5 Central Tendency

Introduction

In the previous units we discussed how data can be collected and organized in a meaningful manner so that we can use it more conveniently for statistical analysis. The frequency table and graphic presentation of data make it more meaningful. This chapter takes you beyond frequency distributions.

Suppose you come to class, the day after a series of examinations. Someone asks, "how much score you expect in this exams?" you may answer like, about 70%, nearly 70%, around 70% etc. Have you thought why you use the words about, nearly, around etc before 70%. It is because you select a representative of the different scores you may get. ie sometimes we use one single value to represent a data. In this way it is easier to compare data of the same type. These representative values are usually known as **averages** or **measures of central tendencies.**

Central Tendency

Consider the distribution of the number of family members of 60 students in a class, we discussed in Chapter 3. What we can see that the observations are more concentrated towards the centre of the distribution. See the table below.

Number of family members	Tally Mark	Frequency
2	II	2
3	卌 I	6
4	卌 IIII	9
5	卌 卌 IIII	14
6	卌 卌 III	13
7	卌 II	7
8	IIII	4
9	II	2
10	II	2
11	I	1
Total		60

Similarly the distribution of body weights of 60 students (Chapter 3) also shows the same property. On examining the frequency distributions we can see that the observations in most distributions show a tendency to cluster around a central value. This property of the observations in a data to cluster or concentrate around a value is known as *Central Tendency*.

Measures of Central Tendency (Averages)

The values which give us an idea about the concentration of observations in the central part of the distribution are known as *Measures of Central Tendency* or *Averages*. It is a single value which represents an entire set of data, around which most of the values of the data cluster. According to *Professor Arthur Lyon Bowley*, averages are, "statistical constants which enable us to comprehend in a single effort, the significance of the whole".

Arthur Lyon Bowley

Sir Arthur Lyon Bowley, born on 6 November 1869 was a British statistician pioneered the use of sampling techniques in social surveys. Bowley's "Elements of Statistics" is generally regarded as the first English-language statistics text-book. It described the techniques of descriptive statistics that would be useful for economists and social sciences.

Desirable properties of a good average

An average should posses the following properties.

(i) Simple and rigid definition.

(ii) Simple to understand and easy to calculate.

(iii) Based on all the observations.

(iv) Least affected by extreme values.

(v) Least affected by fluctuations of sampling.

(vi) Capable of further mathematical treatment.

In 1796, Adolphe Quetelet, a Belgian astronomer, Mathematician, Statistician and Sociologist, investigated the characteristics heights, weights etc of French conscripts to determine the average man. Florence Nightingale, a celebrated British social reformer and statistician, and the founder of modern nursing, was so influenced by Quetelets work that she began collecting and analysing medical records in the military hospitals during the Crimean war. Based on her work, hospitals began keeping accurate records on their patients. Florence Nightingale was the first female member of the Royal Statistical Society.

Various measures of Central Tendencies

The measure of central tendency indicates the location or position of a value to describe the entire data. The various measures of central tendencies are

1. Arithmetic Mean (AM)
2. Median
3. Mode
4. Geometric Mean (GM)
5. Harmonic Mean (HM)

5.1 Arithmetic Mean (AM)

The *'Arithmetic Mean'* (referred to as *'mean'*) is a most common measure of central tendency. The mean is a common measure in which all the values play an equal role. Most of the time when we refer to the 'average' of a data, we are talking about its arithmetic mean. For example, to find the average life of a CFL bulb, the average temperature in a city etc, the average we refer to is the arithmetic mean.

The table below shows the number sales of five retail outlets in a day.

Retailer	:	1	2	3	4	5
Sales (in 1000 Rs)	:	8	23	4	8	2

To find the average sales per a retailer in the day, we sum the values and divide by the number of observations. This average, called the arithmetic mean is computed as

$$AM = \frac{8+23+4+8+2}{5} = \frac{45}{5} = 9$$

ie, the average sales per retailer is Rs. 9000/-.

The arithmetic mean of a set of data is defined as

$$\text{Mean} = \frac{\text{sum of the observations}}{\text{number of observations}}$$

We usually denote mean by the symbol '$\bar{x}$' (read as 'x bar').

$$\bar{x} = \frac{\text{sum of the observations}}{\text{number of observations}}$$

Computation of Arithmetic Mean

(i) Arithmetic Mean from a raw data

Consider a raw data containing n observations $x_1, x_2, x_3, \ldots, x_n$. The mean can be calculated by

$$\bar{x} = \frac{\text{sum of the observations}}{\text{number of observations}}$$

$$= \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$= \frac{\sum x}{n}$$

$$\text{Mean} = \frac{\sum x}{n}$$

The Greek letter $\sum$ represents summation

Remark: The sum of the observations in a series, $\sum x = n\bar{x}$

Illustration 5.1

An umbrella manufacturing company wants to launch a new product in a state. The rainfall (in cms) in the state for the last five years is, 120, 135, 110, 142 and 150. Find the average rainfall of the state for the last five years.

Solution. Average rainfall,

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{120 + 135 + 110 + 142 + 150}{5}$$

$$= 131.4\, cm$$

Illustration 5.2

In the first four class tests, a student got the scores 52, 48, 33 and 27 respectively. (a) Find the mean of the scores.
(b) If in the fifth test, he got a score of 45, find his new mean score.

Solution. (a) The mean score of the first four class tests is

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{52+48+33+27}{4}$$

$$= 40$$

(b) The new mean after 5 class tests is

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{52+48+33+27+45}{5}$$

$$= 41$$

Illustration 5.3

The mean of a group of 100 observations is known to be 50. Later it was discovered that two observations were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Solution. Given that $\bar{x} = 50$ and $n = 100$.

We have the sum of the observations,

$$\sum x = n\bar{x}$$

$$= 100 \times 50$$

$$= 5000$$

But it was wrong, because two observations 192 and 88 were misread as 92 and 8. So the corrected sum of observations is

$$\begin{aligned}\text{corrected sum} &= 5000 - 92 - 8 + 192 + 88 \\ &= 5180 \\ \text{So corrected mean, } \bar{x} &= \frac{\text{corrected sum}}{n} \\ &= \frac{5180}{100} \\ &= 51.80\end{aligned}$$

(ii) Arithmetic Mean from a Discrete Frequency Distribution

A discrete frequency distribution consists of data in which the observations are expressed with their frequencies.

In discrete type distribution, for calculating the mean every item is multiplied by its corresponding frequencies and the total sum of this product is divided by the sum of the frequencies.

The Arithmetic Mean of a Discrete frequency distribution is

$$\bar{x} = \frac{\sum fx}{\sum f}$$

ie.$\bar{x} = \dfrac{\sum fx}{N}$ where $N = \sum f$ is the total frequency

The following steps can be used to find the mean of a given series $x_1, x_2, x_3, \ldots, x_n$ with corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$.

Step 1: Find $f_1x_1, f_2x_2, f_3x_3, \ldots, f_nx_n$

Step 2: Find $\sum fx = f_1x_1 + f_2x_2 + f_3x_3 + \cdots + f_nx_n$

Step 3: Find $\sum f = f_1 + f_2 + f_3 + \cdots + f_n$

Step 4: The mean is obtained by the formula:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Illustration 5.4

The students in a Statistics class were trying to study the heights of participants in a sports meet. They collected the height of 20 participants, as displayed in the table.

Height (in inches)	:	*49*	*53*	*54*	*55*	*66*	*68*	*70*	*80*
No of participants	:	*1*	*2*	*4*	*5*	*3*	*2*	*2*	*1*

Calculate the mean height of the participants.

Solution.

Height (x)	No of participants (f)	fx
49	1	49
53	2	106
54	4	216
55	5	275
66	3	198
70	2	140
80	1	80
Total	$N = 20$	1200

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{1200}{20} \qquad = 60$$

Mean height = 60 inches

Illustration 5.5

A survey is taken by an insurance company to determine how many car accidents the average New Delhi City resident has gotten into in the past 10 years. The company surveyed 200 people who are getting off a train at a subway station. The following table gives the results of the survey.

Number of Accidents	*Number of People*
0	*60*
1	*10*
2	*40*
3	*10*
4	*80*

Calculate the mean number of accidents of this data set.

Solution.

No of accidents (x)	No of people (f)	fx
0	70	0
1	10	10
2	40	80
3	10	30
4	70	280
Total	$N = 200$	400

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{400}{200} \qquad = 2$$

Mean of accidents = 2

(iii) Arithmetic Mean from a Continuous Frequency Distribution

A continuous frequency distribution consists of data that are grouped by classes.

The computation of AM is similar to the computation procedure for the discrete frequency distribution. But, since the data is grouped by classes, we do not know the individual values of every observation. So it is necessary to make an assumption about these values. The assumption is that every observation in a class has a value equal to the midpoint of the class. The formula for computing AM is,

$$\bar{x} = \frac{\sum fx}{N} \text{ where } N = \sum f$$

This arithmetic mean is only an approximation. (Why?)

Illustration 5.6

The table below show the age of 55 patients selected to study the effectiveness of a particular medicine.

Age	*No of Patients*
0-10	*5*
10-20	*7*
20-30	*17*
30-40	*12*
40-50	*5*
50-60	*2*
60-70	*7*

Calculate the mean age of the patients.

Solution. To find mean, prepare the following table

Age	Mid point (x)	No of patients (f)	fx
0-10	5	5	25
10-20	15	7	105
20-30	25	17	425
30-40	35	12	420
40-50	45	5	225
50-60	55	2	110
60-70	65	7	455
Total		$N = 55$	1765

$$\bar{x} = \frac{\sum fx}{\sum f}$$
$$= \frac{1765}{55}$$
$$= 32.09$$

Illustration 5.7

The frequency distribution below represents the weights in kg of parcels carried by a small logistic company. Find the mean weight of parcels.

Weight	*No. of Parcels*
10.0-10.9	*2*
11.0-11.9	*3*
12.0-12.9	*5*
13.0-13.9	*8*
14.0-14.9	*12*
15.0-15.9	*15*
16.0-16.9	*13*
17.0-17.9	*11*
18.0-18.9	*6*
19.0-19.9	*2*

Solution.

Weight	Mid point (x)	No. of Parcels (f)	fx
10.0-10.9	10.45	2	20.90
11.0-11.9	11.45	3	34.35
12.0-12.9	12.45	5	62.25
13.0-13.9	13.45	8	107.60
14.0-14.9	14.45	12	173.40
15.0-15.9	15.45	15	231.75
16.0-16.9	16.45	13	213.85
17.0-17.9	17.45	11	191.95
18.0-18.9	18.45	6	110.70
19.0-19.9	19.45	2	38.90
Total		$N = 67$	1185.65

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{1185.65}{67} \qquad = 17.70$$

Mean weight = 17.70 kg

Mathematical Properties of Arithmetic Mean

The AM of a distribution has the following mathematical properties.

1. The sum of deviations of items in a data from the AM is always Zero.

$$ie \sum (x - \bar{x}) = 0$$

2. The sum of squares of the deviations of the items in a data is the least when the deviation is taken about the Mean.

$$ie \sum (x - a)^2 \text{ is least when } a = \bar{x}$$

3. If the mean of n observations, $x_1, x_2, \ldots, x_n$ is $\bar{x}$ then the mean of the observations, $(x_1 \pm a), (x_2 \pm a), \ldots, (x_n \pm a)$ is $(\bar{x} \pm a)$.

   ie, If each observation is increased by 'a', then the mean also increased by a and if each observation is decreased by 'a', then the mean is also decreased by a.

4. The mean of n observations, $x_1, x_2, \ldots, x_n$ is $\bar{x}$. If each observation is multiplied by $p, p \neq 0$, then the mean of the new observations is $p\bar{x}$.

Activity

Observe the data 10, 25, 17, 22, 20, 35, 28, 42, 68 and 53

(a) Examine whether $\sum(x - \bar{x}) = 0$ for this data.

(b) Examine the admissibility of property 2 by giving some values to a

(c) What happens to the mean by

   (i) adding 3 to all the observations in the data.

(ii) subtracting 3 from all the observations.

(d) What happens to the mean

(i) if all the observations are multiplied by 2.

(ii) if all the observations are divided by 2.

Merits and Demerits of AM

Arithmetic Mean is the measure which has most of the desirable properties of a good measure of central tendency. The following are some of the merits and demerits of it

Merits

1. It has a rigid definition.
2. It is easy to calculate and understand.
3. AM is based upon all the observations.
4. It is least affected by fluctuations of sampling.
5. It is capable of further mathematical treatment.

Demerits

1. AM is highly affected by extreme values.
2. It can't be determined by inspection.
3. It can't be used for qualitative characteristics like, intelligence, honesty, beauty etc.
4. It can't be calculated for open end classes.

Weighted Arithmetic Mean

Usually in computing Arithmetic Mean, equal importance is given to all the observations of the data. However there are cases where all the items are not of equal importance. In other words some items of a series are more important as compared to the other items in the same series. In such cases it becomes important to assign different weights to different items. For example if want to have an idea of the change in the living standards of a certain group of people, the AM we discussed so far can't be used. Because, all the commodities the people used may not be of equal importance. Rice, wheat etc may be more important when compared with sugar, tea, salt etc.

The AM that assign a weight to each observation on its importance related to other is called the weighted arithmetic mean. Weighted AM are widely used in the preparation of consumer and producer price index numbers.

Computation of Weighted Arithmetic Mean

Let $x_1, x_2, \ldots, x_n$ be the observations of a data and $w_1, w_2, \ldots, w_n$ be their corresponding weights. Then the weighted arithmetic mean is given by,

$$\begin{aligned}\bar{x} &= \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w_1 + w_2 + \cdots + w_n} \\ &= \frac{\sum wx}{\sum w}\end{aligned}$$

Illustration 5.8

A student's final scores in Mathematics, Physics, Chemistry and English are respectively 82, 86, 90 and 70. If the respective credits received for these courses are 3, 5, 3, and 1, determine the average score.

Solution. Here the weights associated to the observations 82. 86. 90 and 70 are 3, 5, 3 and 1.

x	:	82	86	90	70
w	:	3	5	3	1

Average,

$$\bar{x} = \frac{\sum wx}{\sum w}$$

$$= \frac{82 \times 3 + 86 \times 5 + 90 \times 3 + 70 \times 1}{3 + 5 + 3 + 1}$$
$$= \frac{1016}{12}$$
$$= 84.67$$

Combined Arithmetic Mean

In a class test taken by 4 boys and 6 girls, the boys obtained a mean score of 35 and the girls obtained a mean score of 75. What would be the mean score of all the students taken together ?

Is it $\frac{35+75}{2} = 55$?

Obviously not, because the number of boys and number of girls are not equal. The mean score of girls has a greater importance than boys (why?).

For all the students,

$$\begin{aligned}
\text{the mean score } &= \frac{\text{Total score of all the students}}{\text{Total number of students}} \\
&= \frac{\text{Total score of boys} + \text{Total score of girls}}{\text{No. of boys} + \text{No. of girls}} \\
&= \frac{4 \times 35 + 6 \times 75}{4 + 6} \\
&= \frac{140 + 450}{10} \\
&= 59
\end{aligned}$$

Therefore, the mean score of the combined group of boys and girls, called the *combined mean is 59.*

If $\bar{x}_1$ and $\bar{x}_2$ are the means of two groups of n_1 and n_2 observations respectively, the mean of the combined group of $n_1 + n_2$ observations is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Illustration 5.9

The mean score obtained in an examination by a group of 100 students was found to be 50. The mean of the scores obtained in the same examination by another group of 200 students was 57. Find the mean of scores obtained by both the groups taken together.

Solution. We are given that

$$\bar{x}_1 = 50 \text{ and } \bar{x}_2 = 57$$
$$n_1 = 100 \text{ and } n_2 = 200$$

We know that the combined mean is given by

$$\begin{aligned}\bar{x} &= \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \\ &= \frac{100 \times 50 + 200 \times 57}{100 + 200} \\ &= 54.67\end{aligned}$$

Illustration 5.10

The mean weight of 150 students in a certain class is 60 kgs. The mean weight of boys in the class is 70 kgs and that of girls is 55 kgs. Find the number of boys and number of girls in the class.

Solution. We are given that

The combined mean, $\bar{x} = 60$ kgs
Mean weight of boys, $\bar{x}_1 = 70$ kgs
Mean weight of girls, $\bar{x}_2 = 55$ kgs
The total no of students = 150

Let there be 'x' boys in the class. Therefore the number of girls in the class is

$150 - x$. We know that,

$$\bar{x} = \frac{n_1\bar{x_1} + n_2\bar{x_2}}{n_1 + n_2}$$

$$\text{ie,}60 = \frac{70x + 55(150 - x)}{150}$$

$$\Rightarrow 9000 = 70x + 8250x - 55x$$

$$\Rightarrow 15x = 750 \Rightarrow x = 50$$

So number of boys in the class is 50 and number of girls in the class is $150 - 50 = 100$.

If there are k groups of sizes $n_1, n_2, \ldots\ldots, n_k$ respectively and $\bar{x_1}, \bar{x_2}, \ldots\ldots, \bar{x_k}$ are their respective means, then the combined mean of $n_1 + n_2 + \ldots + n_k$ observations is given by

$$\bar{x} = \frac{n_1\bar{x_1} + n_2\bar{x_2} + \ldots + n_k\bar{x_k}}{n_1 + n_2 + \ldots + n_k}$$

Know your progress

1. 10 novels were randomly selected and the number of pages were recorded as follows, $415, 398, 497, 399, 402, 405, 395, 412, 407$ and 400. Find the mean of the number of pages.
2. The average of 20 values is calculated to be 25. Later it was discovered that two values 52 and 15 were misread as 42 and 51 respectively. Calculate the corrected value of the mean.
3. The weights of 70 workers in a factory are given below. Find the mean weight of a worker

Weight (in Kgs)	No of workers
60	5
62	10
63	12
65	18
67	15
68	10

4. 30 automobiles were tested for fuel efficiency (in kms/litter). The following frequency distribution was obtained. Calculate the average mileage of the automobiles.

Mileage	No of vehicles
7.5-12.5	3
12.5-17.5	5
17.5-22.5	15
22.5-27.5	5
27.5-32.5	2

5. For 50 antique car owners, the following distribution of the car's ages was obtained. Determine the mean age.

Car age	No of cars
16-18	20
19-21	18
22-24	8
25-27	4

6. The mean mark of 100 students in a class is 39. The mean mark of the boys is 35 while that of girls is 45. Find the number of boys and that of the girls in the class.

7. A candidate obtains the following marks in an examination. English- 46, Economics-58, Accountancy-72 and Statistics-67. It is agreed to give double weights to marks in English and Statistics. What is the mean mark.

5.2 Median

A family has 5 children aged 10, 8, 5, 4 and 12 years. Can you find the age of the middle child? To find the age of the middle child, we arrange the children's ages in ascending order.

$$4, 5, 8, 10, 12$$

The age of the middle child is the middlemost number in the data, which is 8. Here '8' is called the *median* of the five numbers.

Median devides a road into two equal parts

Median is the value of the middlemost observations in the data when the data is arranged in ascending or descending order.

ie, *Median* of a distribution is the value of the variable which divide the distribution into two equal parts. The half of the observations are smaller than or equal to median and half are larger than or equal to median. So median is known as a *positional average*. Median of a data is the middle most observation in the data when the observations are arranged in ascending or descending order of their values.

Computing the Median

(i) Calculation of Median from a raw data

Consider a raw data having 'n' observations. To find the median, first arrange the data in ascending or descending order. Then median is the $\left(\frac{n+1}{2}\right)^{\text{th}}$ item in the data.

Median of a raw data is the $\left(\frac{n+1}{2}\right)^{th}$ item, when the data is arranged in ascending or descending order of magnitude.

Illustration 5.11

Rahna's maths quiz scores in 9 competitions were 88, 97, 87, 92, 90, 88, 93, 98 and 95. What was her median quiz score ?

Solution. Arranging the data in ascending order,

$$87, 88, 88, 90, 92, 93, 95, 96, 98$$

Here the number of observations, $n = 9$.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$
$$= \left(\frac{9+1}{2}\right)^{th} \text{ item}$$
$$= 5^{th} \text{ item}$$

The 5^{th} item in the series is 92. Median quiz score is 92

Illustration 5.12

Anand's family plans a trip from Thiruvananthapuram to Wayanad on their summer vacation. They drove through 8 districts. The following are the petrol prices in the 8 districts on those days. Rs.71.9, Rs.72.3, Rs.72.4, Rs. 72.32, Rs. 73, Rs.73.1, Rs.72.2 and Rs.72.48 What is the median petrol price ?

Solution. Arranging the observations in ascending order,

$$71.9,\ 72.2,\ 72.3,\ 72.32,\ 72.4,\ 72.48,\ 73,\ 73.1$$

Number of observations is 8.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$
$$= \left(\frac{8+1}{2}\right)^{th} \text{ item}$$
$$= 4.5^{th} \text{ item}$$

But there is no item in the series having a position 4.5. So we take the mean of the 4^{th} and 5^{th} items in the series as the median.

$$\begin{aligned}\text{Median} &= \text{Mean of } 4^{th} \text{ and } 5^{th} \text{ item}\\ &= \frac{72.32+72.4}{2}\\ &= 72.36\end{aligned}$$

Median petrol price = Rs. 72.36.

(ii) Median from a Discrete Frequency Distribution

For a discrete frequency distribution median is the observation having cumulative frequency $\frac{N+1}{2}$, when the observations are arranged in ascending order

The following steps can be used to find the median in a discrete distribution.

Step 1: Arrange the data in ascending or descending order of magnitude.

Step 2: Obtain the cumulative frequencies.

Step 3: Determine $\frac{N+1}{2}$, where N is the total frequency.

Step 4: Median is the value for the $\left(\frac{N+1}{2}\right)^{th}$ item of the data.

Illustration 5.13

The following data gives the daily wages of workers in a manufacturing company. Find the median wage.

Daily wage (in 100 Rupees)	:	6	8	10	12	15	18
Number of workers	:	20	14	7	16	12	2

Solution. The given data is in ascending order.

The cumulative frequencies are given by,

Wages (in 100 rupees)	Number of workers (frequency)	Cumulative frequency
6	20	20
8	14	34
10	7	41
12	16	57
15	12	69
18	2	71
	$N = 71$	

Total frequency $N = 71$. So $\frac{N+1}{2} = \frac{72}{2} = 36$

$\therefore$ Median is the value in the data which comes in the 36^{th} position. Which is the value of the item having cumulative frequency 36, which is 10.

$\therefore$ Median = Rs. 1000/-

Illustration 5.14

The table below shows the marks obtained by 42 students in an examination.

Marks	*9*	*20*	*25*	*40*	*50*	*80*
Number of students	*4*	*6*	*11*	*13*	*7*	*2*

Calculate the median mark.

Solution. The data given is in ascending order. The cumulative frequencies are

Marks	Frequency	Cumulative frequency
9	4	4
20	6	10
25	11	21
40	12	33
50	7	40
80	2	42
	$N = 42$	

Total frequency, $N = 42$.

$$\frac{N+1}{2} = \frac{43}{2} = 21.5$$

$\therefore$ Median is the value in the data which comes in the 21.5^{th} position. But there is no observation in the data which has the cumulative frequency, 21.5. Hence we consider the median as the mean of the 21^{th} and 22^{nd} observations. The 21^{th} observation is 25 and 22^{nd} observation is 40

$$\text{ie, Median} = \frac{25+40}{2} = \frac{65}{2} = 32.5$$

Hence median mark is 32.5.

(iii) Median from a Continuous Frequency Distribution

To find median in continuous frequency series, first we have to locate median class. Median class is the class where $(\frac{N}{2})^{th}$ observation lies. Median of a continuous frequency series is given by

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)c}{f}$$

Where l - lower bound (actual class limit) of the median class.
c - class interval of the median class.
f - frequency of the median class and
m - cumulative frequency of the class preceding the median class.

The following steps can be used to determine the median for a continuous frequency series.

Step 1: Convert the inclusive type classes to the exclusive type classes (if any).

Step 2: Obtain the cumulative frequencies.

Step 3: Determine $\frac{N}{2}$, where N is the total frequency.

Step 4: Locate the class having cumulative frequency $\frac{N}{2}$.

Step 5: Find median using the above formula.

Illustration 5.15

The distribution of income of 63 families is,

Income (100 Rupees)	:	*30-40*	*40-50*	*50-60*	*60-70*	*70 -80*	*80-90*	*90 -100*
No of workers	:	*6*	*12*	*18*	*13*	*9*	*4*	*1*

Compute the median income.

Solution. The cumulative frequency table is

Class	Frequency	Cumulative frequency
30-40	6	6
40-50	12	18
50-60	18	36
60-70	13	49
70-80	9	58
80-90	4	62
90-100	1	63
	$N = 63$	

$$\frac{N}{2} = 31.5$$

.

The class having cumulative frequency 31.5 is 50-60.
∴ Median class is 50-60.

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)c}{f}$$

Where, $l = 50, c = 10, f = 18$ and $m = 18$.

$$\begin{aligned} Median &= 50 + \frac{(31.5 - 18)10}{18} \\ &= 50 + \frac{135}{18} \\ &= 57.5 \end{aligned}$$

Illustration 5.16

The table below shows the distribution of marks obtained by 50 students in Economics. Find the median mark.

Marks	:	*10-14*	*15-19*	*20-24*	*25-29*	*30-34*	*35-39*		
Number of students	:	*4*	*6*	*10*	*5*	*7*	*3*		
	:	*40-44*	*45-49*						
	:	*9*	*6*						

Solution. Here the classes are of inclusive type. Before computing median, we have to convert it into exclusive form to get the actual class limits (class bounds).

Let us prepare the cumulative frequency table with the actual class limits.

Marks	Actual class	Frequency	Cumulative frequency
10-14	9.5-14.5	4	4
15-19	14.5-19.5	6	10
20-24	19.5-24.5	10	20
25-29	24.5-29.5	5	25
30-34	29.5-34.5	7	32
35-39	34.5-39.5	3	35
40-44	39.5-44.5	9	44
45-49	44.5-49.5	6	50
		$N = 50$	

Here $N = 50$. $\therefore \frac{N}{2} = 25$

Hence the median class is $24.5 - 29.5$.

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)c}{f}$$

Where $l = 24.5, c = 5, f = 5$ and $m = 20$

$$Median = 24.5 + \frac{(25-20)5}{5}$$
$$= 29.5$$

5.2.1 Graphical location of Median

Median from Ogives

One of the advantages of median than mean is that it can be located graphically. Median is a positional average. There are two methods of locating median graphically.

(i) Presenting the data graphically by one ogive ('less than' or 'more than' ogive).

(ii) Presenting the data graphically by two ogives ('less than' and 'more than' ogives).

(i) Median by one ogive ('less than' or 'more than' ogive).

The steps involved in determining median using 'less than' (or 'more than') ogive are,

Step 1: Draw 'less than' (or 'more than') ogive.

Step 2: Find $\frac{N}{2}$ and mark it on the y-axis. Where N is the total frequency.

Step 3: Draw a perpendicular from $\frac{N}{2}$ to the right to cut the ogive at a point A (say).

Step 4: From the point A, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series.

Illustration 5.17

The table below shows the marks obtained by 100 students in an examination. Locate median graphically.

Marks	:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No of students	:	7	10	21	27	22	9	4

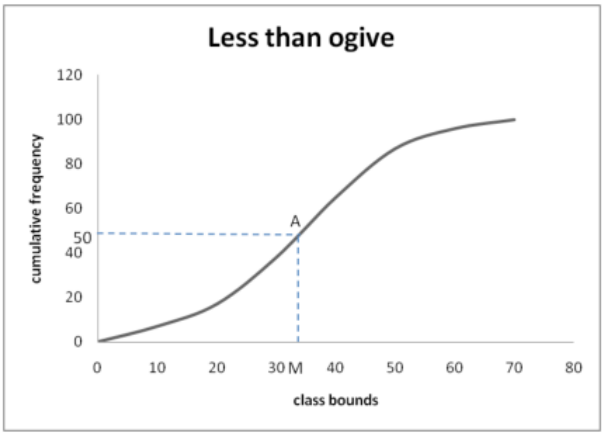
Solution.

(i) Median using 'less than' ogive.

In order to find median using 'less than' ogive, we have to construct the less than frequency table.

Upper bound	Less than cumulative frequency
10	7
20	17
30	38
40	65
50	87
60	96
70	100

$N = 100$ so that $\frac{N}{2} = 50$. Draw a less than ogive.



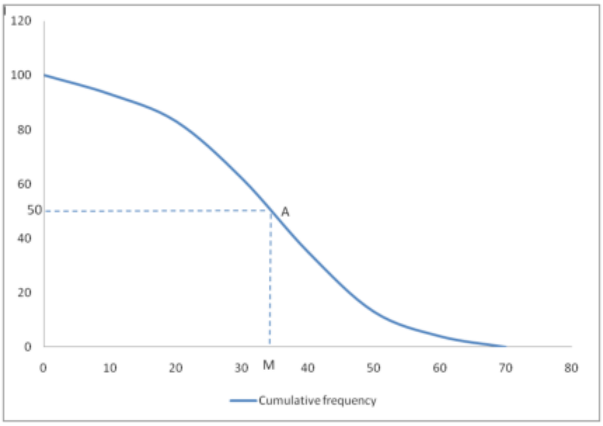
$\therefore$ Median is 34.

(ii) Median using more than ogive.

We have to first prepare a greater than cumulative frequency table.

Lower bound	Greater than cumulative frequency
0	100
10	93
20	83
30	62
40	35
50	13
60	4

Draw a more than ogive.



$\therefore$ Median is 34.

(ii) Median from two ogives ('less than' and 'more than' ogives).

The following are the steps involved in the determination of median from 'less than' and 'more than' ogives by simultaneously drawing them.

Step 1: Draw the two ogives on a paper with the same axis.

Step 2: Mark the point A, where the two ogives intersect.

Step 3: Draw perpendicular from A to the x-axis. The corresponding value on the x-axis would be the median of the data.

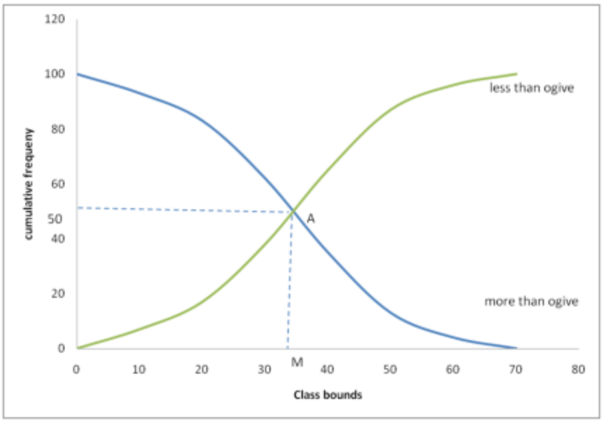
Illustration 5.18

Determine the Median using the data given in ***Illustration 5.17***

Solution. Prepare the two cumulative frequency tables.

Upper bound	Less than cumulative frequency	Lower bound	Greater than cumulative frequency
10	7	0	100
20	17	10	93
30	38	20	83
40	65	30	62
50	87	40	35
60	96	50	13
70	100	60	4

Draw the two ogives simultaneously on the same paper as shown below.



Median = 34.

5.2.2 Merits and demerits of Median

Merits

1. It has a rigid definition.
2. Median is easy to compute. In some cases it be located merely by inspection.
3. It is not affected by the extreme values.

4. It can be calculated for distribution with open end classes.

5. It is the only measure to be used while dealing with qualitative data which can measure quantitatively but can be arranged ascending or descending order of magnitudes.

6. The median may be a better indicator if a set of numbers has an *outlier*. An *outlier* is an extreme value that differs greatly from the other values.

Demerits

1. In some cases median can't be calculated exactly. For example, in the case of even number of observations, we take median as the mean of the two middle terms, as an approximation.

2. It is not based on all the observations.

3. It is affected by the fluctuations of sampling.

4. It is not capable of further mathematical treatments.

Know your progress

1. The manager of a sports shop recorded the number of cricket balls that he sold during seven months. The details are shown below. 132, 121,119, 116, 130, 121, and 131. Calculate median.

2. The weights (in kg) of 10 students are as follows. 31, 35, 27, 29, 32, 43, 37, 41, 34 and 28. Find the median. If the weight of 43 kg is replaced by 48 kg, find the new median.

3. The marks of 43 students are given in the following table. Find the median mark.

Marks	:	20	9	25	50	40	80
No of students	:	6	4	16	7	8	2

4. 80 randomly selected light bulbs were tested to determine their life time (in hours). The following frequency distribution was obtained. Calculate the median life of the bulb.

Life time (in hours)	:	52.5-63.5	63.5-74.5	74.5-85.5
Number of bulbs	:	12	23	18
	:	85.5-96.5	96.5-107.5	107.5-118.5
	:	6	14	5

5. The following is the distribution of marks of 100 students. Calculate the median mark.

Marks	:	30-39	40-49	50-59	60-69
Number of students	:	10	14	26	20
	:	70-79	80-89		
	:	18	12		

5.3 Mode

In everyday life we often apply the concept of majority. For example, in the election to the school parliament, the one who got majority of votes in your class become the class leader. The leader of your class represents your class in the school parliament. ie, Sometimes majority represents the data. *Mode* is the measure which expressing the concept of majority.

> Mode of a data is defined as the value that is repeated most often in the data. It is the observation having the maximum frequency in a data.

Mode is sometimes called the *Fashionable Average* or *Business Average*

Computing the Mode.

(i) Calculating the mode from a raw data.

For a raw data mode is the value which appears most often in the data. ie Mode of a raw data is the observation which appears a maximum number of times in the data.

Illustration 5.19

The ages of six persons who participated in an interview were $20, 21, 21, 24, 25, 24, 21$ *and 27 years. Find the mode of the data.*

Solution. Here the observation 21 appears three times, 24 appears two and all others are appear in a single time. So the value which appears a maximum number of times is 21.

$$\therefore \text{Mode} = 21 \text{ years.}$$

In a distribution, there may be one, two or more than two modes. If in a distribution there are two observations which are appearing a maximum number of times, then both the observations are taken as the modes of the distribution. A distribution which has a single mode is called a unimodal distribution, which has two modes is called a bimodal and has more than two modes is called a multimodal distribution.

Illustration 5.20

Mr. Vijayakumar, the physical education teacher of a school is trying to determine the average height of students in the cricket team of the school. The height of the players in inches are $70, 72, 72, 74, 74, 74, 75, 76, 76, 76$ *and 77. Calculate the mode of the heights.*

Solution. Here the data has two values, 74 and 76, which appears 3 times. All the other values appear less than 3 times.So the data set has two distinct modes 74 and 76.

Illustration 5.21

The generous CEO of a company wants to give all his employees a pay rise.

He is not sure whether to give everyone a straight Rs. 2000 rise or whether to increase the salary by 10%. The mean salary is Rs. 50000, the median is Rs. 20000 and the mode is Rs. 10000. What happens to mean, median and mode if

(a) *everyone at the company is given Rs. 2000 pay rise.*

(b) *the salary is increased by 10%.*

Solution. *Given that Mean, Median and Mode are respectively, Rs. 50000, Rs. 20000 and Rs. 10000.*

(a) *If every employee are given a straight increase of Rs. 2000,*

Let 'x' represents the original value, then the new salary becomes $x + 2000$

$$\begin{aligned} \text{Mean} &= \frac{\sum(x+2000)}{n} \\ &= \frac{\sum x + \sum 2000}{n} \\ &= \frac{\sum x}{n} + \frac{\sum 2000}{n} \\ &= 50000 + 2000 \\ &= 52000. \end{aligned}$$

(b) *ie, by adding 2000 to every observation results an increase of 2000 in mean. Similarly the median and mode are also increased by 2000. If 10% increase in salary is given to all the employees,*

Let 'x' represents the original values, then the new values become 110% of the original values. ie, This time the new observation is obtained by multiplying 1.1 to the actual values.

$$\begin{aligned} \text{Mean} &= \frac{\sum 1.1x}{n} \\ &= 1.1\frac{\sum x}{n} \\ &= 1.1 \times 50000 \\ &= 55000 \end{aligned}$$

Similarly,

$$\text{Median} = 1.1 \times 20000 = 22000$$
$$\text{Mode} = 1.1 \times 10000 = 11000$$

ie, the mean, median and mode are increased by 10%.

(ii) Mode from a discrete frequency distribution

Mode of a discrete frequency distribution is the observation having the maximum frequency.

The mode of a discrete frequency is the observation which appears a maximum number of times. ie, the observation having the highest frequency.

Illustration 5.22

The following distribution shows the sizes of shirts sold on a textile shop in Thiruvananthapuram on a month. Calculate the mode.

Size (in inches)	:	36	38	40	42	44
No of shirts sold	:	15	22	31	30	20

Solution. In the given frequency distribution, the observation having the maximum frequency is 40. Mode is 40.

(iii) Mode from a continuous frequency distribution.

As in the case of median, here also we have to locate a class called modal class to find mode in continous frequency series. Modal class is the class having highest frequency. Mode can be determined by the formula.

$$\text{Mode} = l + \frac{(f_1 - f_0)c}{(f_1 - f_0) + (f_1 - f_2)}$$

$$\text{ie. Mode} = l + \frac{(f_1 - f_0)c}{2f_1 - f_0 - f_2}$$

Where l - lower bound of the modal class.
f_1 - frequency of the modal class.
f_0 - frequency of the preceding class to the modal class.
f_2 - frequency of the succeeding class to the modal class.
c - class interval of the modal class.

For a continuous frequency distribution the calculation of mode involves the following steps.

Step 1: Locate the class having the highest frequency. This class is called the *modal class.*

Step 2: The mode can be determined using the above formula.

Illustration 5.23

For his research on the living standards, a researcher conducted a survey on 100 persons. The distribution of the ages of the persons is attached below.

Age	:	*0-10*	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*
Number of People	:	*12*	*18*	*27*	*20*	*17*	*6*

Determine the mode of this distribution.

Solution. The highest frequency = 27. ∴ The modal class is 20-30.

$$\text{Mode} = l + \frac{(f_1 - f_0)c}{2f_1 - f_0 - f_2}$$

Where $l = 20, c = 10, f_0 = 18, f_1 = 27$ and $f_2 = 20$.

$$\begin{aligned}\therefore \text{Mode} &= 20 + \frac{(27-18)\times 10}{2\times 27 - 18 - 20} \\ &= 20 + \frac{9\times 10}{16} \\ &= 25.625\end{aligned}$$

So Mode is 25.625.

Illustration 5.24

The production per day of a company (in Tons) on 60 days are given below. Calculate the mode.

Production per day	:	*21-22*	*23-24*	*25-26*	*27-28*	*29-30*
Number of days	:	*7*	*13*	*22*	*10*	*8*

Solution. Here the classes are of inclusive type. We have to convert into exclusive class before determining the mode.

Class bounds	Number of days
20.5-22.5	7
22.5-24.5	13
24.5-26.5	22
26.5-28.5	10
28.5-30.5	8

The maximum frequency is 20 so the modal class is 24.5-26.5

$$\text{Mode} = l + \frac{(f_1 - f_0)c}{2f_1 - f_0 - f_2}$$

Where $l = 24.5, c = 2, f_0 = 13, f_1 = 22, f_2 = 10$

$$\begin{aligned}\text{Mode} &= 24.5 + \frac{(22-13)\times 2}{2\times 22 - 13 - 10} \\ &= 25.36\end{aligned}$$

So mode = 25.36 tons.

5.3.1 Graphical location of Mode

Mode from Histogram

Like median, mode can also be located using graph. For locating mode, we use the Histogram. The steps involved in obtaining mode from histogram are

Step 1: Draw a histogram to the given data.

Step 2: Locate modal class (highest bar of histogram).

Step 3: Join diagonally the upper end points of the highest bar to the end points of the adjacent bars.

Step 4: Mark the point of intersection of the diagonals.

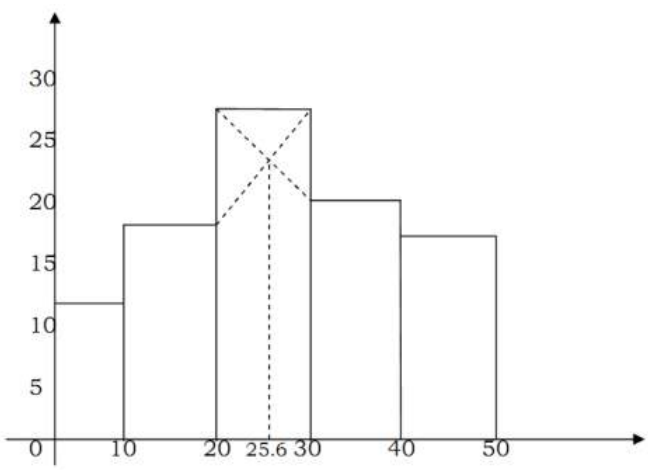
Step 5: Draw perpendicular from this point of intersection to the x-axis.

Step 6: The point where the perpendicular meets the x-axis gives the modal value.

Illustration 5.25

Determine the mode graphically using the data provided in Illustration 5.23

Solution. To locate mode, we have to first draw the histogram.



Mode = 25.6

5.3.2 Merits and demerits of Mode

Like the other measures of central tendency, mode has its own merits and demerits. Mode is widely used in industry.

Merits

1. Mode is easy to calculate and understand. It can sometimes located merely by inspection.
2. Mode is not affected by extreme values.
3. It can be determined for open end classes.
4. Mode is the only average that works with categorical data.

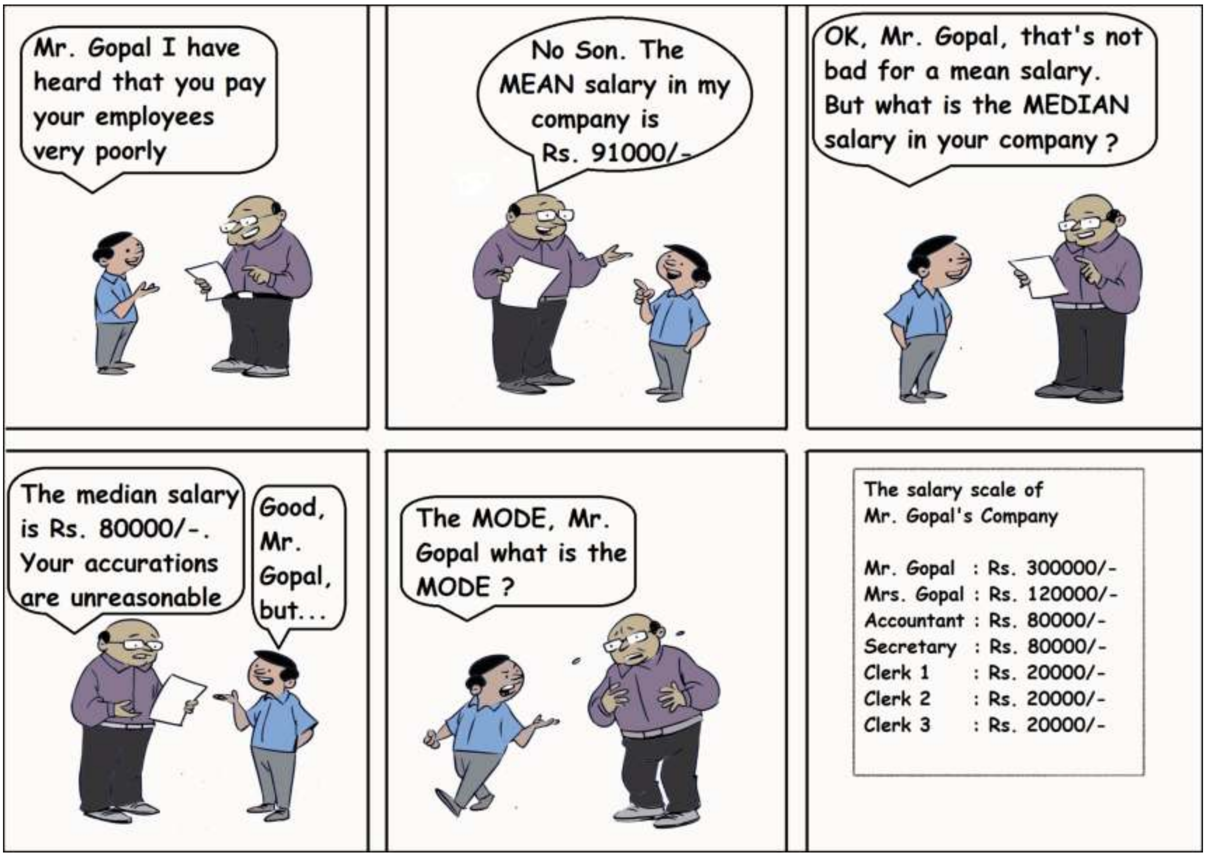
Demerits

1. Mode is sometimes ill-defined. It is not defined rigidly. There are distributions with no mode, one mode, two modes etc.
2. It is not based on all the observations.
3. It is not capable of further mathematical treatments.
4. It is affected by the fluctuations of sampling.

Comparitive table-Mean, Median and Mode

Sl No	Mean	Median	Mode
1.	Defined as the arithmetic average of all the observations.	Defined as the middle value in the data set arranged in ascending or descending order.	Defined as the most frequently occurring value in the data set.

2.	Depend on all the observations.	Doesn't depend on all the observations.	Doesn't depend on all the observations.
3.	Uniquely and comprehensively defined	Can't determine in all the conditions.	Not uniquely defined for multimodal situations.
4.	Affected by extreme values.	Not affected by extreme values.	Not affected by extreme values.
5.	Can be treated mathematically. ie, means of several groups can be combined.	Can't be treated mathematically. ie, medians of several groups can't be combined.	Can't be treated mathematically. ie, modes of several groups can't be combined.
6.	More useful when data is cardinal.	More useful when the data is ordinal.	More useful when the data is nominal.



Empirical relationship among Mean, Median and Mode

For a distribution the empirical relation among mean, median and mode, given by Karl Pearson is,

$$\text{Mean - Mode} = 3(\text{Mean-Median})$$

OR

$$\text{Mode} = 3\ \text{Median-2 Mean}$$

The importance of this relation is that we can estimate the value of any one of them by knowing the values of the other two. However, this relationship is true if the distribution is moderately asymmetric.

Illustration 5.26

From a partially destroyed data, it was obtained that the mode of the distribution is 63 and median is 77. Calculate the mean of the data.

Solution. We have the empirical relation,

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$
$$\text{Mean} - 63 = 3(\text{Mean} - 77)$$
$$2\text{Mean} = 168$$
$$\text{Mean} = 84$$

Know your progress

1. The ages of workers in a firm are, 40, 50, 30, 20, 25, 35, 30, 30, 20 and 30. Find the modal age.
2. A shoe shop has sold 100 pairs of shoes of a particular brand on a certain day with the following distribution. Find the mode of the distribution.

Size of shoe	:	4	5	6	7	8	9	10
No of pairs	:	10	15	20	35	16	3	1

3. The daily wage of workers in a factory are given below. Determine the modal wage.

Wages (Rs.)	:	200-250	250-300	300-350	350-400
Number of workers	:	21	29	19	39
	:	400-450	450-500	500-550	550-600
	:	43	94	73	68

4. Calculate the mode the following distribution

Production per day (in tons)	:	21-22	23-24	25-26	27-28	29-30
Number of days	:	7	13	22	10	8

5.4 Geometric Mean(GM)

We have already studied how the mean, median and mode can be used as tools for obtaining averages. Sometimes, when we are dealing with quantities that changed over a period of time, we need to know an average rate of change, the mean, median or mode is inappropriate, because it may give the wrong answer. Here we need the use of another average called Geometric Mean.

The Geometric mean of 'n' observations of a series is the n^{th} root of the product of the n observations. If there are two observations, then geometric mean is the square root of the product of the two observations. If there are three observations, geometric mean is the cube root of the product of the three observations, etc.

> The Geometric Mean is the n^{th} root of the product of n observations in the data set.

Geometric Mean of a raw data

Consider a raw data of 'n' observations $x_1, x_2, x_3, \ldots, x_n$. The geometric mean of the data is given by,

$$GM = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n}$$
$$= (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{\frac{1}{n}}$$

The geometric mean of n items $x_1, x_2, x_3, \ldots, x_n$ is given by

$$GM = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n} = (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{\frac{1}{n}}$$

Illustration 5.27

A textile has shows the following percentage increase in profit over the last 5 years.

year	:	*2008*	*2009*	*2010*	*2011*	*2012*
Percentage Increase	:	*5*	*10.5*	*9*	*6*	*7.5*

What is the average percentage of increase for the last five years.

Solution. Here the data gives the percentage increase of profits. The average income in 5 years is the GM of $105, 110.5, 109, 106$ and 107.5

$$\begin{aligned} GM &= \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n} \\ &= \sqrt[5]{105 \times 110.5 \times 109 \times 106 \times 107.5} \\ &= 107.58 \end{aligned}$$

So average percentage of increase is 7.58 %.

Illustration 5.28

A town's population is increased at a constant annual rate from 40000 in 2006 to 42436 in 2008.

(a) Find the annual percentage increase.

(b) Find the population size in 2007.

Solution. *Given,*

The population in 2006 = 40000 The population in 2008 = 42436

(a) Le x be the annual rate of increase. Then,

$$\begin{aligned} & 40000 \times (1 + \frac{x}{100}) \times (1 + \frac{x}{100}) = 42436 \\ \Rightarrow \quad & 40000 \times (1 + \frac{x}{100})^2 = 42436 \\ \Rightarrow \quad & (1 + \frac{x}{100}) = 1.03 \Rightarrow x = 3 \end{aligned}$$

Therefore, annual percentage rate of increase = 3%

(b) *Population in 2007 is the GM of 40000 and 42436.*
So Population in 2007 $= \sqrt{40000 \times 42436} = 41200$

OR

Population in $2007 = 40000 \times 1.03 = 41200$

Uses and limitations of Geometric mean

The following are some of the special uses of geometric mean.

1. It is useful for calculating the average percentage increase or decrease, ratios, etc.
2. In the construction of index numbers, geometric mean is considered to be the best average.
3. The importance of GM is that it gives less weightage to the extreme values. Hence the influence of very small and very large values is minimized. In other words, small values get more weightage and large values get less weightage.

Some of the limitations of geometric mean are,

1. If some observations are negative, the Geometric mean will not be calculated.
2. If any one or more observations are zero, the calculation of geometric mean become meaningless because the product of the observations will always zero and hence the GM will be zero.

5.5 Harmonic Mean (HM)

Consider two places A and B.The distance between the places A and B is $30\,km$. A man travelled from A to B on his car at a speed of $60\,Km/hr$ and returns at a speed of $40\,Km/hr$. What will be his average speed?

We know that speed is the ratio between distance travelled and the time taken to travel the distance (ie speed $=$ distance $\div$ time). We are given that

Distance	Speed	Time taken to travel the distance
30 Km	60 Km/hr	$\frac{30}{60} = 0.5\,\text{hr}$
30 Km	40 Km/hr	$\frac{30}{40} = 0.75\,\text{hr}$
Total - 60 Km		1.25 hr

$$\begin{aligned}\text{Average speed} &= \frac{\text{Distance}}{\text{Time}}\\ &= \frac{60}{1.25}\\ &= 48\,\text{Km/hr}\end{aligned}$$

Now, calculate the AM and GM of the speeds, we get

$$\begin{aligned}AM &= \frac{60+40}{2} = 50\,Km/hr\\ GM &= \sqrt{60\times 40} = 48.99\,Km/hr\end{aligned}$$

Clearly, here the AM and GM can't used to determine the average speed, because the average speed is 48 Km/hr. The average we used here to find the average speed is Harmonic Mean. The harmonic mean of 60 and 40 is the reciprocal of the mean of the reciprocals of the observations.

$$\begin{aligned}ieHM &= \frac{1}{\frac{1}{2}\left(\frac{1}{60}+\frac{1}{40}\right)}\\ &= \frac{2\times 60\times 40}{60+40}\\ &= 48\,Km/hr\end{aligned}$$

The Harmonic Mean of a number of observations is the reciprocal of the arithmetic mean of the reciprocals of the given observations.

Harmonic Mean of a raw data

Let $x_1, x_2, x_3, \ldots, x_n$ be the series of 'n' observations. The harmonic mean, HM of this series is given by

$$\begin{aligned}\frac{1}{HM} &= \frac{1}{n}\left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}\right)\\ &= \frac{1}{n}\sum\frac{1}{x}\end{aligned}$$

Therefore

$$HM = \frac{n}{\sum\frac{1}{x}}$$

The harmonic Mean of a raw data is given by

$$HM = \frac{n}{\sum\frac{1}{x}}$$

Illustration 5.29

In a cycle race competition, the speeds of 5 participants are 15 Km/hr, 18 Km/hr, 20 Km/hr, 22 Km/hr and 17 Km/hr. Find the average speed.

Solution. The average speed is best represented by the harmonic mean. The HM of the 5 observations is given by

$$\frac{1}{HM} = \frac{1}{n}\sum\frac{1}{x}$$

From the data given

$$\begin{aligned}\frac{1}{n}\sum\frac{1}{x} &= \frac{1}{5}\left(\frac{1}{15}+\frac{1}{18}+\frac{1}{20}+\frac{1}{22}+\frac{1}{17}\right)\\ &= 0.0553\end{aligned}$$

Therefore

$$HM = \frac{1}{0.0553} = 18.08$$

Average speed = 18.08 Km/hr

Uses and limitations of Harmonic Mean

Harmonic mean is specially used in the computation of average speed under various conditions. In the calculation of HM, smaller values get more weightage. Hence HM is suitable for a highly varying series.

The Harmonic Mean can't be determined if one of the values is zero.

Know your progress

1. Calculate the Geometric and Harmonic means of the following series of weekly expenditure of a batch of students. 125, 130, 75, 10 and 45.
2. Point out the mistake in the following statement, "A person goes x to y on cycle at 20 Km/hr and returns at 24 Km/hr. His average speed was 22 km/hr."
3. A train travels first 300 kilometers at an average speed of 40 Km/hr and further travels the same distance at an average speed of 30 Km/hr. What is the average speed?

Relation among Arithmetic Mean, Geometric Mean and Harmonic Mean

The AM, GM and HM are called the mathematical averages. The relations among them are,

1. $AM \geq GM \geq HM$, for a set of positive values

   When all the observations are the same, then $AM = GM = HM$.

2. $(GM)^2 = AM \times HM$,for two values

   ie, $GM = \sqrt{AM \times HM}$

Activity

For the observations $2, 4, 8, 12$ and 16, examine whether $AM \geq GM \geq HM$.

Illustration 5.30

The AM of two numbers is 10, the GM of those numbers is 8. Find the HM.

Solution. We have,

$$\begin{aligned}(GM)^2 &= AM \times HM \\ 8^2 &= 10 \times HM \\ HM &= \frac{64}{10} \\ &= 6.4\end{aligned}$$

5.6 Partition values- Quartiles, Deciles and Percentiles

Partition values are measures that divide the data into several equal parts. There are three types of partition values-Quartiles, Deciles and Percentiles. Quartiles divide the data into four equal parts, deciles divide the data into ten equal parts while percentiles divide the data into hundred equal parts. The method of computing partition values is similar to the method of computing median.

Quartiles

As mentioned earlier, quartiles are those values which divide a data into four equal parts. There are three quartiles, denoted by Q_1, Q_2 and Q_3. The first quartile Q_1 is the value for which 25% of the observations are below Q_1 and 75% are above it. The third quartile Q_3 is the value for which 75% of the observations are below Q_3 and 25% are above it. For Q_2, 50% are above Q_2 and 50% are below it. ie, Q_2 divides the data into two equal parts. Hence Q_2 is nothing but the Median.

Calculation of Quartiles

(i) Quartiles from a raw data

Let there be 'n' observations. Arrange them in ascending order of magnitude. Then,

$$Q_1 = \text{value of } \left(\frac{n+1}{4}\right)^{\text{th}} \text{ item in the series}$$

$$Q_3 = \text{value of } \left(\frac{3(n+1)}{4}\right)^{\text{th}} \text{ item in the series}$$

Illustration 5.31

The daily wages (in rupees) of 7 workers are 300, 350, 400, 425, 450, 500 and 600. Compute the first and third quartiles.

Solution. The wages in ascending order are, 300, 350, 400, 425, 450, 500, 600. Here $n = 7$

$$\frac{(n+1)}{4} = \frac{(7+1)}{4} = 2.$$

Therefore

$$Q_1 = 2^{\text{nd}} \text{ observation} = 350$$

Also

$$\frac{3(n+1)}{4} = 3 \times 2 = 6$$

Hence

$$Q_3 = 6^{\text{th}} \text{ observation} = 500$$

Illustration 5.32

Compute Q_1 and Q_3 for the given data 9, 13, 14, 7, 12, 17, 8, 10, 6, 15, 18, 21, 20

Solution. Arranging the given data in ascending order, we get 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 20, 21. Here, $n = 13$.

$$Q_1 = \text{Value of } \frac{n+1}{4}^{\text{th}} \text{ item}$$
$$= \text{Value of } \frac{13+1}{4}^{\text{th}} \text{ item, ie, } 3.5^{\text{th}} \text{ item}$$
$$= \text{Value of } 3^{\text{rd}} \text{ item } + 0.5(\text{Value of } 4^{\text{th}} \text{ item } - \text{Value of } 3^{\text{rd}} \text{ item})$$
$$= 8 + 0.5(9 - 8)$$
$$= 8.5$$
$$Q_3 = \text{Value of } \frac{3(n+1)}{4}^{\text{th}} \text{ item}$$
$$= \text{Value of } 3 \times 3.5^{\text{th}} \text{ item, ie, } 10.5^{\text{th}} \text{ item}$$
$$= \text{Value of } 10^{\text{th}} \text{ item} + 0.5 \times (\text{Value of } 11^{\text{th}} \text{ item} - \text{Value of } 10^{\text{th}} \text{ item})$$
$$= 17 + 0.5(18 - 17)$$
$$= 17.5$$

Hence $Q_1 = 8.5$ and $Q_3 = 17.5$

(ii) Quartiles from a discrete frequency distribution

For computing quartiles, prepare the less than frequency table. Let N be the total frequency. Then,

$$Q_1 = \text{observation having cumulative frequency } \frac{(N+1)}{4}$$
$$Q_3 = \text{observation having cumulative frequency } \frac{3(N+1)}{4}$$

Illustration 5.33

The heights in inches of 49 persons are given below

Height	:	*58*	*59*	*60*	*61*	*62*	*63*	*64*	*65*	*66*
No of persons	:	*2*	*3*	*6*	*15*	*10*	*5*	*4*	*3*	*1*

Calculate the first and third quartiles.

Solution. To find the quartiles, we have to prepare the cumulative frequency table

Height	Frequency	Cumulative frequency
58	2	2
59	3	5
60	6	11
61	15	26
62	10	36
63	5	41
64	4	45
65	3	48
66	1	49
	$N = 49$	

Here $N = 49$

$$Q_1 = \text{Observation having cumulative frequency } \frac{N+1}{4}$$
$$= \text{observation having cumulative frequency } 12.5$$
$$= 61$$
$$Q_3 = \text{Observation having cumulative frequency } \frac{3(N+1)}{4}$$
$$= \text{Observation having cumulative frequency } 37.5$$
$$= 63$$

Hence $Q_1 = 61$ and $Q_3 = 63$.

(iii) Quartiles from a continuous frequency distribution

Prepare the cumulative frequency table. Let N be the total frequency. Locate the classes having cumulative frequencies $\frac{N}{4}$ and $\frac{3N}{4}$. These classes are called the quartile classes. Then Q_1 and Q_3 are given by the formula,

$$Q_1 = l_1 + \frac{\left(\frac{N}{4} - m_1\right)c_1}{f_1}$$
$$Q_3 = l_3 + \frac{\left(\frac{3N}{4} - m_3\right)c_3}{f_3}$$

Where l_1 and l_3 are the lower bounds of quartile classes
f_1 and f_3 are the frequencies of the quartile classes
c_1 and c_3 are the class intervals of the quartile classes
m_1 and m_3 are the cumulative frequencies preceding
the quartile classes

Illustration 5.34

The marks of 80 students in an examination are given below. Calculate the lower and upper quartiles.

Marks	:	*0-10*	*10 -20*	*20-40*	*40-60*	*60-80*	*80-100*
No of students	:	*8*	*10*	*22*	*25*	*10*	*5*

Solution. Let us prepare the following cumulative frequency table

Marks	No of students f	Cumulative frequency
0-10	8	8
10-20	10	18
20-40	22	40
40-60	25	65
60-80	10	75
80-100	5	80
	$N = 80$	

Here $N = 80, \frac{N}{4} = 20$ and $\frac{3N}{4} = 60$. Therefore, the quartile classes are 20-40 and 40-60. Then

$$Q_1 = l_1 + \frac{\left(\frac{N}{4} - m_1\right)c_1}{f_1}$$

where $l_1 = 20, c_1 = 20, f_1 = 22$ and $m_1 = 18$. So

$$\begin{aligned} Q_1 &= 20 + \frac{(20-18)\times 20}{22} \\ &= 20 + \frac{2\times 20}{22} \\ &= 21.8 \end{aligned}$$

Also

$$Q_3 = l_3 + \frac{\left(\frac{3N}{4} - m_3\right)c_3}{f_3}$$

where $l_3 = 40, c_3 = 20, f_3 = 25$, and $m_3 = 40$. So

$$\begin{aligned} Q_3 &= 40 + \frac{(60-40)\times 20}{25} \\ &= 20 + \frac{20 \times 20}{25} \\ &= 56 \end{aligned}$$

Hence $Q_1 = 21.8$ and $Q_3 = 56$.

Deciles and Percentiles

Deciles are those values which divide a distribution into ten equal parts. There are 9 deciles.

Percentiles are those values which divide a distribution into hundred equal parts. There are 99 percentiles.

Median is the 5^{th} decile and 50^{th} percentile.

Know your progress

1. Compute the quartiles for following data 13, 14, 7, 12, 17, 8, 10, 6, 15, 18, 21, 20.
2. The following data relate to the sizes of shoes sold at a store during a given week. Find the upper and lower quartiles.

Size of shoes	:	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
Frequency	:	10	18	22	25	40	15	10	8	7

3. Compute Q_1 and Q_3 for the distribution

Marks	:	0-10	10-20	20-30	30-40	40-50
Number of students	:	3	10	17	7	6
	:	50-60	60-70	70-80		
	:	4	2	1		

5.7 Box plot

The **box plot** of a data is a graphical representation based on its quartiles, as well as its smallest and largest values. It attempts to provide a visual image of the data distribution. The box plot is also referred to as *Box and Whisker plot or Box and Whisker diagram*.

A box plot is a graph of a data set that consists of a line extending from the minimum value to the maximum value and a box with lines drawn at the first quartile Q_1, the median, and the third quartile Q_3. The lines extending from the box are called whiskers. The perpendicular line in the box is the median.

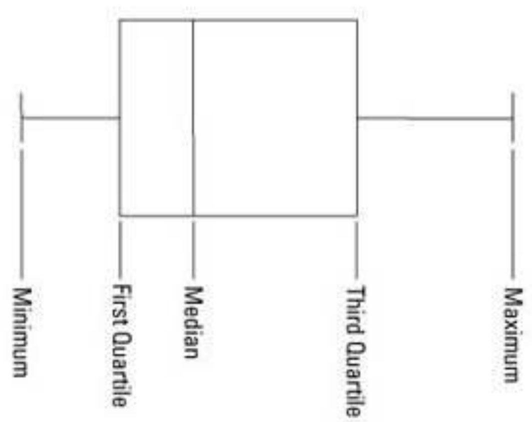


Illustration 5.35
Eleven secretaries were given a test and the scores obtained by them are as follows, 8, 7, 6, 9, 1, 3, 10, 3, 8, 4 and 7. Represent the data using a box plot.

Solution. Arranging the data in ascending order,

$$1, 3, 3, 4, 6, 7, 7, 8, 7, 9, 10$$

Number of observations, $n = 11$. To draw the box plot, we have to determine the following.

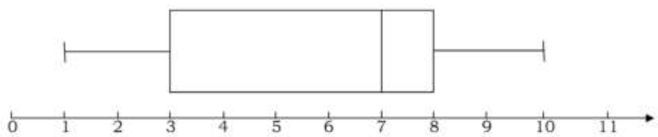
$$\text{Minimum value} = 1$$

$$Q_1 = \frac{n+1}{4}^{\text{th}} \text{ item}$$
$$= \frac{11+1}{4}^{\text{th}} \text{ item}$$
$$= 3^{\text{rd}} \text{ item}$$
$$= 3$$

$$\text{Median} = \frac{n+1}{2}^{\text{th}} \text{ item}$$
$$= 6^{\text{th}} \text{ item}$$
$$= 7$$

$$Q_3 = \frac{3(n+1)}{4}^{\text{th}} \text{ item}$$
$$= 9^{\text{th}} \text{ item}$$
$$= 8$$

$$\text{Maximum value} = 10$$



Let us sum up

Central tendency is the tendency of the observations in a data to concentrate around a central value. A measure of central tendency is a single value which is used to represent an entire set of data. In Statistics there are various measures of central tendencies. They are Arithmetic Mean, Median, mode, Geometric Mean and Harmonic Mean. The AM, GM and HM are also known as the mathematical averages and Median and Mode are known as the positional averages.

The AM of a set of observations is their sum divided by the number of observations. The weighted mean enables us to calculate an average value that takes into account the importance of each value with respect to the overall total. Median of a distribution is the value of the variable which divides it into two equal parts. Mode is the value that is represented most often in the data. Geometric mean is the nth root of the product of n items of a series. It is useful in calculating the average percentage increase or decrease. Harmonic mean of series is the reciprocal of the arithmetic mean of the reciprocals of the observations. It has a specific application in the computation of average speed under various conditions.

AM, GM and HM are mathematical in nature and measure the quantitative characteristics of the data. To measure the qualitative characteristics of the data the other measures, namely median and mode are used.

Partition values are measures which divide the data into several equal parts. Quartiles divide the data into four equal parts, deciles into ten equal parts and percentiles into hundred equal parts.

Learning outcomes

Check your understanding in the following

- Meaning of central tendency and various measures of central tendency.
- Arithmetic mean and calculation of arithmetic mean. Properties, merits and demerits of arithmetic mean.
- Median and calculation of median. Merits and demerits of median.
- Mode and calculation of mode. Merits and demerits of mode.
- Geometric mean-calculation, uses and limitations.
- Harmonic mean-calculation, uses and limitations.
- Partition values-Quartiles, deciles and percentiles.
- Box plot.

Evaluation items

1. The variable value in a series which divides the series into halves is called __________
2. The two ogives intersect at __________
3. Out of all measures of central tendency __________ is the only measure which is not unique.
4. Second quartile is same as __________

5. The Am and HM of a distribution are 10 and 8.1, then GM is (81, 0.81, 9, 2)

6. If each value of a series is multiplied by 10; the median is ________

   (a) not affected, (b) 10 times of the original median value,
   (c) one-tenth of the original median value, (d) increased by 10

7. The GM of two values can be calculated if

   (a) both the values are positive, (b) one of the two values is zero,
   (c) one of them is negative, (d) both of them are zero

8. Find the mean, median and mode

   (a) 12, 17, 17, 39, 41, 44, 54, 67, 82
   (b) 123, 115, 98, 107, 115, 109, 113, 98
   (c) 1.2, 1.4, 1.9, 2.0, 2.4, 3.5, 3.9, 4.3, 5.2

9. The weights of eight babies at birth in kgs are 2.4, 2.8, 3.2, 1.9, 2.7, 4.2, 3.8, 2.2. Find the average weight at birth

10. The frequency of reaction of an individual to a certain stimulant were measured by a psychologist are 0.53, 0.46, 0.5, 0.49, 0.52, 0.53, 0.44, 0.55. Determine the mean reaction time of the individual to the stimulant.

11. For a set of 8 scores, the mean is 5. If seven of these scores are 9, 3, 4, 5, 6, 4, 7, what must be the remaining score?

12. The average income of a person for the first five days of a week is Rs. 350 per day and if he works for the first six days in a week, his average income becomes Rs. 400 per day. Find his income for the sixth day.

13. The average age of 30 students in a class is 17 years. 4 students with an average age of 14.5 years left the class and 5 new students joined the class whose ages were 23, 25, 17.5, 19.5, and 21 years respectively. Calculate the average age of the class.

14. The average of 60 students in a class is 18 years. 7 students left the class, whose average age was 16.25 years and 5 new students joined the class whose ages were respectively 19, 18, 21.5, 23.75 and 14 years respectively. Find out the present average age of the students in the class

15. The mean of 100 items is 49. It was discovered that 3 items which should have been 60, 70 and 80 were wrongly taken as 16, 17 and 18. What is the corrected mean?

16. Ten coins were tossed together and the number of the resulting from them was observed. The operation was performed 1050 times and the frequencies obtained for different number of trials (x) are shown in the following table. Calculate the arithmetic mean

x	:	0	1	2	3	4	5	6	7	8	9	10
f	:	2	8	43	133	207	260	213	120	54	9	1

17. The following table gives the number of children of 150 families in a village.

No of children	:	0	1	2	3	4	5
No of families	:	10	21	55	42	15	7

Find the average number of children per family.

18. The arithmetic mean of the following distribution is 115.61, find out the missing frequency.

Values	:	110	112	113	117	120	125	128	130	Total
Frequency	:	25	?	13	?	14	8	6	2	100

19. The mean of the following series is 68.25, find out the missing value

Wages	:	50	58	60	65	70	–	80	100
Number of workers	:	2	20	5	35	8	10	16	4

20. The following is the data of weekly earnings of 200 persons. Calculate the mean.

Weekly wages	:	1000-1200	1200-1400	1400-1600	1600-1800
Number of persons	:	3	21	35	57
	:	1800-2000	2000-2200	2200-2400	2400-2600
	:	40	24	14	6

21. An agency interviewed 200 persons as part of an opinion poll. The following distribution represents the age of the persons interviewed. Calculate the average age.

Age	:	80 -89	70 -79	60 -69	50 -59	40 -49	30 -39
Frequency	:	2	2	6	20	56	40
	:	20 -29	10 -19				
	:	42	32				

22. Calculate the average temperature from the following data

Temp $(^\circ C)$	:	-40 - -30	-30 - -20	-20 - -10	-10-0
Number of days	:	10	28	30	42
	:	0-10	10-20	20-30	
	:	65	180	10	

23. The following distribution is the ages of 360 patients getting medical treatment in a hospital on a day. Calculate the mean of the ages.

Age	:	10-20	20-30	30-40	40-50	50-60	60-70
No of patients	:	90	50	60	80	50	30

24. The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city. Find the average expenditure.

Expenditure	No of workers	Expenditure	No of workers
100 - 150	24	300 - 350	30
150 - 200	40	350 - 400	22
200 - 250	33	400 - 450	16
250 - 300	28	450 - 500	7

25. A group of 134 girls and 166 boys appeared for an English examination. The boys obtained a mean score of 68.5 and the mean score for all the students was 64.35. Calculate the mean score for the girls.

26. The mean height of 25 male workers in a factory is 161 cms and the mean height of 35 female workers in the same factory is 158 cms. Find the mean height all the workers in the factory.

27. The grades of a student in lab, lecture and recitation parts of a Physics course were 71, 78 and 89 respectively. a) If the weight accorded the grades are 2, 4 and 5 respectively, what is the appropriate average grade? b) What is the average grade if equal weights are assigned?

28. The mark of a student in Economics, Statistics and Commerce are 82, 68 and 89 respectively. If the weights accorded to these marks are 2, 3 and 5 respectively, then a) Which is the appropriate average? b) Calculate the value of the above average

29. Find the median of the observations 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced with 99 and 41 with 43 in the above data, find the new median.

30. The numbers 4, 7, 8, x+1, 2x - 3 , 15, 16, 20 are arranged in ascending order. Find the value of x if the median is 12.5

31. The median of the following observations arranged in ascending order 11, 12, 14, 18, x+2, x+4, 30, 32, 35, 41 is 24. Find x.

32. Find out the median.

Income:	1000	1500	3000	2000	2500	1800
No of persons:	24	26	16	20	6	30

33. Calculate the median.

Values:	1	2	3	4	5	6	7
Frequency:	6	8	10	14	13	9	4

34. The marks out of 60 obtained by 58 students in a certain examination are given below. Calculate the median

Marks	No of students	Marks	No of students
15 - 20	4	40 - 45	8
20 - 25	5	45 - 50	9
25 - 30	11	50 - 65	6
30 - 35	6	65 - 70	4
35 - 40	5		

35. Determine the median wage for the following distribution

Wages	200 - 300	300 - 400	400 - 500	500 - 600	600 - 700
No of labourers	3	5	20	10	5

36. Determine the median from the following distribution

Marks	No of students	Marks	No of students
45 - 50	10	20 - 25	31
40 - 45	15	15 - 20	24
35 - 40	26	10 - 15	15
30 - 35	30	5 - 10	10
25 - 30	42	0 - 5	5

37. The following frequency distribution represents the commission earned (in rupees) by 100 salesmen employed at different branches of a multilevel marketing company.

Commission	No of people	Commission	No of people
150 - 158	5	186 - 194	20
159 - 167	16	195 - 203	15
168 - 176	20	204 - 212	3
177 - 185	21		

Find out the median.

38. Car batteries have a nominal voltage of 12 volts. A manufacturer tested a sample of 160 batteries and obtained the following results.

Voltage	11.8-12.0	12.1-12.3	12.4-12.6	12.7-13.1	13.2-13.4	13.5-13.7
No of batteries	18	26	12	78	18	8

Calculate the median voltage.

39. The following data represent the expenditure (in million of rupees) of 45 municipal corporations. Find the median.

Class	10-20	21-31	32-42	43-53	54-64	65-75
Frequency	2	8	15	7	10	3

40. Obtain the missing frequency of the following distribution where the value of the median is 86.

Class	40-50	50-60	60-70	70-80	80-90	90-100	100-110
Frequency	2	1	6	6	f	12	5

41. Which measure is the most helpful to a shoe maker?

42. The following data gives the sizes of 15 shoes sold at a shop on a particular day 5, 7, 9, 9, 8, 5, 6, 8, 7, 7, 7, 9, 2, 7. Estimate the modal size.

43. The table shows the scores obtained when a die is thrown 60 times. Calculate the mode.

Score :	1	2	3	4	5	6
Frequency:	12	9	8	13	9	9

44. Find the mode of the following distribution

Shoe size :	2	3	4	5	6
Frequency :	8	15	23	20	14

45. The table below shows the number of men in various age groups with some form of paid employment in a village. Find the modal age.

Age	14-20	20-30	30-40	40-50	50-60	60-70	70-90
Frequency	12	14	26	35	23	5	1

46. Calculate the mode of the following distribution.

Age	20-24	25-29	30-34	35-39	40-44	45-49
Frequency	20	24	32	28	20	26

47. The live span of 80 electric bulbs were recorded in hours to the nearest hour and grouped as shown below. Find the mode.

Life (in hours)	660-669	670-679	680-689	690-699	700-709	710-719	720-729	730-739
No ob Bulbs	4	5	12	24	15	10	7	3

48. The frequency table below illustrates the heights of the 87 trees in a public park.

Height (M)	2-5	6-9	10-13	14-17	18-21	22-25	26-29
No of Trees	4	6	14	26	25	7	3

Calculate the mean, median and mode of the heights.

49. A variable has nine values, 4, 11, 25, 37, 11, 26, 35, 11 and p.

a) (i) Which measure of central tendency can be found without knowing the value of p?

(ii) Find out the value of that measure.

b) It is known that p is more than 30, then

(i) Which other measure of central tendency can now be found?

(ii) Find out that measure.

c) If the remaining measure of central tendency has the value 22, find the value of p.

50. Each of the sentences below uses the word average. For each sentence indicate which measure of central tendency the word average refers to.
a) The average car being driven in India is white.
b) The average number of children in 8 classes of a primary school is 32.25.
c) Half number of students secured less than the average marks in an exam.

51. The set of integers p, 13, 18, 29 and 29 has mode 29, mean 20 and median 18. Without calculating the value of p, find out the mean, median and mode of the following distributions
a) p+2, 15, 20, 31, 31.
b) p - 5 , 8, 13, 24, 24
c) 2p, 26, 36, 58, 58
d) p/2 , 6.5, 9, 14.5, 14.5

52. For a set of observations, mode is twice the mean. If median is 23, find out the mean.

53. The value of mean and mode of a frequency distribution are 50 and 45 respectively. Find the median.

54. The average monthly salary of 100 employees of a factory is Rs.4500. If the median salary is found to be Rupees 4900/- per month, find out the appropriate modal value.

55. Find the GM of 4.2 and 16.8.

56. Determine the GM of 4 and 36.

57. Find the GM of 8, 16 and 62.5 .

58. Find the GM of 18, 16, 22, 12

59. 'Kapil & Sons' have shown the following percentage of increase in their business over the last 5 years:

year :	2008	2009	2010	2011	2012
Increase :	7%	8%	10%	12%	18%

Find the average increase in percentage.

60. Find the HM of
a) 2, 3, 6
b) 3.2, 5.2, 4.8, 6.1, 4.2

61. A motor car covered a distance of 50 km in 4 phases, the first phase at 50 k.p.h, the second phase at 20 k.p.h, the third and fourth at 40 k.p.h and 25 k.p.h.respectively. Calculate the average speed.

62. A cyclist travels from his house to school at a speed of 10 km/hr and returns to house at 14 km/hr. Which is the most suitable average to find the average speed? Calculate it.

63. Find the average speed of an aeroplane which flies among the four sides of a square at a speed 100, 200, 300 and 400 km/hr respectively.

64. For the values 8, 6, 4 and 3, prove that $AM>GM>HM$.

65. The following are the scores obtained by 9 students in a class test
38, 7, 43, 25, 20, 15, 12, 18, 11
Calculate the quartiles.

66. Calculate the quartiles.

Marks :	20	30	40	50	60
No of students:	4	16	20	18	11

67. Find the first and third quartiles for the distribution,

Height (In Inches) :	58	59	60	61	62	63	64	65	66
No of person :	2	3	6	15	10	5	4	3	1

68. Calculate the quartiles from the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Students	5	7	8	12	28	22	10	8

69. Find the quartiles of the following distribution.

Scores	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No of Students	1	3	11	21	43	32	9

70. Draw a box plot for the following data.
13, 14, 7, 12, 17, 8, 10, 6, 15, 18, 21, 20.
$(Q_1 = 8.5, Q_3 = 17.5, Q_2 = 13)$

Answers:

1) Median	2) Median	3) Mode
4) Median	5)9	6) b
7) a	8) (a)41.44, 41, 17	8)(b) 109.75, 111, 98 and 115
8)(c)2.87, 2.4, 1.46	9)2.9 Kg.	10) 0.5025
11)2	12) 650	13)18
14) 18.32	15)50.69	16)5.01
17) 2.35	18) 22,10	19)75
20) 1768	21)35.8	22)4.29
23)39.84	24) 266.25	25)59.21

26) 159.25	27) (a). 81.73	27) (b). 79.33
28) (a). Weighted AM,	28 (b). 81.30	29) 58,58
30)9	31) 21	32)1800
33) 4	34)38	35) 467.5
36)27.74	37) 180.36	38)12.80
39) 40.67	40)10	41) Mode
42) Mode=7	43)4	44) 4
45)44.29	46) 32.83	47)695.21
48) Mean=15.64,	48) Median=16	48) Mode=17.19
49) (a) i) Mode,	49) (a) ii) 11	49) (b) i) Median,
49) (b) ii) 25	49) (c) p = 38	50) (a) Mode
50) (b) Mean	50)(c) Median	51) (a) Mean = 22
Median = 20	Mode = 31	51) (b) Mean = 15,
Median = 13,	Mode = 24	51) (c) Mean = 40
Median = 36	Mode = 58	51) (d) Mean = 10,
Median = 9,	Mode = 14.5	52)17.25
53)48.33	54)5700	55)8.40
56)12	57)20	58)16.61
59)10.39%	60) (a) 3	60)(b) 4.484
61)29.63	62)11.67	63)192
64) AM = 5.25	GM = 4.9	HM = 4.57
63) $AM>GM>HM$	65) Q1 = 11.5,	Q2 = 18,
Q3 = 31.5	66) Q1 = 30	Q2 = 40,
Q3 = 50	67) Q1 = 61,	Q3 = 63
68) Q1 = 34.17	Q2 = 46.43,	Q3 = 56.82
69) Q1 = 66.64	Q2 = 75.08	Q3 = 82.94