

Investigate the exponential distribution in R and compare it with the Central Limit Theorem

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Overview

In this exercise, I shall try to compare the exponential distribution with central limit theorem keeping the parameter $\lambda = 0.3$. I choose this because its mean and variance are simple to interpret.

Simulations

Set the simulation variables λ , exponentials, and seed.

```
library('ggplot2')
ECHO=TRUE
set.seed(42)
lambda = 0.3
exponentials = 40
```

Run Simulations with variables

```
cal_mean = NULL
for (i in 1 : 1000) cal_mean = c(cal_mean, mean(rexp(exponentials, lambda)))
```

Sample Mean versus Theoretical Mean comparison

Sample Mean Calculating the mean from the collected sample means.

```
mean(cal_mean)
```

```
## [1] 3.324339
```

Theoretical Mean Theoretical mean is $\text{inverse}(\lambda)$ or λ^{-1}

```
lambda^-1
```

```
## [1] 3.333333
```

Comparison Ideally, there should be a slight difference between the theoretical and calculated values.

```
abs(mean(cal_mean)-lambda^-1)
```

```
## [1] 0.008994455
```

Sample Variance versus Theoretical Variance

Sample Variance Calculated Variance

```
var(cal_mean)
```

```
## [1] 0.2819736
```

Theoretical Variance Theoretical variance $(\lambda * \sqrt{n})^{-2}$.

```
(lambda * sqrt(exponentials))^-2
```

```
## [1] 0.2777778
```

Comparison Difference between both the variances

```
abs(var(cal_mean)-(lambda * sqrt(exponentials))^-2)
```

```
## [1] 0.004195787
```

Distribution

A histogram plot overlayed with normal distribution so that It can represent the concept that any sequence will become normal if we sample a lot of of points. The normal distribution will have a mean of λ^{-1} and standard deviation of $(\lambda * \sqrt{n})^{-1}$, the theoretical normal distribution for the simulations.

```
library(ggplot2)
ggplot(data.frame(y=cal_mean), aes(x=y)) +
  geom_histogram(aes(y=..density..), binwidth=0.2, fill="#0072B2",
                 color="black") +
  geom_function(fun=dnorm, args = c(mean=lambda^-1, sd=(lambda*sqrt(exponentials))^-1),
               size=2) +
  labs(title="Plot of Calculated Means and Overlay Normal Distribution", x="Drawn sample Mean")
```

Plot of Calculated Means and Overlay Normal Distribution

