10 Sampling Techniques

Introduction

In chapter 2, we discussed census and sample survey. The term population is used to mean the totality of items in an investigation. The population may be finite or infinite. If each and every unit in the population is considered in the enquiry, it is called census or complete enumeration. As the population in most enquires are quite large, complete enumeration is not practical or feasible. In such cases, a representative part of the population is taken into consideration. This method is called sampling. Each unit in the population is called a sampling unit. The list of all sampling units of the population is called sampling frame. It is basically, a list from which you can choose your sample

Activity

Look at the following situations. What is your population? What is a sampling unit? How would you develop a sampling frame?

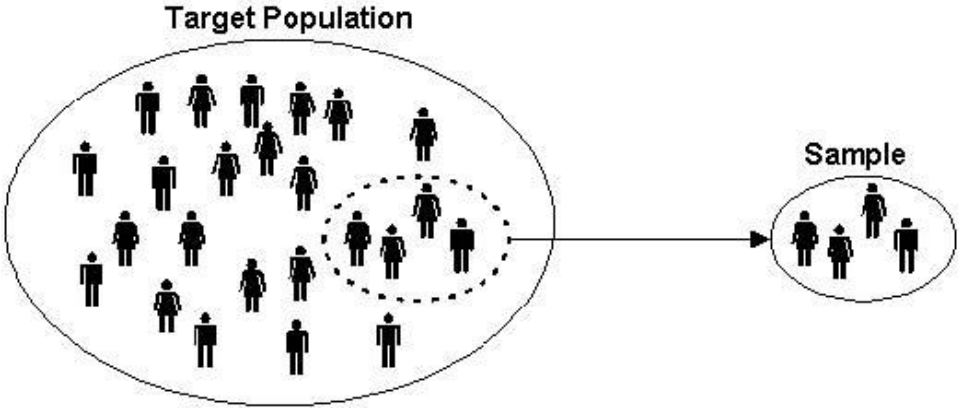
- Monalisa health club wants to conduct a survey to see the facilities that customers expect?
- A chocolate manufacturing company has introduced a new variety of chocolates. They want to check the quality of those chocolates.

Census Survey

The traditional method of acquiring knowledge about an aggregate of individuals is to enumerate them all. Census of population of a country and study of total agricultural production of a country are examples of complete enumeration. The major statistics of agriculture production, distribution of labour force and unemployment have all been based on census approach.

Surveying male smokers between 20 and 40 age who had surgery for throat cancer in a particular hospital during last year, surveying number of printing mistakes in a book, surveying the number of accidents reported in a locality etc. are also examples for census study.

Figure 10.1:



A pioneer in census taking was Pierre-Simon de Laplace. In 1780, he developed the Laplace method of estimating the population of a country. The principle behind his method was to take a census of a few selected communities and determine the ratio of the population to the number of births in these communities. This number would be used to multiply the number of births in the entire country to estimate the number of citizens in the country.

Know your progress

From chapter 2, recall the situations where census and sampling can be implemented.

10.1 Need and Importance of Sampling

In homes, we take out one or two rice grains (any other food item) from the cooking pan to examine whether the grains are fully cooked or not. In clinical laboratory, a few drops of blood are collected to test to know whether the blood has any abnormalities or not.

Whatever observed in the few drops of blood is true for whole blood of the body. In a bulb manufacturing company, one tests the life of few bulbs and comes to a conclusion about the average life of bulbs in the whole lot. Similarly, instead of examining the whole population, which may be difficult or impossible to do, one can examine a representative 3 part of the population. This is called sample. The process of drawing samples is called sampling. These examples reveal that sampling is an age old practice. Now-a-days, sampling methods are extensively used in socio-economic surveys to know the living condition, cost of living index etc of a class of people.

Let us consider the studies to get the following information

- The percentage of Keralites who have access to internet
- Opinion about selection of places for a study tour with the students of your class.
- Any opinion poll on various political leaders of Kerala

In first and third case, it is obvious that interviewing more than 3 crore people is not possible. The process is costly, time consuming and requires a lot of trained investigators. Due to these reasons it is convenient to use sampling method. But in second case, data may be collected through census

10.2 Census and Sampling - Advantages and Disadvantages

Table 10.1: Census:- Advantages and Disadvantages

Advantages	Disadvantages
(i) 100% perfect.	(i) If the population is infinite, the study is impossible.
(ii) For an enquiry, if all the units in the population are to be inspected, census is the only method	(ii) Census method requires more time, money, trained persons etc
(iii) The data obtained by census method may be used for further investigations	(iii) If the units are destroyed in the course of inspection, census is not at all desirable.

Table 10.2: Sampling:- Advantages and disadvantages

Advantages	Disadvantages
(i) Saves time and money	(i) Improper sampling technique may lead to misleading results
(ii) When population is too large or items are destructive in nature, sampling alone can be adopted (eg. Testing the strength of chalks)	(ii) If information is required from each and every unit of population, sampling is inadequate (eg: Leprosy test in school children)
(iii) More accuracy is expected	
(iv) The expected error also can be estimated	

Activity

Find out situations where sampling is preferred to census and vice versa

Errors in survey

The results obtained from statistical studies may not be free from errors. The errors involved in various stages of the study of data may be broadly classified under two heads viz (i) sampling error and (ii) non-sampling error.

10.3 Sampling and Non-Sampling Errors

Sampling errors are seen in sample surveys due to the fact that only a part of the population is used for enquiry. Clearly, sampling errors are absent in census. For example, the estimate of average income of people in certain region obtained on the basis of sampling will not be equal to the true average income.

Sampling errors cannot be completely eliminated but may be minimized by choosing a proper sample of adequate size and a proper sample survey design. Sampling error decreases as sample size increases. Sampling errors can be detected, measured and controlled. Sampling errors arise due to the following

(i) Lack of clarity about the coverage of the population

(ii) Faulty selection of the sample

(iii) Inadequate sample size

(iv) Inappropriate questionnaire

(v) Errors due to substitution

Errors other than sampling errors in a survey are called non-sampling errors. Non sampling errors arise at various stage of observation and processing of data, presentation and printing of tabulated results and are thus present in both census and sampling. Thus the data obtained in complete enumeration, although free from sampling errors would still be subject to non sampling errors. Data obtained in a sample survey would be subject to both sampling and non sampling errors. These errors can be minimized by choosing proper sampling designs, employing efficient investigators and better sampling. Non sampling errors usually increase with increase in sample size. Non sampling errors may arise due to

1. Irrelevant responses to questions
2. Errors in printing and publication of results
3. Errors in data processing

Know your progress

Distinguish between sampling and non sampling errors

Methods of Sampling

Consider a study about the spending habits of students in a class of 60. A sample of 10 students is selected for the study. The investigator can select the sample according to convenience. This type of samping is termed as **Non Probability Sampling**. On the other hand the investigator can select the sample randomly, in which each member of the population has some specified probability of being included in the sample. This method is termed as **Probability Sampling**.

10.4 Non Probability Sampling

In non probability sampling, members are selected from the population in some non- random manner. These include convenience sampling, judgment sampling and quota sampling. These methods are subjective.

Convenience Sampling

A convenience sample is obtained by selecting convenient population units. A sample obtained from readily available lists such as automobile registrations, telephone directories, etc. is a convenience sample, if the sample is drawn according to the convenience of the investigator. The results obtained by this method will not be a representative of the population. This method is very popular in online research and Traditional ″man on the street″ interviews conducted frequently by the visual media.

Judgment Sampling

The investigator exercises his judgment in the choice and includes those items in the sample which he thinks are most typical of the universe with regard to the characteristics under investigation. For example, if a sample of ten students is to be selected from a class of sixty for analyzing the spending habits of students, the investigator would select 10 students who, in his opinion, are

representative of the class. In this sampling the sample is selected with definite purpose in view. If the investigator is experienced and skilled and the sampling carefully applied, then the judgment samples may yield valuable results. This method is also very useful when you need to reach a targeted sample quickly.

Quota Sampling

In this method quotas are set up according to some specified characteristics such as several income groups. Within the quota the selection of sample items depends on personal judgment. For example in an income survey, the interviewers may be told to interview 100 people living in certain area in which 60 are housewives, 25 are regular employees and 15 are businessmen. Within these quotas the interviewer is free to select the people to be interviewed. This method often used in public opinion studies and personal interviews and people are systematically according to some fixed quota.

Activity

Find out similar situations where non probability sampling is appropriate

10.5 Probability Sampling

Probability sampling is the scientific method of selecting samples according to some laws of chance in which each unit in the population has some definite pre-assigned probability of being included in the sample. This method is purely objective.

Different types of probability sampling includes

1. Simple Random Sampling
2. Systematic Sampling
3. Stratified Random Sampling
4. Cluster Sampling

5. Multistage Sampling

There are many situations which demanded probability sampling or non probability sampling or a combination of both. Search them out.

10.6 Simple Random Sampling (SRS)

Simple Random Sampling is a probability sampling in which each unit in the population has an equal chance of being included in the sample. In this case the sampling units are selected at random. Simple random sampling overcome the drawbacks of non-probability sampling viz favouritism, subjectiveness etc. This method is applicable when population is homogeneous.

There are two types of Simple Random Sampling - Simple Random Sampling Without Replacement (SRSWOR) and Simple Random Sampling With Replacement (SRSWR). Suppose you are going to buy orange from a fruit shop. You are selecting five oranges one by one from a basket of oranges without replacing the selected ones. This type of sampling in which all units have an equal chance of being included in the sample is called as simple random sampling without replacement. If the sampling is done by replacing the selected unit it is called simple random sampling with replacement. If a population consists of N units and a sample of n units to be taken, the possible number of samples in SRSWOR is ${}^{N}C_{n}$ and in SRSWR is N^{n}.

Illustration 10.1

If a population consists of 5 numbers 2,3,6,8 and 11

Consider all simple random samples of size 2 that can be drawn

1. *with replacement*
2. *without replacement*

Samples: Using SRSWOR

(2,3), (2,6), (2,8), (2,11), (3,6), (3,8), (3,11), (6,8), (6,11), (8,11)

${}^5C_2 = 10$ *samples*

Samples : Using SRSWR

(2,2), (2,3), (2,6), (2,8), (2,11), (3,2), (3,3), (3,6), (3,8),(3,11), (6,2), (6,3), (6,6), (6,8), (6,11), (8,2), (8,3), (8,6), (8,8), (8,11), (11,2), (11,3), (11,6), (11,8), (11,11)

$5^2 = 25$ *samples*

Activity

Solve the following questions.

1. Suppose we have 5 cards numbered from 1 to 5 and two cards are to be selected, write all possible samples using (i) SRSWOR and (ii) SRSWR
2. A bag contains 10 balls, how many samples of size 3 can be taken in (i) SRSWOR and (ii) SRSWR.

Methods of Sample Selection - SRS

Random samples can be obtained by any of the following methods

(i) Lottery Method

(ii) Random Number Table Method

Lottery Method

The Simplest method of selecting a simple random sample is the lottery method. Suppose we want to select n candidates out of N. We assign the numbers serially starting from 1 to N. Write these numbers (1 to N) on N slips. These slips are made as homogeneous as possible in shape, size, colour etc. These slips are folded and put in a bag and shuffled thoroughly and then n slips are drawn one by one. The n candidates corresponding to the numbers on the selected slips will constitute a random sample. For example, suppose we have to select five students out of 50 to visit an old age home. We assign numbers from 1 to 50 to the students. 50 identical slips are made for these students. These slips folded and put in a box and shuffle thoroughly. Then five slips are

drawn. Suppose the numbers drawn are 44, 6, 28, 39 and 25. Then the students bearing these numbers are selected for visiting the home.

Random Number Table Method

The limitation of lottery method is that it is quite time consuming if the population is large. The most practical and inexpensive method of selecting a random sample consists of the use of Random Number Tables. The random number table are in such a way constructed that each of the digits 0,1,2,3,4,5,6,7,8,9 appears approximately in the same frequency. The digits are also independent. The method of drawing a random sample by this method consists of the following steps.

Let N be the population Size with k digits and n be the Sample Size to be drawn.

- Identify the N units in the population with the numbers from 1 to N
- Select at random, any page of the table and pickup the successive k digit numbers in any row or column or diagonal at random until we get n number of units.
- Discard numbers which are greater than N.
- The population units corresponding to the numbers selected constitute the random sample

Commonly used Random Number Tables are Tippetts Random Number Table, Fisher and Yates Table, Kendall and Babington Smith Table, Rand Corporation table, C.R. Rao, Mitra and Mathai Table. Random Number Generating Programmes are available in Internet, Computer and Calculator.

Know your progress

Explain the selection procedure of a sample of 20 units from a population containing 80 units using random number table.

Activity

Create similar situations in your class where lottery and random number table method can be applied and solve the same using these methods.

10.7 Systematic Sampling

A sampling method in which one unit is selected at random and the remaining units are selected at an interval of predetermined length is called systematic sampling.

Suppose we want to select a systematic sample of 8 units out of 48 units. To do this we first find the sampling interval $k = \frac{48}{8} = 6$. The first unit in the sample is selected by a random number r between 1 and 6. Let it be 3. Then the third unit will be selected to the sample. There after every sixth unit will be selected automatically into the sample. Hence the resulting systematic sample will contain the units with the following serial numbers

3, 9, 15, 21, 27, 33, 39,45

If the population contains 48 items and a sample of 8 items is to be taken, the selection of every 6^{th} $(48 \div 8)^{th}$ item will give the required sample. The first entry (random start) is determined by selecting a number at random between 1 and 6. If the first item obtained in this manner is 3^{rd} then 9^{th}, 15^{th}, 21^{st}, 27^{th}, 33^{rd}, 39^{th} and 45^{th} items will be picked up. This type of sampling in which n samples are taken out of N units and $k = \frac{N}{n}$. A random start is selected from 1 to k. Let it be i, where $1 \leq i \leq k$ then i^{th} , $(i+k)^{th}$,$(i+2k)^{th}$,... comprising of n items are included in the sample with sampling interval as k. This type of sampling is called Systematic Sampling. These type of sampling is done when a complete list of the population is available. In the above example, $k = 6$, the sampling interval and $i = 3$ is the random start. The pictorial representation of above example is given below

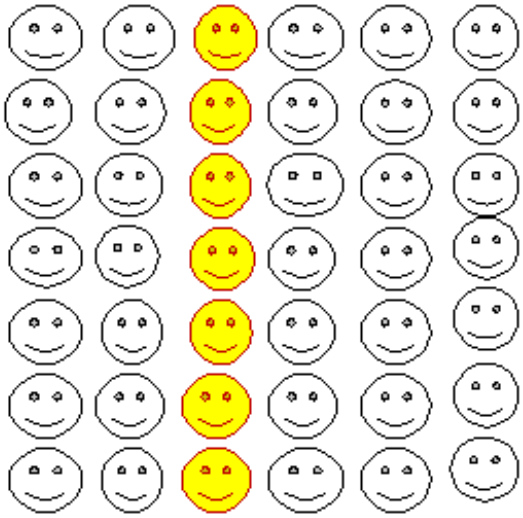


Illustration 10.2

In a class there are 100 students with Roll Numbers from 1 to 100. It is desired to take sample of 10 students. $K = \frac{100}{10} = 10$. *From 1 to 100 roll numbers, the first student between 1 and* k *ie., 1 and 10, will be selected at random and then we will go on taking every* k^{th} *student. Suppose the first student comes out to be* 4^{th}*, the sample would then consist of the following Roll Numbers. 4, 14, 24, 34, 44, 54, 64, 74, 84 and 94.*

10.8 Stratified Random Sampling

Simple Random Sampling is suitable for homogeneous population. When the population is heterogeneous, it is first subdivided into non overlapping exhaustive homogeneous subgroups. These subgroups are called strata . From each stratum, units are selected at random. The number of items taken from each subgroup may be in proportion to its size. This type of sampling is called stratified random sampling. This method is applied so that units within each group are as homogeneous as possible and the group means are as widely different as possible.

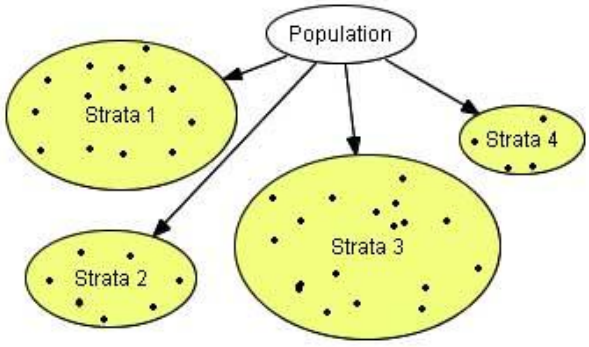


Illustration 10.3

Consider a population which consists of males and females who are smokers or non smokers. The researcher wants to include in the sample, people from all groups-that is, males who smoke, males who do not smoke, female who smoke and female who do not smoke. To accomplish their selection, the researcher divides the population into four subgroups and then selects a random sample from each sub group. This method ensures that the sample is representative on the basis of the characteristics of gender and smoking.

10.9 Cluster Sampling

If we are interested in obtaining the income data in a city, the whole city may be divided into different blocks (clusters) and a Simple Random Sample of required number of blocks is drawn. The individuals of these selected blocks constitute the Cluster Sample. In Cluster Sampling, the total population is divided into some recognizable subdivisions which are termed as clusters and a Simple Random Sample of these clusters is drawn. These clusters are examined completely. This sampling procedure is called Cluster Sampling.

Illustration 10.4

Population	*All school students in the District*
Clusters	*Each school in the district*
Obtain SRS of clusters	*Four schools from the district*
Sample	*Every student in the four schools*

10.10 Multi-Stage Sampling

Selection of a sample of households from a particular State can be done through different stages. The first stage units may be districts, second stage units may be villages in the selected districts and third stage units may be households in the Villages, which are the ultimate units.

State
↓
Districts (First stage)
↓
Villages (Second stage)
↓
Households (Third stage)

Such type of sampling is called Multi-stage Sampling. As the name indicates, multistage sampling refers to a sampling technique which is carried out in various stages. Multi stage sampling consists of sampling first stage units by

some suitable method of sampling. From among the selected first stage units, a sub sample of secondary stage units is drawn by some suitable method of sampling which may be same as or different from the method used in selecting first stage units. Further stages may be added to arrive at a sample of desired sampling units. If the sampling is done only in two stages, it is called Sub-sampling.

Illustration 10.5

Suppose we have to study about the behaviour pattern of marketing of a product in households of a country. Divide the country into different States, States are divided into districts. Districts are divided into cities and towns. These are again divided into wards, and households are selected and study about the marketing of product.

Illustration 10.6

First Stage sampling unit for national surveys are often administrative districts, urban districts or parliamentary constituencies. Within the selected first stage unit one may go direct to the final sampling units, such as individuals, households or addresses, in which case we have a two-stage sample. It would be more usual to introduce intermediate sampling stages, i.e. administrative districts are sub-divided into wards, then polling districts etc.

Probability Sampling Methods and Strategies

Type of Sampling	Selection Strategy
Simple	Each member of the population has an equal probability of being selected.
Systematic	Each member of the population is either assembled or listed, a random start is designated, then members of the population are selected at equal intervals
Stratified	Each member of the population is assigned to a group or stratum, then a simple random sample is selected from each stratum
Cluster	Each member of the population is assigned to a group or cluster, then clusters are selected at random and all members of each selected cluster are included in the sample.
Two stage Sampling	Each member of the population is divided into sub groups, a sample of these groups are selected at random and then a sample of members of each selected subgroups are included in the sample
Multistage sampling	The above stage is extended to multi-levels

Let us sum up

Population is the collection of all observations about which conclusions are to be made. Sample is a part of population. While collecting and processing the data, there may arise two types of errors- sampling error and non-sampling error. Sampling can be done by using non probability or probability sampling. Some of the methods of non probability sampling are convenience sampling, judgment sampling and quota sampling. Some of the methods of probability sampling are simple random sampling - with or without replacement, stratified sampling, systematic sampling, cluster sampling and multistage sampling.

Learning outcomes

After transaction of this unit, the learner:-

- illustrates Census and sampling and their advantages and disadvantages.
- recognises probability and non probability sampling.
- identifies sampling and non sampling errors.
- differentiates SRSWOR and SRSWR, methods of simple random sampling - lottery method and random number table method.
- describes different kinds of sampling - simple random sampling, systematic sampling, stratified random sampling, cluster sampling and multistage sampling.

Evaluation Items

1. Census study involves ———

   (a) 50% subjects of the population

   (b) Each and every subject comprising the population

   (c) Any Number of subjects

   (d) None of the above

2. If a doctor wants to assess the efficacy of a drug on the patients of gastroentitis, then which sampling procedure should he follow?

   (a) Simple random sampling with replacement

   (b) Simple random sampling without replacement

   (c) Judgment sampling

   (d) None of the above

3. From a well shuffled pack of cards, a card is drawn blindly. Its colour is noted and replaced. This process is continued 5 times. This type of sampling is known as

   (a) Sampling with replacement

   (b) Sampling without replacement

   (c) Convenience sampling

(d) Non random sampling

4. Sample study is inevitable because:

   (a) It is not possible to study an infinite population

   (b) It is not possible to test all units of the population if they are perished under observation

   (c) A population study requires too much time and rewources

   (d) All the above

5. There are more chances of Non sampling errors than sampling errors in case of

   (a) Studies of large sample

   (b) Inefficient investigators

   (c) Complete enumeration

   (d) All the above

6. Which type of sampling technique is used in following situations

   (a) Trees in a forest

   (b) Houses in blocks

   (c) Entries in a register which are in serial order

7. Which of the following sampling designs will be categorized as non probability sampling?

   (a) Quota sampling

   (b) Convenience sampling

   (c) Judgment sampling

   (d) All the above

8. Errors other than sampling errors are termed as —

9. The sampling procedure in which population is first divided into homogeneous groups and then a sample is drawn from each group is called—

10. Stratification is appropriate when population is —

11. A sample consists of— of population

12. If a10Cn investigator selects districts from a State, Panchayat Samities from districts and farmers from Panchayat Samities, then such a sampling procedure is known as—

13. Suppose there are 10 students in your class. You want to select 3 out of them. How many distinct samples are possible?

14. Discuss how you would you use the lottery method to select 3 students out of 25 in your class using simple random sampling with replacement and without replacement

15. Explain the procedure for selecting a random sample of 10 students out of 60 in your group by using random number tables

16. Do the errors in sample studies are always greater than that of complete enumeration? Justify your answer

17. A population consists of four numbers 3,7,11 and 15. consider all simple random samples of size 2 that can be drawn (i) with replacement & (ii) without replacement from this population

18. Suggest three situations when sampling is more suitable than census

19. Distinguish between sampling errors and Non sampling errors

20. If a survey is conducted to estimate the crop production in villages and on farms, which type of sampling is preferred?

21. For each of the following sampling plans, suggest methods of sampling plans

(a) A librarian wants to estimate the proportion of the damaged books in the library. He decide to select a book per shelf as sample by measuring 12 inches from the left edge of each shelf and selecting the book in that location

(b) Political surveyors visit 200 houses to collect the details of eligible voters in each house whom they intend to vote for

22. Give three situations where non sampling errors arise.

Answers:

1) b 2) c 3) a 4) d 5) d

6 (a) systematic sampling (b) cluster sampling (c) systematic sampling

7) d 8) non sampling errors 9) stratified random sampling 10) hetrogeneous

11) representative part 12) multi-stage sampling 13 $10C_3$

Glossary

Arithmetic Mean	:	The sum of observations divided by the number of observations.
Bar Diagram	:	Diagrammatic representation of data using bars proportional to the frequencies
Bivariate Frequency Table	:	Frequency distribution of a bivariate data in rows and columns
Box plot	:	Box plot is the graphical representation of data based on its quartiles. It is also known as box and whisker plot.
Census	:	Data collected from each and every unit of the populations
Central Tendency	:	Tendency of the observations in a data to cluster around a central value is called central tendency.
Chronological Classification	:	Classification based on time.
Classification	:	Arrangement of items according to some attributes
Cluster Sampling	:	Choosing a cluster of items as a unit.
Coefficient of QD	:	Relative measure of dispersion based on Quartiles.
Coefficient of variation	:	Relative measure of dispersion based on standard deviation.
Combined mean	:	The mean of a combined group two or more sets taken together.
Conditional probability	:	Probability of an event conditioned by another event

Continuous variables	:	Variables take any values within a specified range.
Covariance	:	It indicates strength of linear relationship between two variables.
CSO	:	Central Statistical Office
Cumulative Frequency	:	Number of observations less than or greater than a particular value.
Cumulative Frequency Table	:	Tabular representation of cumulative frequencies
Data	:	Any measurement, result, fact or observation which gives information.
Deciles	:	The values of a data which divide the distribution into ten equal parts are called deciles.
Dichotomy	:	Classification into two disjoint groups
Discrete variable	:	Variables take countable number of values.
Enumeration	:	The process of data collection by enumerator.
Enumerator	:	The person deputed by the investigator to collect data from field.
Equally likely events	:	Two or more events having an equal chance of occurrence.
Event	:	Subset of a sample space.
Frequency	:	Number of repetitions of an observation
Frequency curve	:	Joining the points of a frequency polygon by a freehand smoothed curve
Frequency polygon	:	Graphical device for understanding the shapes of distributions.
Frequency Table	:	Tabular representation of frequencies

Geographical Classification	:	Classification based on location.
Geometric Mean	:	GM is the n th root of the product of n observations in a data.
Harmonic Mean	:	HM of a number of observations is the reciprocal of the AM of the reciprocals of the observations.
Histogram	:	Graphical representation of frequency distribution using adjacent vertical bars.
Independent events	:	Occurrence of one event does not affect the occurrence of the other
Investigator	:	The person authorised to make investigation.
ISI	:	Indian Statistical Institution
Kurtosis	:	Measure of Peakedness
Lepto Kurtic	:	Highly peaked curve.
Manifold Classification	:	Classification by considering more than one attribute at a time
Mean deviation	:	Arithmetic mean of the absolute deviations of observations from their average
Median	:	The middlemost observation in the data which divides the distribution into two equal parts, when the data is arranged in ascending or descending order.
Meso Kurtic	:	Curve which is moderately peaked.
Mode	:	Mode of a data is the value that is repeated most often in the data.
Moments	:	Represent a convenient and unifying method of summarising certain descriptive statistical measures

Multistage Sampling	:	Sampling in various stages.
Mutually exclusive events	:	Events which cannot occur together.
Non Sampling Error	:	Errors other than sampling error.
NSSO	:	National Sample Survey Office
Ogives	:	Curves obtained by plotting cumulative frequencies.
Percentage Frequency	:	Frequency in terms of the percentage of the total frequency
Percentage Frequency Table	:	Tabular representation of percentage frequencies
Percentiles	:	The values of a data which divide the distribution into hundred equal parts are called deciles.
Pie diagram	:	Circle divided in to various segments proportional to the frequencies.
Platy Kurtic	:	Curve which is flat topped.
Population	:	All elements whose characteristics are being studied
Probability	:	A numerical measure of the possibility of an event.
Probability Sampling	:	All units have specified probability of being included in the sample
Qualitative Classification	:	Classification based on the quality
Qualitative data	:	Data which can be observed but cannot be numerically measured.
Quantitative Classification	:	Classification based on quantity
Quantitative data	:	Variables which can be numerically measured.
Quartile deviation	:	Half of the difference between third quartile and first quartile
Quartiles	:	The values of a data which divide the distribution into four equal parts are called deciles.

Random experiment	:	Experiment having more than one possible result.
Range	:	Difference between the highest and lowest values.
Relative Frequency	:	Ratio of frequency to the total frequency
Relative Frequency Table	:	Tabular representation of relative frequencies
Sample	:	Representative part of the population
Sample space	:	The set of all possible outcomes of a random experiment.
Sampling	:	Studying the population by using samples.
Sampling Error	:	Errors due to sampling.
Scatter plots	:	Diagrammatic representation of bivariate data
Simple event	:	The basic possible outcome of a random experiment.
Simple Random Sampling	:	All units have equal chance or probability of being included in the sample
Skewness	:	Lack of symmetry
Standard deviation	:	Positive square root of the arithmetic mean of the squares ofdeviations of the observations from their arithmetic mean
Statistical Investigation	:	Collection, organization, analysis and interpretation of data according to well defined procedure.
Stratified Sampling	:	Sampling by dividing the population into strata.
Symmetric distribution	:	Data distributed equally on either sides of the mode.

Systematic Sampling	:	Sampling by systematic manner.
Tabulation	:	Presentation of data in rows and columns.
Variance	:	Square of standard deviation
Weighted AM	:	The AM that assign a weight to each observation on its importance related to other is called weighted AM.

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.1 Appendix-1

Many softwares are available in the market for statistical data analysis. Spreadsheet, SPSS, Statistica, Minitab, R etc.. are some examples. Among these R is free software programming language and software environment for statistical data analysis and graphics. Some of the R – Codes are given below.

Frequency Distribution of Qualitative Data

Example 1

Following are school types of 54 schools in an education district.

A C CC H HH C C D DDDDD E A AA B BBB C E EEEE G G A AB A A

D D E F FFF G D D A A B G GGG H

Obtain the frequency distribution of school types.

R Codes

```
>school=c("A","C", "C", "C", "H", "H", "H", "C",
"C", "D", "D", "D", "D", "D", "D", "E", "A", "A",
"A", "B", "B", "B", "B", "C", "E", "E", "E",
"E","E","G", "G", "A", "A", "B", "A", "A", "D",
"D","E", "F", "F", "F", "F", "G", "D","D", "A",
"A", "B", "G", "G", "G", "G", "H") # Enter the raw
data as a vector
>table(school) # Prepare the frequency
distribution for school #type
```

Output:

School	A	B	C	D	E	F	G	H
	10	6	6	10	7	4	7	4

Remark: To get a fancy output one can use the R Codes:

```
>mytable=table(school) # Prepare the frequency
table

>cbind(mytable) Prepare a fancy frequency
table(try!)
```

Bar Diagrams

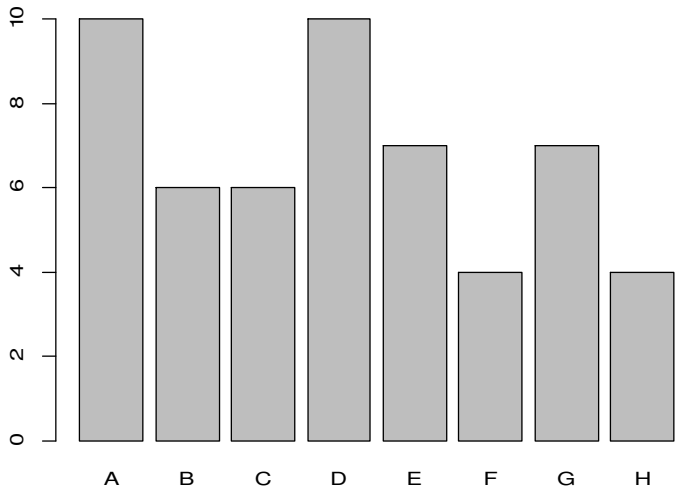
Example 2(Simple Bar Diagram)

Draw the bar graph of the variable school types in example 1.

R Codes

```
>school
>mytable=table(school)
>barplot(mytable)
```

Output:



Remark: To get a fancy output one can use the R Codes:

```
>mytable=table(school)

>barplot(mytable,xlab="School Type",ylab="No.of
Schools", main="BAR DIAGRAM",
col=c("red", "yellow", "green","violet",
```

```
"orange","blue", "pink", "cyan")) # Try!
```

Example 3(Multiple and Subdivided Bar diagrams)

Following table shows the number of students admitted in different faculties in a university indifferent years:

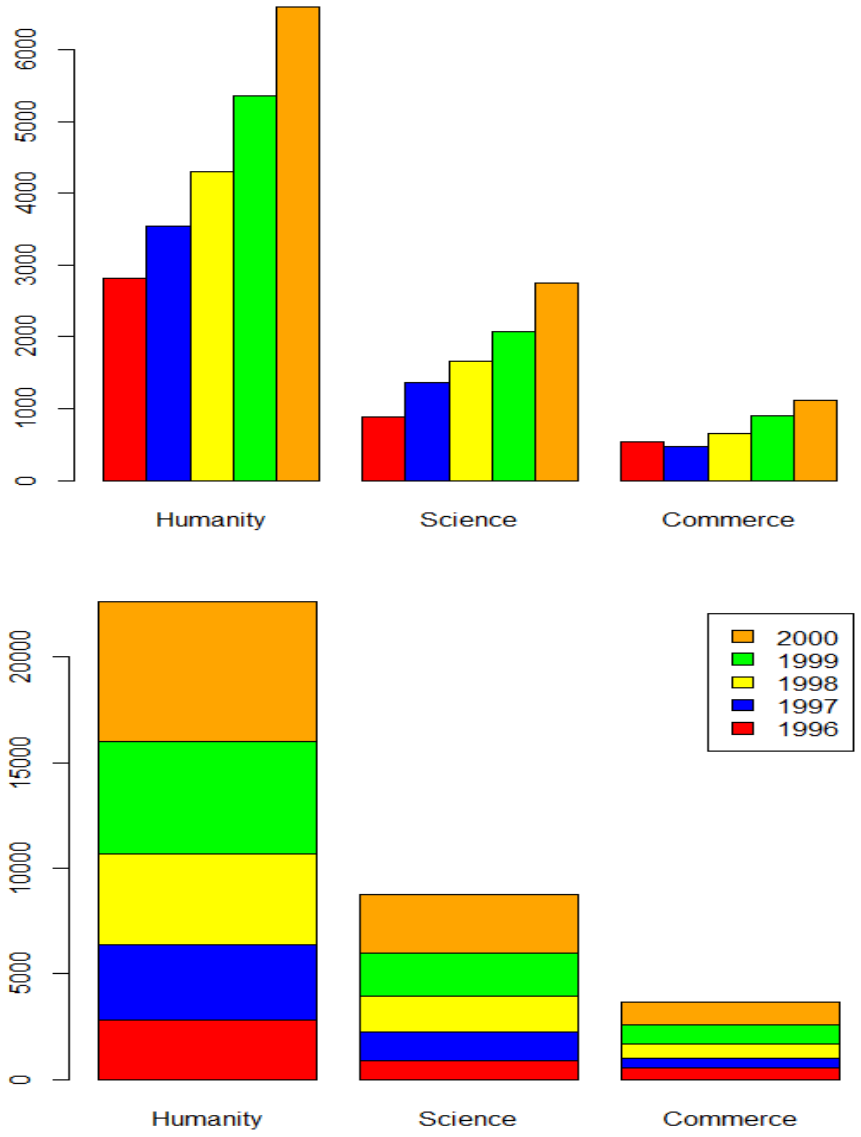
Sl.Number	Year	Humanity	Science	Commerce
1	1996	2810	890	540
2	1997	3542	1363	471
3	1998	4301	1662	652
4	1999	5362	2071	895
5	2000	6593	2752	1113

R Codes

```
>no.stud=matrix(c(2810,890,540,3542,1363,471,4301,
1662,652,5362,2071,895,6593,2752,1113), byrow=T,
ncol=3)

# Entries( that is number of students) into matrix
form; enter #values by row and number of columns
is 3;

>rownames(no.stud)=c("1996","1997","1998","1999","
2000")

# define row names

>colnames(no.stud)=c("Humanity","Science","Commerc
e")

 # define column names

>no.stud      # Print table in the matrix form

>barplot(no.stud,
col=c("red","blue","yellow","green","orange"),

legend = rownames(no.stud))
```

```
# Gives subdivided bar diagram with legends as row
names.

>barplot(no.stud,beside=T,col=c("red","blue","yell
ow","green","orange")) # Gives multiple bar
diagram in which bars side by #side.
```

Outputs



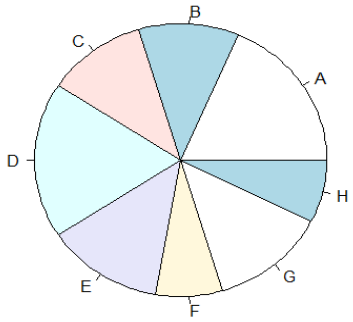
Pie Diagram

Example 4(For qualitativedata)

Draw Pie diagram for the school types in example 1.

```
>school #print the values of the qualitative
variable 'school'

>mytable=table(school)

>pie(mytable)    # pie diagram for the frequency
table of #school.

>pie(mytable,label=c("A","B","C","D","E","F","G","
H"))
```

Output



Example 5 (For quantitative data)

The salesof an appliance in 5 cities in October, 2013 is given in lakhs(Rs.) is given bellow:

Kannur	Kozhikode	Cochi	Kollam	Thiruvanathapuram
78.5	98.75	135,75	65.5	82.45

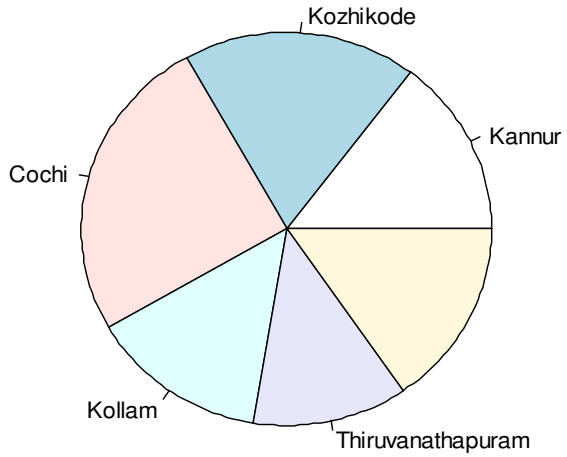
Draw the pie diagram for the sales data

Answer

R Codes

```
>sales.data=c(78.5,  98.75,  135,75,  65.5, 82.45)
>names=c("Kannur"," Kozhikode"," Cochi", "Kollam"
, "Thiruvanathapuram")
>pie(sales.data,label=names)
```

Output



Frequency Distribution of Quantitative variables

Example6 : The daily numbers computer stoppages are observed over 30 days at a school computing center. Prepare the frequency distribution and draw the line diagram for the following data.

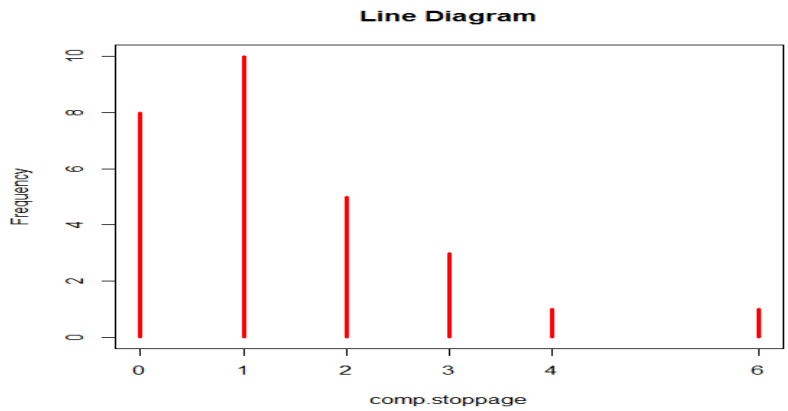
Daily Numbers of Computer Stoppages

3	1	1	0	1	0	1	1	0	2	2
0	0	0	1	2	1	2	0	0	1	6
4	3	3	1	2	1					

R Codes

```
>comp.stoppage=c(3,1,1,0,1,0,1,1,0,2,2,0,0,0,1,2,1
,2,0,0,1,6,4,3,3,1,2,1)

# Enter the raw data as a vector

> stoppage=table(comp.stoppage)

#Prepare the frequency table(Try the table
output!)

>plot(stoppage, type = "h", col = "red",
lwd=3,ylab=Frequency main="Line Diagram")

#Draw the line diagram;type="h" adds vertical
lines and lwd=3 #decides the thickness of vertical
lines.
```

Output



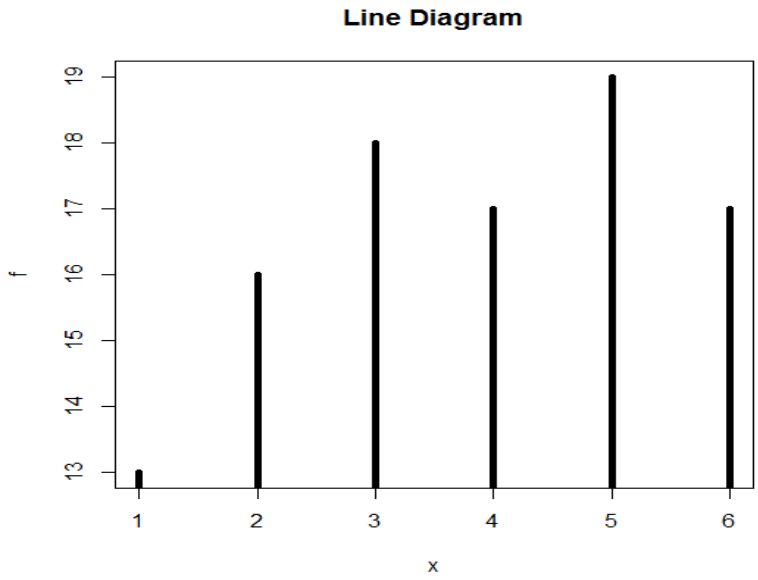
Example7.(When values and frequencies are directly given)

The number (X) obtained when a die is tossed 100 times. is tabulates as

X:	1	2	3	4	5	6
f:	13	16	18	17	19	17

R Codes

```
>x=c(1,2,3,4,5,6)

>f=c(13,16,18,17,19,17)

>plot(x,f,type="h",lwd=4,xlab="x",ylab="f",main="L
ine Diagram")
```



Histogram

Example 8

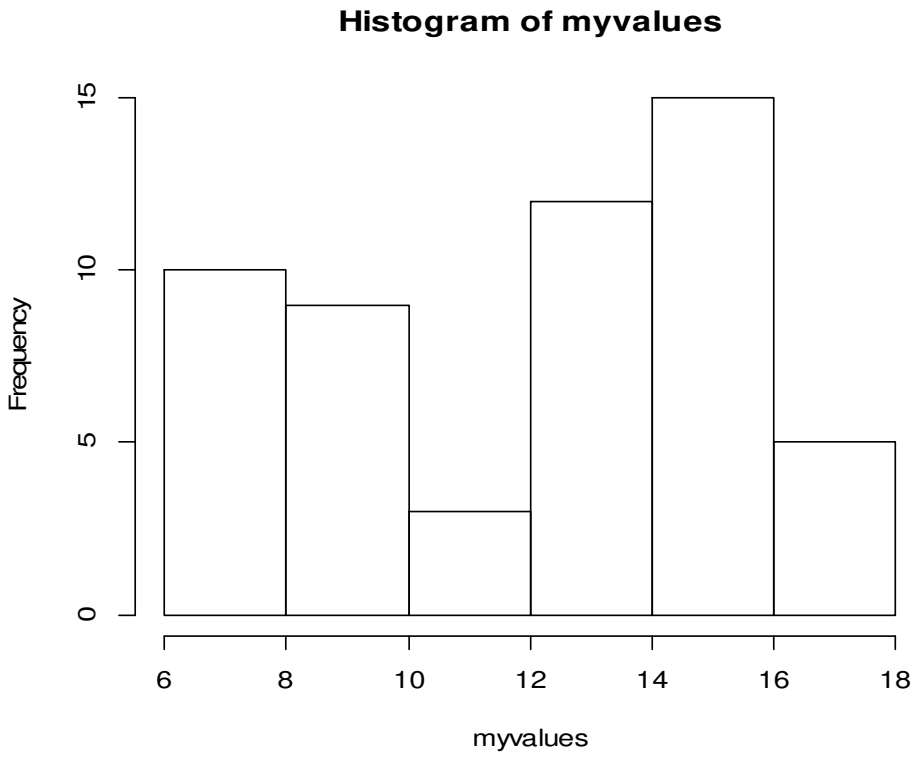
Obtain the Histogram for the data

8	16	13	16	15	16	17	16	12	18	13	15
	15	14	14	15	15	14	10	12	15	15	8
	6	9	8	9	6	14	14	15	10	14	6
	13	17	10	13	17	10	10	15	6	10	8
	14	6	13	12	10	8	16	15	17		

R Codes

```
>myvalues=c(8,16, 13,16,  15, 16,  17, 16,  12, 18,  13,
15,  15, 14,  14, 15,  15, 14,  10, 12,  15,  15, 8, 6,
9, 8, 9, 6, 14, 14, 15, 10, 14, 6,  13, 17, 10, 13,  17, 10,  10,
15,  6, 10, 8, 14, 6,  13, 12,  10, 8, 16, 15, 17)
```

```
>hist(myvalues) # Default output; break points set
automatically
```



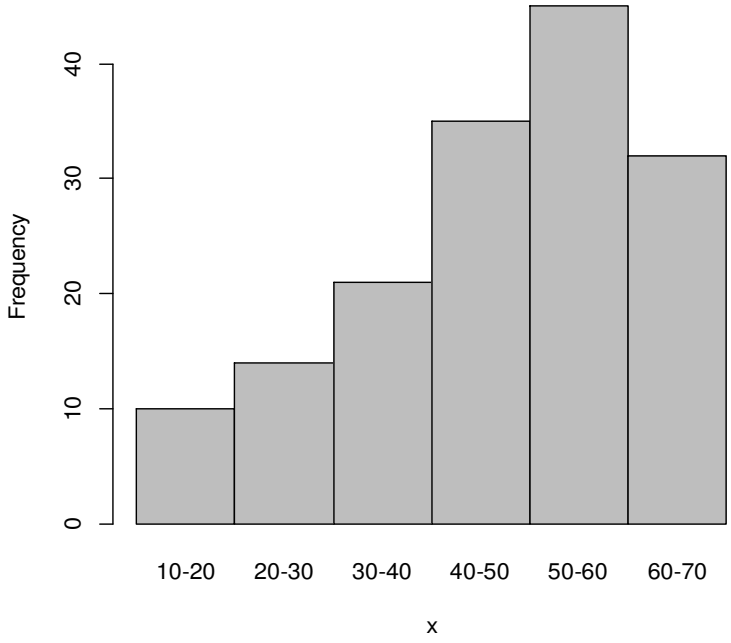

```
>hist(myvalues,breaks=c(5,6.5,8,9.5,11,12.5,14,15.
5,17,18.5),col="red") # Fancy; break points are
manualy set.(Try!)
```

Example 8(a) Draw the histograms for the following frequency distributions

1. Class: 10-20 20-30 30-40 40-50 50-60 60-70
   Freq.: 10 14 21 35 45 32

R Codes

```
>x=c("10-20","20-30","30-40","40-50","50-
60","60-70")
> f=c(10,14,21,35,45,32)
>
barplot(f,names=x,space=0,xlab="x",ylab="Frequ
ency") # It #is a trick of drwing bardiagram
with 0 space between bars.
```



2. Midvalue: 5 10 15 20 25 30
   Frequency: 1 8 14 21 18 32

R Codes

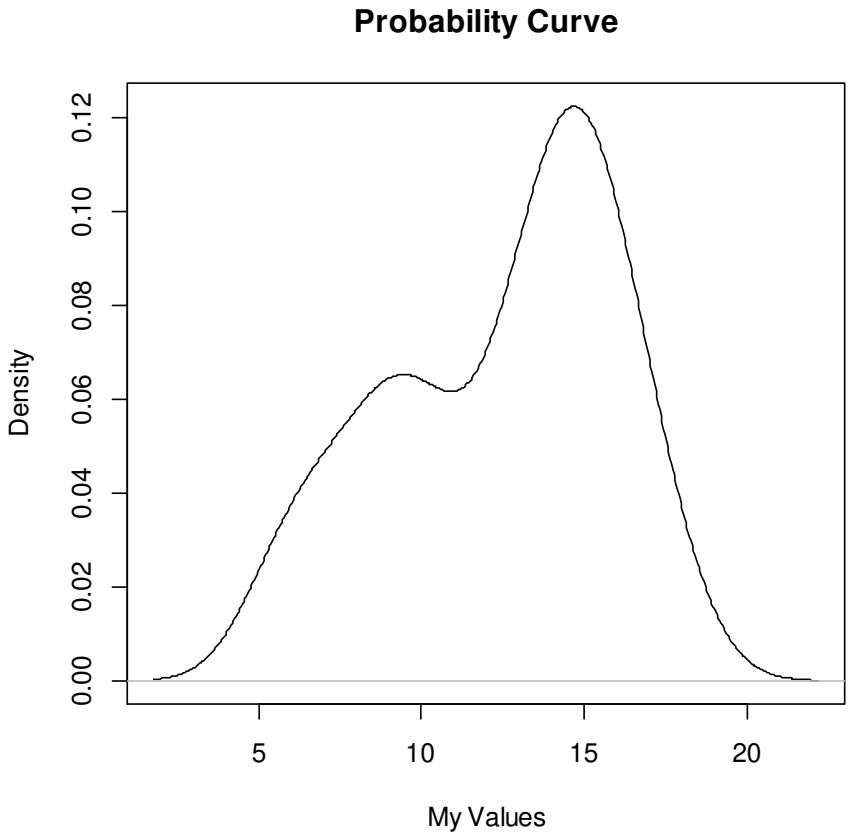
```
>x=c("5","10","15","20","25","30")
>f=c(1,8,14,21,18,32)
>barplot(f,names=x,space=0)# Try!
```

Probability Curve (probability Density Plot)

Example 9 Draw the probability curve for the data given in example 8

R Codes

```
>myvalues=c(8,16, 13,16,  15, 16,  17, 16,  12, 18,  13,
15,  15, 14,  14, 15,  15, 14,  10, 12,  15,  15, 8, 6,
9, 8, 9, 6, 14, 14, 15, 10, 14, 6,  13, 17, 10, 13,  17, 10,  10,
15,  6, 10, 8, 14, 6,  13, 12,  10, 8, 16, 15, 17)

>plot(density(myvalues))# default output(Try!)

>plot(density(myvalues),xlab="My
Values",main="Probability Curve")# Output shown
```



Scattar Diagram

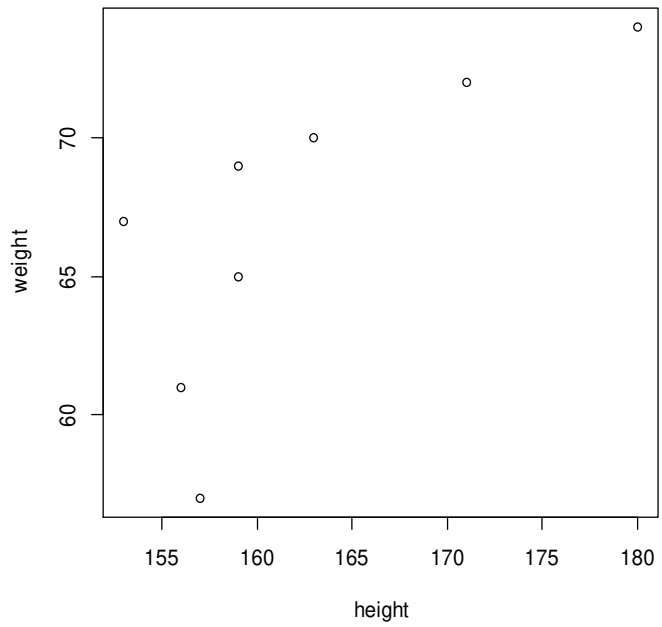
Example10 Draw Ascattar diagram to the bivariate data given bellow and comment on the plot.

Height(Cm): 157 159 163 156 171 180 153 159

Weight(Kg): 57 65 70 61 72 74 67 69

R Codes/ Outputs

```
>height=c(157,159,163,156,171,180,153,159)
>weight=c(57,65,70,61,72,74,67,69)
>plot(height,weight)
```



Descriptive Statistics

Example10 Find the common measures of central tendencies and dispersions for the data in example 9(myvalues)

R Codes/ Outputs

```
>mean(myvalues) # Arithematic Mean
[1] 12.46296
>median(myvalues) # Median
[1] 13.5
>sd(myvalues) # Standard Deviation
[1] 3.457084
var(myvalues)
[1] 11.95143
```

Quantiles(Quartiles,Deciles,Percentilesetc)

Example 11 Find the three quartiles, third and seventh deciles and 14th, 23rd and 72th percentiles for the data in example 9(myvalues)

R Codes/ Outputs

```
>quantile(myvalues) # Default is minimum, Q1, Q2,
Q3 and maximum

     0%     25%     50%       75%      100%
     6.0    10.0    13.5      15.0     18.0

>quantile(myvalues,probs=c(.3,.7,.14,.23,.72))#
deciles and #percentiles of specified order.

     30%    70%     14%       23%      72%
     10     15      8         10       15
```

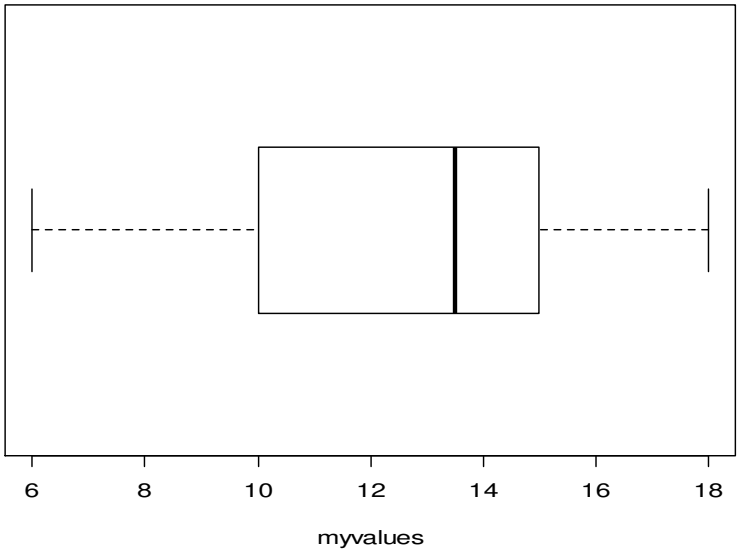
Box and Whisker Plot

Example 12 Draw the box and whisker plot for the data in example 9(myvalues)

R Codes

```
>boxplot(myvalues,horizontal=T,xlab="myvalues")
```

Output



Try the code: `>boxplot(myvalues)`

Range and inter Quartile Range

Example 13 Find the range and inter quartile range for the data in example 9(myvalues)

R Codes/ Outputs

```
>range(myvalues) # Wrong code!! See the output
[1]  6 18
>range=max(myvalues)-min(myvalues)
>range
[1] 12
>IQR(myvalues)# Inter Quartile Range; IQR=Q3-Q1
[1] 5
```

.2 Appendix-2

CENSUS- 2011

POPULATION AND DECADAL GROWTH RATE

As per 2011 Provisional Population Figures, Rural Population in Kerala is 17,455,506. Out of this 8,403,706 are males and 9,051,800 are females whereas urban population in this state is 15,932,171. Out of this, 7,617,584 are males and 8,314,587 are females. The decadal decline of rural population was –25.96%, whereas the urban population has grown by 92.72%.

TRENDS IN RURAL AND URBAN CLASSIFICATION OF POPULATION IN KERALA

The State has now 52.30 percent rural population in 2011 Census as against 74.04 per cent in 2001 Census. The huge growth in urban population during the past decade 2001-2011(92.72 %) could be attributed squarely to the manifold increase in number of Towns in the State between 2001 & 2011 from 159 to 520. 47.72% of the total population of Kerala are from Urban. Ernakulam is the most urbanised district (68.07%) and Wayanad (3.87%) is the least urbanised district of the State.

POPULATION IN THE AGE GROUP 0-6 AGE

Total Population in the age group 0-6 is 3,322,247. Out of this males are 1,695,935 and females are 1,626,312. Rural Population in the age group 0- 6 is 1,747,512. Males are 891,668 and females are 855,844. Urban Population of the age group 0- 6 is 1,574,735. Out of this 804,267 are males and 770,468 are females. Percentage of rural population in the age group 0- 6 to the total rural population is 10.01 and that of urban is 9.88

SEX RATIO (FEMALES PER 1000 MALES)

The Overall sex ratio of Kerala is 1084 females per 1000 males. Whereas, sex ratio of rural area is 1077 and that of urban area is 1091.

CHILD SEX RATIO (0-6 YEARS)

Child sex ratio in respect of 0-6 age population in Kerala is 959. In rural areas it is 960, whereas, sex ratio of 0-6 age population in urban areas is 958.

LITERACY

Total number of literates in Kerala is 28,234,227and total literacy rate is 93.91%. Among these, literates in Rural area is 14,595,727 and that in Urban area is 13,638,500 .The numbers of male literates in Rural area is 7,158,427 and the number of male literates in Urban area is 6,597,461. Female literates in rural areas are 7,437,300 and that in urban area is 7,041,039. Literacy rate in the rural area is 92.92% and that of urban area is 94.99 %. The gender gap in literacy in rural area of the State is found to be 4.55%; whereas that in urban area is 3.5%

Figures at a glance and Data Sheet showing district level break up of Rural & Urban of the state is attached.

(See overleaf)

Table-1

Distrubution of Population, Decadal Growth Rate, Sex-Ratio and Population Density

State/ District Code	State/ District	Population 2011			Percentage decadal growth rate of population		Sex- Ratio* (Number of Females per 1000 Males)		Population density per sq. km.	
		Persons	Males	Females	1991-01	2001-11	2001	2011	2001	2011
1	2	3	4	5	6	7	8	9	10	11
32	**Kerala**	**3,33,87,677**	**1,60,21,290**	**1,73,66,387**	+9.43	+4.86	1058	1084	819	859
01	Kasaragod	13,02,600	6,26,617	6,75,983	+12.37	+8.18	1047	1079	604	654
02	Kannur	25,25,637	11,84,012	13,41,625	+6.98	+4.84	1090	1133	812	852
03	Wayanad	8,16,558	4,01,314	4,15,244	+16.14	+4.60	995	1035	366	383
04	Kozhikode	30,89,543	14,73,028	16,16,515	+9.89	+7.31	1057	1097	1228	1318
05	Malappuram	41,10,956	19,61,014	21,49,942	+17.09	+13.39	1066	1096	1021	1158
06	Palakkad	28,10,892	13,60,067	14,50,825	+9.88	+7.39	1066	1067	584	627
07	Thrissur	31,10,327	14,74,665	16,35,662	+8.66	+4.58	1092	1109	981	1026
08	Ernakulam	32,79,860	16,17,602	16,62,258	+9.35	+5.60	1019	1028	1012	1069
09	Idukki	11,07,453	5,51,944	5,55,509	+7.03	-1.93	993	1006	259	254
10	Kottayam	19,79,384	9,70,140	10,09,244	+6.86	+1.32	1025	1040	885	896
11	Alappuzha	21,21,943	10,10,252	11,11,691	+5.39	+0.61	1079	1100	1492	1501
12	Pathanamthitta	11,95,537	5,61,620	6,33,917	+3.84	-3.12	1094	1129	468	453
13	Kollam	26,29,703	12,44,815	13,84,888	+7.38	+1.72	1069	1113	1038	1056
14	Thiruvananthapuram	33,07,284	15,84,200	17,23,084	+9.76	+2.25	1060	1088	1476	1509

**For calculation of sex ratio, others have been considered as males.*

.4 Appendix-4

Table-3

Sex-Ratio for State and Districts : 1901-2011

State/ District Code	State/ District	*Sex-ratio (Number of females per 1000 males)											
		1901	1911	1921	1931	1941	1951	1961	1971	1981	1991	2001	2011
1	2	3	4	5	6	7	8	9	10	11	12	13	14
32	**Kerala**	**1004**	**1008**	**1011**	**1022**	**1027**	**1028**	**1022**	**1016**	**1032**	**1036**	**1058**	**1084**
01	Kasaragod	1060	1053	1050	1040	1039	1046	1026	998	1020	1026	1047	1079
02	Kannur	1060	1079	1121	1106	1110	1074	1048	1033	1040	1049	1090	1133
03	Wayanad	805	815	786	804	835	838	903	922	949	966	995	1035
04	Kozhikode	1009	1022	1038	1032	1044	1019	1007	1004	1020	1027	1057	1097
05	Malappuram	1017	1020	1037	1059	1062	1055	1057	1041	1052	1053	1066	1096
06	Palakkad	1042	1057	1069	1079	1079	1085	1077	1056	1056	1061	1066	1067
07	Thrissur	1004	1009	1051	1075	1082	1105	1093	1081	1100	1085	1092	1109
08	Ernakulam	985	990	969	994	994	1008	999	988	997	1000	1019	1028
09	Idukki	839	842	850	834	875	909	914	937	963	975	993	1006
10	Kottayam	965	969	947	966	966	987	988	991	1001	1003	1025	1040
11	Alappuzha	986	987	986	997	1003	1022	1026	1025	1043	1051	1079	1100
12	Pathanamthitta	986	987	949	975	986	996	1011	1019	1056	1062	1094	1129
13	Kollam	987	988	989	1006	1013	997	996	1000	1022	1035	1069	1113
14	Thiruvananthapuram	996	990	981	1003	1017	1010	1005	1008	1030	1036	1060	1088

**For calculation of sex ratio, others have been considered as males.*

Table-4

Population in the Age-Group 0-6, Number of Literates and Literacy Rate for State and Districts : 2011

State/ District Code	State/ District	Total Population			Population in age group 0-6			Number of literates			Literacy rate#		
		P	M	F	P	M	F	P	M	F	P	M	F
1	2	3	4	5	6	7	8	9	10	11	12	13	14
32	**Kerala**	**3,33,87,677**	**1,60,21,290**	**1,73,66,387**	**33,22,247**	**16,95,935**	**16,26,312**	**2,82,34,227**	**1,37,55,888**	**1,44,78,339**	**93.91**	**96.02**	**91.98**
01	Kasaragod	13,02,600	6,26,617	6,75,983	1,49,280	76,149	73,131	10,36,289	5,17,031	5,19,258	89.85	93.93	86.13
02	Kannur	25,25,637	11,84,012	13,41,625	2,65,276	1,35,189	1,30,087	21,56,575	10,22,972	11,33,603	95.41	97.54	93.57
03	Wayanad	8,16,558	4,01,314	4,15,244	89,720	45,776	43,944	6,49,186	3,30,093	3,19,093	89.32	92.84	85.94
04	Kozhikode	30,89,543	14,73,028	16,16,515	3,23,511	1,64,800	1,58,711	26,34,493	12,76,384	13,58,109	95.24	97.57	93.16
05	Malappuram	41,10,956	19,61,014	21,49,942	5,52,771	2,81,958	2,70,813	33,28,658	16,08,229	17,20,429	93.55	95.78	91.55
06	Palakkad	28,10,892	13,60,067	14,50,825	2,88,366	1,46,947	1,41,419	22,32,190	11,19,360	11,12,830	88.49	92.27	84.99
07	Thrissur	31,10,327	14,74,665	16,35,662	2,89,126	1,48,428	1,40,698	26,89,229	12,86,141	14,03,088	95.32	96.98	93.85
08	Ernakulam	32,79,860	16,17,602	16,62,258	2,89,281	1,48,047	1,41,234	28,61,509	14,27,572	14,33,937	95.68	97.14	94.27
09	Idukki	11,07,453	5,51,944	5,55,509	1,00,107	51,132	48,975	9,28,774	4,74,988	4,53,786	92.20	94.84	89.59
10	Kottayam	19,79,384	9,70,140	10,09,244	1,68,563	86,113	82,450	17,45,694	8,59,038	8,86,656	96.40	97.17	95.67
11	Alappuzha	21,21,943	10,10,252	11,11,691	1,86,022	95,556	90,466	18,63,558	8,95,476	9,68,082	96.26	97.90	94.80
12	Pathanamthitta	11,95,537	5,61,620	6,33,917	91,501	46,582	44,919	10,70,120	5,03,171	5,66,949	96.93	97.70	96.26
13	Kollam	26,29,703	12,44,815	13,84,888	2,38,062	1,21,481	1,16,581	22,42,757	10,76,509	11,66,248	93.77	95.83	91.95
14	Thiruvananthapuram	33,07,284	15,84,200	17,23,084	2,90,661	1,47,777	1,42,884	27,95,195	13,58,924	14,36,271	92.66	94.60	90.89

#Literacy rate is the percentage of literates to total population aged 7 years and above.

.6 Appendix-6

Random Number Table

4139	8518	2724	7450	8584	7837	0080	0695	3538	7663	7943	9759	1656	0654	6315	9332
2938	6809	0507	9413	7383	3472	5207	5313	4993	5954	3070	1722	3111	8945	1442	1295
4393	5100	5634	4032	8838	1763	0334	7277	3792	4245	8197	3685	4566	7236	9225	3258
5848	3391	0761	5995	0293	0054	5461	1895	5247	2536	3324	8304	3365	5527	4352	7877
7302	1682	5888	0614	9092	8345	0588	3859	6702	0827	8452	0267	4820	3818	9479	9840
6102	9973	1015	2577	0547	6636	8371	8477	8157	9118	3579	4886	6275	2109	4606	4459
7557	8264	8798	7196	2002	4927	3498	0441	6956	7409	1361	6849	7730	0400	9733	6422
9011	3899	3925	9159	0801	3218	8625	2404	8411	3044	6488	1468	6529	8691	7516	1041
7811	2190	9052	1122	2256	1509	3752	7022	9866	1335	1615	3431	7984	6982	2643	3004
9266	0481	4179	5741	3711	9800	8879	8986	8665	9626	6743	8050	9439	2617	7770	7623
0721	8772	9306	7704	2510	8091	4006	3604	0120	7917	1870	0013	8238	0908	2897	9586
9520	7063	7089	2323	3965	6382	1789	5568	1575	6208	9652	1976	9693	9199	8024	1549
0975	5354	2216	4286	5420	4673	6916	0186	0374	4499	4779	6595	1148	7490	3151	6168
2430	3645	7343	8905	6875	2964	2043	2150	1829	2790	9906	8558	9947	5781	0934	8131
1229	1936	2470	0868	5674	1255	7170	4113	3284	1081	5033	3177	1402	4072	6061	2750
2684	0227	7597	2831	7129	9546	2297	8732	2083	9372	0161	5140	2857	2363	1188	4713
6756	8144	3979	6476	6328	9426	0133	0667	9065	3872	6796	5367	2310	8826	9279	3231
8719	9599	2270	1603	8292	0881	5769	5794	1028	2671	5087	0494	4273	0281	4914	8358
3338	8398	0561	9386	2910	9680	4059	0921	2991	4126	3378	5621	8892	1736	3205	3485
5301	9853	8852	4513	4874	1135	2350	6048	7610	5581	1669	0748	0855	0535	1496	8612
9920	1308	4487	9640	9492	2590	0641	1176	9573	4380	9960	5875	5474	1990	9787	3739
1883	0107	2778	4767	1456	1389	8932	8958	4192	5835	8251	3658	7437	3445	8078	1522
3846	1562	1069	9894	3419	2844	7223	4085	6155	7290	6542	8785	2056	2244	6369	6649
8465	3017	9360	5021	8037	4299	5514	9212	0774	8745	4833	3912	4019	3699	4660	1776
0428	1816	7651	2804	0001	3098	3805	4339	2737	7544	3124	9039	5982	5154	2951	6903
5047	3271	5942	7931	4619	4553	2096	9467	4700	8999	1415	4166	0601	3953	1242	2030
7010	4726	4233	3058	6583	6008	0387	4594	9319	0454	9706	1949	2564	5408	9533	9813
1629	3525	2524	8185	1201	4807	8678	2376	1282	9253	5341	7076	7183	6863	7824	4940
3592	4980	0815	3312	3165	6262	6969	7503	5901	0708	3632	2203	9146	5662	6115	0067
5555	6435	9106	1095	5128	7717	5260	2630	7864	2163	1923	7330	3765	7117	4406	5194
0174	5234	7397	6222	9747	9172	3551	7758	2483	0962	0214	2457	5728	8572	2697	0321
2137	6689	5688	1349	1710	7971	1842	2885	4446	2417	8505	0240	0347	0026	0988	5448
7308	3996	3930	1473	8843	4077	8376	0792	3543	9978	3330	0619	1407	6387	4103	3609
2435	5959	5385	9764	3970	8696	9831	9083	1326	1941	4784	8910	9190	1006	2902	1900
7562	0578	4184	8055	1753	0659	8630	4718	6453	6560	6239	7201	4317	2969	4357	0191
2689	2541	5639	6346	6880	5278	0085	3009	1580	8523	5039	5492	9444	4932	5812	8482
0471	4504	7094	4637	2007	7241	1540	1300	6707	3142	6493	3783	4571	9551	4611	6773