Walk or run in the rain: Which speed is best?

Wang Yuzhe, Stanley Zheng

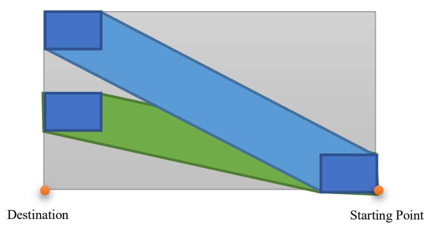
March 26, 2023

Abstract

We analysis which speed is the best strategy which allows the object contact least rain when it is moving from a position to another. The ground is flat and the object is moving straightly at a constant speed.

1 A Simple Approach

Consider an initial space right angle coordination Oxyz. Let the starting point of the object be the origin and the direction of the velocity of the object be the positive direction of X-axis. The velocity of the object **v** = $(v_x, 0,0)$ and the velocity of the rain u = (u_x, u_y, u_z). Let the distance between starting point be *d* so that the time object traveled t = $\frac{d}{v_x}$. Then, given the coordination Oxyz a velocity which is the same as the velocity of the rain. The velocity of the object $v' = (v - u_x, -u_y, -u_z)$. The speed of the object v' $= \sqrt{(v-u_x)^2 + u_y^2 + u_z^2}$. In this coordination, the rain stays static and the volume that the object passed is the volume which the object is able to contact with the rain. The track:



Let S be the area of the plane whose natural vector is parallel to the velocity of the object under the coordination which is with rain's velocity. In reference to the object, the area facing to x, y, z direction is S_x, S_y, S_z. Suppose the ratio of rain's volume and the space's volume is ω, then the volume of the rain which contact with the object is Q = ωSv't.

2 Solving the Problem

$$\vec{v'} \cdot \vec{n_x} = \mid \vec{v'} \mid \cdot \mid \vec{n_x} \mid \cdot cos\theta_x$$

$$cos\theta_x = \frac{\vec{v'} \cdot \vec{n_x}}{\mid \vec{v'} \mid \cdot \mid \vec{n_x} \mid}$$

$$cos\theta_x = \frac{u_x - v}{\sqrt{(u_x - v)^2 + u_y^2 + u_z^2}}$$

as the same method, we know

$$cos\theta_y = \frac{\vec{v'} \cdot \vec{n_y}}{\mid \vec{v'} \mid \cdot \mid \vec{n_y} \mid} \qquad cos\theta_y = \frac{u_y}{\sqrt{(u_x - v)^2 + u_y^2 + u_z^2}}$$

$$cos\theta_z = \frac{\vec{v'} \cdot \vec{n_z}}{\mid \vec{v'} \mid \cdot \mid \vec{n_z} \mid} \qquad cos\theta_z = \frac{u_z}{\sqrt{(u_x - v)^2 + u_y^2 + u_z^2}}$$

$$S = S_x \cdot cos\theta_x + S_y \cdot cos\theta_y + S_z \cdot cos\theta_z$$

$$Q = \omega \cdot S \cdot \mid \vec{v'} \mid \cdot t$$

So,

$$Q = \omega \cdot \left(\frac{S_x \cdot (u_x - v) + S_y \cdot u_y + S_z \cdot u_z}{\sqrt{(u_x - v)^2 + u_y^2 + u_z^2}}\right) \cdot \sqrt{(u_x - v)^2 + u_y^2 + u_z^2} \cdot \frac{d}{v}$$

So, we can conclude that Q is only related to v and u_x, u_y, u_z.

And if we choose a certain $\vec{u}$, then we can calculate the best speed in this model. So, I use python in Jupyter write the whole process, we can easily change parameters and get the answer we want.

Detail please see the .ipynb document, and because it needs to set the environment to execute the document, or you can only read. I am glad to show you how to do this if you ask.