5.4 1, 2, 7, 9, 10, 36
5.5 4, 6, 8, 12, 21, 26, 33, 39, 41
5.6 5, 6, 12, 39, 42

Review ex: 25, 27

5.4. 1. $\int_a^b f(x)dx + \int_b^c f(x)dx + \int_c^a f(x)dx = \int_a^c f(x)dx + \int_c^a f(x)dx = 0$

2. $\int_0^2 3f(x)\,dx + \int_1^3 3f(x)\,dx - 2\int_0^3 f(x)dx - \int_1^2 3f(x)dx = \int_0^3 f(x)dx$

7. $\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{2-t^2}\,dt$ this is a half circle with radius $\sqrt{2}$ $\frac{1}{2}\pi R^2 = \frac{1}{2}\pi(\sqrt{2})^2 = \pi$

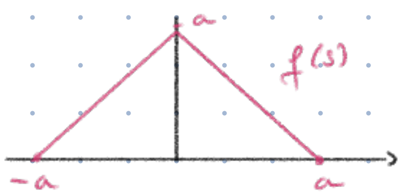
circle $x^2+y^2 = R^2 \Rightarrow y = \pm\sqrt{R^2-x^2}$

9. $\int_{-\pi}^{\pi} \sin(x^3)\,dx = 0$ (odd function, symmetric domain around 0)

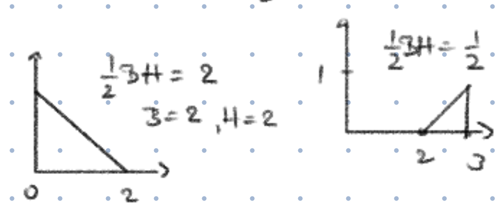
10. $\int_{-a}^{a} (a-|s|)\,ds = a^2$

area of the triangle $= \frac{1}{2}BH = a^2$

$B = 2a$
$H = a$



36. $\int_0^3 |2-x|dx = \int_0^2 (2-x)dx + \int_2^3 (x-2)dx = 2+\frac{1}{2} = \frac{5}{2}$



5.5 4. $\int_{-2}^{-1}\left(\frac{1}{x^2}-\frac{1}{x^3}\right)dx = \left[-\frac{1}{x}+\frac{1}{2x^2}\right]_{-2}^{-1} = \left(1+\cancel{\frac{1}{2}}\right)-\left(\cancel{\frac{1}{2}}+\frac{1}{8}\right) = \frac{7}{8}$

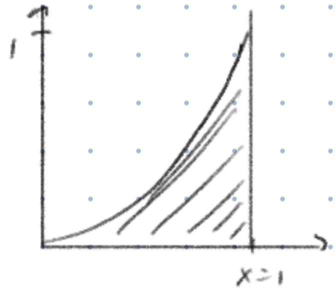
6. $\int_1^2\left(\frac{2}{x^3}-\frac{x^3}{2}\right)dx = \left[-\frac{1}{x^2}-\frac{x^4}{8}\right]_1^2 = \left(-\frac{1}{4}-\frac{16}{8}\right)+\left(+1+\frac{1}{8}\right) = -1-\frac{1}{8} = -\frac{9}{8}$

8. $\int_4^9 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \left[\frac{2}{3} x\sqrt{x} - 2\sqrt{x}\right]_4^9 = \left(\frac{2}{3} 27 - 6\right) - \left(\frac{2}{3} 8 - 4\right) = \frac{32}{3}$

12. $\int_0^{2\pi} (1 + \sin(u))\, du = \int_0^{2\pi} du = 2\pi$

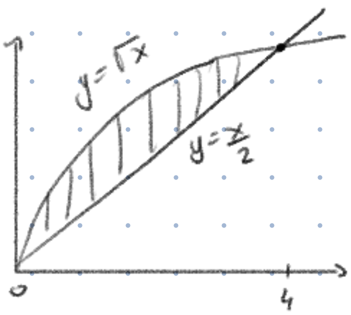
$\hookrightarrow \int_0^{2\pi} \sin(u)\, du = \left[-\cos(u)\right]_0^{2\pi} = -1 + 1 = 0$

21.



$\int_0^1 x^4 dx = \left|\frac{x^5}{5}\right|_0^1 = \frac{1}{5}$

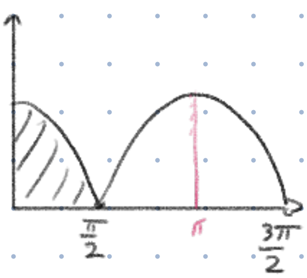
26.



intersection? $\sqrt{x} = \frac{x}{2} \Leftrightarrow x = \frac{x^2}{4} \Rightarrow x = 4$

$\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \left[\frac{2}{3} x\sqrt{x} - \frac{x^2}{4}\right]_0^4 = \frac{16}{3} - 4 = \frac{4}{3}$

33. $\int_0^{3\pi/2} |\cos(x)|\, dx = 3 \int_0^{\pi/2} \cos(x)\, dx = 3 \left[\sin(x)\right]_0^{\pi/2} = 3$



39. $\frac{d}{dx} \int_2^x \frac{\sin(t)}{t}\, dt = \frac{\sin(x)}{x}$

41. $\frac{d}{dx} \int_{x^2}^0 \frac{\sin(t)}{t}\, dt = -\frac{d}{du} \int_0^u \frac{\sin(t)}{t} \cdot \frac{du}{dx} = -2x \frac{\sin(u)}{u} = -2x \frac{\sin(x^2)}{x^2} = -\frac{2\sin(x^2)}{x}$

$u = x^2$

5.6 5, 6, 12, 39, 42

5. $\int \frac{x\,dx}{(4x^2+1)^5} = \frac{1}{8}\int \frac{du}{u^5} = \frac{1}{8}\frac{-1}{4u^4} = \frac{-1}{32}\frac{1}{(4x^2+1)^4} + C$

$u = 4x^2+1$, $du = 8x\,dx$

6. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx = 2\int \sin(u)\,du = -2\cos(u)+C = -2\cos(\sqrt{x}) + C$

$u = \sqrt{x}$, $du = \frac{dx}{2\sqrt{x}}$

12. $\int \frac{\ln(t)}{t}dt = \int u\,du = \frac{u^2}{2}+c = \frac{1}{2}(\ln(t))^2 + C$

$u = \ln(t)$, $du = \frac{dt}{t}$

39. $\int_0^4 x^3(x^2+1)^{-1/2}dx = \frac{1}{2}\int_1^{17}\frac{(u-1)}{\sqrt{u}}du = \frac{1}{2}\int_1^{17}\sqrt{u}\,du - \frac{1}{2}\int_1^{17}\frac{du}{\sqrt{u}} = \frac{1}{2}\frac{2}{3}\Big[u\sqrt{u}\Big]_1^{17} - \Big[\sqrt{u}\Big]_1^{17}$

$x^2 = u-1$ $\quad u = x^2+1$, $du = 2x\,dx$ $\quad x=0 \to u=1$, $x=4 \to u=17$

$= \frac{1}{3}(17\sqrt{17}-1) - (\sqrt{17}-1)$

$= \frac{14}{3}\sqrt{17} + \frac{2}{3}$

42. $\int_{\pi/4}^{\pi}\sin^5 x\,dx = -\int_{\sqrt{2}/2}^{-1}(1-u^2)^2du = \int_{-1}^{\sqrt{2}/2}(1-2u^2+u^4)du = \Big[u - \frac{2}{3}u^3 + \frac{u^5}{5}\Big]_{-1}^{\sqrt{2}/2}$

$u = \cos(x)$, $du = -\sin(x)dx$, $\sin^4(x) = (1-\cos^2 x)^2$ $\quad x = \frac{\pi}{4} \to u = \frac{\sqrt{2}}{2}$, $x = \pi \to u = -1$

$= \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{40}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)$

$= \sqrt{2}\left(\frac{21}{40} - \frac{1}{6}\right) + \left(\frac{6}{5} - \frac{2}{3}\right)$

$= \sqrt{2}\,\frac{43}{120} + \frac{8}{15}$

Review ex.

25. $\int_0^4 \sqrt{9t^2+t^4}\,dt = \int_0^4 t\cdot\sqrt{9+t^2}\,dt = \frac{1}{2}\int_9^{25}\sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}\Big[u\sqrt{u}\Big]_9^{25} = \frac{1}{3}(125-8)$

$u = 9+t^2$, $du = 2t\,dt$ $\quad x=0 \to u=9$, $x=4 \to u=25$

$= \frac{117}{3} = 39$