4.3 : 2, 6, 8, 15, 17, 19, 26
4.4 : 14, 16, 38, 39, 42

4.3 2. $\lim_{x \to 2} \frac{\ln(2x-3)}{x^2-4} \overset{H}{=} \lim_{x \to 2} \frac{\frac{2}{2x-3}}{2x} = \frac{1}{2}$

6. $\lim_{x \to 1} \frac{x^{1/3}-1}{x^{2/3}-1} \overset{H}{=} \lim_{x \to 1} \frac{\frac{1}{3}x^{-2/3}}{\frac{2}{3}x^{-1/3}} = \frac{1}{2}$

8. $\lim_{x \to 0} \frac{1-\cos(x)}{\ln(1+x^2)} \overset{H}{=} \lim_{x \to 0} \frac{\sin(x)}{\frac{2x}{1+x^2}} = \lim_{x \to 0} \frac{(1+x^2)\cdot\sin(x)}{2x}$

$\overset{H}{=} \lim_{x \to 0} \frac{\cos(x)(1+x^2) + 2x\sin(x)}{2} = \frac{1}{2}$

15. $\lim_{x \to 0} \frac{x-\sin(x)}{x-\tan(x)} \overset{H}{=} \lim_{x \to 0} \frac{1-\cos(x)}{1-\frac{1}{\cos^2(x)}} = \lim_{x \to 0} \frac{\cos^2(x)-\cos^3(x)}{\cos^2(x)-1}$

$= \lim_{x \to 0} \frac{\cos^2(x)(\cancel{1-\cos(x)})}{-(\cancel{1-\cos(x)})(1+\cos(x))} = \lim_{x \to 0} \frac{\cos^2(x)}{-(1+\cos(x))} = -\frac{1}{2}$

17. $\lim_{x \to 0^+} \frac{\sin^2(x)}{\tan(x)-x} \overset{H}{=} \lim_{x \to 0^+} \frac{2\sin(x)\cdot\cos(x)}{\frac{1}{\cos^2(x)}-1} = \lim_{x \to 0^+} \frac{2\sin(x)\cos^3(x)}{1-\cos^2(x)}$

$= \lim_{x \to 0^+} \frac{2\cos^3(x)}{\sin(x)} = +\infty$ ($\sin(x) \to 0$ and $\sin(x) > 0$ if $x > 0$)

19. $\lim_{t \to \frac{\pi}{2}} \frac{\sin(t)}{t} = \frac{2}{\pi}$ (not an indeterminate form)

26. $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln(x)} \right) = \lim_{x \to 1^+} \frac{x\cdot\ln(x) - (x-1)}{(x-1)\cdot\ln(x)} \overset{H}{=} \lim_{x \to 1^+} \frac{\ln(x)}{\ln(x) + (1-\frac{1}{x})}$

$\overset{H}{=} \lim_{x \to 1^+} \frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \frac{1}{2}$

4.4 14, 16, 38, 39, 52

14. $f(x) = |x^2-x-2|$ on $[-3,3]$ $\qquad f(x) = |(x-2)(x+1)|$

- end points : $f(-3) = |9+3-2| = 10$
  $f(3) = |9-3-2| = 4$

- singular points
$$f(2) = f(-1) = 0$$
- critical points $\quad f'(x) = \begin{cases} 2x-1 & x < -1 \text{ or } x > 2 \\ -2x+1 & -1 < x < 2 \end{cases}$

$f'(x) = 0$ for $x = \frac{1}{2}$, $f(\frac{1}{2}) = |\frac{1}{4} - \frac{1}{2} - 2| = 2.25$

$\Rightarrow$ absolute maximum : -3, $f(-3) = 10$
absolute minimum : $-1, 2$, $f(-1) = f(2) = 0$

local maximum at $x = \frac{1}{2}$ $\quad$ ($f'(x) > 0$ for $x < \frac{1}{2}$
$f'(x) < 0$ $\quad x > \frac{1}{2}$)

endpoint $x = 3$ is also a local maximum ($f'(x) > 0$ for $x < 3$)

16. $f(x) = (x+2)^{2/3}$

$\lim_{x \to \pm\infty} f(x) = +\infty$, so there cannot be an absolute maximum.

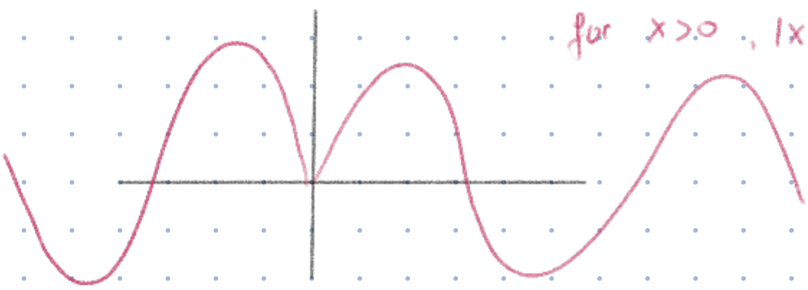
$f(x) = \left(\sqrt[3]{x+2}\right)^2 \geq 0$, so at $x = -2$, $f(x) = 0$ is an absolute min.

$$f'(x) = \frac{2}{3}(x+2)^{-1/3}$$

$\rightarrow f'(x) < 0$ for $x < -2$
> 0 $\quad x > -2$

$x = -2$ is a singular point ($f'(-2)$ DNE)

38. $f(x) = \sin(|x|)$



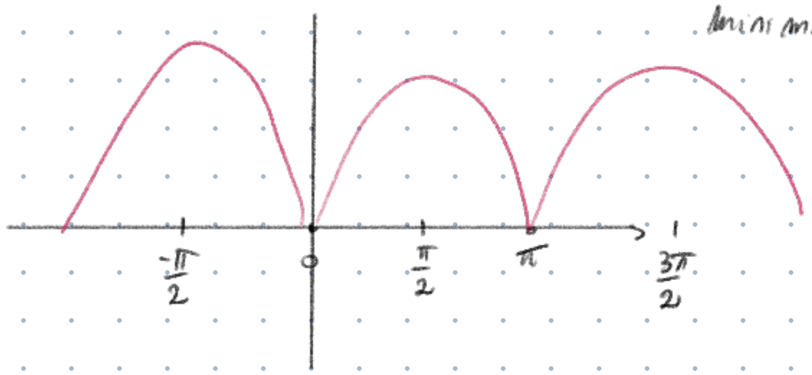
for $x > 0$, $|x| = x$, $\sin(|x|) = \sin(x)$

absolute ~~maxima~~ minima at
$\frac{3\pi}{2}$ / $\frac{\pi}{2} + 2k\pi$, $k > 0$
$-\frac{3\pi}{2}$ / $-\frac{\pi}{2} + 2k\pi$, $k < 0$

for $x < 0$, $|x| = -x$, so $\sin(|x|) = -\sin(x)$

39. $f(x) = |\sin(x)|$

absolute maxima at $\frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$
minima at $k\pi, \quad k \in \mathbb{Z}$



42. $f(x) = \frac{x}{\sqrt{x^4+1}}$

$\lim_{x \to \pm\infty} f(x) = 0$ since $f(1) = \frac{1}{\sqrt{2}} > 0$ and $f(-1) = \frac{-1}{\sqrt{2}} < 0$

we have an absolute minimum and maximum.

$$f'(x) = \frac{\sqrt{x^4+1} - \frac{4x^3 \cdot x}{2\sqrt{x^4+1}}}{x^4+1} = \frac{(x^4+1) - 2x^4}{(\sqrt{x^4+1})^3} = \frac{1-x^4}{\sqrt{x^4+1}^3}$$

$f'(x) = 0$ at $x = \pm 1$, $f(x)$ increases for $-1 < x < 1$
decreases $x < -1$ and $x > 1$

$\longrightarrow$ we have an absolute maximum at $x = 1$
minimum at $x = -1$