

Problem Statement

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1 Equations

In this section, we summarize the variables and partial differential equations that will be used to implement Physics Informed Neural Networks. The following convention is adopted from [1]. (Standard notation for space, time, velocity and pressure is used and not defined here.)

Specific terms for multiphase flows using the Continuum Surface Force approach:

σ = Surface tension coefficient

κ = Curvature of the interface

n = Surface normal of the interface

δ = Dirac delta function (1 at the interface and 0 everywhere else)

ϕ = Junction angle of the side channel relative to the main channel

α = Volume Fraction

Where,

$$\begin{aligned} n &= \nabla \alpha \\ \hat{n} &= \frac{n}{|n|} = \frac{\nabla \alpha}{|\nabla \alpha|} \\ \kappa &= -\nabla \cdot \hat{n} \end{aligned}$$

Variables used for non-dimensionalizing the conservation equations:

W_c = Main channel width

W_d = Side channel width

U_c = Average inlet velocity in the main channel

U_d = Average inlet velocity in the outlet channel

μ_c = Molecular viscosity of the continuous phase

μ_d = Molecular viscosity of the dispersed phase

The definitions of the non-dimensional variables:

$$x^* = \frac{x}{W_c}; \quad y^* = \frac{y}{W_c}; \quad u^* = \frac{u}{U_c}; \quad t^* = \frac{U_c}{W_c} t; \quad p^* = \frac{W_c}{\mu_c U_c} p; \quad \kappa^* = W_c \kappa; \quad n^* = W_c n$$

Bulk properties of the fluid are calculated as the volume weighted average of the two flows:

$$\begin{aligned} \rho &= \alpha \rho_c + (1 - \alpha) \rho_d \\ \mu &= \alpha \mu_c + (1 - \alpha) \mu_d \end{aligned}$$

Non-dimensional conservation equations for the continuous phase (Star notation (*) is dropped from this point onwards):

$$\begin{aligned} \nabla \cdot u &= 0 \\ Re \left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] &= \nabla \cdot \{ -pI + [\nabla u + (\nabla u)^T] \} + \frac{1}{Ca} \kappa n \delta \\ \frac{\partial \alpha}{\partial t} + (u \cdot \nabla) \alpha &= 0 \end{aligned}$$

Momentum Equation rewritten for the dispersed phase:

$$\gamma Re \left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = \nabla \cdot \{ -pI + \beta [\nabla u + (\nabla u)^T] \} + \frac{1}{Ca} \kappa n \delta$$

Where,

$$Re = \frac{\rho_c U_c W_c}{\mu_c}; \quad Ca = \frac{\mu_c U_c}{\sigma}; \quad \gamma = \frac{\rho_d}{\rho_c}; \quad \beta = \frac{\mu_d}{\mu_c}$$

2 PDEs, IC and BCs in Cartesian form

The continuity equation can be represented as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1)$$

The N-S equation for X -direction can be represented as follows (using the continuous phase convention):

$$Re \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{Ca} \delta \frac{\partial \alpha}{\partial x} \nabla \cdot \hat{n} \quad (2)$$

Similarly, the N-S equation for Y-direction is:

$$Re \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{1}{Ca} \delta \frac{\partial \alpha}{\partial y} \nabla \cdot \hat{n} \quad (3)$$

Where,

$$\nabla \cdot \hat{n} = \left(\frac{\frac{\partial^2 \alpha}{\partial x^2} + \frac{\partial^2 \alpha}{\partial y^2}}{\sqrt{\left(\frac{\partial \alpha}{\partial x} \right)^2 + \left(\frac{\partial \alpha}{\partial y} \right)^2}} \right)$$

The convection-diffusion equation for the volume fraction

$$\frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} + v \frac{\partial \alpha}{\partial y} = 0 \quad (4)$$

The initial conditions could be random, because we are interested in behavior after long time. (To be discussed)

For boundary conditions, the three inlets are provided with fully developed Poiseuille flow with the following average velocities. Additionally, extra terms have been defined.

$$\begin{aligned} U_c &= 1 \\ U_d &= \frac{w_f}{2\lambda(1-w_f)} \\ w_f &= \frac{Q_{d1} + Q_{d2}}{Q_c} \\ \lambda &= \frac{W_d}{W_c} \end{aligned}$$

The walls of the microfluidic channel are assigned with no-slip condition.

$$u = 0; \quad v = 0 \quad (5)$$

For adjusting the normal of the interface near the walls, wall adhesion boundary condition is used. For this study, $\theta_{wa} = 180$

$$\hat{n} = \hat{n}_{wa} \cos \theta_{wa} + \hat{\tau}_{wa} \sin \theta_{wa} \quad (6)$$

References

- [1] Ich-Long Ngo, Sang Woo Joo, and Chan Byon. “Effects of Junction Angle and Viscosity Ratio on Droplet Formation in Microfluidic Cross-Junction”. In: *Journal of Fluids Engineering* 138.5 (Jan. 2016). 051202. ISSN: 0098-2202. DOI: [10.1115/1.4031881](https://doi.org/10.1115/1.4031881). eprint: https://asmedigitalcollection.asme.org/fluidsengineering/article-pdf/138/5/051202/6195577/fe_138_05_051202.pdf. URL: <https://doi.org/10.1115/1.4031881>.