EXERCISES 1.2

1. Find: (a) $\lim_{x\to -1} f(x)$, (b) $\lim_{x\to 0} f(x)$, and (c) $\lim_{x\to 1} f(x)$, for the function f whose graph is shown in Figure 1.13.

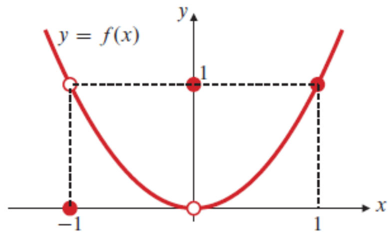

Figure 1.13

2. For the function $y = g(x)$ graphed in Figure 1.14, find each of the following limits or explain why it does not exist.
(a) $\lim_{x\to 1} g(x)$, (b) $\lim_{x\to 2} g(x)$, (c) $\lim_{x\to 3} g(x)$

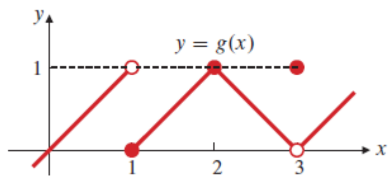

Figure 1.14

In Exercises 3–6, find the indicated one-sided limit of the function g whose graph is given in Figure 1.14.

3. $\lim_{x\to 1-} g(x)$

4. $\lim_{x\to 1+} g(x)$

5. $\lim_{x\to 3+} g(x)$

6. $\lim_{x\to 3-} g(x)$

In Exercises 7–36, evaluate the limit or explain why it does not exist.

7. $\lim_{x\to 4} (x^2 - 4x + 1)$

8. $\lim_{x\to 2} 3(1-x)(2-x)$

9. $\lim_{x\to 3} \dfrac{x+3}{x+6}$

10. $\lim_{t\to -4} \dfrac{t^2}{4-t}$

11. $\lim_{x\to 1} \dfrac{x^2-1}{x+1}$

12. $\lim_{x\to -1} \dfrac{x^2-1}{x+1}$

13. $\lim_{x\to 3} \dfrac{x^2-6x+9}{x^2-9}$

14. $\lim_{x\to -2} \dfrac{x^2+2x}{x^2-4}$

15. $\lim_{h\to 2} \dfrac{1}{4-h^2}$

16. $\lim_{h\to 0} \dfrac{3h+4h^2}{h^2-h^3}$

17. $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

18. $\lim_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$

19. $\lim_{x\to \pi} \dfrac{(x-\pi)^2}{\pi x}$

20. $\lim_{x\to -2} |x-2|$

21. $\lim_{x\to 0} \dfrac{|x-2|}{x-2}$

22. $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$

23. $\lim_{t\to 1} \dfrac{t^2-1}{t^2-2t+1}$

24. $\lim_{x\to 2} \dfrac{\sqrt{4-4x+x^2}}{x-2}$

25. $\lim_{t\to 0} \dfrac{t}{\sqrt{4+t}-\sqrt{4-t}}$

26. $\lim_{x\to 1} \dfrac{x^2-1}{\sqrt{x+3}-2}$

27. $\lim_{t\to 0} \dfrac{t^2+3t}{(t+2)^2-(t-2)^2}$

28. $\lim_{s\to 0} \dfrac{(s+1)^2-(s-1)^2}{s}$

29. $\lim_{y\to 1} \dfrac{y-4\sqrt{y}+3}{y^2-1}$

30. $\lim_{x\to -1} \dfrac{x^3+1}{x+1}$

31. $\lim_{x\to 2} \dfrac{x^4-16}{x^3-8}$

32. $\lim_{x\to 8} \dfrac{x^{2/3}-4}{x^{1/3}-2}$

33. $\lim_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4}\right)$

34. $\lim_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{1}{x^2-4}\right)$

35. $\lim_{x\to 0} \dfrac{\sqrt{2+x^2}-\sqrt{2-x^2}}{x^2}$

36. $\lim_{x\to 0} \dfrac{|3x-1|-|3x+1|}{x}$

EXERCISES 1.3

Find the limits in Exercises 1–10.

1. $\lim_{x\to \infty} \dfrac{x}{2x-3}$

2. $\lim_{x\to \infty} \dfrac{x}{x^2-4}$

3. $\lim_{x\to \infty} \dfrac{3x^3-5x^2+7}{8+2x-5x^3}$

4. $\lim_{x\to -\infty} \dfrac{x^2-2}{x-x^2}$

5. $\lim_{x\to -\infty} \dfrac{x^2+3}{x^3+2}$

6. $\lim_{x\to \infty} \dfrac{x^2+\sin x}{x^2+\cos x}$

7. $\lim_{x\to \infty} \dfrac{3x+2\sqrt{x}}{1-x}$

8. $\lim_{x\to \infty} \dfrac{2x-1}{\sqrt{3x^2+x+1}}$

9. $\lim_{x\to -\infty} \dfrac{2x-1}{\sqrt{3x^2+x+1}}$

10. $\lim_{x\to -\infty} \dfrac{2x-5}{|3x+2|}$

In Exercises 11–32 evaluate the indicated limit. If it does not exist, is the limit ∞, $-\infty$, or neither?

11. $\lim_{x\to 3} \dfrac{1}{3-x}$

12. $\lim_{x\to 3} \dfrac{1}{(3-x)^2}$

13. $\lim_{x\to 3-} \dfrac{1}{3-x}$

14. $\lim_{x\to 3+} \dfrac{1}{3-x}$

15. $\lim_{x\to -5/2} \dfrac{2x+5}{5x+2}$

16. $\lim_{x\to -2/5} \dfrac{2x+5}{5x+2}$

17. $\lim_{x\to -(2/5)-} \dfrac{2x+5}{5x+2}$

18. $\lim_{x\to -(2/5)+} \dfrac{2x+5}{5x+2}$

19. $\lim_{x\to 2+} \dfrac{x}{(2-x)^3}$

20. $\lim_{x\to 1-} \dfrac{x}{\sqrt{1-x^2}}$

21. $\lim_{x\to 1+} \dfrac{1}{|x-1|}$

22. $\lim_{x\to 1-} \dfrac{1}{|x-1|}$

23. $\lim_{x\to 2} \dfrac{x-3}{x^2-4x+4}$

24. $\lim_{x\to 1+} \dfrac{\sqrt{x^2-x}}{x-x^2}$

25. $\lim_{x\to \infty} \dfrac{x+x^3+x^5}{1+x^2+x^3}$

26. $\lim_{x\to \infty} \dfrac{x^3+3}{x^2+2}$

31. $\lim_{x\to \infty} \dfrac{1}{\sqrt{x^2-2x}-x}$

32. $\lim_{x\to -\infty} \dfrac{1}{\sqrt{x^2+2x}-x}$

33. What are the horizontal asymptotes of $y = \dfrac{1}{\sqrt{x^2-2x}-x}$? What are its vertical asymptotes?

34. What are the horizontal and vertical asymptotes of $y = \dfrac{2x-5}{|3x+2|}$?

EXERCISES 1.4

Exercises 1–3 refer to the function g defined on $[-2, 2]$, whose graph is shown in Figure 1.33.

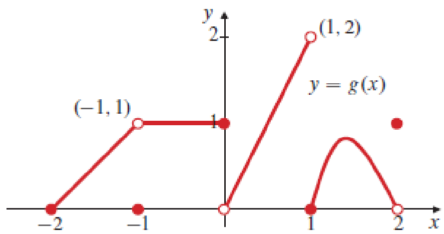

Figure 1.33

1. State whether g is (a) continuous, (b) left continuous, (c) right continuous, and (d) discontinuous at each of the points $-2, -1, 0, 1$, and 2.
2. At what points in its domain does g have a removable discontinuity, and how should g be redefined at each of those points so as to be continuous there?
3. Does g have an absolute maximum value on $[-2, 2]$? an absolute minimum value?

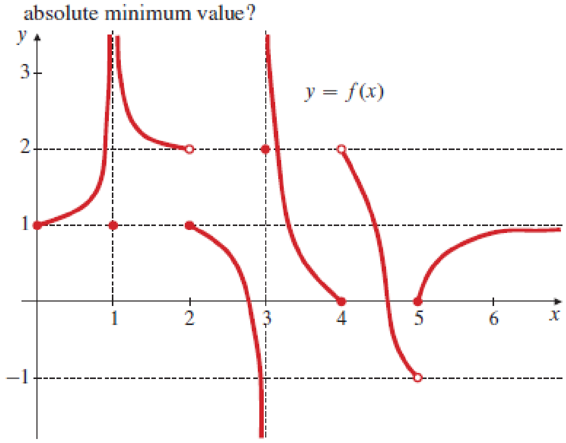

Figure 1.34

4. At what points is the function f, whose graph is shown in Figure 1.34, discontinuous? At which of those points is it left continuous? right continuous?
5. Can the function f graphed in Figure 1.34 be redefined at the single point $x = 1$ so that it becomes continuous there?
6. The function $\operatorname{sgn}(x) = x/|x|$ is neither continuous nor discontinuous at $x = 0$. How is this possible?

In Exercises 7–12, state where in its domain the given function is continuous, where it is left or right continuous, and where it is just discontinuous.

7. $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

8. $f(x) = \begin{cases} x & \text{if } x < -1 \\ x^2 & \text{if } x \ge -1 \end{cases}$

9. $f(x) = \begin{cases} 1/x^2 & \text{if } x \ne 0 \\ 0 & \text{if } x = 0 \end{cases}$

10. $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 0.987 & \text{if } x > 1 \end{cases}$

11. The least integer function $\lceil x \rceil$ of Example 11 in Section P.5.
12. The cost function $C(t)$ of Exercise 53 in Section 1.3.

In Exercises 13–16, how should the given function be defined at the given point to be continuous there? Give a formula for the continuous extension to that point.

13. $\dfrac{x^2 - 4}{x - 2}$ at $x = 2$

14. $\dfrac{1 + t^3}{1 - t^2}$ at $t = -1$

15. $\dfrac{t^2 - 5t + 6}{t^2 - t - 6}$ at 3

16. $\dfrac{x^2 - 2}{x^4 - 4}$ at $\sqrt{2}$

17. Find k so that $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ k - x^2 & \text{if } x > 2 \end{cases}$ is a continuous function.
18. Find m so that $g(x) = \begin{cases} x - m & \text{if } x < 3 \\ 1 - mx & \text{if } x \ge 3 \end{cases}$ is continuous for all x.
19. Does the function x^2 have a maximum value on the open interval $-1 < x < 1$? a minimum value? Explain.
20. The Heaviside function of Example 1 has both absolute maximum and minimum values on the interval $[-1, 1]$, but it is not continuous on that interval. Does this violate the Max-Min Theorem? Why?

Exercises 21–24 ask for maximum and minimum values of functions. They can all be done by the method of Example 9.

21. The sum of two nonnegative numbers is 8. What is the largest possible value of their product?
22. The sum of two nonnegative numbers is 8. What is (a) the smallest and (b) the largest possible value for the sum of their squares?
23. A software company estimates that if it assigns x programmers to work on the project, it can develop a new product in T days, where
$$T = 100 - 30x + 3x^2.$$
How many programmers should the company assign in order to complete the development as quickly as possible?
24. It costs a desk manufacturer $\$(245x - 30x^2 + x^3)$ to send a shipment of x desks to its warehouse. How many desks should it include in each shipment to minimize the average shipping cost per desk?

Find the intervals on which the functions $f(x)$ in Exercises 25–28 are positive and negative.

25. $f(x) = \dfrac{x^2 - 1}{x}$

26. $f(x) = x^2 + 4x + 3$

27. $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$

28. $f(x) = \dfrac{x^2 + x - 2}{x^3}$

29. Show that $f(x) = x^3 + x - 1$ has a zero between $x = 0$ and $x = 1$.
30. Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$.