1. $\int_1^2\int_0^4 2xy\,dy\,dx = \int_1^2 16x\,dx = \left[8x^2\right]_1^2 = 32-8 = 24$

$A(x) = \int_0^4 2xy\,dy = \left[xy^2\right]_0^4 = 16x$

7. $\int_0^1\int_0^1 \frac{y}{1+xy}\,dx\,dy = \int_0^1 \ln(1+y)\,dy = \int_1^2 \ln(u)\,du = \left[u\ln(u)\right]_1^2 - \int_1^2 du = 2\ln(2) - 1$

$u = 1+y$, $du = dy$

$z = \ln(u)$, $dz = \frac{du}{u}$, $dv = du$, $v = u$

$A(y) = \int_0^1 \frac{y}{1+xy}\,dx = \int_0^1 \frac{d(yx)}{1+xy} = \left[\ln|1+xy|\right]_0^1 = \ln(1+y) - \cancel{\ln(1)}$
0

12. $\int_\pi^{2\pi}\int_0^\pi (\sin(x) + \cos(y))\,dx\,dy = \int_\pi^{2\pi}(2+\pi\cos(y))\,dy = 2\pi + \pi\left[\sin(y)\right]_\pi^{2\pi} = 2\pi$
0

$A(y) = \int_0^\pi \sin(x)\,dx + \int_0^\pi \cos(y)\,dx = \left[-\cos(x)\right]_0^\pi + \pi\cos(y) = 2 + \pi\cos(y)$

14. $\int_{-1}^{2}\int_{+1}^{2} x\ln(y)\,dy\,dx = \int_{-1}^{2}(2\ln(2)-1)\,x\,dx = (2\ln(2)-1)\left[\frac{x^2}{2}\right]_{-1}^{2} = (2\ln(2)-1)(2-\frac{1}{2})$
$= \frac{3}{2}(2\ln(2)-1)$

$A(x) = \int_{+1}^{2} x\ln(y)\,dy = x\underbrace{\int_{+1}^{2}\ln(y)\,dy}_{\text{see ex. 7}} = x\left[y\ln(y)-y\right]_{+1}^{2} = 2x\ln(2) - x$

27. $\int_0^1\int_0^1 (2-x-y)\,dx\,dy = \int_0^1 (\frac{3}{2}-y)\,dy = \left[\frac{3}{2}y - \frac{y^2}{2}\right]_0^1 = \frac{3}{2} - \frac{1}{2} = 1$

$A(y) = \int_0^1 (2-x-y)\,dx = \left[2x - \frac{x^2}{2} - xy\right]_0^1 = 2 - \frac{1}{2} - y = \frac{3}{2} - y$

30. $\int_0^2\int_0^1 (4-y^2)\,dx\,dy = \int_0^2 (4-y^2)\underbrace{\int_0^1 dx}_{=1}\,dy = \int_0^2 (4-y^2)\,dy = \left[4y - \frac{y^3}{3}\right]_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$