

$$1. f(0)=0, f'(0)=3, f'(-1)=2, g(0)=-1, g'(0)=2$$

$$\begin{aligned} \text{then } \frac{d}{dx}(f(x+g(x))) \Big|_{x=0} &= f'(0+g(0)) \cdot \frac{d}{dx}(x+g(x)) \Big|_{x=0} \\ &= \underbrace{f'(-1)}_2 \cdot (1 + \underbrace{g'(0)}_2) = 2 \cdot (1+2) = 6 \end{aligned}$$

$$2. f(x) = \begin{cases} ae^{bx} & \text{for } x > 0 \\ 2x-1 & \text{for } x \leq 0 \end{cases}$$

$$A. \text{ Continuous? } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0^+} ae^{bx} = \lim_{x \rightarrow 0^-} (2x-1) = -1$$

$$\Rightarrow a = -1$$

$$B. \text{ Differentiable? } f'_+(0) = f'_-(0) \quad (\text{left and right derivative are equal})$$

$$\Rightarrow abe^{b \cdot 0} = 2$$

$$\Rightarrow b = -2$$

$$\downarrow a = -1, e^{b \cdot 0} = 1$$

For $a = -1$ and $b = -2$, $f(x)$ is continuous and differentiable at $x=0$

