Read me

Inspired by the method on the construction of Monte Carlo estimator in Jiang [1], we consider the tail probabilities of extremal scaled eigenvalues of β-Jacobi ensemble.

A β-Jacobi ensemble $\mathcal{J}_n(p_1,p_2)$ is a set of random variables $\lambda := (\lambda_1, \lambda_2, \cdots, \lambda_n) \in [0,1]^n$ with joint probability density function

$$f_n^{p_1,p_2}(x_1,\cdots,x_n) = C_n^{p_1,p_2} \prod_{1\le i<j\le n} |x_i - x_j|^\beta \prod_{i=1}^n x_i^{r_{1,n}-1}(1-x_i)^{r_{2,n}-1}, \tag{1}$$

where $p_1, p_2 \ge n, r_{i,n} := \frac{\beta(p_i - n + 1)}{2}$ for $i = 1,2$ and $\beta > 0$, and the normalizing constant $C_n^{p_1,p_2}$ is given by

$$C_n^{p_1,p_2} = \prod_{j=1}^n \frac{\Gamma\left(1+\frac{\beta}{2}\right)\Gamma\left(\frac{\beta(p-n+j)}{2}\right)}{\Gamma\left(1+\frac{\beta j}{2}\right)\Gamma\left(\frac{\beta(p_1-n+j)}{2}\right)\Gamma\left(\frac{\beta(p_2-n+j)}{2}\right)}$$

with $p = p_1 + p_2$.

Let $(\lambda_1, \lambda_2, \cdots, \lambda_n)$ be the β-Jacobi ensemble $\mathcal{J}_n(p_1,p_2)$ with joint density function (1). For simplicity, set

$$X_i = \frac{(p_1+p_2)\lambda_i - p_1}{\sqrt{np_1}}.$$

Let $X_{(1)} < \cdots < X_{(n)}$ be the order statistics of $X_1, \cdots, X_n$.

In [2], we construct asymptotically efficient importance sampling estimators F_n related to $X_{(n)}$, which are all constructed by the "three-step Peeling" technique ([1]). In order to evaluate the actual performance of our algorithm, we conducted numerical stud in the Asym_IS.R. The details of the algorithm are presented in [2].

References

[1] Jiang, T., Leder, K., and Xu, G. (2017). Rare-event analysis for extremal eigenvalues of white wishart matrices. *Annals of Statistics*, **45**(4), 1609-1637.

[2] Ma, Y-T. and Wang, S-Y.. Rare-event analysis for extremal eigenvalues of β-Jacobi matrices at any temperatures.