would mean it can reduce or shift a significant amount of load quickly and maintain that shift for a substantial period, if needed.

Flexibility is increasingly important as renewable energy penetration rises, given that sources like wind and solar are inherently variable. Flexibility can help maintain a stable and reliable energy system by aligning demand with renewable energy production without relying solely on traditional, fossil-fuel-based backup power sources.

Characterizing flexibility involves identifying and quantifying the capacity of various users or systems to change their energy demand in response to external signals. Flexibility characterization allows energy providers to map out which parts of the demand side can be mobilized under different circumstances, such as for balancing supply during renewable generation dips or peaks.

B. Flexibility function

The flexibility function is a digital twin used to characterize the flexibility potential of energy systems. It captures how a system can adjust its operations in response to varying external conditions, such as changes in price or demand. The flexibility function integrates various parameters and functions that describe the system's flexibility behavior. The flexibility function helps in optimizing performance and ensuring efficient resource utilization by providing information about the current and future modes of the energy system. This information is of great importance for Distribution System Operators (DSOs) and aggregators for energy system management and decision-making, especially in scenarios involving renewable resources and uncertainty [9], [10].

Nonlinear dynamics of the price-demand relationship, proposed in [17], that characterize the system's flexibility dynamic is in the following stochastic differential equation form

$$dX_t = \frac{1}{C}(D_t - B_t)dt + \sigma_x dW_t, \tag{1}$$

$$\delta_t = \ell\Big(f(X_t;\alpha) + g(U_t;\beta); k\Big), \tag{2}$$

$$D_t = B_t + \delta_t \eta \Big(\mathbb{1}_{\delta_t>0}(1-B_t) + \mathbb{1}_{\delta_t<0} B_t\Big), \tag{3}$$

where $X \in [0,\ 1] \subset \mathbb{R}$ is the state of charge, $B \in [0,\ 1] \subset \mathbb{R}$ is the baseline demand, $U \in [0,\ 1] \subset \mathbb{R}$ is the energy price, $\delta \in [0,\ 1] \subset \mathbb{R}$ is the demand change, $D \in [0,\ 1] \subset \mathbb{R}$ is the expected demand, C is the amount of flexible energy, and $\eta \in [0,\ 1]$ is the proportion of flexible demand. Also, β and α are vectors of parameters $\beta_1, \dots, \beta_7$, and $\alpha_1, \dots, \alpha_4$, respectively. The set of equation (1)-(3) is known as nonlinear *flexibility function*. These equations are constructed based on the normalized parameters between 0 and 1 and contain stochastic terms. W is a Wiener process, and σ_X represents process noise. Moreover, the function $\mathbb{1}_{\delta_t<0}$ is equal to 1 when $\delta_t < 0$ and 0 otherwise, and the function $\mathbb{1}_{\delta_t>0}$ is equal to 1 when $\delta_t > 0$ and 0 otherwise. Nonlinear functions involved in the flexibility function are also given below:

$$g(U;\beta) = \beta_1 Is_1(U) + \dots + \beta_7 Is_7(U), \tag{4}$$

$$f(X;\alpha) = \Big(1 - 2X + \alpha_1(1-(2X-1)^2)\Big)\Big(\alpha_2 + \alpha_3(2X-1)^2 + \alpha_4(2X-1)^6\Big), \tag{5}$$

$$\ell(X,U) = -1 + \frac{2}{1+e^{-k\big(f(X;\alpha)+g(U;\beta)\big)}}, \tag{6}$$

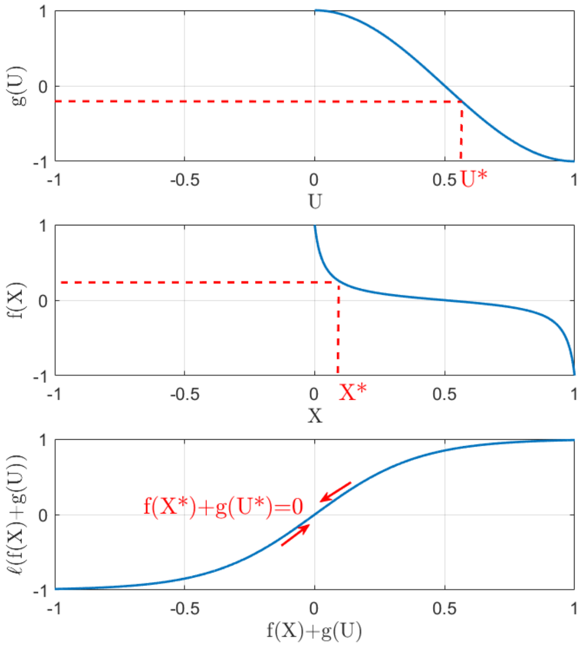

Fig. 2. Typical g and f functions, reflecting the effect of the state of charge and price in the digital twin (4)-(5), and the function ℓ that links these two functions.

where Is_1, ..., Is_7 are I-spline functions [24]. The parameters $\beta_1, \ldots, \beta_7$, $\alpha_1, \ldots$, and α_4 need to be identified based on the historical data measured from the energy system. These parameters can be identified using different approaches to maximize the likelihood between the final flexibility function and the data [25]. In the following, we may occasionally omit the parameters α and β in the notation when defining the functions f and g, respectively.

The nonlinear functions f and g play a crucial role in defining the model, serving as a digital twin that accurately captures the behavior of the energy system. Figure 2 illustrates examples of typical $g(.)$ and $f(.)$ functions as well as the $\ell(.)$ function. While the function f projects the effect of the state of charge on the price-demand relationship, g reflects the impact of price. Together, they shape the dynamics of the flexibility function through their interaction within the logistic function, ℓ, reflecting both the consumer's demand habits and responsiveness to price changes.

As it is seen in Figure 2, the functions $f(.)$ and $g(.)$ are designed such that they are monotonically decreasing, while $\ell(.)$ is designed to be monotonically increasing. For a price, U^*, the state of charge reaches a value, X^*, such that $\ell(X^*, U^*)$ converges to zero. For further details, please see [18].

Remark 1: Note that the original formulation of the flexibility function [17] includes a state-dependent diffusion term in the state equation. In this study, to simplify the derivation of the recursive identification method, we disregarded the state-dependency of the diffusion term, as can be seen in (1).

The relationship between price and demand in energy systems is highly dynamic, influenced by numerous factors that evolve over time, creating fluctuations in consumption patterns and pricing responses. A change in the controller of an energy management system, for instance, introduces new strategies for balancing supply and demand. Similarly, changes in residence, with new occupants bringing different energy habits, can alter household demand patterns, as each family or individual may have unique preferences for heating, cooling, or appliance use. Seasonal changes also play a significant role. Other factors, such as holidays, price jumps, and evolving lifestyles, add further complexity. As a result, this dynamic mapping between price and demand (Flexibility Function) needs to be constantly adjusting to real-world variations, reflecting both consumption behaviors and other factors like price changes.

In the following section, we propose a methodology to identify and adjust the parameters of the flexibility function recursively over time.

III. Adaptive Digital Twin

At first, we assume that we have access to the measurement data of B_t, U_t, and X_t, at each time instant. The state equation of the flexibility function can be written in a compact form as

$$dX_t = F(X_t, B_t, U_t; \theta)dt + \sigma_x dW_t, \tag{7}$$

where $\theta = [\alpha_1, \ldots, \alpha_4, \beta_1, \ldots, \beta_7]^\top$. It is noted that the subscript t is removed from α_i and β_i for better readability. Moreover, $F(.)$ is the drift coefficient and can be written as

$$\begin{aligned} F(X_t, B_t, U_t; \theta) &= \frac{1}{C}\delta_t \eta\Big(\mathbb{1}_{\delta_t>0}(1-B_t) + \mathbb{1}_{\delta_t<0}B_t\Big) \\ &= \frac{1}{C}\ell\Big(f(X_t,\alpha) + g(U_t,\beta)\Big) \\ &\quad \times \eta\Big(\mathbb{1}_{\delta_t>0}(1-B_t) + \mathbb{1}_{\delta_t<0}B_t\Big). \end{aligned} \tag{8}$$

We use the Euler-Maruyama method [26]–[28] to discretize (7). The resulting discrete-time system is

$$X_{t+1} = X_t + F(X_t, B_t, U_t; \theta)\Delta t + \sigma_x \sqrt{\Delta t}\epsilon_t, \tag{9}$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$. This implies that

$$X_{t+1}|X_t \sim \mathcal{N}\Big(X_t + F(X_t, B_t, U_t; \theta)\Delta t,\ \sigma_x^2\Delta t\Big). \tag{10}$$

Having the measurements X_i, B_i and U_i for time instants $i = 1, \ldots, t$, the likelihood function can be defined as the joint probability of Gaussian density functions as

$$L = \prod_{i=1}^{t} \frac{1}{\sqrt{2\pi\sigma_x^2\Delta t}} \exp\left(-\frac{\Big(X_{i+1} - X_i - F(X_i, B_i, U_i; \theta)\Delta t\Big)^2}{2\sigma_x^2\Delta t}\right), \tag{11}$$

and the log-likelihood function can be obtained as

$$\log(L) = -\frac{t}{2}\log(2\pi\sigma_x^2\Delta t) - \sum_{i=1}^{t} \frac{\left(X_{i+1} - X_i - F(X_i, B_i, U_i; \theta)\Delta t\right)^2}{2\sigma_x^2\Delta t}. \quad (12)$$

Assuming that σ_X is constant, the parameter estimate, $\hat{\theta}$, can then be found by maximizing the log-likelihood function, or equivalently,

$$\hat{\theta} = \arg\min_\theta \mathcal{J}(\theta), \quad (13)$$

where

$$\mathcal{J}(\theta) = \sum_{i=1}^{t} \frac{1}{2\sigma_x^2\Delta t}\left(X_{i+1} - X_i - F(X_i, B_i, U_i; \theta)\Delta t\right)^2. \quad (14)$$

The optimization problem (13) is unconstrained and convex. Therefore, a unique optimal solution, θ^*, always exists [29]. Moreover, the optimal solution satisfies $\nabla\mathcal{J}(\theta^*) = 0$, where ∇ is the gradient operator and can be calculated as $\nabla\mathcal{J}(\theta) = [\frac{\partial\mathcal{J}}{\partial\alpha_1}, ..., \frac{\partial\mathcal{J}}{\partial\beta_7}]^\top$. Taylor series expansion of $\mathcal{J}(\theta)$ around θ_t is obtained as

$$\mathcal{J}(\theta) \approx \mathcal{J}(\theta_t) + \nabla\mathcal{J}(\theta_t)^\top(\theta - \theta_t) + \frac{1}{2}(\theta - \theta_t)^\top H(\theta_t)(\theta - \theta_t), \quad (15)$$

where $H(\theta)$ is the Hessian matrix and its i, jth element is calculated as $\frac{\partial^2\mathcal{J}}{\partial\theta_i\partial\theta_j}$, and where θ_i and θ_j are the ith and jth elements of θ, accordingly. Using the properties $\frac{\partial a^\top x}{\partial x} = a$ and $\frac{\partial(x-x_0)^\top W(x-x_0)}{\partial x} = 2W(x - x_0)$, and setting the gradient of Taylor expansion of $\nabla\mathcal{J}$ to zero, we get

$$\nabla\mathcal{J}(\theta) \approx \nabla\mathcal{J}(\theta_t) + H(\theta_t)(\theta - \theta_t) = 0, \quad (16)$$

which leads to the following recursive formula

$$\theta_{t+1} = \theta_t - H(\theta_t)^{-1}\nabla\mathcal{J}(\theta_t). \quad (17)$$

Moreover, using (14), the gradient and Hessian of $\mathcal{J}(\theta)$ can be calculated as

$$\nabla\mathcal{J}(\theta_t) = \sum_{i=1}^{t} \frac{-1}{\sigma_x^2}\nabla F_i\left(X_{i+1} - X_i - F_i\Delta t\right) \quad (18)$$

and

$$H_t(\theta) = \sum_{i=1}^{t} \frac{-1}{\sigma_x^2}\nabla^2 F_i\ (X_{i+1} - X_i - F_i\Delta t) + \sum_{t=1}^{t} \frac{1}{\sigma_x^2}\nabla F_i \nabla F_i^\top \Delta t, \quad (19)$$

where F_i denotes $F(X_i, B_i, U_i; \theta_t)$ for brevity. In the vicinity of the optimal point, the first term in (19) is close to zero [30], [31]. By disregarding this term, we get

$$H_t(\theta) = \sum_{i=1}^{t} \frac{1}{\sigma_x^2}\nabla F_i \nabla F_i^\top \Delta t, \quad (20)$$

which is a positive definite function. The Hessian matrix, $H_t(\theta)$, can then be written in the form of a recursive formula by rewriting (20) as

$$\begin{aligned} H_t(\theta_t) &= \sum_{i=1}^{t-1} \frac{\lambda_t}{\sigma_x^2}\nabla F_i \nabla F_i^\top \Delta t + \frac{1}{\sigma_x^2}\nabla F_t \nabla F_t^\top \Delta t, \\ &= \lambda_t H_{t-1}(\theta_t) + \frac{\Delta t}{\sigma_x^2}\nabla F_t \nabla F_t^\top \end{aligned} \quad (21)$$

where λ_t is added as a forgetting factor. For a more detailed explanation of how to choose a proper forgetting factor, refer to [31].

Remark 2: In this study, we assume that the state of charge is directly accessible. However, in some applications, this may not be feasible. In such cases, since D_t and B_t are accessible, one can use the expression $X_t = X_0 + \int_0^t \frac{1}{C}(D_t - B_t)dt$ to estimate the state of charge. Alternatively, methods such as nonlinear Kalman filtering techniques can be employed for this estimation.

Suppose that the baseline, B_t, the price, U_t, and the state of charge, X_t, are accessible, the proposed method for the adaptive digital twin for price-demand dynamics of energy systems that is capable of identifying and adjusting its parameters recursively is given as

$$\begin{aligned} \mathcal{J}(\hat{\theta}_t) &= \sum_{i=1}^{t} \frac{1}{2\sigma_x^2\Delta t}\left(X_{i+1} - X_i - F(X_i, B_i, U_i; \hat{\theta}_t)\Delta t\right)^2 \\ \nabla\mathcal{J}(\hat{\theta}_t) &= \sum_{i=1}^{t} \frac{-1}{\sigma_x^2}\nabla F_i\left(X_{i+1} - X_i - F_i\Delta t\right) \\ H_t(\hat{\theta}_t) &= \lambda_t H_{t-1}(\hat{\theta}_t) + \frac{\Delta t}{\sigma_x^2}\nabla F_t \nabla F_t^\top, \\ \hat{\theta}_{t+1} &= \hat{\theta}_t - H(\hat{\theta}_t)^{-1}\nabla\mathcal{J}(\hat{\theta}_t), \end{aligned} \quad (22)$$

where F is defined in (1)-(3), and $\hat{\theta}$ is an estimate of θ.

Remark 3: Historical demand data can be used to find the baseline data. The average demand for each hour of the day can be found for weekdays and weekends as the baseline.

Remark 4: The Persistency of Excitation (PE) condition is a key assumption required to identify system parameters successfully. For a more detailed explanation, refer to [32], [33].

IV. SIMULATION RESULTS

In this section, we present the simulation results of the proposed recursive parameter identification method for the flexibility function. The results demonstrate the effectiveness of our approach in accurately estimating the parameters across varying conditions.

In this simulation, we examine two scenarios. Initially, the energy system exhibits low sensitivity to extreme values of the state of charge and limited responsiveness to price fluctuations. Over time, however, the system shifts to become more sensitive to extreme state-of-charge values and more responsive to changes in price. The scenarios outlined above are realistic. Analyzing demand across various districts reveals that sensitivity to the state of charge may be lower during certain periods. Furthermore, after crises or sudden price increases, sensitivity to prices tends to rise significantly.

Throughout the simulation, we assume that the total capacity of flexible energy remains constant at $C = 2.97$. However, the parameters f and g vary over time, represented as θ_t, as defined in Section III. We assume that

$$\begin{cases} \theta_t \in \Omega_1 & \text{if} \quad t < t_{change}, \\ \theta_t \in \Omega_2 & \text{if} \quad t \geq t_{change}, \end{cases} \tag{23}$$

where t_{change} is the time instant when the price-demand dynamic changes. Moreover, the sets of true values, Ω_1 and Ω_2, are chosen as $\Omega_1 = [\alpha_1 = 0.5, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0, \beta_1 = 0.085, \beta_2 = 0.2, \beta_3 = 0.006, \beta_4 = 0.53, \beta_5 = 0, \beta_6 = 0.089, \beta_7 = 0.089]$ and $\Omega_2 = [\alpha_1 = 0.7, \alpha_2 = 0, \alpha_3 = 1.2, \alpha_4 = 0.05, \beta_1 = 0.09, \beta_2 = 0.3, \beta_3 = 0.01, \beta_4 = 0.48, \beta_5 = 0.02, \beta_6 = 0.06, \beta_7 = 0.09]$.

Figure 3 demonstrates the shape of the functions f and g in the flexibility function before and after flexibility dynamic change. These functions reflect the effect of the state of charge and price on the price-demand dynamics. For instance, the top panel shows the difference between f before and after the dynamic change. It is seen that after the dynamic change (yellow line) f changes with a higher slope over the extreme values of x, that is, x values close to 1 and 0. This shows that the system is more sensitive to the extreme values of the state of charge after the dynamic change. The bottom panel demonstrates the change in g before and after the dynamic change. It is seen that after the dynamic change (red line) g has higher values for low prices while it has lower values for high prices. This shows that the system acts smartly and is more sensitive to price signals. This kind of behavior can be due to the employment of an advanced incentive-based controller in the system or due to tuning the model predictive controller.

Figure 4 shows the parameters of the digital twin for energy flexibility. It is seen that the parameters consistently converge to the true parameter values, showcasing its robustness in handling noise and model uncertainties. It is seen that the convergence occurs relatively quickly. In this figure, the true parameters are shown with dashed lines and the estimations are given with solid lines. In this figure the estimated parameters are denoted as $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4, \hat{\beta}_5, \hat{\beta}_6, \hat{\beta}_7$. Initially, their values are set to 0.5.

V. CONCLUSIONS

In conclusion, this paper presented an adaptive digital twin framework tailored for capturing the time-varying dynamics of flexibility in energy systems. The proposed adaptive identification method effectively estimates and adjusts the parameters of the flexibility function, responding dynamically to shifts in demand patterns. Simulation results validate the robustness and responsiveness of the approach, demonstrating its accuracy in parameter identification both before and after changes in flexibility dynamics. These findings underscore the potential of the adaptive digital twin as a reliable tool for real-time management in evolving

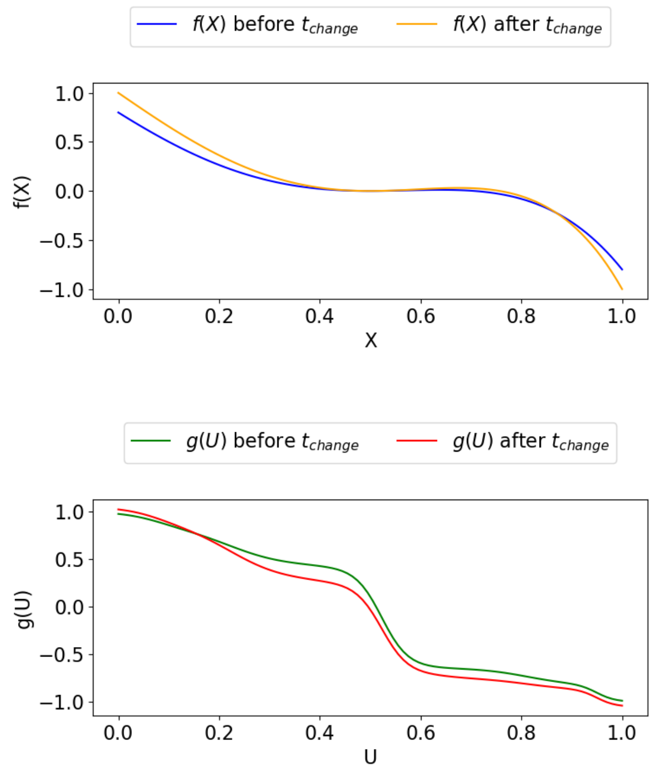

Fig. 3. f (top) and g (bottom) functions before and after the change in the parameter set from Ω_1 to Ω_2.

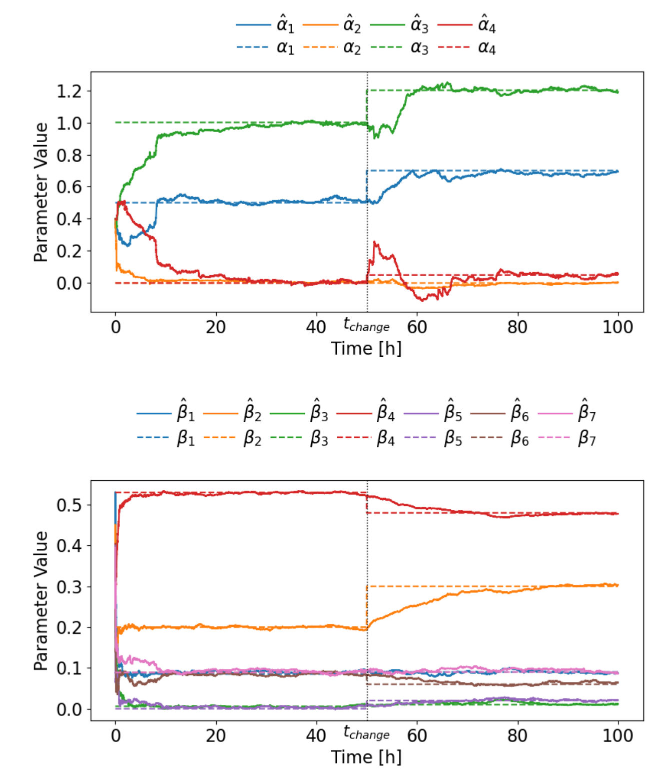

Fig. 4. Recursive identification of parameters of f and g functions before and after change in the parameter set at t_{change}.

energy systems, paving the way for more resilient, demand-responsive infrastructures.

ACKNOWLEDGMENT

This work is supported by Sustainable plus energy neighbourhoods (syn.ikia) (H2020 No. 869918), ELEXIA (Horizon Europe No. 101075656), ARV (H2020 No. 101036723), SEEDS (Horizon Europe No. 101138211), SEM4Cities (IFD Project No. 0143-0004), IEA EBC - Annex 81 - Data-Driven Smart Buildings (EUDP Project No. 64019-0539), and IEA EBC - Annex 82 - Energy Flexible Buildings Towards Resilient Low Carbon Energy Systems (EUDP Project No. 64020-2131).

REFERENCES

[1] "State of the climate in europe 2022," *WMO: World Meteorological Organisation*, 2023.

[2] L. Á. Flórez, T. Péan, and J. Salom, "Hourly based methods to assess carbon footprint flexibility and primary energy use in decarbonized buildings," *Energy and Buildings*, vol. 294, p. 113213, 2023.

[3] J. Le Dréau, R. A. Lopes, S. O'Connell, D. Finn, M. Hu, H. Queiroz, D. Alexander, A. Satchwell, D. Österreicher, B. Polly, *et al.*, "Developing energy flexibility in clusters of buildings: a critical analysis of barriers from planning to operation," *Energy and Buildings*, p. 113608, 2023.

[4] R. Li, A. J. Satchwell, D. Finn, T. H. Christensen, M. Kummert, J. Le Dréau, R. A. Lopes, H. Madsen, J. Salom, G. Henze, *et al.*, "Ten questions concerning energy flexibility in buildings," *Building and Environment*, vol. 223, p. 109461, 2022.

[5] T. Q. Péan, J. Salom, and R. Costa-Castelló, "Review of control strategies for improving the energy flexibility provided by heat pump systems in buildings," *Journal of Process Control*, vol. 74, pp. 35–49, 2019.

[6] N. Mahdavi, J. H. Braslavsky, M. M. Seron, and S. R. West, "Model predictive control of distributed air-conditioning loads to compensate fluctuations in solar power," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 3055–3065, 2017.

[7] N. Mahdavi and J. H. Braslavsky, "Modelling and control of ensembles of variable-speed air conditioning loads for demand response," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4249–4260, 2020.

[8] A. Schledorn, S. Charousset-Brignol, R. G. Junker, D. Guericke, H. Madsen, and D. F. Dominković, "Frigg 2.0: Integrating price-based demand response into large-scale energy system analysis," *Applied Energy*, vol. 364, 2024.

[9] H. Madsen, G. Tsaousoglou, T. K. S. Ritschel, S. Tohidi, H. Binder, H. L. Frandsen, and R. G. Junker, "Incentivising and activating multi-purpose flexibility for the future power system," in *Incentives and Digitalizarion for Flexibility in the Green Transition*, pp. 28–42, Danish Utility Regulator, 2024.

[10] H. Madsen, S. S. Tohidi, R. Ebrahimy, M. Banaei, T. K. Ritschel, and N. Mahdavi, "Recent trends in demand-side flexibility," in *Energy Informatics Academy Conference*, pp. 167–184, Springer, 2024.

[11] S. Stinner, K. Huchtemann, and D. Müller, "Quantifying the operational flexibility of building energy systems with thermal energy storages," *Applied Energy*, vol. 181, pp. 140–154, 2016.

[12] R. De Coninck and L. Helsen, "Quantification of flexibility in buildings by cost curves–methodology and application," *Applied Energy*, vol. 162, pp. 653–665, 2016.

[13] O. Corradi, H. Ochsenfeld, H. Madsen, and P. Pinson, "Controlling electricity consumption by forecasting its response to varying prices," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 421–429, 2012.

[14] G. Dorini, P. Pinson, and H. Madsen, "Chance-constrained optimization of demand response to price signals," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2072–2080, 2013.

[15] R. G. Junker, A. G. Azar, R. A. Lopes, K. B. Lindberg, G. Reynders, R. Relan, and H. Madsen, "Characterizing the energy flexibility of buildings and districts," *Applied energy*, vol. 225, pp. 175–182, 2018.

[16] D. F. Dominković, R. G. Junker, K. B. Lindberg, and H. Madsen, "Implementing flexibility into energy planning models: Soft-linking of a high-level energy planning model and a short-term operational model," *Applied Energy*, vol. 260, p. 114292, 2020.

[17] R. G. Junker, C. S. Kallesøe, J. P. Real, B. Howard, R. A. Lopes, and H. Madsen, "Stochastic nonlinear modelling and application of price-based energy flexibility," *Applied Energy*, vol. 275, p. 115096, 2020.

[18] S. S. Tohidi, T. K. Ritschel, G. Tsaousoglou, U. H. Thygesen, and H. Madsen, "Stability analysis of nonlinear stochastic flexibility function in smart energy systems," *arXiv preprint arXiv:2405.15099*, 2024.

[19] S. S. Tohidi, H. Madsen, D. Calì, and T. K. Ritschel, "Optimal price signal generation for demand-side energy management," *Smart Energy*, p. 100173, 2025.

[20] G. Tsaousoglou, R. Junker, M. Banaei, S. S. Tohidi, and H. Madsen, "Integrating distributed flexibility into TSO-DSO coordinated electricity markets," *IEEE Transactions on Energy Markets, Policy and Regulation*, 2023.

[21] S. S. Tohidi, H. Madsen, G. Tsaousoglou, and T. K. Ritschel, "Adaptive flexibility function in smart energy systems: A linearized price-demand mapping approach," in *2024 European Control Conference (ECC)*, pp. 1315–1320, IEEE, 2024.

[22] K. S. Narendra and A. M. Annaswamy, *Stable adaptive systems*. Courier Corporation, 2012.

[23] E. Lavretsky and K. A. Wise, "Robust and adaptive control with output feedback," *Robust and Adaptive Control: With Aerospace Applications*, pp. 417–449, 2013.

[24] J. O. Ramsay, "Monotone regression splines in action," *Statistical science*, pp. 425–441, 1988.

[25] S. S. Tohidi, D. Cali, M. Tamm, J. Ortiz, J. Salom, and H. Madsen, "From white-box to grey-box modelling of the heat dynamics of buildings," in *E3S Web of Conferences*, vol. 362, p. 12002, EDP Sciences, 2022.

[26] U. H. Thygesen, *Stochastic Differential Equations for Science and Engineering*. CRC Press, 2023.

[27] D. J. Higham, "An algorithmic introduction to numerical simulation of stochastic differential equations," *SIAM Review*, vol. 43, no. 3, pp. 525–546, 2001.

[28] D. J. Higham and P. E. Kloeden, *An introduction to the numerical simulation of stochastic differential equations*. SIAM, 2021.

[29] J. Nocedal and S. J. Wright, *Numerical optimization*. Springer, 2006.

[30] L. Ljung and T. Söderström, *Theory and practice of recursive identification*. MIT press, 1983.

[31] H. Madsen, *Time series analysis*. Chapman and Hall/CRC, 2007.

[32] P. Ioannou and B. Fidan, *Adaptive control tutorial*. SIAM, 2006.

[33] K. J. Åström and B. Wittenmark, *Adaptive control*. Courier Corporation, 2013.