Solve the equations in Exercises 11–14 for x.

11. $2^{x+1} = 3^x$

12. $3^x = 9^{1-x}$

13. $\dfrac{1}{2^x} = \dfrac{5}{8^{x+3}}$

14. $2^{x^2-3} = 4^x$

Find the domains of the functions in Exercises 15–16.

15. $\ln \dfrac{x}{2-x}$

16. $\ln(x^2 - x - 2)$

Solve the inequalities in Exercises 17–18.

17. $\ln(2x-5) > \ln(7-2x)$

18. $\ln(x^2 - 2) \le \ln x$

In Exercises 19–48, differentiate the given functions. If possible, simplify your answers.

19. $y = e^{5x}$

20. $y = xe^x - x$

21. $y = \dfrac{x}{e^{2x}}$

22. $y = x^2 e^{x/2}$

23. $y = \ln(3x - 2)$

24. $y = \ln|3x - 2|$

25. $y = \ln(1 + e^x)$

26. $f(x) = e^{(x^2)}$

27. $y = \dfrac{e^x + e^{-x}}{2}$

28. $x = e^{3t} \ln t$

29. $y = e^{(e^x)}$

30. $y = \dfrac{e^x}{1 + e^x}$

31. $y = e^x \sin x$

32. $y = e^{-x} \cos x$

33. $y = \ln \ln x$

34. $y = x \ln x - x$

35. $y = x^2 \ln x - \dfrac{x^2}{2}$

36. $y = \ln|\sin x|$

37. $y = 5^{2x+1}$

38. $y = 2^{(x^2-3x+8)}$

39. $g(x) = t^x x^t$

40. $h(t) = t^x - x^t$

41. $f(s) = \log_a(bs + c)$

42. $g(x) = \log_x(2x + 3)$

43. $y = x^{\sqrt{x}}$

44. $y = (1/x)^{\ln x}$

45. $y = \ln|\sec x + \tan x|$

46. $y = \ln|x + \sqrt{x^2 - a^2}|$

47. $y = \ln\left(\sqrt{x^2 + a^2} - x\right)$

48. $y = (\cos x)^x - x^{\cos x}$

49. Find the nth derivative of $f(x) = xe^{ax}$.

50. Show that the nth derivative of $(ax^2 + bx + c)e^x$ is a function of the same form but with different constants.

51. Find the first four derivatives of e^{x^2}.

52. Find the nth derivative of $\ln(2x + 1)$.

53. Differentiate (a) $f(x) = (x^x)^x$ and (b) $g(x) = x^{(x^x)}$. Which function grows more rapidly as x grows large?

54. Solve the equation $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = a$, where $a > 0$. The exponent tower goes on forever.

Use logarithmic differentiation to find the required derivatives in Exercises 55–57.

55. $f(x) = (x-1)(x-2)(x-3)(x-4)$. Find $f'(x)$.

56. $F(x) = \dfrac{\sqrt{1+x}(1-x)^{1/3}}{(1+5x)^{4/5}}$. Find $F'(0)$.

57. $f(x) = \dfrac{(x^2-1)(x^2-2)(x^2-3)}{(x^2+1)(x^2+2)(x^2+3)}$. Find $f'(2)$. Also find $f'(1)$.

58. At what points does the graph $y = x^2 e^{-x^2}$ have a horizontal tangent line?

59. Let $f(x) = xe^{-x}$. Determine where f is increasing and where it is decreasing. Sketch the graph of f.

60. Find the equation of a straight line of slope 4 that is tangent to the graph of $y = \ln x$.

61. Find an equation of the straight line tangent to the curve $y = e^x$ and passing through the origin.

62. Find an equation of the straight line tangent to the curve $y = \ln x$ and passing through the origin.

63. Find an equation of the straight line that is tangent to $y = 2^x$ and that passes through the point $(1, 0)$.

64. For what values of $a > 0$ does the curve $y = a^x$ intersect the straight line $y = x$?

65. Find the slope of the curve $e^{xy} \ln \dfrac{x}{y} = x + \dfrac{1}{y}$ at $(e, 1/e)$.

66. Find an equation of the straight line tangent to the curve $xe^y + y - 2x = \ln 2$ at the point $(1, \ln 2)$.

67. Find the derivative of $f(x) = Ax \cos \ln x + Bx \sin \ln x$. Use the result to help you find the indefinite integrals

$$\int \cos \ln x \, dx \text{ and } \int \sin \ln x \, dx.$$

68. Let $F_{A,B}(x) = Ae^x \cos x + Be^x \sin x$. Show that $(d/dx)F_{A,B}(x) = F_{A+B,B-A}(x)$.

69. Using the results of Exercise 68, find (a) $(d^2/dx^2)F_{A,B}(x)$ and (b) $(d^3/dx^3)e^x \cos x$.

70. Find $\dfrac{d}{dx}(Ae^{ax} \cos bx + Be^{ax} \sin bx)$ and use the answer to help you evaluate

(a) $\displaystyle\int e^{ax} \cos bx \, dx$ and (b) $\displaystyle\int e^{ax} \sin bx \, dx$.

71. Prove identity (ii) of Theorem 2 by examining the derivative of the left side minus the right side, as was done in the proof of identity (i).

72. Deduce identity (iii) of Theorem 2 from identities (i) and (ii).

73. Prove identity (iv) of Theorem 2 for rational exponents r by the same method used for Exercise 71.

74. Let $x > 0$, and let $F(x)$ be the area bounded by the curve $y = t^2$, the t-axis, and the vertical lines $t = 0$ and $t = x$. Using the method of the proof of Theorem 1, show that $F'(x) = x^2$. Hence, find an explicit formula for $F(x)$. What is the area of the region bounded by $y = t^2$, $y = 0$, $t = 0$, and $t = 2$?

75. Carry out the following steps to show that $2 < e < 3$. Let $f(t) = 1/t$ for $t > 0$.

(a) Show that the area under $y = f(t)$, above $y = 0$, and between $t = 1$ and $t = 2$ is less than 1 square unit. Deduce that $e > 2$.

(b) Show that all tangent lines to the graph of f lie below the graph. *Hint:* $f''(t) = 2/t^3 > 0$.

(c) Find the lines T_2 and T_3 that are tangent to $y = f(t)$ at $t = 2$ and $t = 3$, respectively.

(d) Find the area A_2 under T_2, above $y = 0$, and between $t = 1$ and $t = 2$. Also find the area A_3 under T_3, above $y = 0$, and between $t = 2$ and $t = 3$.

(e) Show that $A_2 + A_3 > 1$ square unit. Deduce that $e < 3$.