Exercises Tutorial 2 (12-04-2024)

Exercise 1

Determine the value of c so that each of the following functions can serve as a probability distribution of the discrete random variable X:

a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$

b) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2$

Exercise 2

Classify the following random variables as discrete or continuous:

a) X: the number of automobile accidents per year in Virginia

b) Y: the length of time to play 18 holes of golf

c) M: the amount of milk produced yearly by a particular cow

d) N: the number of eggs laid each month by a hen

e) P: the number of building permits issued each month in a certain city

f) Q: the weight of grain produced per acre

Exercise 3

Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value of W.

Exercise 4

A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

Exercise 5

The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function of X.

Exercise 6

The shelf life, in days, for bottles of a certain prescribed medicines a random variable having the density function

$$f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3} & x > 0, \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of

a) at least 200 days

b) anywhere from 80 to 120 days

Exercise 7

The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year in a continuous random variable X that has a density function

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \leq x \leq 2, \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

a) less than 120 hours

b) between 50 and 100 hours

Exercise 8

A continuous random variable X that can assume values between $x = 1$ and $x = 3$ has a density function given by

$$f(x) = \tfrac{1}{2}$$

a) Show that the area under the curve is equal to 1.

b) Find $P(2 < X < 2.5)$

c) Find $P(X \leq 1.6)$

Exercise 9

A continuous random variable X that can have values between $x = 2$ and $x = 5$ has a density function given by

$$f(x) = \tfrac{2(1+x)}{27}$$

Find

a) $P(X < 4)$

b) $P(3 \leq X < 4)$

Exercise 10

The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5} & 0 < x < 1, \\ 0 & \text{elsewhere} \end{cases}$$

a) Show that $P(0 < X < 1) = 1$

b) Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

Exercise 11

An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3\,x^{-4} & x > 1, \\ 0 & \text{elsewhere} \end{cases}$$

a) Verify that this is a valid density function

b) Find $F(x)$

c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

Exercise 12

Based on extensive testing, it is determined by the manufacturer of a washing machine that the time Y (in years) before a major repair is required is characterized by the probability density function

$$f(y) = \begin{cases} \frac{1}{4}e^{\frac{-y}{4}} & y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

a) Critics would certainly consider the product a bargain if it is unlikely to require a major repair before the sixth year. Comment on this by determining $P(Y > 6)$.

b) What is the probability that a major repair occurs in the first year?

Exercise 13

Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The probability density function that characterizes the proportion Y that make a profit is given by

$$f(y) = \begin{cases} k\,y^4(1-y)^3 & 0 \le y \le 1, \\ 0 & \text{elsewhere} \end{cases}$$

a) What is the value of k that renders the above a valid density function?

b) Find the probability that at most 50 % of the firms make a profit in the first year?

c) Find the probability that at most 80 % of the firms make a profit in the first year?

Exercise 14

In the senior year of a high school graduating class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history but neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three. Randomly select a student from the class and find the probabilities of the following events.

a) A person enrolled in psychology takes all three subjects.

b) A person not taking psychology is taking both history and mathematics.

Exercise 15

The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; the probability that both the oil and the filter need changing is 0.14.

a) If the oil has to be changed, what is the probability that a new oil filter is needed?

b) If a new oil filter is needed, what is the probability that the oil has to be changed?

Exercise 16

The probability that a vehicle entering the Luray Caverns has Canadian licence plates is 0.12; the probability that it is a camper is 0.28; the probability that it is a camper with Canadian licence plate is 0.09. What is the probability that

a) a camper entering the Luray Caverns has Canadian licence plates?

b) a vehicle with Canadian licence plates entering the Luray Caverns is a Camper?

c) a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper?