GIJS SCHOENMAKERS
OTTO D'HUYS

NUM. METH.

PBS ⟶ SIMUL. ⎫
MACH. LEARN. ⎬⟶ HCI ⎫
NLP ⎭ ⎬⟶ REC. SYS.

TOPICS: 1. FUNCTIONS & CONTINUITY ⎫
2. LIMITS ⎬⟶ T1
3/4 DIFFERENTIATION
5/6 INTEGRATION ⎤⟶ T2, T3
7 SEQ. & SERIES ⎤ T4 / T5
8/9 MULTIVARIABLE ⎤ T6, T7
10 Q&A.

⟶ BONUS ASSIGNMENT.

34 HOURS   4 ECTS = 4·28 = 112 HOURS

⟹ 78 HOURS WORK OUTSIDE LECTURES/TUTORIALS.

FUNCTION

$$f: D \longrightarrow S$$

DOMAIN   CODOMAIN $= \mathbb{R}$.

CONVENTION
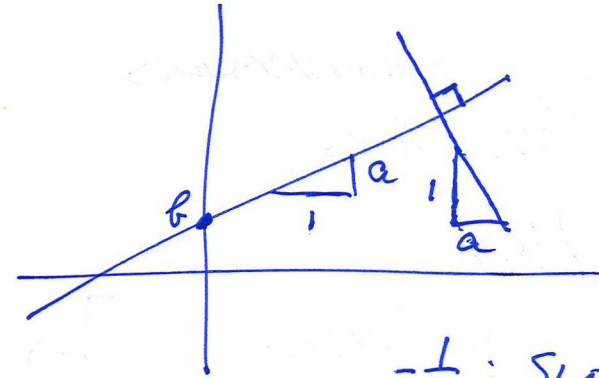
- LINEAR FUNCTION: $f(x) = ax + b$

  SLOPE   Y-INTERCEPT

  $ax + c$

$-\frac{1}{a}$: SLOPE OF PERPENDICULAR LINE

- ABSOLUTE VALUE : $f(x) = |x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$

  DISTANCE FROM 0

$$|x \cdot y| = |x| \cdot |y|$$

$$|x + y| \leq |x| + |y|$$

$|x - c|$: DISTANCE FROM x TO c.

$|x - c| < \delta$: $-\delta < x - c < \delta$

- $f(x) = \sqrt{x}$: DOMAIN $[0, \infty)$

$(a, b) = \{x \in \mathbb{R} : a < x < b\}$

OPEN INTERVAL

$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

POLYNOMIALS:

$$P(x) = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

WITH $a_n \neq 0$.

DEGREE n.

ROOTS: r SUCH THAT $f(r) = 0. = P(r)$

THEN $P(x) = (x-r) \cdot Q(x)$ DEGREE $n-1$.

RATIONAL FUNCTION:

$$f(x) = \frac{P(x)}{Q(x)} \quad \text{FOR 2 POL. P \& Q.}$$

$$f(x) = \frac{x^2 - 2x - 8}{x + 2} \qquad \mathcal{D} = \mathbb{R} \setminus \{-2\}$$

· $\ln(x)$: DOMAIN = $(0, \infty)$

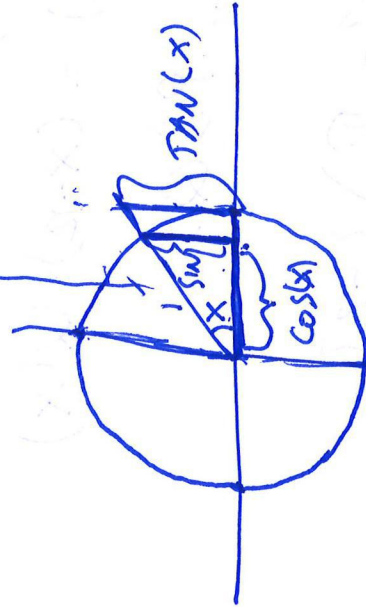$\ln|x|$ $\mathbb{R} \setminus \{0\}$.   LOGARITHMIC FUNCTION

TRIGONOMETRIC FUNCTION: $SIN(x), COS(x), TAN(x)$
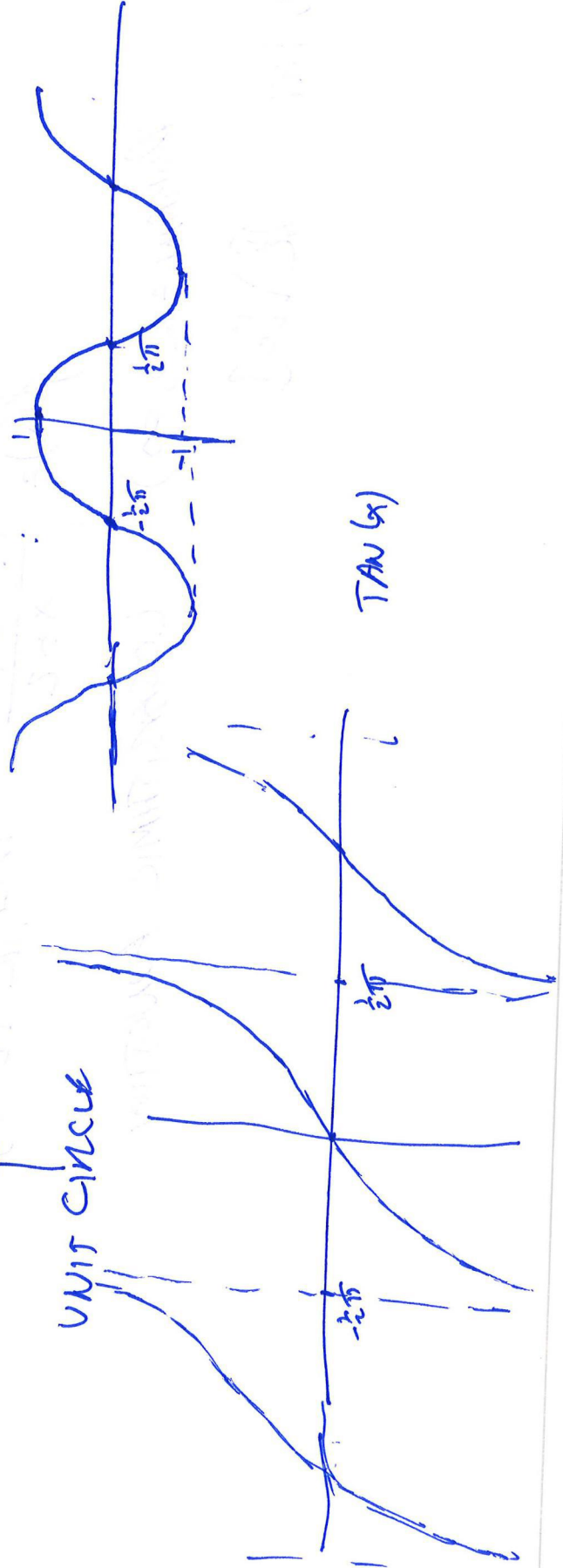
HYPOTHENUSE

$$COS(\theta) = \frac{A}{C}$$

$$SIN(\theta) = \frac{B}{C}$$

$$TAN(\theta) = \frac{SIN(\theta)}{COS(\theta)} = \frac{B}{A}$$

B OPPOSITE

A ADJACENS

VOM: $SIN(x) \rightarrow \mathbb{R}$

$COS(x) \rightarrow \mathbb{R}$.

TAN(x)

$SIN(x)$

$COS(x)$

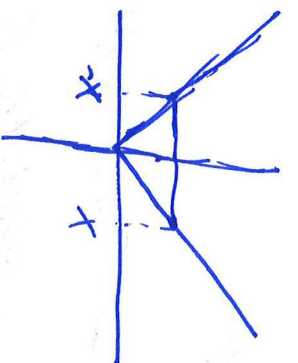UNIT CIRCLE

TAN (x)

' EVEN / ODD FUNCTION
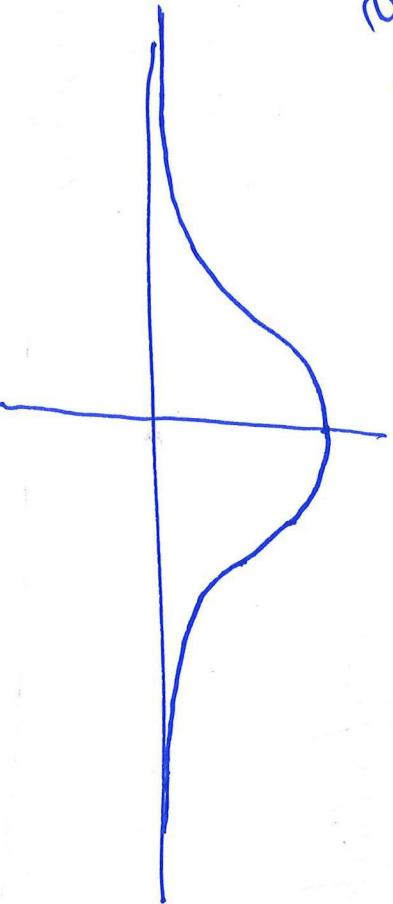
EVEN : $f(-x) = f(x)$ E.G. $|x|$

$$f(x) = x^2$$
$$\cos(x)$$
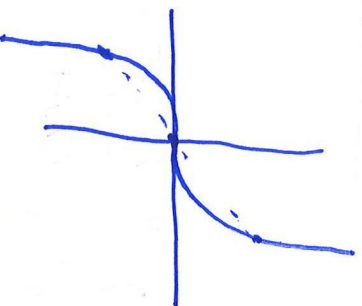
$$f(x) = e^{-x^2}$$

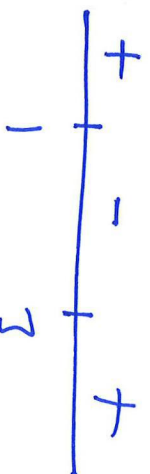ODD : $f(-x) = -f(x)$ E.G. $f(x) = x$

$$f(x) = \sin(x)$$
$$f(x) = x^3$$

• COMPOSITE FUNCTIONS :

$$f \circ g(x) \quad , \quad f(g(x))$$

$$f(x) = \sqrt{x^2 - 4x + 3}$$

DOMAIN : $x^2 - 4x + 3 \geq 0$

$$(x-1)(x-3) \geq 0$$

DOMAIN : $(-\infty, 1] \cup [3, \infty)$

$$\begin{array}{ccc} + & - & + \\ \hline & 1 & 3 \end{array}$$

③

CONTINUITY:

$f(x)$ IS CONTINUOUS AT INTERIOR POINT c OF D IF
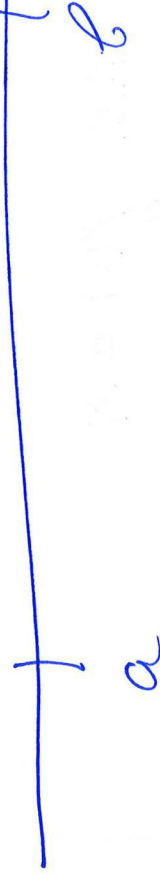
(WHENEVER x APPROACHES c, $f(x)$ APPROACHES $f(c)$.

$$\lim_{x \to c} f(x) = f(c)$$

$D = [a, b]$
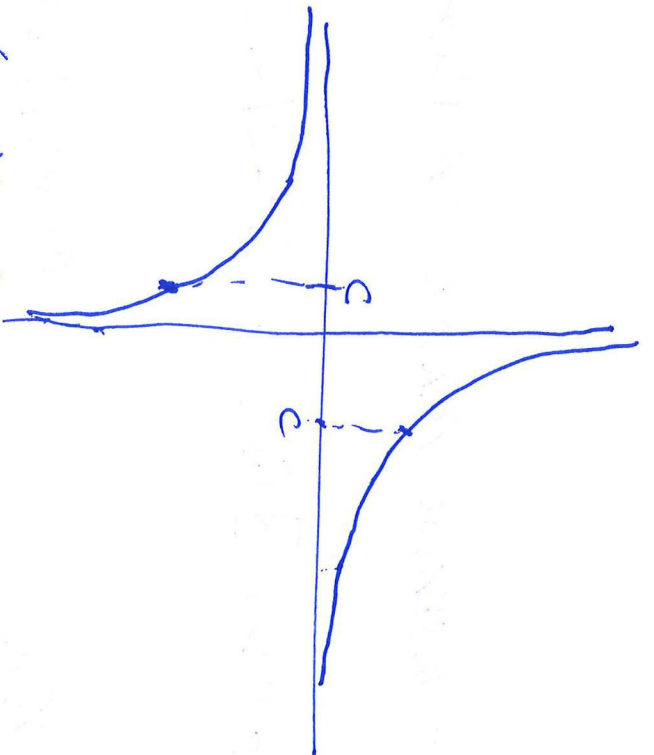
a, b: END POINTS

ALL $x \in (a, b)$: INTERIOR POINTS.

f IS RIGHT CONTINUOUS AT c IF $f(x) \to f(c)$ AS LONG AS $x \to c$ FROM THE RIGHT.

f IS LEFT " " " " " " " LEFT.

f IS RIGHT CONT. AT a
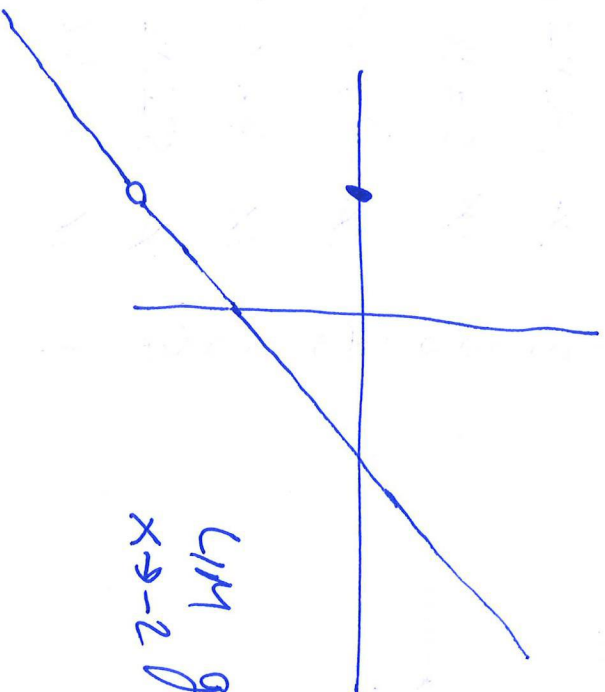
f " LEFT " " b.

$f(x) = \dfrac{1}{x}$ IS CONTINUOUS.

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

EXAMPLE: $f(x) = \dfrac{x^2 - 2x - 8}{x+2} = \dfrac{(x-4)(x+2)}{x+2} = x-4$

$$g(x) = \begin{cases} \dfrac{x^2-2x-8}{x+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$$

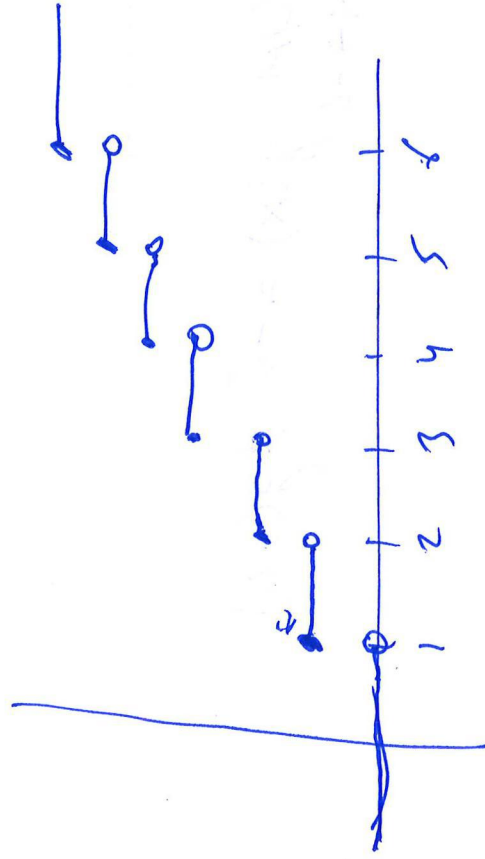$\lim\limits_{x \to -2} g(x) = -6 \neq g(-2)$

FORMAL DEFINITION :

f IS CONTINUOUS AT INTERIOR POINTS c OF DOMAIN IF

$$\forall x \in D : \left[\forall \varepsilon > 0 \;\; \exists \delta > 0 : \; |x-c| < \delta \Rightarrow |f(x)-f(c)| < \varepsilon \right]$$

EXAMPLE : DIE ROLL

$$F(x) = P(X \leq x) = \begin{cases} 0 & \text{IF } x < 1 \\ \frac{1}{6} & \text{IF } 1 \leq x < 2 \\ \frac{2}{6} & \text{IF } 2 \leq x < 3 \\ \frac{3}{6} & \text{IF } 3 \leq x < 4 \\ \frac{4}{6} & \text{IF } 4 \leq x < 5 \\ \frac{5}{6} & \text{IF } 5 \leq x < 6 \\ 1 & \text{IF } x \geq 6 \end{cases}$$

$$P(X=1) = P(X=2) = \ldots = P(X=6) = \frac{1}{6}$$

RIGHT CONTINUOUS
BUT NOT
LEFT CONTINUOUS