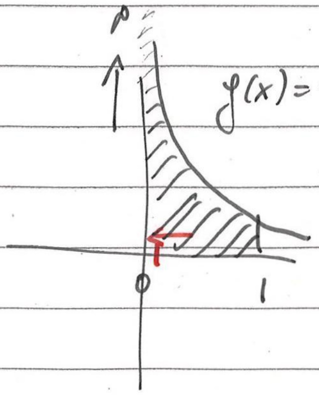
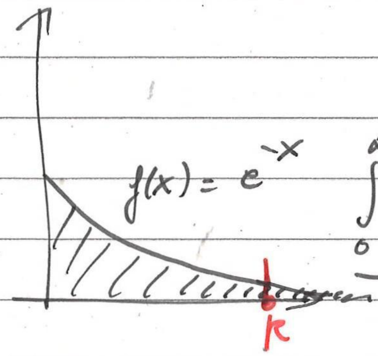
Improper integrals



$\int_0^1 \frac{dx}{x}$



$\int_0^\infty e^{-x} dx$ $\longrightarrow \infty$

type I improper integral — type II improper integral

* improper integrals might converge / diverge to ∞
  diverge

* how to calculate improper integrals

$$\int_0^\infty e^{-x} dx = \lim_{R \to \infty} \int_0^R e^{-x} dx = \lim_{R \to \infty} \left(\left[-e^{-x} \right]_0^R \right)$$

$$= \lim_{R \to \infty} \left(-e^{-R} + 1 \right) = 1$$

converging (limit exists)

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{R \to \infty} \left(\int_1^R \frac{dx}{x^2} \right) = \lim_{R \to \infty} \left(\left[\frac{-1}{x} \right]_1^R \right)$$

$$= \lim_{R \to \infty} \left(-\frac{1}{R} + 1 \right) = 1$$

$$\int_1^{+\infty} \frac{dx}{\sqrt{x}} = \lim_{R \to \infty} \left(\int_1^R \frac{dx}{\sqrt{x}} \right) = \lim_{R \to \infty} \left(\left[2\sqrt{x} \right]_1^R \right) =$$

$$= \lim_{R \to \infty} \left(2\sqrt{R} - 2 \right) = \infty$$ diverging to ∞

$$\int_0^\infty \sin(x) dx = \lim_{R \to \infty} \left(\int_0^R \sin(x) dx \right) = \lim_{R \to \infty} \left(\left[-\cos(x) \right]_0^R \right)$$

$$= \lim_{R \to \infty} \left(-\cos(R) + 1 \right)$$ DNE diverging

* ~~Exm~~ $\displaystyle\int_0^1 \frac{1}{x^2}\,dx = \lim_{a\to 0^+}\left(\int_a^1 \frac{dx}{x^2}\right) = \lim_{a\to 0^+}\left(\left[\frac{-1}{x}\right]_a^1\right)$

$\displaystyle = \lim_{a\to 0^+}\left(\frac{1}{a} - 1\right) = +\infty$ <u>diverging</u> to $+\infty$

$\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{a\to 0^+}\left(\int_a^1 \frac{dx}{\sqrt{x}}\right) = \lim_{a\to 0^+}\left(2\sqrt{x}\Big|_a^1\right) = \lim_{a\to 0^+}\left(2 - 2\sqrt{a}\right)$

$= 2$

<u>converging</u>

* if $f(x) \leq g(x)$ (on our domain of interest)
  - if $\displaystyle\int_A^B f(x)\,dx$ diverges (to $+\infty$), then $\displaystyle\int_A^B g(x)\,dx$ diverges
  - if $\displaystyle\int_A^B g(x)\,dx$ converges, then $\displaystyle\int_A^B f(x)\,dx$ converges

$\displaystyle\int_0^1 \frac{dx}{x\sqrt{1-x}}$

observe: $\displaystyle\frac{1}{x} \leq \frac{1}{x\sqrt{1-x}}$ on $(0,1)$

and $\displaystyle\int_0^1 \frac{dx}{x} = \lim_{a\to 0^+}\int_a^1 \frac{dx}{x} = \lim_{a\to 0^+}\left(\ln(x)\Big|_a^1\right)$

$\displaystyle = \lim_{a\to 0^+} -\ln(a) = +\infty$

so $\displaystyle\int_0^1 \frac{dx}{x\sqrt{1-x}}$ diverges!