Course Code: GRA 65515, Quantitative Risk And Asset Management.
To be answered in groups of 1-3.
Instructions:

- You do NOT need to add a bibliography or reference list.
- You should upload only one file, a pdf containing a short paper (max length 20 pages), not the source code.
- The answers do not need to be long, but try to use precise language and present your results in an easily readable format (e.g. in small tables when appropriate).
- Report three decimals.

Take-home exam, Sping 2025.

Question 1 (35 points). Each sub-question gives 5 points.

For this question, we are going to simulate data from a simple stochastic volatility model with leverage (meaning that volatility is affected by past returns). The data-generating-process is

$$r_t = \beta e^{0.5h_t}\varepsilon_t, \tag{1}$$

where r_t is a daily log returns, and log volatility changes through time according to

$$h_t = \phi h_{t-1} + \rho\sigma\varepsilon_{t-1} + \sigma\sqrt{1-\rho^2}u_t, \tag{2}$$

with $\varepsilon_t \sim N(0,1)$, and $u_t \sim N(0,1)$.

Simulate one million observations, then throw away the first 10000. Use the following parameters: $\beta = 0.01, \phi = 0.99, \sigma = 0.1, \rho = -0.8$. Then answer the following questions:

1. What is the mean and std of daily log returns?
2. Compute the skew and kurtosis of cumulative log returns at the following horizons: 1,20,250. Report them in a table.
3. For the same horizons, report the VaR at the 95% and 99% level (i.e. the 5% and 1% of the distributions). Add them to the table.
4. What do the parameters ϕ and ρ represent?
5. Plot the histogram of yearly cumulative log returns.
6. Briefly comment on the exercise (what was its purpose?) and its results. What is generating the skeweness?

7. How would the distribution of monthly cumulative log returns of your portfolio look like if you aimed at a constant portfolio volatility? (Assume that you can estimate h_t accurately at time t).

Question 2 (65 points). Subquestions 1-7 give 7 points, subquestion 8 gives 9. The file *Portfolios_formed_on_OP_daily.csv* has total daily returns (not logs) for portfolio of firms sorted from lowest to highest profitability (from the Kenneth French database). Notice that this is profitability in the recent past, not projected future profitability. Assume the interest rate is constant at 7% annual, and, in the questions below, interpret "returns" as meaning excess returns. (That is, cumulative returns, Sharpe ratios etc... should be computed on excess returns). Consider portfolio of quintiles (i.e. "Lo 20", ""Qnt 2","Qnt 3","Qnt 4","Hi 20").

1. For each of the give portfolios, compute the average annualized return, the annualized standard deviation, the annualized Sharpe ratio. (In a table. 1% return can be printed as either 1 or 0.01, just clarify). (Annualized means that the daily stastistics are multiplied by $\times 252$ for returns and $\times\sqrt{252}$ for std).

2. Plot the cumulative returns (not log returns) for the first and last quintile, assuming the portfolio is reinvested every day.

3. The first quintile barely delivers any wealth (in excess of the safe interest rate) at the end of over 60 years. How can that be, when the average annualized excess return is positive?

4. Briefly comment on the results.

5. Now compute the (approximate) market daily returns as the equal-weight average of the returns of the 5 portfolios. Plot the log cumulative returns of the portfolio (reinvested at each day), with time on the x-axis.

6. We now consider investing in each of the five portfolios. Assume that we can borrow at the same 7% interest rate, and that there are no limits on shorting or leverage. Use the historical mean and the covariance matrix to compute half-Kelly weights and report them. Also report the sum of the weights and the sum of the absolute value of the weights. Comment briefly.

7. A recently proposed alternative to mean-variance is Enhanced Portfolio Optimization, which can be seen as merging elements of mean-variance and volatility weighting. EPO implies replacing the covariance matrix V with $V_{epo} = \alpha V + (1-\alpha)D$, where D is a diagonal matrix with $D[i,i] = V[i,i]$. The portfolio weights are then found with the mean-variance formula, which for us using half-Kelly is $w = 0.5V_{epo}^{-1}m$, where $m = E(R)$. Values of α can be cross-validated, but we'll take $\alpha = 1/3$. Notice that the resulting V_{epo} is used only to select the weights, and should

not be used to compute any risk measure. Compute the weights, and also report the sum of the weights and the sum of the absolute value of the weights. Comment briefly.

8. The low profitability portfolio may generate lower returns because it has a lower correlation with the market. Is that the case? Consider monthly cumulative log returns. (Approximate the market as the equal-weight average of the 5 portfolio from question 5). Or perhaps the 1st quintile could be less left skewed, and therefore less risky in a sense. Is that the case? Compute the 5% percentile divided by std for cumulative monthly log returns to answer that question.

9. Using the weights from (7), compute the Sharpe ratio of the portfolio.