1.2 2-6, 21, 22, 25, 28, 34, 36
1.3 19, 22, 33
1.4 1-2, 17,

1.2 2-6. $\lim_{x\to 1} g(x)$ does not exist, since $\lim_{x\to 1^+} g(x) = 0$ and $\lim_{x\to 1^-} g(x) = 1$

$\lim_{x\to 2} g(x) = 1$

$\lim_{x\to 3} g(x) = 0 = \lim_{x\to 3^+} g(x) = \lim_{x\to 3^-} g(x)$

21. $\lim_{x\to 0} \frac{|x-2|}{x-2} = \lim_{x\to 0} \frac{2-x}{x-2} = -1$

since x approaches 0, $x<2$, so $|x-2| = |2-x| = 2-x$

22. $\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$ (since $x>2$, $|x-2| = x-2$)

$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{2-x}{x-2} = -1$ (since $x<2$, $|x-2| = 2-x$)

=> left and right limit are different, so $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist.

25. $\lim_{t\to 1} \frac{t^2-1}{t^2-2t+1} = \lim_{t\to 1} \frac{(\cancel{t-1})(t+1)}{(t-1)^{\cancel{2}}} = \lim_{t\to 1} \frac{t+1}{t-1}$ does not exist

- $\lim_{t\to 1^+} \frac{t+1}{t-1} = +\infty$
- $\lim_{t\to 1^-} \frac{t+1}{t-1} = -\infty$

} left and right limit are different.

28. $\lim_{s\to 0} \frac{(s+1)^2-(s-1)^2}{s} = \lim_{s\to 0} \frac{(\cancel{s^2}+2s+\cancel{1})-(\cancel{s^2}-2s+\cancel{1})}{s} = \lim_{s\to 0} \frac{4s}{s} = 4$

34. $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{1}{x^2-4}\right) = \lim_{x\to 2} \frac{x+2-1}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x+1}{x+2} \cdot \frac{1}{x-2}$ does not exist.

$\frac{3}{4}$

- $\lim_{x\to 2^+} \frac{1}{x-2} = +\infty$
- $\lim_{x\to 2^-} \frac{1}{x-2} = -\infty$

} left and right limit are different

36. $\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} = \lim_{x\to 0} \frac{(\cancel{1}-3x)-(\cancel{1}+3x)}{x} = -6$

since $x \approx 0$, $|3x-1| = 1-3x$
$|3x+1| = 1+3x$

1.3. 19 $\lim_{x\to 2^+} \frac{x}{(2-x)^3} = -\infty$
$(2-x)^3 < 0$ if $x > 2$

22. $\lim_{x\to 1^-} \frac{1}{|x-1|} = +\infty$ $(|x-1| > 0)$

33. Asymptotes of $f(x) = \frac{1}{\sqrt{x^2-2x}-x}$

$$\lim_{x\to+\infty} \frac{1}{\sqrt{x^2-2x}-x} = \lim_{x\to+\infty} \frac{\sqrt{x^2-2x}+x}{(\cancel{x^2}-2x)-\cancel{x^2}} =$$

$$\lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1-\frac{2}{x}}+\cancel{x}}{-2\cancel{x}} = \lim_{x\to+\infty} \frac{\sqrt{1-\frac{2}{x}}+1}{-2} = -1$$

horizontal asymptote $y=-1$ (one-sided)

$$\lim_{x\to-\infty} \frac{1}{\sqrt{x^2-2x}-x} = 0 \longrightarrow \text{horizontal asymptote } y=0$$

$\hookrightarrow$ $\sqrt{x^2-2x} \to +\infty$
$-x \to +\infty$
as $x \to -\infty$

$$\lim_{x\to 0^-} \frac{1}{\underbrace{\sqrt{x^2-2x}}_{>0}\ \underbrace{-x}_{>0}} = +\infty \longrightarrow \text{vertical asymptote } x=0$$

there are no other vertical asymptotes, since

$\sqrt{x^2-2x}-x=0 \iff x=\sqrt{x^2-2x} \Rightarrow x^2 = x^2-2x$
$\Rightarrow x=0$

1.4 #2

-2 : continuous / right continuous (this is the same for an end point)

-1 : discontinuous, removable discontinuity
$g(-1) = 1$ would make the function continuous

0 : left continuous (discontinuous)

1 : right continuous (discontinuous)

2 : removable discontinuity
$g(2) = 0$ would make the function continuous

17. $f(x) = \begin{cases} x^2 & x \leq 2 \\ k - x^2 & x > 2 \end{cases}$

$f(2) = 4 = \lim_{x \to 2^+} (k - x^2) = k - 4 \Rightarrow k = 8$