

$O(n^3)$ solution is simple.

You look at every pair (i,j) and compute the sum of elements between i and j and then take the maximum of the sums.

Obviously, this is not the best solution.

Next improvement is $O(n^2)$ when you notice that you don't need to sum up all the elements between i and j . You can just add $A[j]$ to the sum you have already calculated in the previous loop from i to $j-1$.

However, we are looking for something better than N^2 .

For $O(n)$ solution can you look at optimal subarray and deduce some observations from it?
If you start taking every element greedily when should you stop?