12.1 : 1, 2, 3, 4, 19, 20
12.3 : 1, 2, 3, 4, 8, 17, 18, 19
12.4 : 1, 3
12.5 : 2, 8, 9, 11

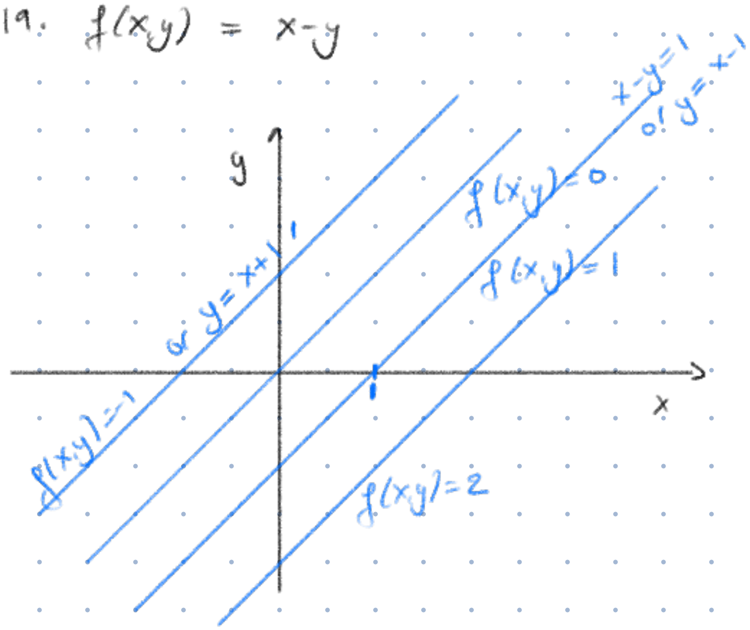
1. $f(x,y) = \frac{x+y}{x-y}$ domain $\{(x,y) \in \mathbb{R}^2 : x \neq y\}$

2. $f(x,y) = \sqrt{xy}$ domain $\{(x,y) \in \mathbb{R} : (x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0)\}$

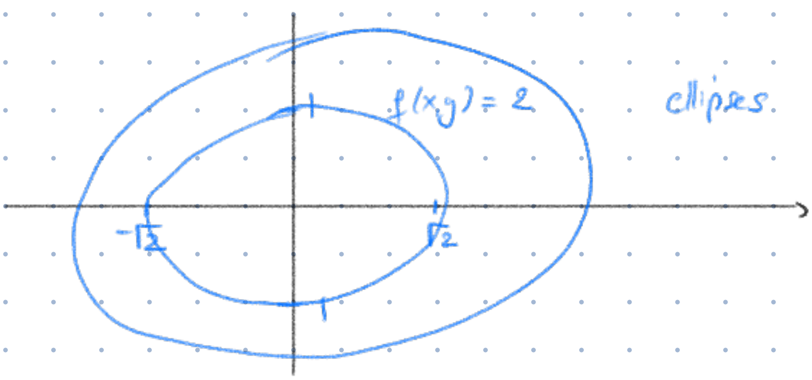
3. $f(x,y) = \frac{x}{x^2+y^2}$ domain $\mathbb{R}^2 \setminus \{(0,0)\}$

4. $f(x,y) = \frac{xy}{x^2-y^2}$ domain $\{(x,y) \in \mathbb{R} : x \neq y \wedge x \neq -y\}$

19. $f(x,y) = x-y$



20. $f(x,y) = x^2 + 2y^2$



12.3 1-4, 8, 17-19

1. $f(x,y) = x - y + 2$

$\frac{\partial f}{\partial x} = 1 \quad , \frac{\partial f}{\partial y} = -1 \qquad \forall (x,y) \in \mathbb{R}^2$

2 $f(x,y) = xy + x^2$

$\frac{\partial f}{\partial x} = y + 2x \qquad \frac{\partial f}{\partial x}(2,0) = 4 \qquad \frac{\partial f}{\partial y} = x \quad , \frac{\partial f}{\partial y}(2,0) = 2$

3. $f(x,y,z) = x^3 y^4 z^5 \qquad \frac{\partial f}{\partial x} = 3x^2 y^4 z^5 \qquad \frac{\partial f}{\partial y} = x^3 \, 4y^3 \, z^5 \qquad \frac{\partial f}{\partial z} = x^3 y^4 5z^4$

$= 0$ for $x=0$ $\qquad = 0$ for $x=0$ $\qquad = 0$ for $x=0$

4. $g(x,y,z) = \frac{xz}{y+z} \qquad \frac{\partial f}{\partial x} = \frac{z}{y+z} \qquad \frac{\partial f}{\partial y} = \frac{-xz}{(y+z)^2} \qquad \frac{\partial f}{\partial z} = \frac{x(y+z) - xz}{(y+z)^2}$

$= \frac{1}{2} \qquad = \frac{-1}{4} \qquad = \frac{1}{4}$

8. $f(x,y) = \frac{1}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} = \frac{4}{5}$ at $(-3,4)$

$= \frac{-3}{5}$ at $(-3,4)$

17. $f(x,y) = \frac{x}{x^2+y^2} \qquad \frac{\partial f}{\partial x} = \frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} \qquad \frac{\partial f}{\partial x}(1,2) = \frac{4-1}{5^2} = \frac{3}{25}$

$f(1,2) = \frac{1}{5} \qquad \frac{\partial f}{\partial y} = \frac{-2xy}{(x^2+y^2)^2} \qquad \frac{\partial f}{\partial y}(1,2) = \frac{-2 \cdot 1 \cdot 2}{5^2} = \frac{-4}{25}$

tangent plane : $z = \frac{1}{5} + \frac{3}{25}(x-1) - \frac{4}{25}(y-2)$

$= \frac{2}{5} + \frac{3x}{25} - \frac{4y}{25}$

18. $f(x,y) = ye^{-x^2} \qquad \frac{\partial f}{\partial x} = -2xy\,e^{-x^2} \qquad \frac{\partial f}{\partial y} = e^{-x^2}$

$f(0,1) = 1 \qquad = 0$ at $x=0 \qquad = 1$ at $x=0$

tangent plane $z = 1 + 0\cdot(x-0) + 1\cdot(y-1)$
$= y$

19. $f(x,y) = \ln(x^2+y^2)$ $\quad \frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2}$ $\quad \frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2}$

$f(1,-2) = \ln(5)$ $\quad = \frac{2}{5}$ at $(1,-2)$ $\quad = \frac{-4}{5}$ at $(1,-2)$

tangent plane: $z = \ln(5) + \frac{2}{5}(x-1) - \frac{4}{5}(y+2)$
$= \ln(5) - 2 + \frac{2x}{5} - \frac{4y}{5}$

12.4 1. $z = x^2 + x^2y^2$ $\quad \frac{\partial z}{\partial x} = 2x + 2xy^2$ $\quad \frac{\partial^2 z}{\partial x^2} = 2 + 2y^2$ $\quad \frac{\partial^2 z}{\partial x \partial y} = 4xy$

$\frac{\partial z}{\partial y} = 2x^2y$ $\quad \frac{\partial^2 z}{\partial y^2} = 2x^2$

3. $w = x^3y^3z^3$

$\frac{\partial w}{\partial x} = 3x^2y^3z^3$ $\quad \frac{\partial^2 w}{\partial x^2} = 6xy^3z^3$ $\quad \frac{\partial^2 w}{\partial y \partial z} = 9x^3y^2z^2$

$\frac{\partial w}{\partial y} = 3x^3y^2z^3$ $\quad \frac{\partial^2 w}{\partial y^2} = 6x^3yz^3$ $\quad \frac{\partial^2 w}{\partial x \partial z} = 9x^2y^3z^2$

$\frac{\partial w}{\partial z} = 3x^3y^3z^2$ $\quad \frac{\partial^2 w}{\partial z^2} = 6x^3y^3z$ $\quad \frac{\partial^2 w}{\partial x \partial y} = 9x^2y^2z^3$

12.5 2, 8, 9, 11

2. $\frac{\partial w}{\partial t} = \cancel{\frac{\partial w}{\partial x}\frac{\partial x}{\partial t}} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{dz}{dt}$ → "normal" derivative, since z only depends on t.

0

$\frac{\partial x}{\partial t} = 0$, since x only depends on s

8. $\frac{dz}{dt} = \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x}\left(\frac{\partial x}{\partial t} + \frac{\partial x}{\partial y}\frac{dy}{dt}\right) + \frac{\partial z}{\partial y}\frac{dy}{dt}$ (method 1)

$= xy^2 + ty^2\left(1 + \frac{2t}{y+t^2} + \frac{1}{y+t^2}\cdot e^t\right) + 2txy\cdot e^t$

⇒ substitute $x = t + \ln(y+t^2)$, $y = e^t$

$z(t) = t\cdot(t + \ln(e^t + t^2))\, e^{2t} \to \frac{dz}{dt} = \ldots$ (calculate derivative)

9. $\frac{\partial}{\partial x} f(2x, 3y) = 2 f_1(2x, 3y)$

11. $\frac{\partial}{\partial x} f(y^2, x^2) = 2x f_2(y^2, x^2)$