EXERCISES 4.6

1. Figure 4.43 shows the graphs of a function f, its two derivatives f' and f'', and another function g. Which graph corresponds to each function?

2. List, for each function graphed in Figure 4.43, such information that you can determine (approximately) by inspecting the graph (e.g., symmetry, asymptotes, intercepts, intervals of increase and decrease, critical and singular points, local maxima and minima, intervals of constant concavity, inflection points).

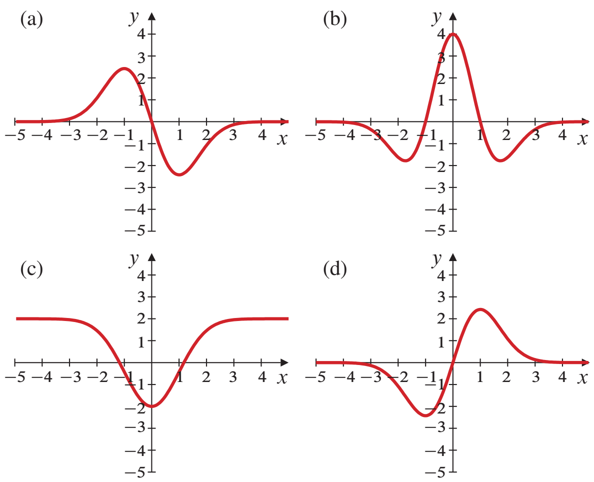

Figure 4.43

3. Figure 4.44 shows the graphs of four functions:

$$f(x) = \frac{x}{1-x^2}, \qquad g(x) = \frac{x^3}{1-x^4},$$

$$h(x) = \frac{x^3 - x}{\sqrt{x^6+1}}, \qquad k(x) = \frac{x^3}{\sqrt{|x^4-1|}}.$$

Which graph corresponds to each function?

4. Repeat Exercise 2 for the graphs in Figure 4.44.

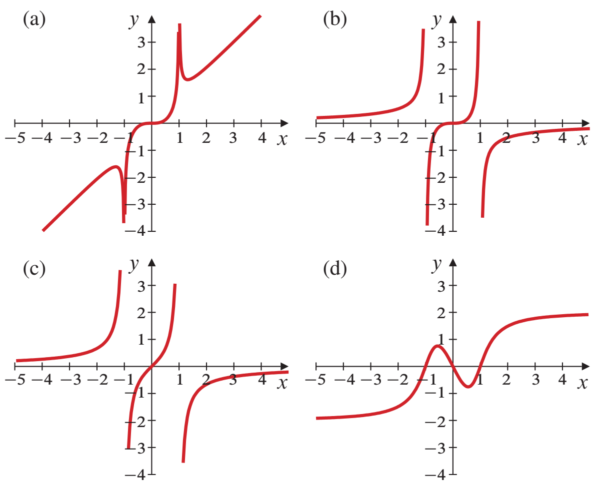

Figure 4.44

In Exercises 5–6, sketch the graph of a function that has the given properties. Identify any critical points, singular points, local maxima and minima, and inflection points. Assume that f is continuous and its derivatives exist everywhere unless the contrary is implied or explicitly stated.

5. $f(0) = 1$, $f(\pm 1) = 0$, $f(2) = 1$, $\lim_{x\to\infty} f(x) = 2$, $\lim_{x\to-\infty} f(x) = -1$, $f'(x) > 0$ on $(-\infty, 0)$ and on $(1, \infty)$, $f'(x) < 0$ on $(0, 1)$, $f''(x) > 0$ on $(-\infty, 0)$ and on $(0, 2)$, and $f''(x) < 0$ on $(2, \infty)$.

6. $f(-1) = 0$, $f(0) = 2$, $f(1) = 1$, $f(2) = 0$, $f(3) = 1$, $\lim_{x\to\pm\infty}(f(x) + 1 - x) = 0$, $f'(x) > 0$ on $(-\infty, -1)$, $(-1, 0)$ and $(2, \infty)$, $f'(x) < 0$ on $(0, 2)$, $\lim_{x\to-1} f'(x) = \infty$, $f''(x) > 0$ on $(-\infty, -1)$ and on $(1, 3)$, and $f''(x) < 0$ on $(-1, 1)$ and on $(3, \infty)$.

In Exercises 7–39, sketch the graphs of the given functions, making use of any suitable information you can obtain from the function and its first and second derivatives.

7. $y = (x^2-1)^3$

8. $y = x(x^2-1)^2$

9. $y = \dfrac{2-x}{x}$

10. $y = \dfrac{x-1}{x+1}$

11. $y = \dfrac{x^3}{1+x}$

12. $y = \dfrac{1}{4+x^2}$

13. $y = \dfrac{1}{2-x^2}$

14. $y = \dfrac{x}{x^2-1}$

15. $y = \dfrac{x^2}{x^2-1}$

16. $y = \dfrac{x^3}{x^2-1}$

17. $y = \dfrac{x^3}{x^2+1}$

18. $y = \dfrac{x^2}{x^2+1}$

19. $y = \dfrac{x^2-4}{x+1}$

20. $y = \dfrac{x^2-2}{x^2-1}$

21. $y = \dfrac{x^3-4x}{x^2-1}$

22. $y = \dfrac{x^2-1}{x^2}$

23. $y = \dfrac{x^5}{(x^2-1)^2}$

24. $y = \dfrac{(2-x)^2}{x^3}$

25. $y = \dfrac{1}{x^3-4x}$

26. $y = \dfrac{x}{x^2+x-2}$

27. $y = \dfrac{x^3-3x^2+1}{x^3}$

28. $y = x + \sin x$

29. $y = x + 2\sin x$

30. $y = e^{-x^2}$

31. $y = xe^x$

32. $y = e^{-x}\sin x, \ (x \ge 0)$

33. $y = x^2e^{-x^2}$

34. $y = x^2e^x$

35. $y = \dfrac{\ln x}{x}, \ (x > 0)$

36. $y = \dfrac{\ln x}{x^2}, \ (x > 0)$

37. $y = \dfrac{1}{\sqrt{4-x^2}}$

38. $y = \dfrac{x}{\sqrt{x^2+1}}$

39. $y = (x^2-1)^{1/3}$

! **40.** What is $\lim_{x\to0+} x\ln x$? $\lim_{x\to0} x\ln|x|$? If $f(x) = x\ln|x|$ for $x \ne 0$, is it possible to define $f(0)$ in such a way that f is continuous on the whole real line? Sketch the graph of f.

41. What straight line is an asymptote of the curve $y = \dfrac{\sin x}{1+x^2}$? At what points does the curve cross this asymptote?