EXERCISES 12.1

Specify the domains of the functions in Exercises 1–10.

1. $f(x, y) = \dfrac{x+y}{x-y}$
2. $f(x, y) = \sqrt{xy}$
3. $f(x, y) = \dfrac{x}{x^2+y^2}$
4. $f(x, y) = \dfrac{xy}{x^2-y^2}$
5. $f(x, y) = \sqrt{4x^2+9y^2-36}$

Sketch some of the level curves of the functions in Exercises 19–26.

19. $f(x, y) = x - y$
20. $f(x, y) = x^2 + 2y^2$
21. $f(x, y) = xy$
22. $f(x, y) = \dfrac{x^2}{y}$
23. $f(x, y) = \dfrac{x-y}{x+y}$
24. $f(x, y) = \dfrac{y}{x^2+y^2}$
25. $f(x, y) = xe^{-y}$
26. $f(x, y) = \sqrt{\dfrac{1}{y} - x^2}$

EXERCISES 12.4

In Exercises 1–6, find all the second partial derivatives of the given function.

1. $z = x^2(1 + y^2)$
2. $f(x, y) = x^2 + y^2$
3. $w = x^3y^3z^3$
4. $z = \sqrt{3x^2 + y^2}$
5. $z = x\,e^y - y\,e^x$
6. $f(x, y) = \ln\bigl(1 + \sin(xy)\bigr)$

EXERCISES 12.3

In Exercises 1–10, find all the first partial derivatives of the function specified, and evaluate them at the given point.

1. $f(x, y) = x - y + 2, \quad (3, 2)$
2. $f(x, y) = xy + x^2, \quad (2, 0)$
3. $f(x, y, z) = x^3y^4z^5, \quad (0, -1, -1)$
4. $g(x, y, z) = \dfrac{xz}{y+z}, \quad (1, 1, 1)$
5. $z = \tan^{-1}\left(\dfrac{y}{x}\right), \quad (-1, 1)$
6. $w = \ln(1 + e^{xyz}), \quad (2, 0, -1)$
7. $f(x, y) = \sin(x\sqrt{y}), \quad \left(\dfrac{\pi}{3}, 4\right)$
8. $f(x, y) = \dfrac{1}{\sqrt{x^2+y^2}}, \quad (-3, 4)$
9. $w = x^{(y \ln z)}, \quad (e, 2, e)$
10. $g(x_1, x_2, x_3, x_4) = \dfrac{x_1 - x_2^2}{x_3 + x_4^2}, \quad (3, 1, -1, -2)$

In Exercises 11–12, calculate the first partial derivatives of the given functions at $(0, 0)$. You will have to use Definition 4.

11. $f(x, y) = \begin{cases} \dfrac{2x^3 - y^3}{x^2 + 3y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$
12. $f(x, y) = \begin{cases} \dfrac{x^2 - 2y^2}{x - y}, & \text{if } x \neq y \\ 0, & \text{if } x = y. \end{cases}$

In Exercises 13–22, find equations of the tangent plane and normal line to the graph of the given function at the point with specified values of x and y.

13. $f(x, y) = x^2 - y^2$ at $(-2, 1)$
14. $f(x, y) = \dfrac{x-y}{x+y}$ at $(1, 1)$
15. $f(x, y) = \cos(x/y)$ at $(\pi, 4)$
16. $f(x, y) = e^{xy}$ at $(2, 0)$
17. $f(x, y) = \dfrac{x}{x^2+y^2}$ at $(1, 2)$
18. $f(x, y) = y\,e^{-x^2}$ at $(0, 1)$
19. $f(x, y) = \ln(x^2 + y^2)$ at $(1, -2)$
20. $f(x, y) = \dfrac{2xy}{x^2+y^2}$ at $(0, 2)$
21. $f(x, y) = \tan^{-1}(y/x)$ at $(1, -1)$
22. $f(x, y) = \sqrt{1 + x^3y^2}$ at $(2, 1)$
23. Find the coordinates of all points on the surface with equation $z = x^4 - 4xy^3 + 6y^2 - 2$ where the surface has a horizontal tangent plane.
24. Find all horizontal planes that are tangent to the surface with equation $z = xye^{-(x^2+y^2)/2}$. At what points are they tangent?

EXERCISES 12.5

In Exercises 1–4, write appropriate versions of the Chain Rule for the indicated derivatives.

1. $\partial w/\partial t$ if $w = f(x, y, z)$, where $x = g(s, t)$, $y = h(s, t)$, and $z = k(s, t)$
2. $\partial w/\partial t$ if $w = f(x, y, z)$, where $x = g(s)$, $y = h(s, t)$, and $z = k(t)$
3. $\partial z/\partial u$ if $z = g(x, y)$, where $y = f(x)$ and $x = h(u, v)$
4. dw/dt if $w = f(x, y)$, $x = g(r, s)$, $y = h(r, t)$, $r = k(s, t)$, and $s = m(t)$
5. If $w = f(x, y, z)$, where $x = g(y, z)$ and $y = h(z)$, state appropriate versions of the Chain Rule for $\dfrac{dw}{dz}$, $\left(\dfrac{\partial w}{\partial z}\right)_x$, and $\left(\dfrac{\partial w}{\partial z}\right)_{x,y}$.
6. Use two different methods to calculate $\partial u/\partial t$ if $u = \sqrt{x^2 + y^2}$, $x = e^{st}$, and $y = 1 + s^2 \cos t$.
7. Use two different methods to calculate $\partial z/\partial x$ if $z = \tan^{-1}(u/v)$, $u = 2x + y$, and $v = 3x - y$.
8. Use two methods to calculate dz/dt given that $z = txy^2$, $x = t + \ln(y + t^2)$, and $y = e^t$.

In Exercises 9–12, find the indicated derivatives, assuming that the function $f(x, y)$ has continuous first partial derivatives.

9. $\dfrac{\partial}{\partial x} f(2x, 3y)$
10. $\dfrac{\partial}{\partial x} f(2y, 3x)$
11. $\dfrac{\partial}{\partial x} f(y^2, x^2)$
12. $\dfrac{\partial}{\partial y} f\Bigl(yf(x, t), f(y, t)\Bigr)$