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Let probability measure $\tilde{\nu}_{\gamma,\sigma}$ has density function

$$8\tilde{h}_{\gamma,\sigma}(x)=\begin{cases}\frac{\sigma\sqrt{\gamma}}{1+\sigma}h_{\gamma,\sigma}\left(\frac{\sigma}{1+\sigma}(\sqrt{\gamma}x+1)\right), & 0<\sigma\gamma\leq 1\\ \sqrt{\gamma}h_{\gamma}(1+\sqrt{\gamma}x), & \sigma=0,0<\gamma\leq 1;\\ \frac{1+\sigma}{2\pi}\sqrt{\frac{4}{1+\sigma}-x^2}, & \gamma=0,\sigma\geq 0.\end{cases}$$

Here h_γ is the density function of the well-known Marchenko-Pastur law ν_γ and $h_{\gamma,\sigma}$ is the density function of the Wachter law $\nu_{\gamma,\sigma}$, which are given as follows

$$h_{\gamma,\sigma}(x)=\frac{1+\sigma}{2\pi\sigma\gamma}\frac{\sqrt{(x-u_1)(u_2-x)}}{x(1-x)}1_{[u_1,u_2]}(x);$$

$$h_\gamma(x)=\frac{1}{2\pi\gamma x}\sqrt{(x-\gamma_1)(\gamma_2-x)}1_{\gamma_1\leq x\leq\gamma_2},$$

where $\gamma_{1,2}=(\sqrt{\gamma}\pm 1)^2$ and $u_{1,2}=\frac{\sigma}{1+\sigma}\left(\sqrt{1-\frac{\sigma\gamma}{1+\sigma}}\pm\sqrt{\frac{\gamma}{1+\sigma}}\right)^2$.

The gif.R offer a GIF to show the fluctuations of the density function $\widetilde{h}_{\gamma,\sigma}$ as γ and σ vary.

The sigto0.gif shows the fluctuation of the density function $\widetilde{h}_{\gamma,\sigma}$ when the parameter $\gamma=0.5$ and σ goes from 1 to 0.

The gamto0.gif shows the fluctuation of the density function $\widetilde{h}_{\gamma,\sigma}$ when the parameter $\sigma=0.5$ and γ goes from 0.9 to 0.