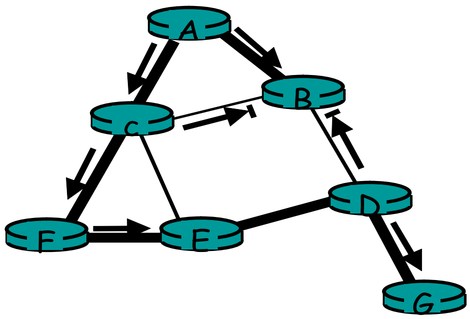
HW 8 Ans Sp. 2015

Problem 1



The thicker shaded lines represent
The shortest path tree from A to all
destination. Other solutions are
possible, but in these solutions, B
can not route to either C or D from A.

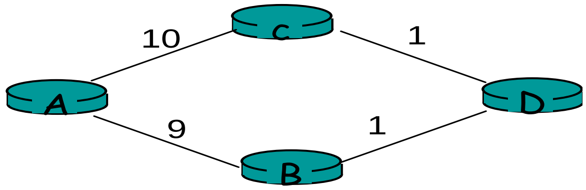
Note that this question does not assume link weights are all 1. The idea is to **create** a shortest path tree such that the shortest paths from A, C and D back to the source A do not pass through B. The tree outlined in bold above is one such tree. If link weights were all 1, then RPF on the minimum cost tree would be different.

Problem 2

The center-based tree for the topology shown in the original figure connects A to C; B to C; E to C; and F to C (all directly). D connects to C via E, and G connects to C via D, E. In this case, the cost is minimum, but not unique. Other centers can be chosen so that the cost is minimum. As in problem 1, if length weights were not all 1, in general we cannot say that a particular center based tree is minimal cost.

Problem 3

Dijkstra’s algorithm for the network below, with node A as the source, results in a least-unicast-cost path tree of links AC, AB, and BD, with an overall free cost of 20. The minimum spanning tree contains links AB, BD, and DC, at a cost of 11. .



Problem 4

a) $32 - 4 = 28$ bits are available for multicast addresses. Thus, the size of the multicast address space is $N = 2^{28}$.

b) The probability that two groups choose the same address is $\frac{1}{N} = 2^{-28} = 3.73 \cdot 10^{-9}$

c) The probability that 1000 groups all have different addresses is

$$\frac{N \cdot (N-1) \cdot (N-2) \cdots (N-999)}{N^{1000}} = \left(1 - \frac{1}{N}\right)\left(1 - \frac{2}{N}\right) \cdots \left(1 - \frac{999}{N}\right)$$

Ignoring cross-product terms, this is approximately equal to

$$1 - \left(\frac{1 + 2 + \cdots + 999}{N}\right) = 1 - \frac{999 \cdot 1000}{2N} = 0.998$$

The probability that they interfere is 1-0.998.

Problem 5

a) 011
b) 110
c) 110
d) 011

Problem 6

a)

$$E(p) = Np(1-p)^{N-1}$$

$$E'(p) = N(1-p)^{N-1} - Np(N-1)(1-p)^{N-2}$$

$$= N(1-p)^{N-2}((1-p) - p(N-1))$$

$$E'(p) = 0 \Rightarrow p^* = \frac{1}{N}$$

b)

$$E(p^*) = N\frac{1}{N}(1-\frac{1}{N})^{N-1} = (1-\frac{1}{N})^{N-1} = \frac{(1-\frac{1}{N})^N}{1-\frac{1}{N}}$$

$$\lim_{N\to\infty}(1-\frac{1}{N}) = 1 \qquad \lim_{N\to\infty}(1-\frac{1}{N})^N = \frac{1}{e}$$

Thus

$$\lim_{N\to\infty} E(p^*) = \frac{1}{e}$$

c)

$$E(p) = Np(1-p)^{2(N-1)}$$

$$E'(p) = N(1-p)^{2(N-2)} - Np\,2(N-1)(1-p)^{2(N-3)}$$

$$= N(1-p)^{2(N-3)}((1-p) - p\,2(N-1))$$

$$E'(p) = 0 \Rightarrow p^* = \frac{1}{2N-1}$$

$$E(p^*) = \frac{N}{2N-1}(1-\frac{1}{2N-1})^{2(N-1)}$$

$$\lim_{N\to\infty} E(p^*) = \frac{1}{2}\cdot\frac{1}{e} = \frac{1}{2e}$$ by using the fact that $$\lim_{N\to\infty}(1-\frac{1}{N})^N = \frac{1}{e}$$

Problem 7

a) p(some node succeeds in a slot) = 3 p(1-p)2
p(no node succeeds in a slot) = 1 - 3 p(1-p)2
Hence, p(first success occurs in slot 2) = p(no node succeeds in first slot) p(some node succeeds in 2rd slot) = (1 - 3 p(1-p)2) * 3 p(1-p)2

b) (1 – p(B))2 p(B)
where, p(B) = probability that B succeeds in a slot
p(B) = p(B transmits and A does not and C does not)

= p(B transmits) p(A does not transmit) p(C does not transmit)
= p(1 – p) (1 – p) = p(1 – p)2

Hence, p(B succeeds for first time in slot 3)
= (1 – p(B))2 p(B) = (1 – p(1 – p)2)2 p(1 – p)2

c) p(A succeeds in slot 4) = p(1-p)2
p(B succeeds in slot 4) = p(1-p)2
p(C succeeds in slot 4) = p(1-p)2

p(either A or B or C succeeds in slot 4) = 3 p(1-p)2
(because these events are mutually exclusive)

d) efficiency = p(success in a slot) =3 p(1-p)2

Problem 8

Wait for 512*K bit times. For 10 Mbps and 100 Mbps, this wait is

$$\frac{512 \times K bits}{10 \times 10^6 bps}$$

$$\frac{512 \times K bits}{10 \times 10^7 bps}$$

Note that the exact value will depend on the value of K chosen randomly.

Problem 9.

a) Yes, both stations can randomly pick the same value of K.
b)

Time, t	Event
0	A and B begin transmission
225	A and B detect collision
273	A and B finish transmitting jam signal
273+225 = 498	B 's last bit arrives at A ; A detects an idle channel
498+96=594	A starts transmitting
273+512 = 785	B returns to Step2
	B must sense idle channel for 96 bit times before it transmits
594+225=819	A’s transmission reaches B

Because A's retransmission reaches B before B's scheduled retransmission time (785+96), B refrains from transmitting while A retransmits. Thus A and B do not collide. Thus the factor 512 appearing in the exponential backoff algorithm is sufficiently large.

Problem 10

A frame size is 1000*8+64 = 8064 bits, as 64-bit preamble is added into the frame.
We want $1/(1+5a) = .7$ or, equivalently, $a = 0.08571428\quad 571 = t_{prop}/t_{trans}$. Let's assume $t_{prop} = d/(2\times 10^8)$ m/sec and $t_{trans} = (8064\ \text{bits})/(10^8\ \text{bits/sec}) = 80.64\,\mu$ sec. Solving for d we obtain $d = 1382.4$ meters.

For transmitting station A to detect whether any other station transmitted during A's interval, t_{trans} must be greater than $2t_{prop} = 2\times 1382.4 \backslash\ \text{m}/2\times 10^8\ \text{m/sec} = 13.824\ \mu$ sec. Because $13.824 < 80.64$, A will detect B's signal before the end of its transmission.

Problem 11

a)

$$\frac{1000\ m}{2\cdot 10^8\ m/\sec} + 4\cdot\frac{20\ bits}{10\times 10^6\ bps}$$

$$= (5\times 10^{-6} + 8\times 10^{-6})\sec$$
$$= 13\ \mu\ \sec$$

b)
First note, the transmission time of a single frame is given by 1500/(10Mbps)=150 micro sec, longer than the propagation delay of a bit.

- At time $t = 0$, both A and B transmit.
- At time $t = 13\,\mu\sec$, both A and B detect a collision, and then abort.
- At time $t = 26\,\mu\sec$ last bit of B's aborted transmission arrives at A.
- At time $t = 39\,\mu\sec$ first bit of A's retransmission frame arrives at B.
- At time $t = 39\ \mu\sec + \frac{1500\ bits}{10\times 10^6\ bps} = 189\ \mu\sec$ A's packet is completely delivered at B.

c) The line is divided into 5 segments by the switches, so the propagation delay between switches or between a switch and a host is given by $\frac{1000\ m/5}{2\cdot 10^8\ m/\sec} = 1.0\,micro\ \sec$.
The delay from Host A to the first switch is given by 150microsec (transmission delay), longer than propagation delay. Thus, the first switch will wait 153=150+1.0+ 2.0 (note, 2.0 is processing

delay) till it is ready to send the frame to the second switch. Note that the store-and-forward delay at a switch is 150 microsec. Similarly each of the other 3 switches will wait for 153 microsec before ready for transmitting the frame.

The total delay is:

153*4 + 150+1.0=763 micro sec.