2.1 12, 13, 16, 19, 21-24
2.2 1-6, 25-27
2.3 33-36
2.4 ; 22-28
2.5 : 11, 15, 28, 58
2.6 : 1, 19

2.1 12. tangent line to $f(x) = \frac{1}{x}$ at $(a, \frac{1}{a})$.

$$y = mx + b \qquad m = f'(a) = -\frac{1}{a^2}$$

$$\frac{1}{a} = -\frac{1}{a^2} \cdot a + b \Rightarrow b = \frac{2}{a}$$

solution: $y = -\frac{x}{a^2} + \frac{2}{a}$

13. $f(x) = \sqrt{|x|}$ does not have a tangent line, since

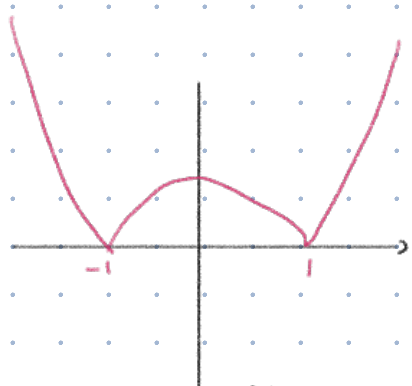
$$\lim_{h \to 0^+} \frac{f(h) - f(0)}{h} = \lim_{h \to 0^+} \frac{\sqrt{h} - 0}{h} = +\infty$$

$$\lim_{h \to 0^-} \frac{f(h) - f(0)}{h} = \lim_{h \to 0^-} \frac{\sqrt{|h|} - 0}{h} = -\infty$$

$\Rightarrow \lim_{h \to 0} \frac{f(h) - f(0)}{h}$ DNE $\rightarrow$ the tangent line cannot exist.

16. $f(x) = |x^2 - 1|$

$\rightarrow f(x) = x^2 - 1$ for $x \geq 1$ or $x \leq -1$
$= 1 - x^2$ for $-1 < x < 1$



$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{(1+h)^2 - 1}{h}$$

$$= \lim_{h \to 0^+} \frac{2h + h^2}{h} = 2$$

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{1 - (1+h)^2}{h} = -2$$

$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ DNE
$\hookrightarrow$ no tangent line.

19. Slope of the tangent line of $f(x) = x^3$ at $x = a$

(a) $= f'(a) = 3a^2$

(b) $f'(x) = 3x^2 = 3 \Rightarrow x = \pm 1$

↳ tangent lines at $(1,1)$ and $(-1,-1)$

at $(1,1)$: $y = \underbrace{3}_{m=f'(1)}x - 2$ at $(-1,-1)$ = $y = \underbrace{3}_{m=f'(-1)}x + 2$

21. $f(x) = x^3 - x + 1$

tangent line has slope 2?

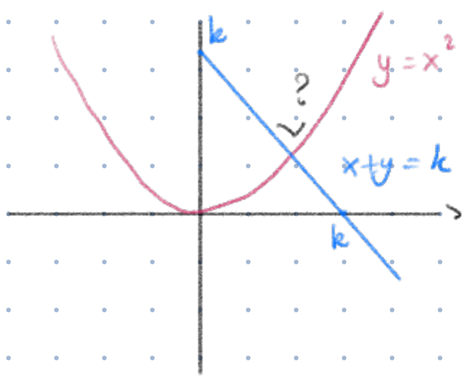
$f'(x) = 3x^2 - 1 = 2 \Leftrightarrow 3x^2 = 3 \Leftrightarrow x = \pm 1$

22. $f(x) = \frac{1}{x}$

tangent line perpendicular to $y = 4x - 3$ → slope: $-\frac{1}{4}$

$f'(x) = -\frac{1}{x^2} = -\frac{1}{4} \Rightarrow x = \pm 2$

23. $x + y = k$ and $y = x^2$ perpendicular at intersection?



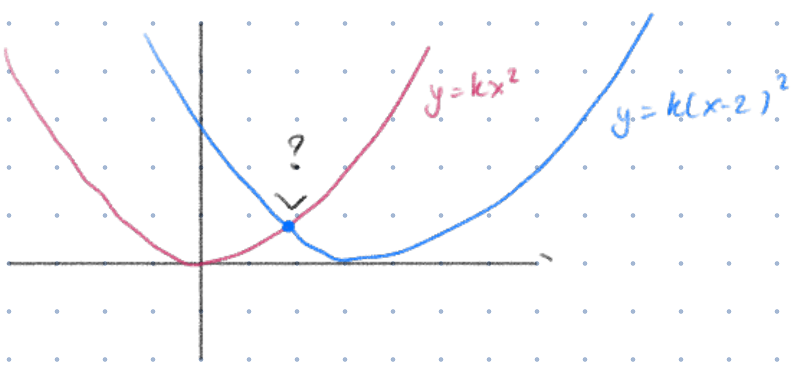
At intersection: $y = k - x = x^2$

perpendicular: $\underbrace{-1}_{\text{slope line}} = \underbrace{-\frac{1}{2x}}_{\frac{-1}{\text{slope parabola}}}$

$\Rightarrow x = \frac{1}{2}$

$\Rightarrow k - \frac{1}{2} = \frac{1}{4} \Rightarrow k = \frac{3}{4}$

24.



At intersection: $\cancel{k}x^2 = \cancel{k}(x-2)^2$

$\cancel{x^2} = \cancel{x^2} - 4x + 4$

$x = 1$

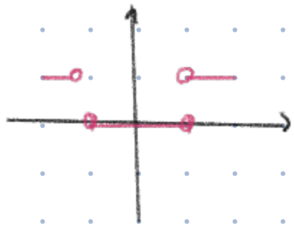
perpendicular at $x=1$: $2kx = \frac{-1}{2k(x-2)}$

$\Leftrightarrow 2k = \frac{1}{2k}$

$\Rightarrow k = \pm\frac{1}{2}$

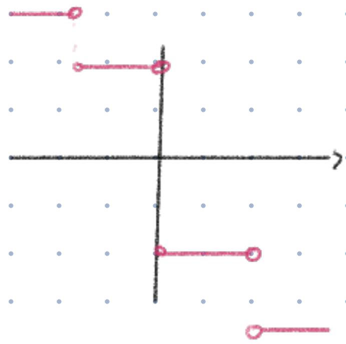
2.2

1)



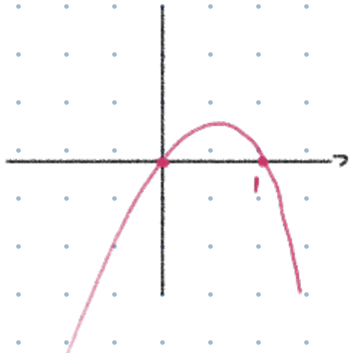
5) not differentiable at ± 1

2)

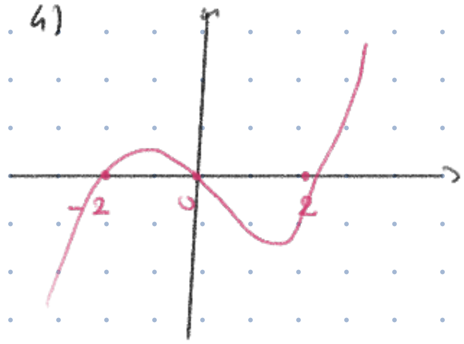


6) not differentiable at 0 and ± 1

3)



4)



25) $f(x) = x \cdot sgn(x) = |x|$ (at $x \neq 0$)

$\rightarrow$ if $f(0) = 0$, $f(x) = |x|$ is continuous.

$\rightarrow$ no, it is not differentiable, since $\lim\limits_{h\to 0^+} \underbrace{\frac{f(h)-f(0)}{h}}_{+1} \neq \lim\limits_{h\to 0^-} \underbrace{\frac{f(h)-f(0)}{h}}_{-1}$

$$\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^\pm} \frac{h \cdot sgn(h) - 0}{h} = \lim_{h\to 0^\pm} sgn(h) = +1$$
$= -1$

26) $g(x) = x^2 \cdot sgn(x) = x \cdot |x|$ (at $x \neq 0$)

$\rightarrow f(0) = 0$ makes $f(x)$ continuous.

$$\lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2 sgn(h) - 0}{h} = \lim_{h\to 0} h \cdot sgn(h) = 0$$

so $f(x)$ is differentiable at $x=0$ and $f'(0) = 0$

27) $h(x) = |x^2 + 3x + 2| = |(x+1)(x+2)|$

$= (x+1)(x+2)$ for $x \geqslant -1$ or $x \leqslant -2$ $\rightarrow f'(x) = 2x + 3$

$= -(x+1)(x+2)$ for $-2 < x < -1$ $\quad f'(x) = -2x - 3$

At $x=-1$, $f'_+(-1) = 2(-1)+3 = 1$ $\rightarrow f'(1)$ does not exist.

$f'_-(-1) = -2(-1)-3 = -1$

$x = -2$ $f'_-(-2) = 2(-2)+3 = -1$ $\rightarrow f'(2)$ does not exist.

$f'_+(-2) = -2(-2)-3 = +1$

2.3 33. $\frac{d}{dx}\left(\frac{x^2}{f(x)}\right)\Big|_{x=2} = \frac{2x \cdot f(x) - x^2 f'(x)}{(f(x))^2}\Big|_{x=2} = \frac{2\cdot 2\cdot f(2) - 2^2 f'(2)}{(f(2))^2} = \frac{8-4\cdot 3}{2^2} = -1$

34. $\frac{d}{dx}\left(\frac{f(x)}{x^2}\right)\Big|_{x=2} = \frac{f'(x)\cdot x^{\cancel{2}} - 2\cancel{x} f(x)}{x^{\cancel{4}\,3}}\Big|_{x=2} = \frac{f'(2)\cdot 2 - 2f(2)}{2^3} = \frac{6-4}{8} = \frac{1}{4}$

35. $\frac{d}{dx}(x^2 f(x))\Big|_{x=2} = (2x f(x) + x^2 f'(x))\Big|_{x=2} = 2\cdot 2\cdot f(2) + 2^2 f'(2) = 20$

36. $\frac{d}{dx}\left(\frac{f(x)}{x^2+f(x)}\right)\Big|_{x=2} = \frac{f'(x)\cdot(x^2+\cancel{f(x)}) - (2x+\cancel{f'(x)})f(x)}{(x^2+f(x))^2}\Big|_{x=2}$

$= \frac{f'(2)\cdot 2^2 - 2\cdot 2\, f(2)}{(2^2+f(2))^2} = \frac{12-8}{(4+2)^2} = \frac{1}{9}$

2.4 22. $\frac{d}{dt}(f(2t+3)) = 2f'(2t+3)$

23. $\frac{d}{dx}(f(5x-x^2)) = (5-2x)\cdot f'(5x-x^2)$

24. $\frac{d}{dx}\left[f\left(\frac{2}{x}\right)\right]^3 = 3\left(f\left(\frac{2}{x}\right)\right)^2 \cdot f'\left(\frac{2}{x}\right)\cdot \frac{-2}{x^2}$

25. $\frac{d}{dx}\sqrt{3+2f(x)} = \frac{1}{\cancel{2}\sqrt{3+2f(x)}}\cdot \cancel{2} f'(x) = \frac{f'(x)}{\sqrt{3+2f(x)}}$

26. $\frac{d}{dt} f(\sqrt{3+2t}) = f'(\sqrt{3+2t})\cdot\frac{\cancel{2}}{\cancel{2}\sqrt{3+2t}} = \frac{f'(\sqrt{3+2t})}{\sqrt{3+2t}}$

27. $\frac{d}{dx} f(3+2\sqrt{x}) = f'(3+2\sqrt{x})\cdot\frac{\cancel{2}}{\cancel{2}\sqrt{x}} = \frac{f'(3+2\sqrt{x})}{\sqrt{x}}$

28. $\frac{d}{dx}\left(f(2f(3f(x)))\right) = f'(2f(3f(x))) \cdot 2f'(3f(x)) \cdot 3 f'(x)$

2.5 11, 15, 28, 58

11. $\frac{d}{dx}(\sin(\pi x^2)) = 2\pi x \cdot \cos(\pi x^2)$

15. $\frac{d}{dx}(\cos(x+\sin(x))) = -\sin(x+\sin(x))\,(1+\cos(x))$

28. $\frac{d}{dt}(t\sin(t) + \cos(t)) = \sin(t) + t\cdot\cos(t) - \sin(t) = t\cos(t)$

58.
$$f(x) = \begin{cases} ax+b & x<0 \\ 2\sin(x) + 3\cos(x) & x \geq 0 \end{cases}$$

continuous $\lim_{x\to 0^-} f(x) = b = f(0) = 3 \Rightarrow b = 3$
$= 2\sin(0) + 3\cos(0)$

differentiable $f'_-(0) = a = f'_+(0) = 2 \Rightarrow a = 2$
$= 2\cos(0) - 3\sin(0)$

2.6 1, 19. 1. $y = (3-2x)^7 \longrightarrow y' = 7(3-2x)^6(-2) = -14(3-2x)^6 \longrightarrow y'' = -84(3-2x)^5(-2) = 168(3-2x)^5$

$\longrightarrow y''' = 168\cdot 5\cdot(-2)(3-2x)^4 = -1680(3-2x)^4$

19. $f(x) = \cos(ax) \longrightarrow f'(x) = -a\sin(ax) \longrightarrow f''(x) = -a^2\cos(ax)$

$f^{(2n)}(x) = (-1)^n a^{(2n)} \cos(ax)$

$f^{(2n+1)}(x) = (-1)^{n+1} a^{2n+1} \sin(ax)$