

Q7

Sol. DFT uses recursive function.

For an array of size  $n = 2^m$

DFT of  $n$  size array = DFT of  $\frac{n}{2}$  size array

+ Number  $\times$  DFT of  
 $\frac{n}{2}$  size array

At this step,

No. of operations are  $\frac{n}{2}$  (For

$\frac{n}{2}$  (For multiply a number  $W_N^k$  with each member of  $\frac{n}{2}$  size array)

$\frac{n}{2}$  (For summing each element of  $2 \times \frac{n}{2}$  size array).

Total operations at this steps =  $\frac{n}{2} + \frac{n}{2}$

=  $n$

Now, similarly to compute DFT of  $\frac{n}{2}$  size array.

We need  $\frac{n}{2}$  Number of operations

But, we have to compute 2 DFT of  $\frac{n}{2}$  size.



Thus, total number of operation in  
calculating ~~of all need~~  $\frac{n}{2}$  size array

after calculating DFT of all total 2  
numbers of  $\frac{n}{2}$  size array  $= 2 \times \frac{n}{2}$

$$= n$$

Similarly, to compute the DFT of  $\frac{n}{4}$

size array, we need to perform  
 $\frac{n}{4}$  numbers of operation. But

we need 4 DFT of  $\frac{n}{4}$  size array

Thus, total numbers of operations  $= 4 \times \frac{n}{4}$

$$= n$$

Similarly, going to last minimum size,  
we need only 1 operation to perform  
to compute DFT of 1 size array.

But we need to calculate DFT of  $n$  such  
1 size array. Thus, number  
of operations need intotal  $= n$ .

Total of operation needed overall

$$= n + n + n + \dots \text{m times}$$

$$\text{But } n = 2^m \Rightarrow m = \log_2 n$$

$$= n + n + n + \dots \log_2 n \text{ times}$$

$$= n \log_2 n$$

Thus, algorithm is  $O(n \log_2 n)$