14.2 1, 2, 3, 8, 9, 13, 19

1. $\int_0^1 dx \int_0^x (xy + y^2)\,dy = \int_0^1 \frac{5x^3}{6}\,dx = \left[\frac{5x^4}{24}\right]_0^1 = \frac{5}{24}$

$\int_0^x (xy + y^2)\,dy = \left|\frac{xy^2}{2} + \frac{y^3}{3}\right|_0^x = \frac{x^3}{2} + \frac{x^3}{3} = \frac{5x^3}{6}$

2. $\int_0^1 \int_0^y (xy + y^2)\,dx\,dy = \frac{3}{2}\int_0^1 y^3\,dy = \left[\frac{3}{2}\frac{y^4}{4}\right]_0^1 = \frac{3}{8}$

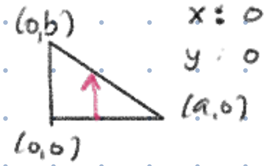
$\int_0^y (xy + y^2)\,dx = \left[\frac{x^2 y}{2} + y^2 x\right]_0^y = \frac{y^3}{2} + y^3 = \frac{3}{2}y^3$

3. $\int_0^\pi \int_{-x}^{x} \cos(y)\,dy\,dx = 2\int_0^\pi \sin(x)\,dx = -2\left[\cos(x)\right]_0^\pi = -2(\cos(\pi) - \cos(0)) = 4$

$\int_{-x}^{x} \cos(y)\,dy = \left[+\sin(y)\right]_{-x}^{x} = 2\sin(x)$

8. $\iint_T (x - 3y)\,dA$

T: triangle with vertices $(0,0)$, $(a,0)$, $(0,b)$



$x: 0 \to a$
$y: 0 \to -\frac{b}{a}x + b$

line $y = -\frac{b}{a}x + b$

$\int_0^a \int_0^{b - \frac{bx}{a}} (x - 3y)\,dy\,dx$

$\int_0^{b - \frac{b}{a}x} (x - 3y)\,dy = \left[xy - \frac{3y^2}{2}\right]_0^{b - \frac{b}{a}x} = bx\left(1 - \frac{x}{a}\right) - \frac{3}{2}b^2\left(1 - \frac{x}{a}\right)^2$

$b\int_0^a x\left(1 - \frac{x}{a}\right)dx - \frac{3}{2}b^2\int_0^a \left(1 - \frac{x}{a}\right)^2 dx = b\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a + \frac{3}{2}b^2 a\int_1^0 u^2\,du$

$u = 1 - \frac{x}{a}$ $\quad x = 0 \to u = 1$
$du = -\frac{dx}{a}$ $\quad x = a \to u = 0$

$= b\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a + \frac{\cancel{3}}{2}b^2 a\left|\frac{u^3}{\cancel{3}}\right|_0^1 = \frac{a^2 b}{6} + \frac{ab^2}{2}$

9. $\iint_R xy^2\,dA$

$x: 0 \to 1$
$y: x^2 \to \sqrt{x}$

$\iint_R xy^2\,dA = \int_0^1 \int_{x^2}^{\sqrt{x}} xy^2\,dy\,dx$

$= \int_0^1 x \int_{x^2}^{\sqrt{x}} y^2\,dy\,dx$

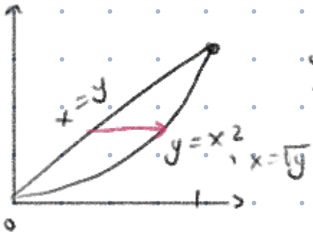
$$\int_{x^2}^{\sqrt{x}} y^2\,dy = \left[\frac{y^3}{3}\right]_{x^2}^{\sqrt{x}} = \frac{1}{3}\left(x^{\frac{3}{2}} - x^6\right)$$

$$\frac{1}{3}\int_0^1 \left(x^{5/2} - x^7\right)dx = \frac{1}{3}\left(\frac{2}{7}x^{7/2} - \frac{x^8}{8}\right) = \frac{1}{3}\left(\frac{2}{7} - \frac{1}{8}\right) = \frac{3}{56}$$

integration by parts

13. $$\iint_R \frac{x}{y} e^y\,dA = \int_0^1\int_y^{\sqrt{y}} \frac{x}{y} e^y\,dx\,dy = \int_0^1 \frac{e^y}{y}\int_y^{\sqrt{y}} x\,dx\,dy = \frac{1}{2}\int_0^1 e^y(1-y)\,dy = \frac{1}{2}\left[e^y(2-y)\right]_0^1$$

$$= \frac{1}{2}(e-2)$$



$y : 0 \to 1$
$x : y \to \sqrt{y}$

$$\int_y^{\sqrt{y}} x\,dx = \left[\frac{x^2}{2}\right]_y^{\sqrt{y}} = \frac{y}{2} - \frac{y^2}{2} = \frac{y}{2}(1-y)$$

19. $$\iint_R (1-x^2)\,dA = \int_0^1\int_0^x (1-x^2)\,dy\,dx = \int_0^1 (1-x^2)\underbrace{\int_0^x dy}_{x}\,dx = \int_0^1 x(1-x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{4}$$

R: $x : 0 \to 1$
$y : 0 \to x$