- $\lim_{x \to 0} \frac{|x-1| - |2x+1|}{x} = \lim_{x \to 0} \frac{(\cancel{1}-x) - (2x+\cancel{1})}{x} = \lim_{x \to 0} \frac{-3x}{x} = -3$

since $x \to 0$, $1-x > 0$ and $2x+1 > 0$, both for $x>0$ and $x<0$

- $\lim_{x \to \infty} \sqrt{x^2 - x} - x = \lim_{x \to +\infty} \frac{(\sqrt{x^2-x} - x)(\sqrt{x^2-x} + x)}{\sqrt{x^2-x} + x}$

$= \lim_{x \to \infty} \frac{(\cancel{x^2} - x) - \cancel{x^2}}{\sqrt{x^2-x} + x} = \lim_{x \to \infty} \frac{-x}{x\sqrt{1-\frac{1}{x}} + x} = \lim_{x \to \infty} \frac{-1}{\sqrt{1-\frac{1}{x}}+1} = -\frac{1}{2}$

- $\lim_{x \to -\infty} \underset{\downarrow \atop +\infty}{\sqrt{x^2-x}} \underbrace{-x}_{+\infty} = +\infty$

- $\lim_{x \to 3^-} \left(\frac{1}{x-3} + \frac{6}{9-x^2}\right) = \lim_{x \to 3^-} \left(\frac{-(x+3) + 6}{9-x^2}\right) = \lim_{x \to 3^-} \frac{\cancel{3-x}}{(\cancel{3-x})(3+x)} = \frac{1}{6}$

- $\lim_{x \to 1} \frac{x^2-1}{|x-1|}$ DNE : left limit $\neq$ right limit.

left limit : $\lim_{x \to 1^-} \frac{(\cancel{x-1})(x+1)}{\cancel{1-x} \; -1} = \lim_{x \to 1^-} -(x+1) = -2$

$x<1$
$1-x>0$

right limit : $\lim_{x \to 1^+} \frac{(\cancel{x-1})(x+1)}{\cancel{x-1}} = 2$

$x>1$
$x-1>0$