MASTER MUNDUS MIR / M1 ROC

Assignment - Dynamic Modeling - 2024-2025

All answers must be justified.

Ball regulator

The ball regulator (Fig. 1), also known as the centrifugal ball regulator system (S), is represented by the model shown in Fig. 1. The mechanical device consists of two spherical balls B_1 and B_2 of mass M and connected to a vertical axis by two rods of length L =distance OB_1 = distance OB_2. Rods A_1C and A_2C are of length a, and distances OA_1 and OA_2 are also equal to a. The joints at O, $A1$ and $A2$ are considered as perfect pivot joints (no friction). The joint at C is also a perfect sliding-pivot joint. Points O and C are connected by a spring of stiffness constant k, zero length at rest and negligible mass. Distance OC is named r. All the rods have a negligible mass w.r.t the spherical balls.
OA_1A_2C forms a rhombus which remains constantly in the vertical Oxz plane. (S) can rotate around axis (OC) with scalar angular velocity $\Omega = \dot{\phi}$. Point O is fixed to the axis, while point C can slide without friction along (Oz). Let $R(O, xyz)$ be the reference frame in rotational motion relative to the world reference frame $R_0(O, x_0y_0z_0)$, assumed to be Galilean, with $\vec{\Omega}(R/R0) = \dot{\phi}\vec{z}$.

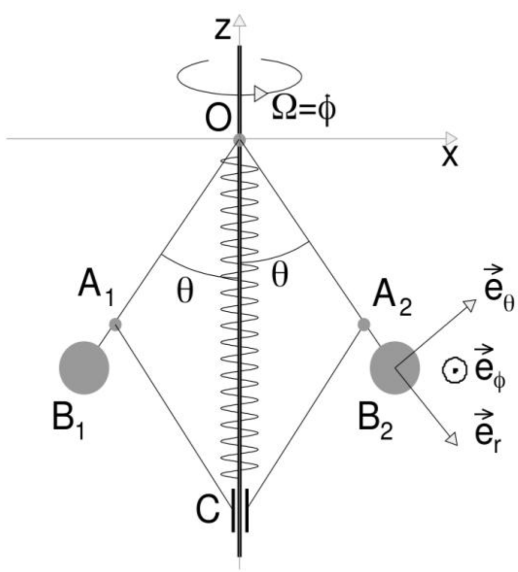

FIGURE 1 – Ball regulator

The gravity constant is noted g.
The moment of inertia of the sphere with respect to its diameter will be designated by I_s.

Questions

1. The set of generalized coordinates is (r, θ, ϕ). Give the geometric constraint(s) of the system (S)[1]. How many independent degrees of freedom does the system have? Give a brief description of them.
2. Now, the new set of coordinates is $q = (q_1, q_2) = (\theta, \phi)$. Give the direct geometric model that expresses the Cartesian coordinates in R_0 of the ball center B_2 as a function of the generalized coordinates ($\vec{x}$ is the same direction as $\vec{r}_{A_1A_2}$, and $\phi = \widehat{(\vec{x}_0, \vec{x})}$).
3. Calculate the Jacobian matrix that gives the variations of the Cartesian coordinates of B_2 as as function of the variations of the generalized coordinates.
4. Express the generalized forces/torques, namely $Q_\theta(\vec{W})$ and $Q_\phi(\vec{W})$ relative to the weight $\vec{W}$ of each of the spheres.
5. Express the generalized forces/torques, namely $Q_\theta(\vec{S})$ and $Q_\phi(\vec{S})$ relative to the spring force $\vec{S}$.
6. What are the expressions of $Q_\theta(\vec{R}_O)$ and $Q_\phi(\vec{R}_O)$, where $\vec{R}_O$ is the reaction force at O of the vertical axis on the system?
7. What are the expressions of $Q_\theta(\vec{R}_C)$ and $Q_\phi(\vec{R}_C)$, where $\vec{R}_C$ is the reaction force at C of the vertical axis on the system?
8. Identify the conservative forces exerted on the device, and give the potential energy they derive from. The total potential energy U will be set to 0 for $r = 0$ and $\theta = \pi/2$.
   N.B : a force $\vec{F}$ that derives from a potential energy E can be expressed as $\vec{F} = -\vec{\nabla}E$. The virtual elementary work is linked to the differential of energy by $\delta W = \vec{F}.d\vec{M} = -\vec{\nabla}E.d\vec{M} = -dE$, where $d\vec{M}$ is the displacement vector of the application point of the force (or angular displacement vector in case of a moment of force).
9. Calculate the translational kinetic energy T_t
10. Calculate the rotational kinetic energy T_r (use the moment of inertia of the sphere with respect to its diameter).
11. Proof that Lagrangian $L = (ML^2 + I_s).\dot{\theta}^2 + (ML^2 \sin^2\theta + I_s)\dot{\phi}^2 + 2.MgL.\cos\theta - 2.k.a^2.\cos^2\theta$
12. Calculate the partial derivatives of the Lagrangian to be used for the differential equations of Lagrange.
13. Express the differential equations of Lagrange.
14. Use the matrix format to present the equations of Lagrange.
15. The system is driven in pivoting rotation with constant angular velocity Ω_0. Express $\cos\theta$ as a function of Ω_0, a, k, M, L and g.
16. From the previous motion state, imagine that the pivoting velocity is suddenly decreased to an angular velocity Ω_0^- due to some kind of disturbance, how will the system evolve? Same question if the velocity is set to Ω_0^+. Explain. Indication : use the 1st differential equation to get the sign of $\ddot{\theta}$ after replacing $\dot{\phi}$ by $\Omega_0^{+/-}$, then deduce the sign of $\ddot{\phi}$ using the second equation.
17. The system is now pivoting at constant angular speed Ω_0, which enables the device to be in equilibrium in the vertical plane. Represent the forces exerted on the right part of the system on the figure in the case of equilibrium in the vertical plane of the device, including the inertial centrifugal force due to the pivoting rotation, the weight and the reaction forces from the vertical axis. Make sure the force polygon is closed. Indication : rods A_1C and A_2C have no mass, so the reaction force at C from the axis is along the rod.

1. (S) does not include the spring