6.1 1, 2, 10, 13
6.2 4, 11, 13, 7, 10
6.5 1, 10, 15, 19, 22

6.1 1. $\int x \cdot \cos(x)\,dx = x \cdot \sin(x) - \int \sin(x)\,dx = x\sin(x) + \cos(x) + C$

$u = x \quad du = dx$
$dv = \cos(x)\,dx \quad v = \sin(x)$

2. $\int (x+3)e^{2x}\,dx = \frac{1}{2}(x+3)e^{2x} - \frac{1}{2}\int e^{2x}\,dx = \frac{1}{2}(x+3)e^{2x} - \frac{e^{2x}}{4} + C$

$u = x+3 \quad du = dx$
$dv = e^{2x}\,dx \quad v = \frac{1}{2}e^{2x}$

10. $\int x^5 e^{-x^2}\,dx = \frac{1}{2}\int u^2 e^{-u}\,du = -\frac{1}{2}u^2 e^{-u} + \int u e^{-u}\,du = -\frac{1}{2}u^2 e^{-u} - ue^{-u} + \int e^{-u}\,du$

$u = x^2$
$du = 2x\,dx$

$y = u^2 \quad dy = 2u\,du$
$dv = e^{-u}\,du \quad v = -e^{-u}$

$y = u \quad dy = du$
$dv = e^{-u}\,du \quad v = -e^{-u}$

$= -\frac{1}{2}u^2 e^{-u} - ue^{-u} - e^{-u} + C = -\frac{1}{2}x^4 e^{-x^2} - x^2 e^{-x^2} - e^{-x^2} + C$

13. $\int e^{2x}\sin(3x)\,dx = -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{3}\int e^{2x}\cos(3x)\,dx$

$u = e^{2x} \quad du = 2e^{2x}\,dx$
$dv = \sin(3x)\,dx \quad v = -\frac{1}{3}\cos(3x)$

$u = e^{2x} \quad du = 2e^{2x}\,dx$
$dv = \cos(3x)\,dx \quad v = \frac{1}{3}\sin(3x)$

$= -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x) - \frac{4}{9}\int e^{2x}\sin(3x)\,dx$

$\Rightarrow \int e^{2x}\sin(3x)\,dx = \frac{9}{13}\left(-\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x)\right) + C$

6.2 4, 7, 10, 11, 13

4. $\int \frac{x^2}{x-4}\,dx = \int \frac{(x^2-4x) + (4x-16) + 16}{x-4}\,dx = \int \left(x + 4 + \frac{16}{x-4}\right)dx$

$= \frac{x^2}{2} + 4x + 16\ln|x-4| + C$

7. $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\left(\int \frac{dx}{a-x} + \int \frac{dx}{a+x}\right) = \frac{1}{2a}\Big(\ln|a-x| + \ln|a+x|\Big) + C$

$$\frac{1}{a^2-x^2} = \frac{A}{a-x} + \frac{B}{a+x}$$

$$A(a+x) + B(a-x) = 1$$

$$a(A+B) = 1 \Rightarrow A = B = \frac{1}{2a}$$

$$x(A-B) = 0 \Rightarrow A = B$$

10. $\displaystyle\int \frac{x\,dx}{3x^2+8x-3} = \frac{1}{10}\int \frac{dx}{3x-1} + \frac{3}{10}\int \frac{dx}{x+3} = \frac{1}{30}\ln|3x-1| + \frac{3}{10}\ln|x+3| + C$

$$3x^2+8x-3 = (3x-1)(x+3)$$

$$\frac{x}{3x^2+8x-3} = \frac{A}{3x-1} + \frac{B}{x+3}$$

$$A(x+3) + B(3x-1) = x$$

$$3A - B = 0 \rightarrow B = 3A$$

$$A + 3B = 1 \rightarrow 10A = 1 \Rightarrow A = \frac{1}{10}, \quad B = \frac{3}{10}$$

11. $\displaystyle\int \frac{x-2}{x^2+x}\,dx = -2\int \frac{dx}{x} + 3\int \frac{dx}{x+1} = -2\ln|x| + 3\ln|x+1| + C$

$$\frac{x-2}{x^2+x} = \frac{A}{x} + \frac{B}{x+1}$$

$$A(x+1) + Bx = x-2$$

$$A + B = 1 \rightarrow B = 3$$

$$A = -2$$

13. $\displaystyle\int \frac{dx}{1-6x+9x^2} = \int \frac{dx}{(3x-1)^2} = \frac{1}{3}\,\frac{-1}{(3x-1)} + C$

$u = 3x-1$, $\displaystyle\int \frac{du}{u^2} = -\frac{1}{u} + C$

$du = 3dx$

6.5 1, 10, 15, 19, 22

1. $\displaystyle\int_2^\infty \frac{dx}{(x-1)^3} = \int_1^\infty \frac{du}{u^3} = \lim_{a\to+\infty}\int_1^a \frac{du}{u^3} = \lim_{a\to\infty}\left[\frac{-1}{2u^2}\right]_1^a = \frac{1}{2} - \underbrace{\lim_{a\to\infty}\frac{1}{2a^2}}_{\to\ \text{to } 0}$

$u = x-1$ $\qquad = \frac{1}{2}$

$x=2 \rightarrow u=1$

$dx = du$

10. $\int_0^{\infty} x e^{-x}\, dx = \lim_{a\to\infty} \int_0^a x e^{-x}\, dx = \lim_{a\to\infty} \left[(-xe^{-x})\Big|_0^a + \int_0^a e^{-x}\, dx \right]$

$u = x \to du = dx$
$dv = e^{-x}dx \to v = -e^{-x}$

$= \lim_{a\to\infty} \left[(-xe^{-x})\Big|_0^a - e^{-x}\Big|_0^a \right)$

$= 1 - \lim_{a\to\infty} \underbrace{(ae^{-a} - e^{-a})}_{\text{to } 0} = 1$

15. $\int_0^{\pi/2} \tan x\, dx = \lim_{a\to\frac{\pi}{2}^-} \int_0^a \tan(x)\, dx$

$= \lim_{a\to\frac{\pi}{2}^-} \int_0^a \frac{\sin(x)}{\cos(x)}\, dx = \lim_{b\to 0^+} -\int_1^b \frac{du}{u} = \lim_{b\to 0^+} -\ln|u|\Big|_1^b$

$u = \cos(x)$
$du = -\sin(x)dx$
$a \to \frac{\pi}{2}^- \Leftrightarrow \underbrace{\cos(a)}_{b} \to 0^+$

$= \lim_{b\to 0^+} (-\ln(b)) = +\infty$

19. $\int_{-\infty}^{+\infty} \frac{x}{1+x^2}\, dx$

$\longrightarrow$ we first evaluate $\int_0^{+\infty} \frac{x}{1+x^2}\, dx = \lim_{a\to\infty} \int_0^a \frac{x\,dx}{1+x^2} = \lim_{a\to\infty} \frac{1}{2}\int_1^a \frac{du}{u}$

$u = 1+x^2$
$du = 2x\,dx$

$= \lim_{a\to\infty} \frac{1}{2}(\ln(a) - \ln(1))$

$= +\infty$

$\hookrightarrow$ since $\int_0^{+\infty} \frac{x\,dx}{1+x^2} = +\infty$ and $\int_{-\infty}^{0} \frac{x\,dx}{1+x^2} = -\int_0^{+\infty} \frac{x\,dx}{1+x^2}$, we say that the integral diverges (as it consists of 2 infinite areas.)

22. $\int_{-\infty}^{+\infty} e^{-|x|}\, dx = 2\int_0^{+\infty} e^{-x}\, dx = 2$

even function.