46770 Integrated energy grids

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Lecture 8 – Join capacity and dispatch optimization in one node

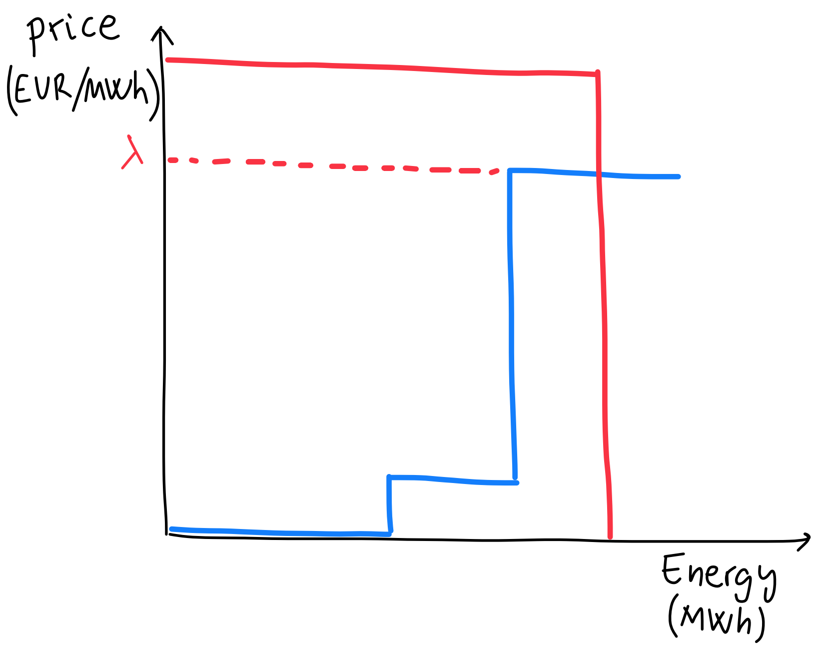
Types of optimization problems and course structure

	One node	Network			
		Power		Gas flow (Lecture 6)	Heat flow (Lecture 7)
One time step	Economic dispatch or One-node dispatch optimization (Lecture 2)	Linearized AC power flow (Lecture 4)	AC power flow (Lecture 5)		
Multiple time steps	Multi-period optimization Join capacity and dispatch optimization in one node (Lecture 8)	Join capacity and dispatch optimization in a network (Lecture 10)			

Review of Lecture 2

Economic dispatch example

	Wind	Solar	Gas
Variable cost o_s (EUR/MWh)	0	5	50
Installed Capacity G_s (MW)	2	1	1
CF_s	0.5	0.5	1
Calculate generation g_s (assuming demand d = 1.5 MWh)	1	0.5	0
Calculate generation g_s (assuming demand d = 2 MWh)	1	0.5	0.5



Economic dispatch is also called merit-order dispatch (because we rank the generators based on their merit)

Economic dispatch is used to decide which generators produce energy in a market.

Economic dispatch or one-node dispatch optimization (II)

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_s. The economic dispatch consists in calculating the optimal dispatch (how much energy is being produced by each generator g_s) to supply the demand d in a certain hour while minimizing the total system cost.

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_s}$$

To find a solution, we start by building the Lagrangian function

$$\mathcal{L}(x, \lambda, \mu) = \sum_s o_s g_s - \lambda \left(\sum_s g_s - d\right) - \sum_s \overline{\mu_s}(-g_s + G_s)$$

We derive the Lagrangian and make the derivative equal to zero

$$\frac{\partial \mathcal{L}}{\partial g_s} = o_s - \lambda^* + \overline{\mu_s^*} = 0$$

The inequality constraint can be binding ($\overline{\mu_s^*} > 0$) when the installed capacity is limiting the generation or not-binding ($\overline{\mu_s^*} = 0$).

The most expensive generator s whose capacity is not binding sets the price because for that generator $s1$ $\quad \overline{\mu_{s1}^*} = 0 \quad \lambda^* = o_{s1}$

Economic dispatch or one-node dispatch optimization (III)

$$
\left\{
\begin{aligned}
&\min_{g_s} \sum_s o_s g_s \\
&\text{subject to:} \\
&\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda \\
&g_s \geq 0 \;\leftrightarrow\; \underline{\mu_s} \\
&-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_s}
\end{aligned}
\right.
$$

λ represents the change in the objective function at the optimal solution with respect to a small change in the constraint.

Small change in constraint : $d^* = d^* + 1\ \mathrm{MWh}$

Change in objective function :

$$System\ cost^* = System\ cost + \Delta System\ cost$$

λ represents the cost of 1 MWh, i.e. the electricity price

So far, this set of equations excludes the consideration of any network constraints (e.g. line limits), and any additional security constraints, and additional generation constraints (CO_2 emissions, ramp limits …)

Learning goals for this lecture

- Describe the optimal generator and demand behaviors and their relation with maximizing welfare.
- Explain how renewable and backup generators recover their capital and variable costs via market revenues.
- Interpret the meaning of Lagrange/KKT multipliers associated with the constraints in join capacity and dispatch optimization problems and analyze their values.
- Formulate join capacity and dispatch optimization on a computer.

The first part of this lecture reviews the electricity market and shows why maximizing profits for independent generators is equivalent to maximizing social welfare.

For a comprehensive discussion and demonstration that the market-clearing problem obtains the Nash equilibrium solution (i.e. no market player desires to deviate from the market-clearing results), check the nice DTU course 46755 Renewables in Electricity Markets

Dispatch optimization in one node

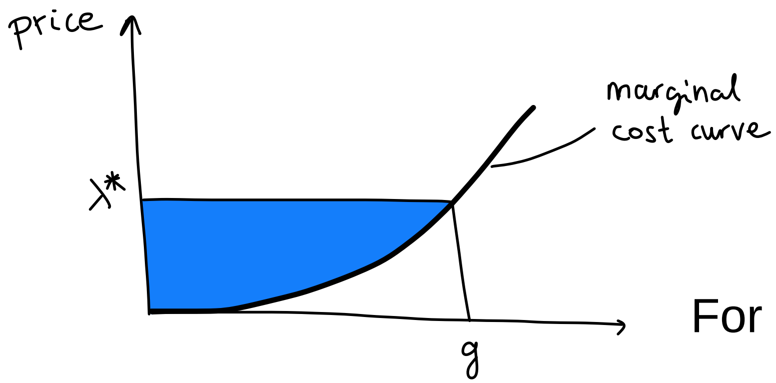
Optimal generator behavior

If we assume that a generator is a price-taker (it cannot influence the market price λ) which wants to maximize its profits (revenues minus expenditures)

$$\max_{g}[\lambda g - C(g)]$$

where g is the energy generated and $C(g)$ the cost curve of the generator

The generation g^* that maximizes the generator's profits fulfills $\lambda - \left.\frac{\partial C}{\partial g}\right|_{g=g^*} = 0 \quad \rightarrow \quad \lambda = \left.\frac{\partial C}{\partial g}\right|_{g=g^*}$



The marginal cost curve $\frac{\partial C}{\partial g}$ is the supply curve for a competitive firm. It shows for each generation g at which price the generator is willing to supply

For a generator with constant marginal costs $C(g) = o_s \cdot g \quad \rightarrow \quad \lambda = \left.\frac{\partial C}{\partial g}\right|_{g=g^*} = o_s$

The maximum generation can be constrained, e.g. by the installed capacity

$g_s \leq G_s \leftrightarrow \overline{\mu_s}$

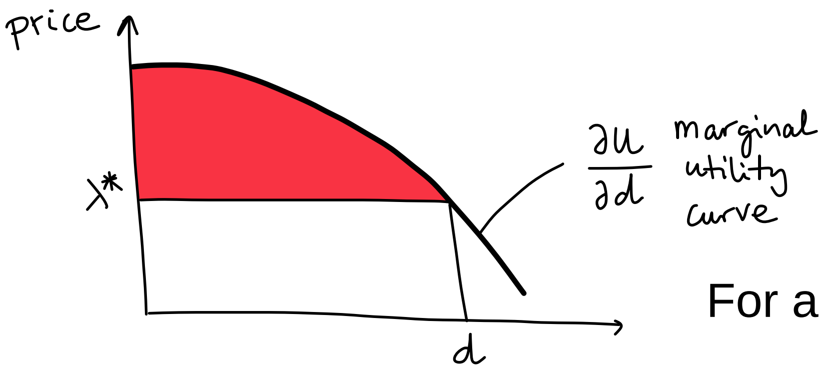
Optimal consumer behavior

If we assume that a consumer is a price-taker (it cannot influence the market price λ) which wants to maximize its net utility or profits (utility minus expenditures)

$$\max_{d}[U(d) - \lambda d]$$

where d is the energy consumed and $\mathrm{U}(d)$ the utility curve of the consumer

The consumption d^* that maximizes the consumer's net utility $\left.\frac{\partial U}{\partial d}\right|_{d=d^*} - \lambda = 0 \quad \rightarrow \quad \lambda = \left.\frac{\partial U}{\partial d}\right|_{d=d^*}$



The marginal utility curve $\frac{\partial U}{\partial d}$ is the demand curve for a rational consumer. It shows for each demand d at which price the consumer is willing to buy.

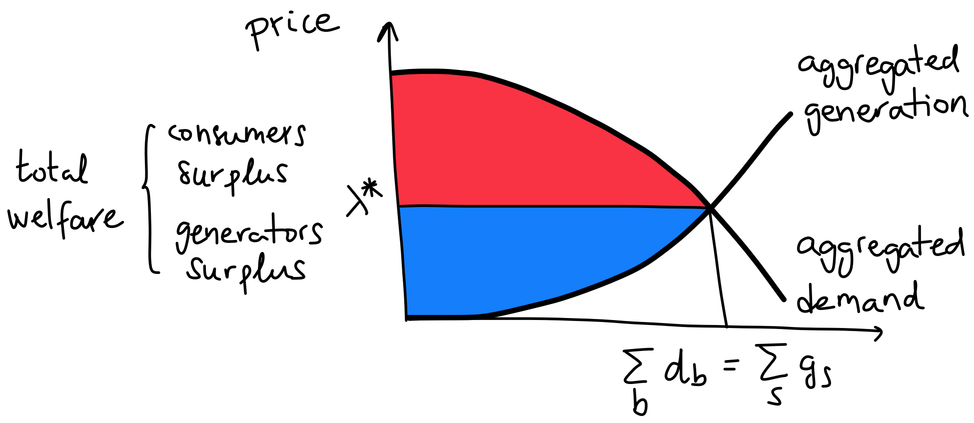
For a consumer with constant marginal utility $\mathrm{U}(d) = o_b \cdot d_b \rightarrow \quad \lambda = \left.\frac{\partial U}{\partial d}\right|_{d=d^*} = o_b$

The maximum demand can be constrained, e.g. by the capacity of the electrical machinery in the factory $\quad d_b \leq D_b \leftrightarrow \overline{\mu_b}$

Dispatch optimization in one node

The total welfare (consumers and generator surplus) is maximized at the point where aggregated marginal cost (offer) and aggregated marginal utility (demand) meet

$$\begin{cases} \max_{d_b,\, g_s} \left[\sum_b U_b\,(d_b) - \sum_s C_s\,(g_s) \right] \\ \text{subject to:} \\ \sum_b d_b - \sum_s g_s = 0 \leftrightarrow \lambda \end{cases}$$



$$0 = \frac{\partial \mathcal{L}}{\partial d_b} = \frac{\partial f}{\partial d_b} - \lambda \frac{\partial h}{\partial d_b} = \frac{\partial U}{\partial d_b} - \lambda^* = 0 \quad \rightarrow \quad \lambda^* = \left.\frac{\partial U_b}{\partial d_b}\right|_{b=b^*}$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_s} = \frac{\partial f}{\partial g_s} - \lambda \frac{\partial h}{\partial g_s} = -\frac{\partial C}{\partial g_s} + \lambda^* = 0 \quad \rightarrow \quad \lambda^* = \left.\frac{\partial C_s}{\partial g_s}\right|_{g=g^*}$$

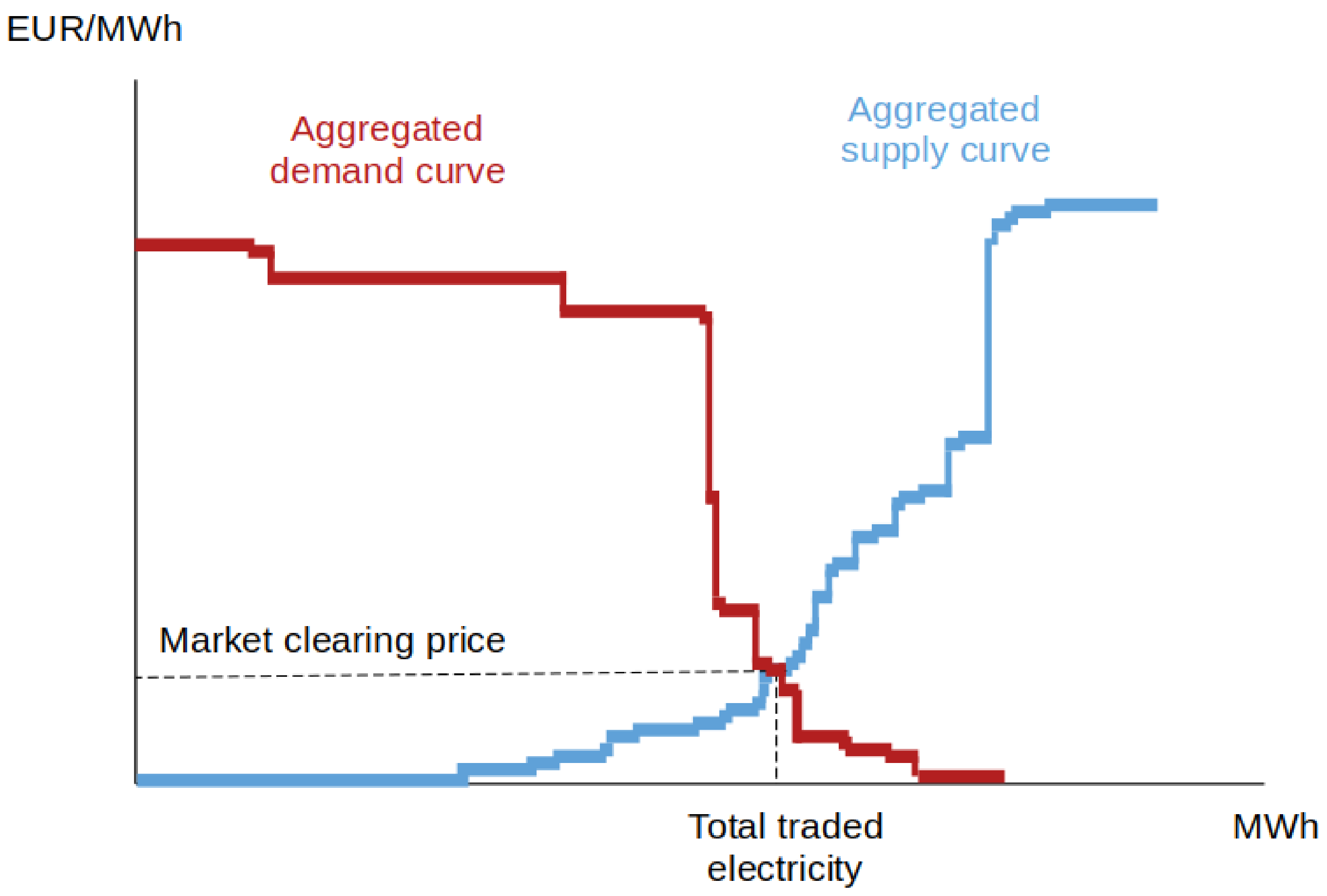
At the optimal point of maximum total economic welfare, we get the same result as if everyone maximizes their own welfare separately → This is a fundamental results in microeconomics that justifies electricity markets

Dispatch optimization in one node – Electricity markets

The electricity market is the same as markets for coffee, wood, or any other goods

There are a few relevant differences:

- Supply and demand of electricity must be equal at any time step
- Demand is typically inelastic
- The resulting economic optimal solution must be compatible with power flow equations on the networks

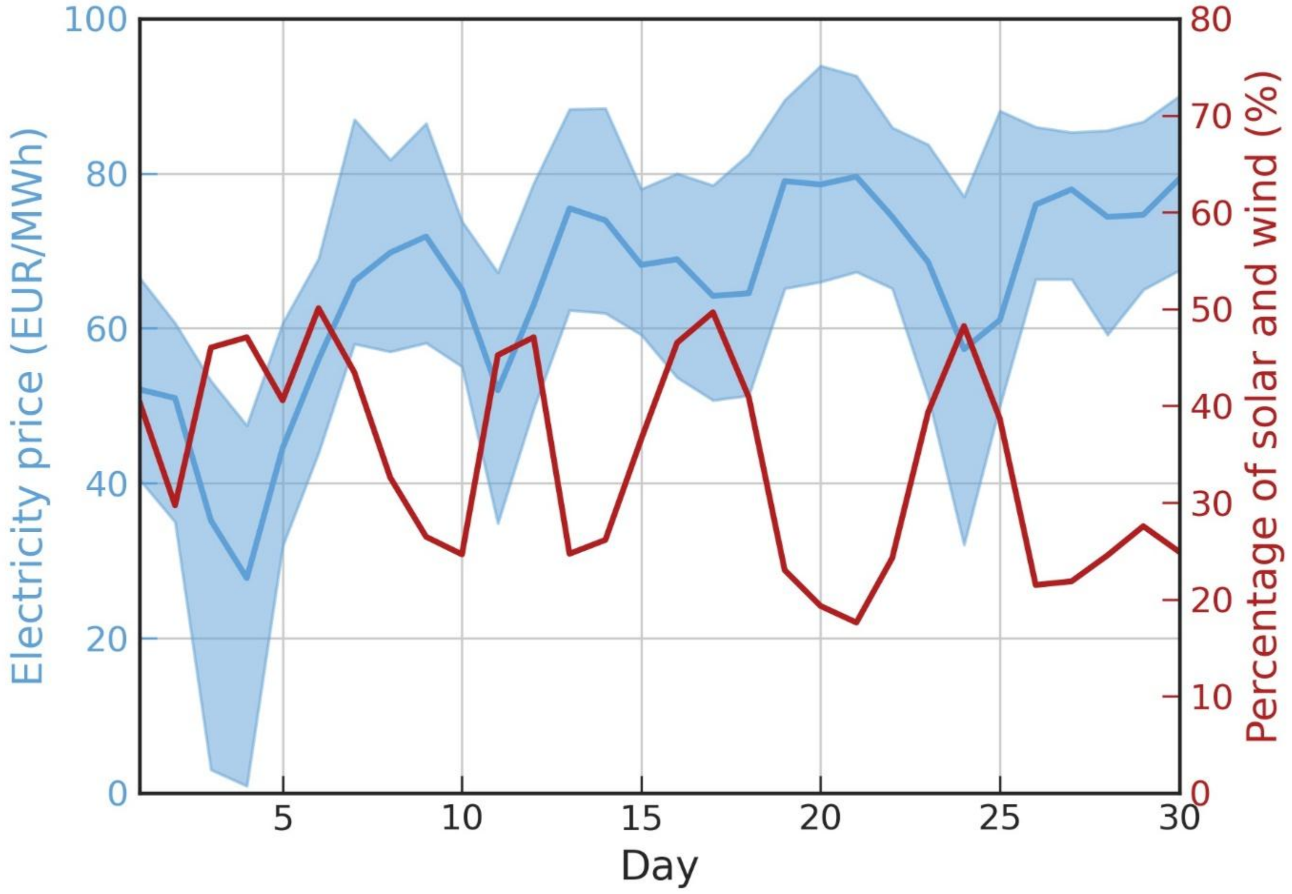


Ref: Victoria and Gallego, 2024, Economics of Photovoltaic Solar Energy (open-license figure)

For a comprehensive discussion on electricity markets, check the nice DTU course 46755 Renewables in Electricity Markets

Dispatch optimization in one node – Electricity markets

Increasing penetration of renewable generation reduces the market price. This is known as the merit-order effect.



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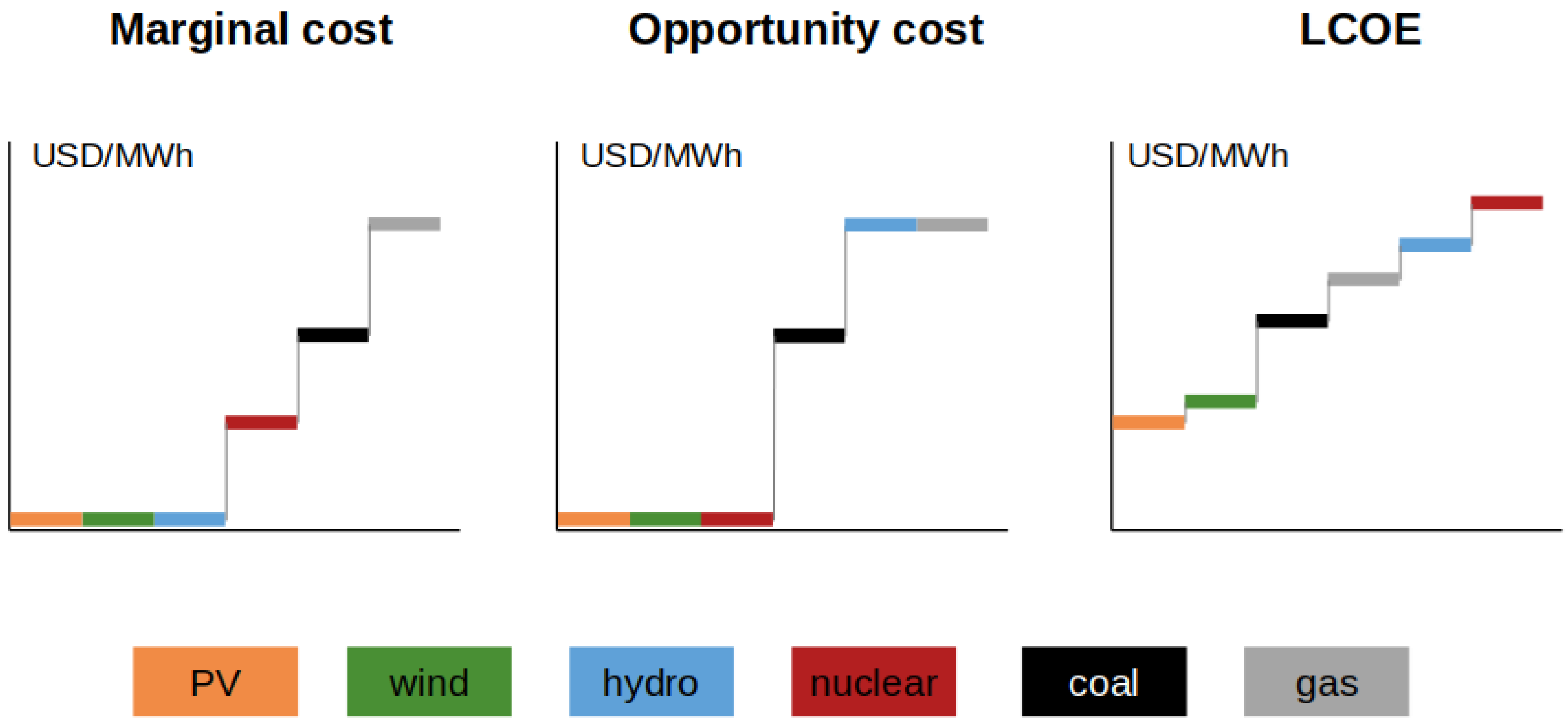
Dispatch optimization in one node – Electricity markets

Optimal generator behaviour is not only impacted by marginal generation cost but also by opportunity cost.



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Maximizing welfare

At the optimal point of maximum total economic welfare, we get the same result as if everyone maximizes their own welfare separately → This is a fundamental results in microeconomics that justifies electricity markets

Welfare can be maximized with decentralized markets:

1º) If the market price is equal to the Lagrange/KKT multiplier of the supply-demand balance constraint

2º) All consumers are price-takers under perfect competition

Maximizing welfare is equivalent to minimizing total cost

For the sake of simplicity, we can assume inelastic demand (i.e., it does note respond to price), consumers with linear utility and generators with linear costs

$$\left\{\begin{array}{l} \max_{g_s} \left[o_d D - \sum_s o_s \, g_s \right] \\ \text{subject to:} \\ D - \sum_s g_s = 0 \leftrightarrow \lambda \end{array}\right.$$

In this simple case, since D is a constant, maximizing welfare is equivalent to minimizing aggregated cost.

Maximizing welfare is equivalent to minimizing total cost

We can assume now elastic demand and represent demand elasticity or load shedding by adding a dummy generator so that $d = D - g_d$

$$\left\{\begin{array}{l} \max\limits_{g_s} \left[o_d D - o_d g_d - \sum\limits_{s} o_s \, g_s \right] \\ \text{subject to:} \\ \\ \qquad D - \sum\limits_{s} g_s = 0 \leftrightarrow \lambda \end{array}\right.$$

We can group the dummy generator with the rest of generators and again we can see that maximizing welfare is equivalent to minimizing aggregated cost.

Join capacity and dispatch optimization in one node

Join capacity and dispatch optimization in one node

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$\left\{\begin{array}{l} \min_{g_{s,t},\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s\, g_{s,t}\right] \\ \text{subject to:} \\ \quad \sum_s g_{s,t} - d_t \; = 0 \leftrightarrow \lambda_t \\ \quad -g_{s,t} + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}} \end{array}\right.$$

Energy balance in every time step

Generation limited by installed capacity in every generator and time step

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda_t^* + \overline{\mu_{s,t}^*} = 0 \quad \rightarrow \quad \lambda_t^* = \left.\frac{\partial C_s}{\partial g_s}\right|_{g=g^*}$$

The optimal solution for the dispatch is the same as “without capacity optimization” → For every time step t, the generator needed so that the suppy curve intersects the demand sets the price $\lambda_t^* = \left.\frac{\partial C_s}{\partial g_s}\right|_{g=g^*} = o_s$

Join capacity and dispatch optimization in one node

We can also derive the Lagrangian with respect to the installed capacity G_s of every generator

$$\min_{g_{s,}\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s \, g_{s,t} \right]$$

subject to:

$$\sum_s g_{s,t} - d_t \; = 0 \leftrightarrow \lambda_t$$

$$-g_{s,t} + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}}$$

From previous slide

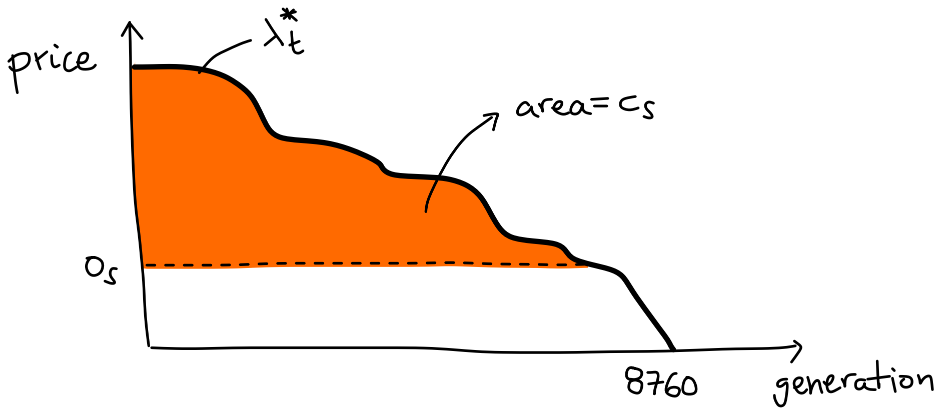
$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = \frac{\partial f}{\partial G_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial G_s} - \sum_j \mu_j \frac{\partial g_j}{\partial G_s} = c_s - \sum_t \overline{\mu^*_{s,t}} \cdot (1) = 0 \quad \rightarrow \quad c_s = \sum_t \overline{\mu^*_{s,t}} = \sum_t \lambda^*_t - o_s$$

The level of investment in generator capacity is optimal when the sum of the gap between the electricity price λ and the generator marginal cost o_s is equal to the capital cost of the added generation capacity.

Join capacity and dispatch optimization in one node

$$c_s = \sum_t \overline{\mu^*_{s,t}} = \sum_t \lambda^*_t - o_s$$

The level of investment in generator capacity is optimal when the sum of the gap between the electricity price λ and the generator marginal cost o_s is equal to the capital cost of the added generation capacity.

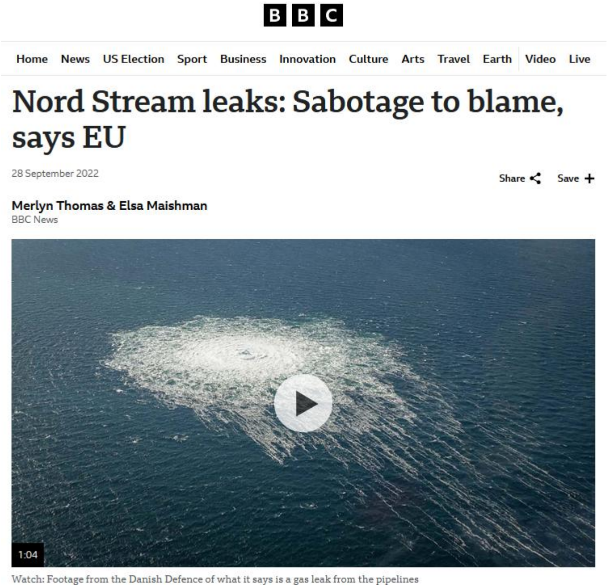


The price of the electricity market must be higher than the variable cost for the most expensive generator for several time steps. In those time steps, the expensive generator can cover its capital cost.

High electricity prices in energy markets

High electricity prices:

- Are needed to ensure that the most expensive generators recover their capital costs
- Create concerns in consumes and whole society
- Are a sign of scarcity (renewable droughts, gas scarcity …)
- Are a consequence of elastic demand (and could be smoothened by demand elasticity and additional electricity demand from other sectors)
- Can indicate market power (assumption of perfect competition is not valid anymore)
- How do we pay for capacities that ensure resilience but are rarely used throughout the year?



This is a hot discussion topic for European energy markets, some interesting angles:

- Fabra 2018, A primer on capacity mechanisms, Energy Economics
- Brown et al. 2024, Price formation without fuel costs https://arxiv.org/abs/2407.21409

Problems for this lecture

Problems 8.1 (**Group 16**)

Problems 8.2 (**Group 17**)

Feedback from mid-term survey