Name- Ritik Saxena
Department - DHEP

Computational Physics Final exam

Q1
Sol.

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ -9 \end{bmatrix}$$

Writing the numbers in given decimal computer

$$\begin{bmatrix} 0.4\times10 & 0.1\times10 & 0.2\times10 \\ 0.2\times10 & 0.4\times10 & -0.1\times10 \\ 0.1\times10 & 0.1\times10 & -0.3\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.9\times10 \\ -0.5\times10 \\ -0.9\times10 \end{bmatrix}$$

$$R_1 \to \frac{R_1}{0.4\times10}$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0.2\times10 & 0.4\times10 & -0.1\times10 \\ 0.1\times10 & 0.1\times10 & -0.3\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.5\times10 \\ -0.9\times10 \end{bmatrix}$$

$$R_2 \to R_2 - 0.2\times10 R_1$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0 & +0.35\times10 & -0.2\times10 \\ 0.1\times10 & 0.1\times10 & -0.3\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.96\times10 \\ -0.9\times10 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 0.1\times10 \times R_1$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0 & 0.35\times10 & -0.2\times10 \\ 0 & 0.7\times10^0 & -0.35\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.96\times10^2 \\ -0.11\times10^2 \end{bmatrix}$$

$$R_2 \rightarrow \frac{R_2}{0.35\times10}$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0 & 0.1\times10 & -0.57\times10^0 \\ 0 & 0.7\times10^0 & -0.35\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.27\times10 \\ -0.11\times10^2 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 0.7\times10^0 R_2$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0 & 0.1\times10 & -0.57\times10^0 \\ 0 & \cancel{0.1\times10}\ 0 & -0.31\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.27\times10 \\ -0.9\times10 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-0.31\times10}$$

$$\begin{bmatrix} 0.1\times10 & 0.25\times10^0 & 0.5\times10^0 \\ 0 & 0.1\times10 & -0.57\times10^0 \\ 0 & 0 & 0.1\times10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.23\times10 \\ -0.27\times10 \\ 0.29\times10 \end{bmatrix}$$

Thus, we have

$$(0.1\times10)x_1 + (0.25\times10^0)x_2 + (0.5\times10^0)x_3 = 0.23\times10$$

$$(0.1\times10)x_2 - (0.57\times10^0)x_3 = -0.29\times10$$

$$(0.1\times10)x_3 = 0.29\times10$$

$x_3 = 0.29 \times 10$

$x_2 = -0.29 \times 10 + 0.57 \times 10^0 \times 0.29 \times 10$
$= -0.10 \times 10$

$x_1 = 0.23 \times 10 - (0.5 \times 10^0) \times 0.29 \times 10$
$- (0.25 \times 10^0) \times -0.10 \times 10$
$= 0.11 \times 10$

Thus,

$x_1 = 0.11 \times 10 = 1.1$
$x_2 = -0.10 \times 10 = -1.0$
$x_3 = 0.29 \times 10 = 2.9$

Q3
Sol For tridiagonal $n \times n$ matrix A
$Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & \cdot & 0 \\ a_{21} & a_{22} & a_{23} & 0 & \cdot & 0 \\ 0 & & & & & 0 \\ 0 & & & & & 0 \\ \cdot & \cdot & \cdot & 0 & a_{n,n-2}\ a_{n,n-1} & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Here code is written such that operation on zero element of matrix are not ~~perform~~ performed.

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Let the number of steps for $n \times n$ matrix be $S(n)$. Then,
~~S(n) =~~

Number of steps in making a_{11} to be 1
= (3 divisions) = 3

Number of steps in making a_{21} to 0
= 3 multiplication + ~~multipli~~ 3 subtraction
= 6

Then, we have perform operation on $(n-1) \times (n-1)$ matrix

So, Number of steps in solving $(n-1) \times (n-1)$ matrix $= S(n-1)$

Then, number of steps in getting value of x_1 = 1 multiplication + 1 substraction
= 2

So,

$$S(n) = 3 + 6 + S(n-1) + 2$$

$$S(n) - S(n-1) = 11$$

$$T(n) = 11$$

$$S(n) = \Sigma T(n)$$

$$= \Sigma 11$$

$$= 11n$$

Thus, we have $O(n)$ ~~adagent~~ operations.

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Q4
Sol. (e) Because it has roughly constant pdf. Thus it must ~~had~~ have dirac delta at frequency = 0.

(c) Min & max ~~wo~~ value of k depends upon sampling rate. In our ~~cdo~~ case,
Min value = -0.5
~~Max value ≈ 0.5~~
Max value $= 0.499023437$
≈ 0.5

Q5
Sol. My 3 criteria will be
① Documentation – Documentation of library should be clear. There should be example for function. Input & output parameters should be clearly mentioned.

② Numerical stability and order of alogrithms – The function should be numerically ~~stabl~~ stable & ~~order of algorithm will be~~ good alogrithms should be used which has ~~ver~~ low order ~~a~~ O(n).

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(3) Variety of alogrithms - There should a vast of alogrithms available. Also, alogrithms for ~~various tasks~~ ~~sho~~ as many task as possible should be there.

Q6
Sol. As the rate of $\frac{dy_2}{dx} \approx -2 \times \frac{dy_1}{dx}$

we expect for same initial value.

$$y_1 \sim -2y_2$$

~~This~~ is which we can see from our plot.

Q9
Sol Singular values of $\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ = 2.44948974 & 1

Singular values of $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ are 2, 1, 1

Q7
Sol. For seed to appear again at some point we choose seed less than modulus ~~than~~ in standard ~~for~~ ~~a large~~ alogrithm then seed can reappear at some point

If we choose seed = modulus
~~Then~~ in standard ~~odo~~ linear congruential generator alogrithm then seed will never appear again

Q2

Sol. (a) numpy.fft.fftn, ~~nump~~

(b) scipy.linalg.qr

(c) numpy.random.lognormal

(d) scipy.integrate.DOP853.

(e) numpy.linalg.svd

(f)

(g) scipy.integrate.ode()

(h) mcint.integrate in mcint library

(i) ~~sci~~ scipy.integrate.solve_bvp.

(j) numpy.linalg.eig.

Q8

Sol To ~~ge~~ get relative error, run code. There was 100 mesh point so I couldn't write relative error by hand.