DTU

46770 Integrated energy grids

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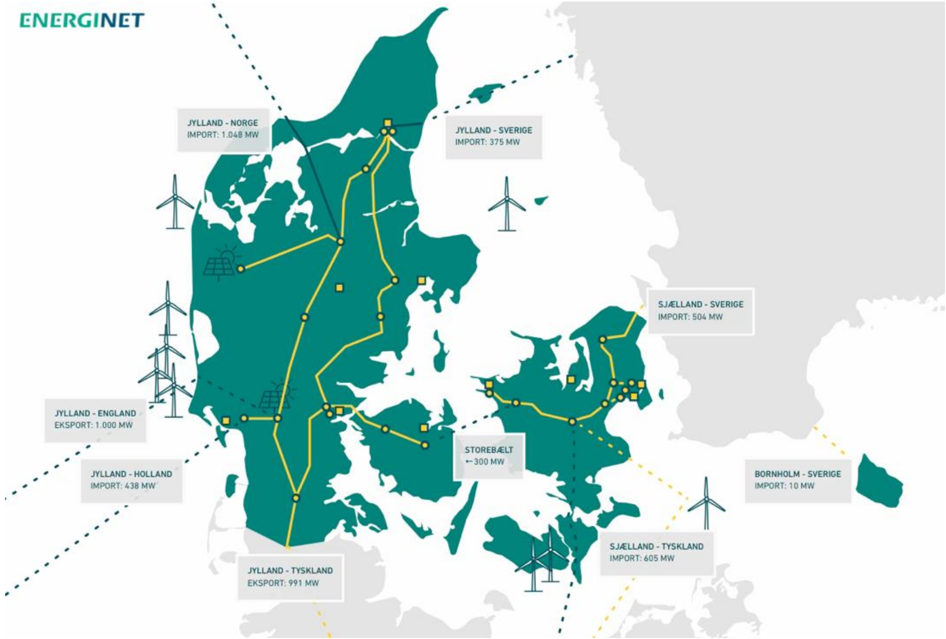
Lecture 10 - Capacity expansion and dispatch in a network

Learning goals

- Discuss benefits and risks of interconnection and market integration
- Calculate how transmission infrastructure recovers its cost
- Implement previous learnings about networks, power flow and capacity and dispatch optimisation
- Illustrate how energy storage recovers its cost
- Operate energy storage short-term and model it in long-term planning
- Build a mathematical model of a renewable-based power system

Layers of electricity networks

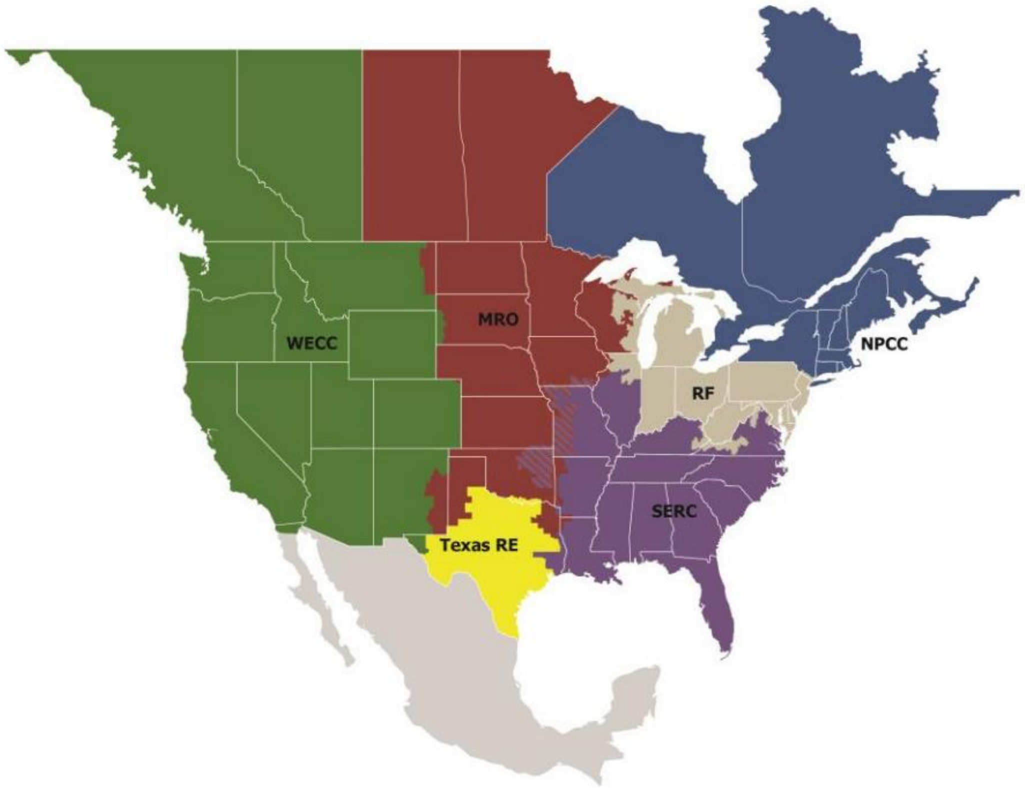
Source: ENTSO-E



energinet.dk/energisystemet-lige-nu/ Source: Energinet

Everything is bigger in Texas…

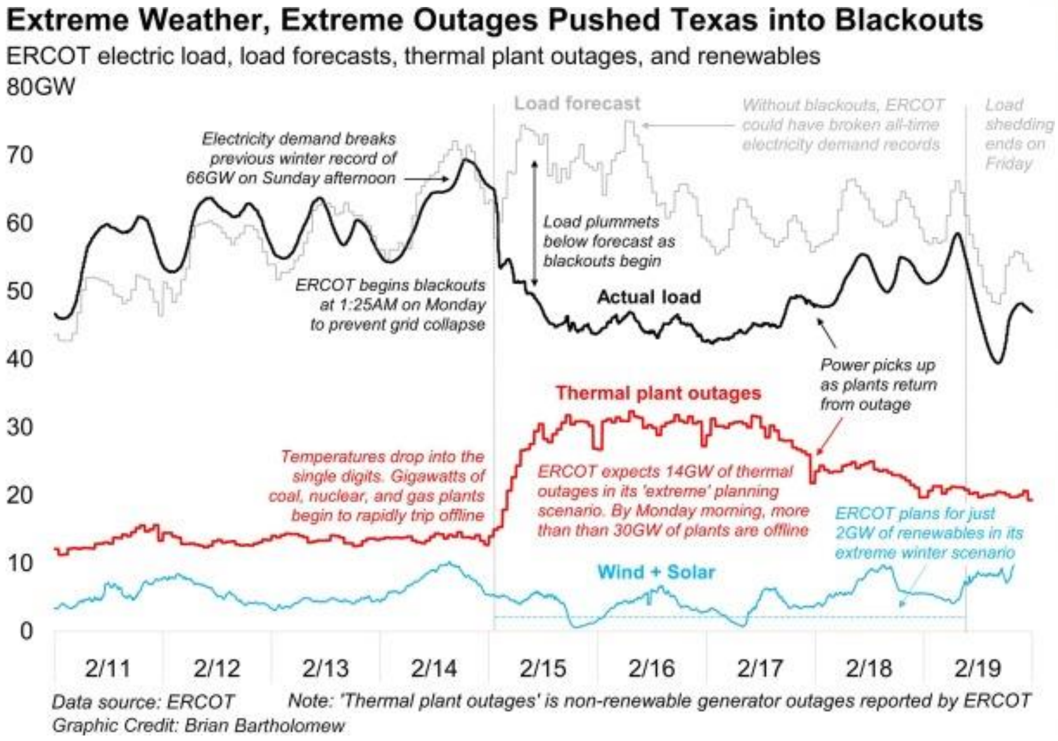
- …also the (self-imposed) problems: example of winter storm Uri in February 2021
- Simultaneous spikes in demand and failure of generation led to a catastrophic failure of the power system
  - 57 deaths
  - $195 bn in property damage
  - Power outages for several days with rolling blackouts
  - ”minutes away from grid failure that could have left Texas in the dark for weeks”

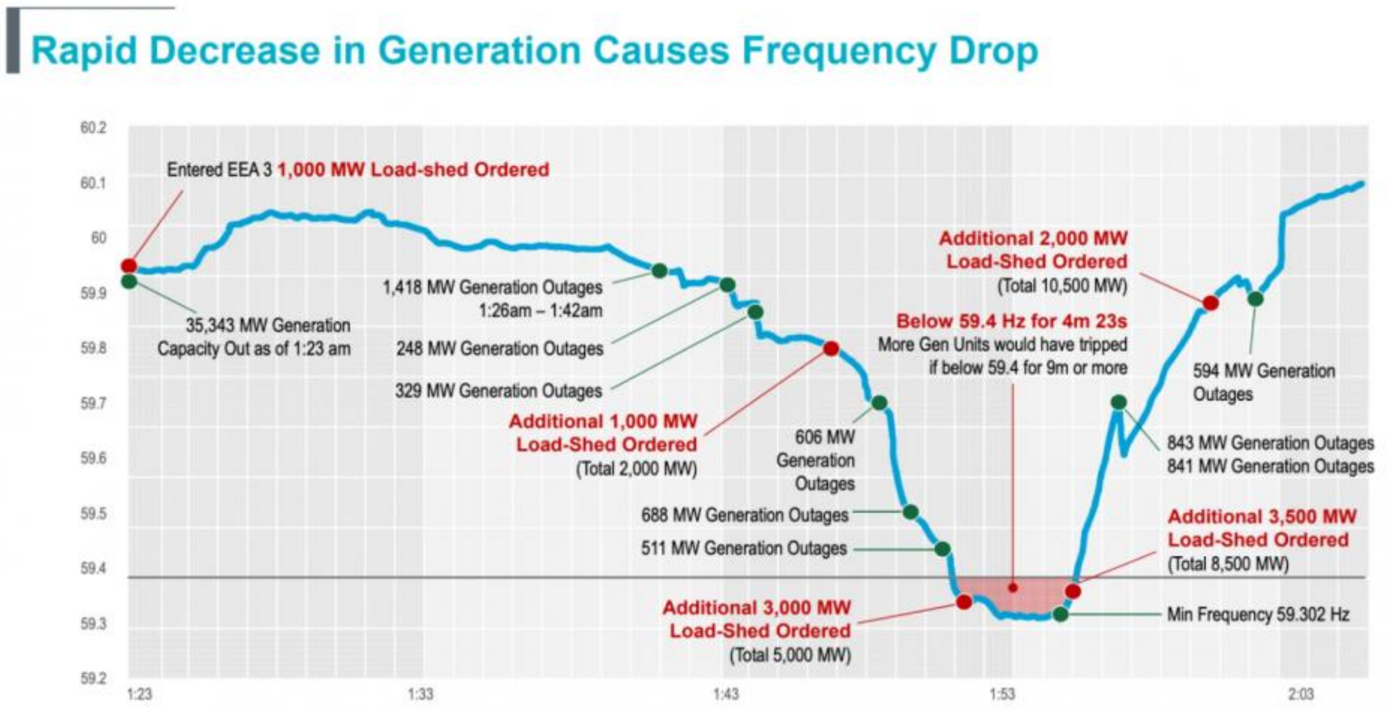


More info about the timeline

Meteorological conditions

Texas freeze and its ramifications on the power system





[Research on the Texas freeze]

Recommended watching: [https://www.youtube.com/watch?v=Zcrsgdl_hP0]

[Increasing resilience of the power grid in Texas]

Outages are rare, but not impossible in Europe

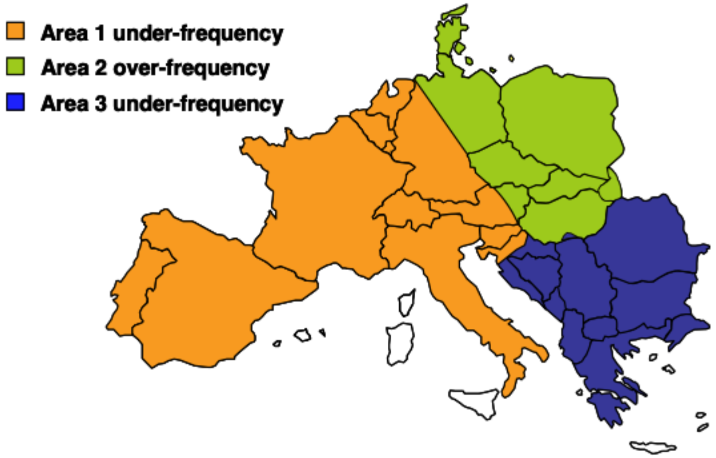


Figure 4: Schematic map of UCTE area split into three areas

Source: ENTSO-E

Figure 1 – Geographic area affected by the incident of 21 June 2024 (in black).

Source: ENTSO-E

Blackouts in 2006 routine disconnection cut power for 15 mn Europeans for a few hours

Grid incident (2024) cut off parts of the Balkan from the remaining European grid

Weighing off large-scale integration and modelling of different energy systems

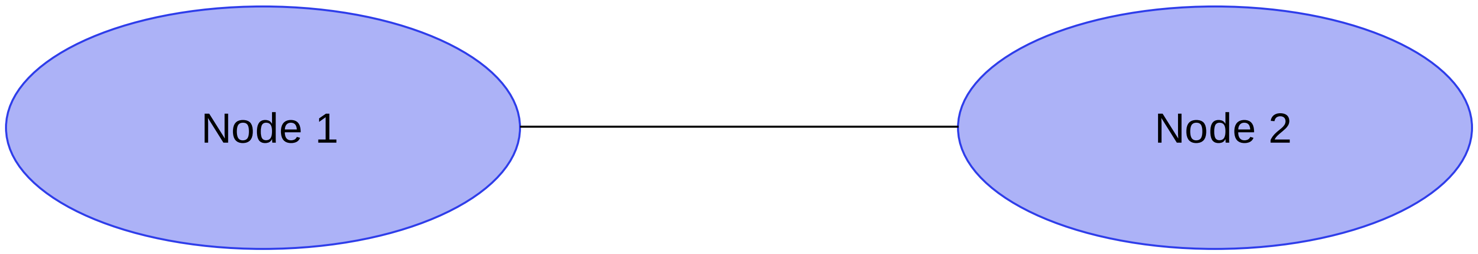
Advantages

- More reliability and security
- Transport of renewable, smoothing production
- Price signals
- Decentralised operation
- Modelling: closer to reality

Disadvantages

- Decentralised operation
- Price differences / equity considerations
- Reliance on energy policy of other regions
- Supranational/federal regulation, lack of national/regional control
- Modelling: computationally more difficult, discussions in Lectures 3-5

Two-node example

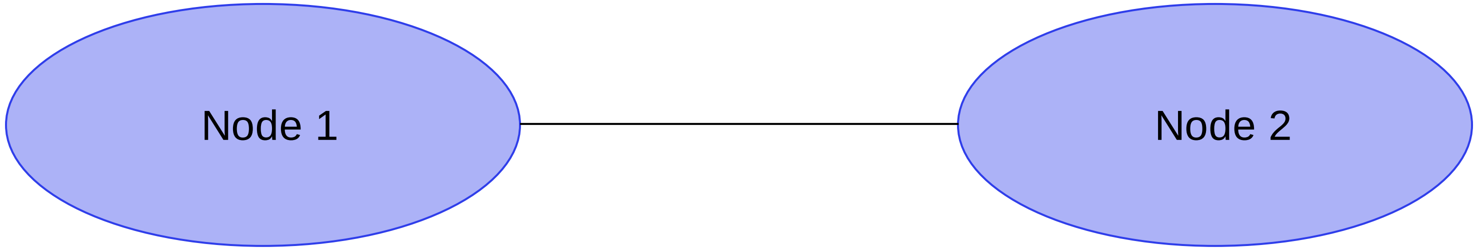


	Node 1	Node 2
Generators	Wind $G_{W,1}$ Fossil $G_{F,1}$	Wind $G_{W,2}$ Fossil $G_{F,2}$
Demand	$(d_1)_t$	$(d_2)_t$
Dispatch	$(g_{W,1})_t, (g_{F,1})_t$	$(g_{W,2})_t, (g_{F,2})_t$

Transmission capacity F_{ℓ}

Transmission flow $(f_{\ell})_t$

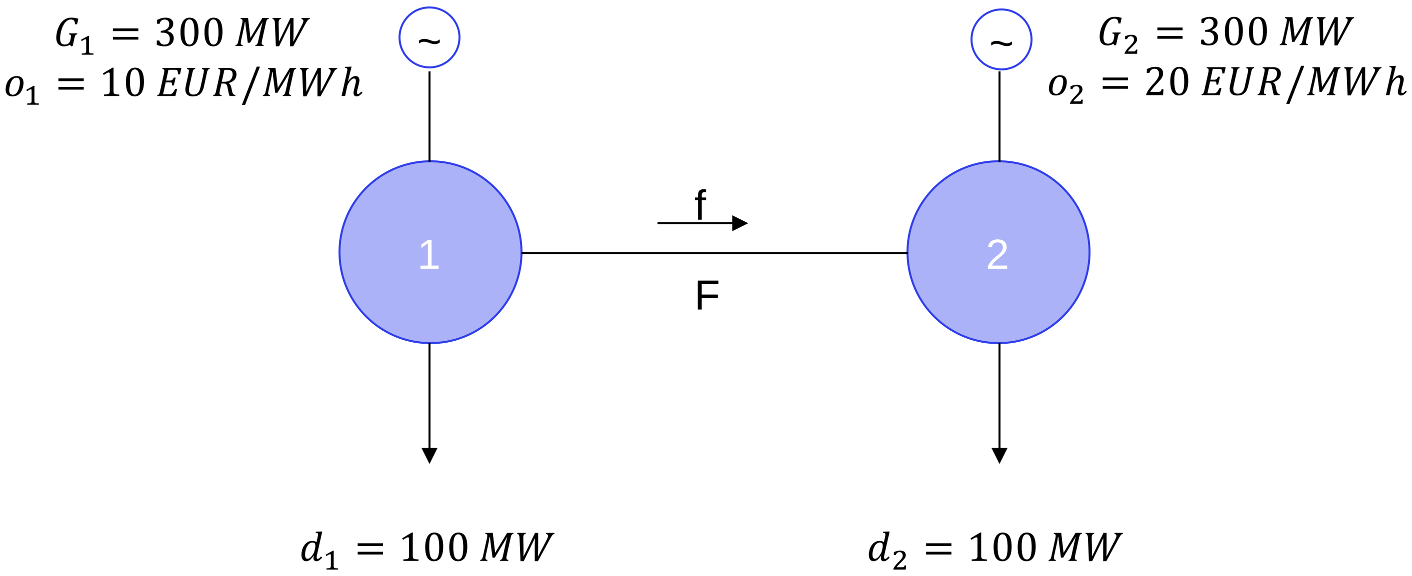
Two-node example



Constraints

	Node 1	Node 2	Constraints
Generators	Wind $G_{W,1}$ Fossil $G_{F,1}$	Wind $G_{W,2}$ Fossil $G_{F,2}$	≥ 0
Demand	$(d_1)_t$	$(d_2)_t$	Energy balance equation
Dispatch	$(g_{W,1})_t, (g_{F,1})_t$	$(g_{W,2})_t, (g_{F,2})_t$	≥ 0, limited by gen. cap.
	Transmission capacity F_ℓ		≥ 0
	Transmission flow $(f_\ell)_t$		Limited by transmission cap.

Congestion rent: example I



F = 0

$$\min o_1 g_1 + o_2 g_2$$

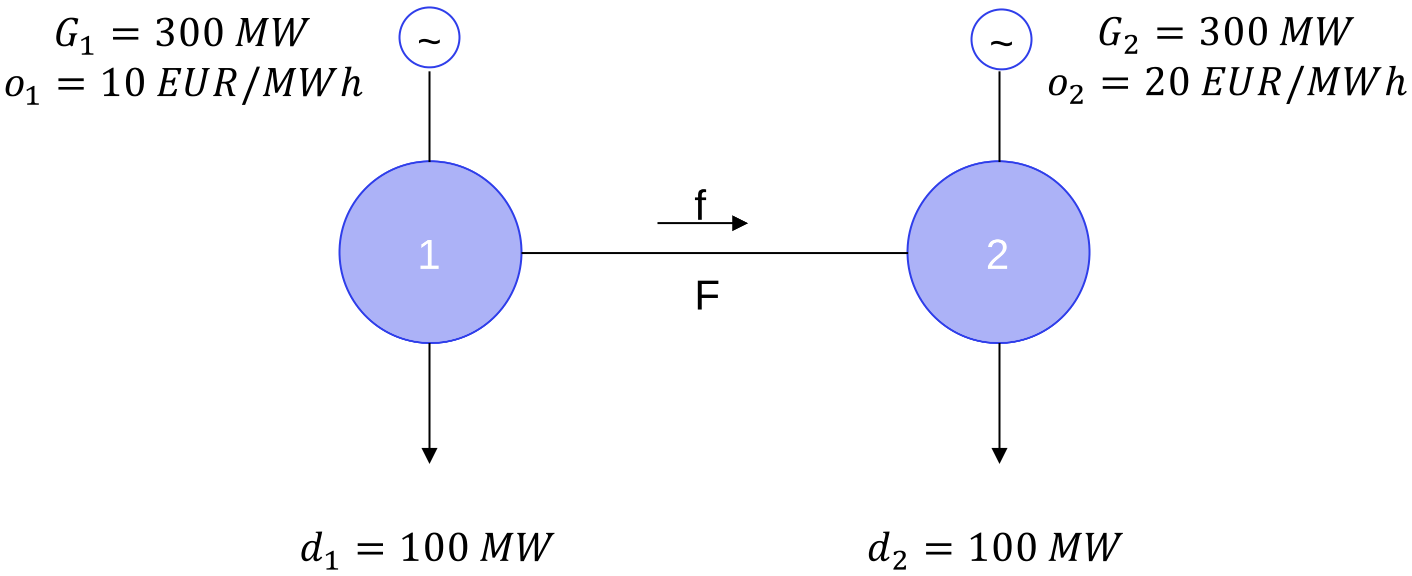
s.t.

$$g_i \leq G_i \longleftrightarrow \overline{\mu}_i$$
$$-g_i \leq 0 \longleftrightarrow \underline{\mu}_i$$
$$f \leq F \longleftrightarrow \overline{\mu}_f$$
$$-f \leq F \longleftrightarrow \underline{\mu}_f$$
$$g_i - d_i - f = 0 \longleftrightarrow \lambda_i$$

$$f^* = 0$$
$$g_i^* = d_i$$
$$0 = \overline{\mu}_i^*$$ (non-binding capacity constraints)

$$0 = \frac{\partial L}{\partial g_i} = o_i - \lambda_i \Rightarrow \lambda_i^* = o_i$$ (price set by generators)

Congestion rent: example II



$$F = \infty$$
$$g_1^* = d_1 + f^* = 200\ MW$$
$$g_2^* = d_2 - f^* = 0\ MW$$

$$\min\ o_1 g_1 + o_2 g_2$$

s.t.

$$g_i \le G_i \longleftrightarrow \overline{\mu}_i$$
$$-g_i \le 0 \longleftrightarrow \underline{\mu}_i$$
$$f \le F \longleftrightarrow \overline{\mu}_f$$
$$-f \le F \longleftrightarrow \underline{\mu}_f$$
$$g_i - d_i - f = 0 \longleftrightarrow \lambda_i$$

$$0 = \overline{\mu_1^*} = \overline{\mu_2^*} = \underline{\mu_1^*}$$

$$0 = \frac{\partial L}{\partial g_1} = o_1 - \lambda_1 \Rightarrow \lambda_1^* = o_1$$

$$0 = \frac{\partial L}{\partial g_2} = o_2 - \lambda_2 - \underline{\mu_2^*}(-1) \Rightarrow \underline{\mu_2^*} = \lambda_2^* - o_2$$

$$0 = \frac{\partial L}{\partial f} = -\lambda_1^*(-1) - \lambda_2^* - \cancel{\overline{\mu_f}} - \cancel{\underline{\mu_f^*}}(-1) \Rightarrow \lambda_1^* = \lambda_2^*$$

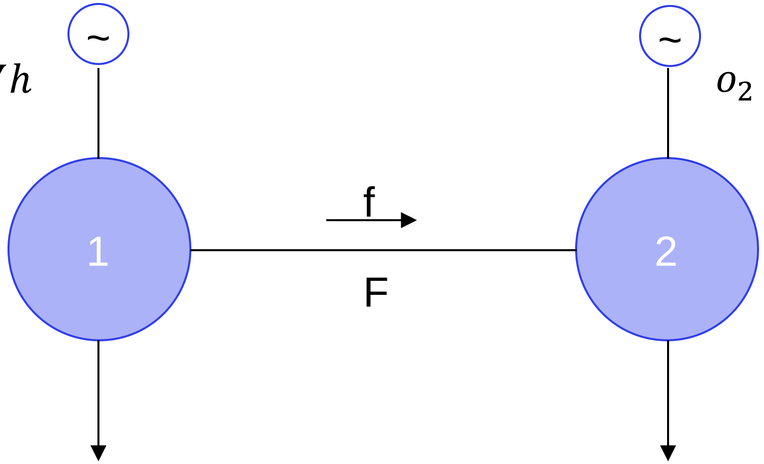
both = 0

Price equalization! Equivalent to merging both nodes.

Congestion rent: example III

$G_1 = 300\ MW$
$o_1 = 10\ EUR/MWh$

$G_2 = 300\ MW$
$o_2 = 20\ EUR/MWh$



$d_1 = 100\ MW$ $\quad$ $d_2 = 100\ MW$

$F = 50\ MW$
$g_1^* = d_1 + f^* = 150\ MW$
$g_2^* = d_2 - f^* = 50\ MW$

$$\min o_1 g_1 + o_2 g_2$$

s.t.

$$\begin{aligned}
g_i &\leq G_i & \longleftrightarrow \quad & \overline{\mu}_i \\
-g_i &\leq 0 & \longleftrightarrow \quad & \underline{\mu}_i \\
f &\leq F & \longleftrightarrow \quad & \overline{\mu}_f \\
-f &\leq F & \longleftrightarrow \quad & \underline{\mu}_f \\
g_i - d_i - f &= 0 & \longleftrightarrow \quad & \lambda_i
\end{aligned}$$

$$f^* = 50\ MW$$
$$0 = \overline{\mu_i^*} = \underline{\mu}^*{}_i$$

$\lambda_1^* = 10\ EUR/MWh$
$\lambda_2^* = 20\ EUR/MWh$

$$0 = \frac{\partial L}{\partial g_i} = o_i - \lambda_i \Rightarrow \lambda_i^* = o_i$$

= 0 (non-binding)

$$0 = \frac{\partial L}{\partial f} = -\lambda_1(-1) - \lambda_2 - \overline{\mu_f} - \underline{\mu}_f(-1)$$

$$\Rightarrow \overline{\mu_f^*} = \lambda_1^* - \lambda_2^* = 10 - 20 = -10\ EUR/MWh$$

Congestion rent mathematically

Case	Consumer expenses EUR/h	Generator income EUR/h	Price difference EUR/MWh	Flow MW	Congestion rent EUR
$F = 0$	3000	3000	10	0	0
$F = 50\ MW$	3000	2500	10	50	500
$F = \infty$	2000	2000	0	100	0

$$\text{Congestion rent} = \Delta\lambda \cdot f = \text{Price difference} \cdot \text{Power flow}$$

Consumer expenses and generator income depend on the nodal price.

Congestion rent is the difference between the total payment of consumers and total revenues of producers. It is the income of the network operator for transferring energy from a low-price node to a high-price node.

As congestion rent is the only income for the network operators, this is how it recovers capital costs.

Mathematical formulation

Capacity and dispatch optimisation in a network

$$\min \underbrace{\sum_{s,i} c_{s,i} \cdot G_{s,i} + \boxed{\sum_{l} c_l \cdot F_l}}_{\text{investment costs}} + \underbrace{\sum_{s,t,i} o_{s,i} \cdot g_{s,i,t}}_{\text{operational costs}} \qquad (1)$$

Minimizing system cost or maximizing social welfare

such that:

$$\begin{array}{rlll}
\sum_s g_{s,t,i} - d_{t,i} - \boxed{\sum_l K_{il} \cdot f_{l,t}} & = 0 & \leftrightarrow \lambda_{i,t} & \text{for all } i, t \\
g_{s,t,i} - G_{s,i} & \leq 0 & \leftrightarrow \mu_{i,s,t} & \text{for all } i, s, t, \\
f_{l,t} & \leq F_l & \leftrightarrow \overline{\mu_{l,t}} & \text{for all } l, t, \\
-f_{l,t} & \leq F_l & \leftrightarrow \underline{\mu_{l,t}} & \text{for all } l, t, \\
\sum_c C_{lc} \cdot x_l \cdot f_l & = 0 & \leftrightarrow \lambda_c & \text{for all cycles } c,
\end{array} \qquad (2)$$

Balance constraint (gen., demand, im-/export) must be fulfilled at all times

Electricity generation limited by installed capacity

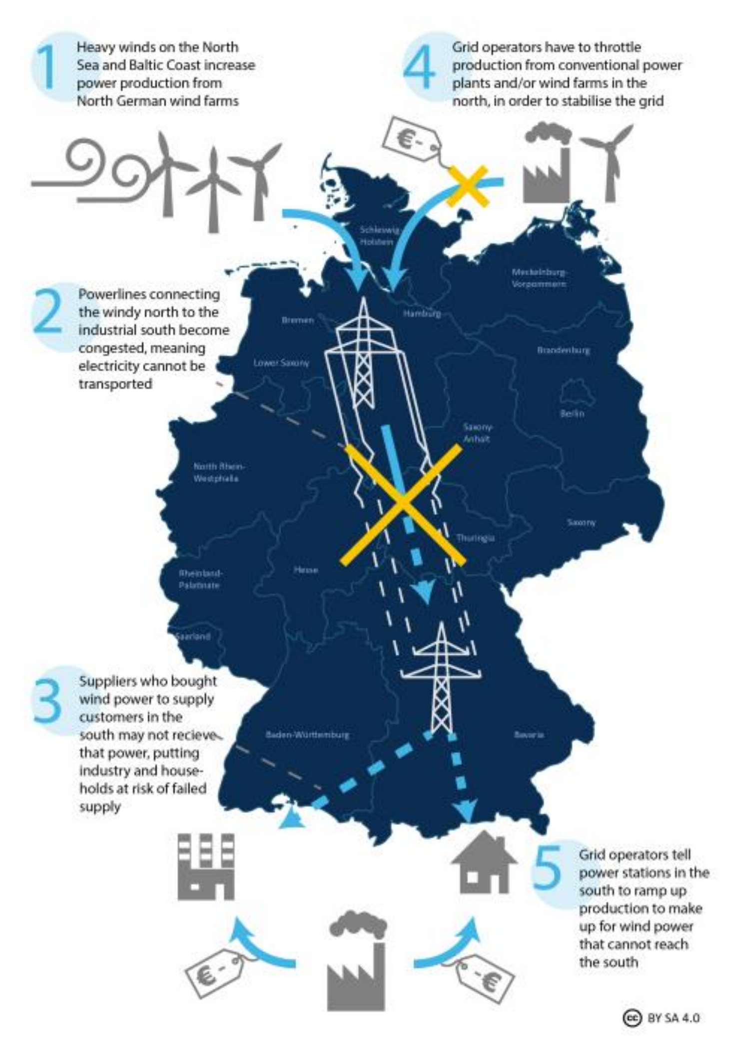
Flow through links are limited by maximum capacity (recall discussions about net-transfer-capacities/linearized AC power flow)

Kirchhoff's voltage law for every cycle

Symbols	Description
i, t, s, l	Indices for node, time step, generator, line
$G_{s,i}, F_l$	Investment variable of generator s at node i, line l
$g_{s,i,t}, f_{l,t}$	Operational variable of generator g at node i, line l at time t
$c_{s,i}, c_l$	Investment costs of generator g at node i, line l
$o_{s,i}$	Operational costs of generator g at node i
$d_{i,t}$	Demand at node i at time step t
$f_{l,t}$	Transmission flow through line l at time t
$\lambda_{i,t}$	Electricity price at node i at time t
$\mu_{i,s,t}, \mu_{l,t}, \lambda_c$	other KKT multipliers
K_{il}	Incidence matrix
c_{lc}	Cycle matrix

Figure 1: Formulation of an energy system optimisation model in terms of a LP.

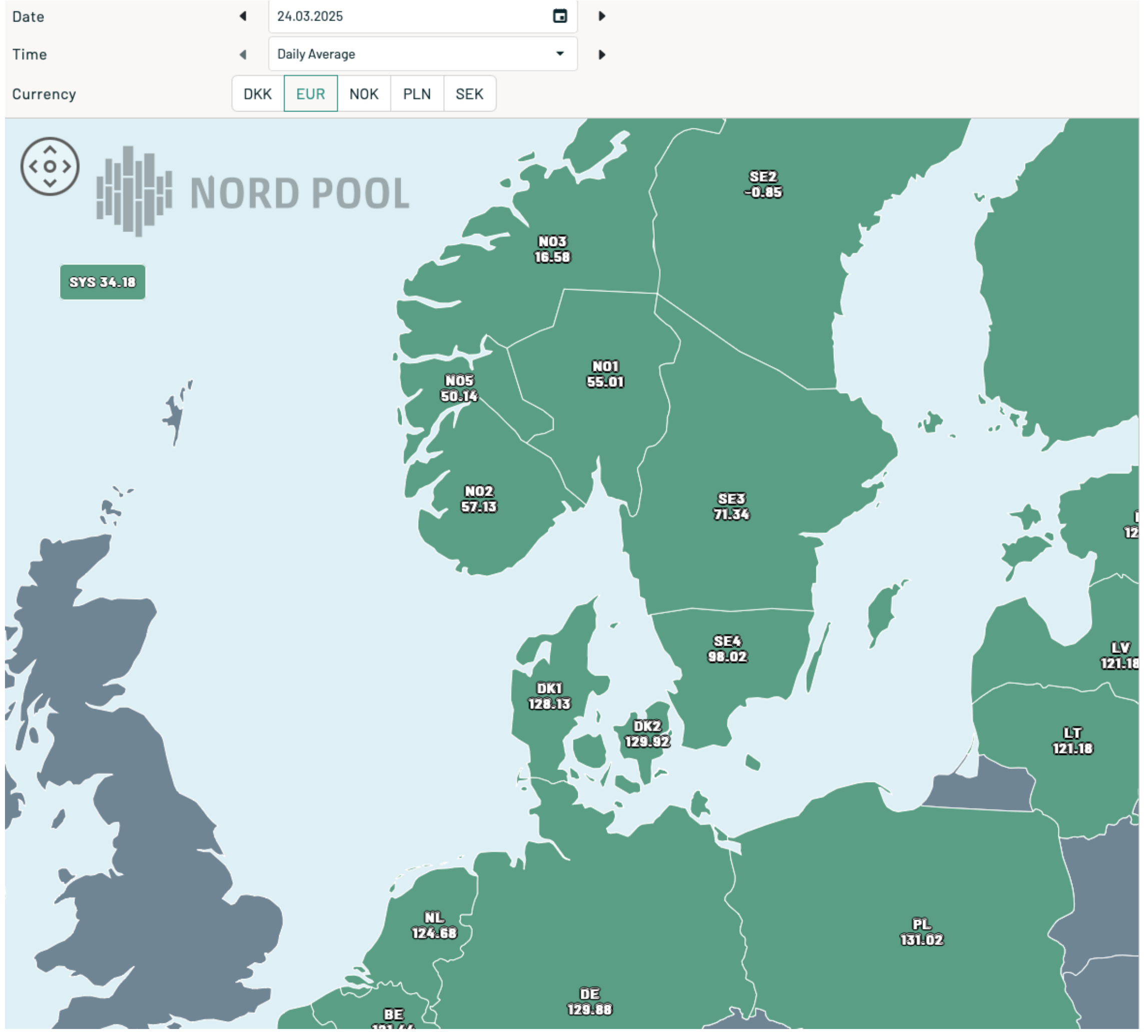
Physical infrastructure and market design



Source: CLEW, 2016

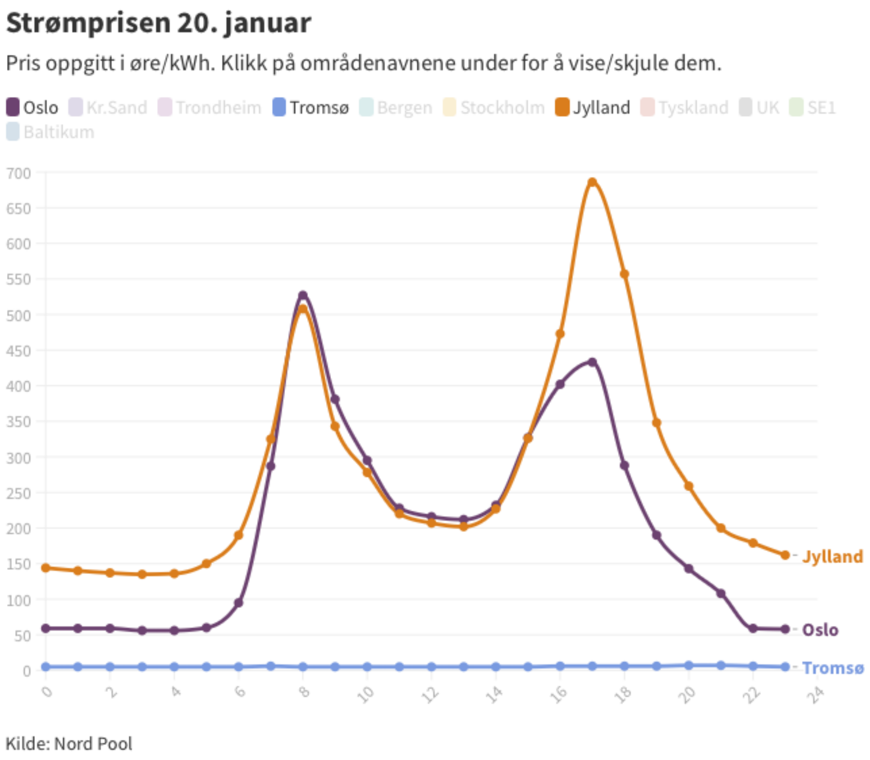
- Mismatch of renewable and transmission capacity build-out in Germany
    - Large opposition against transmission power-lines
- In 2023, approx. 4% of electricity production (~19 TWh) was curtailed, mostly wind power
- Grid operators need to align supply and demand according to transmission grid, not necessarily how market was cleared
- Significant costs for consumers via grid fees (2023: 3.1 bn EUR)
    - Compensation for curtailment and re-dispatch
    - Misguided market incentives
    - Grid fees are higher where most investment in transmission grid is done
- Political decisions vs economic or technical factors

Market zones and nodes in a network



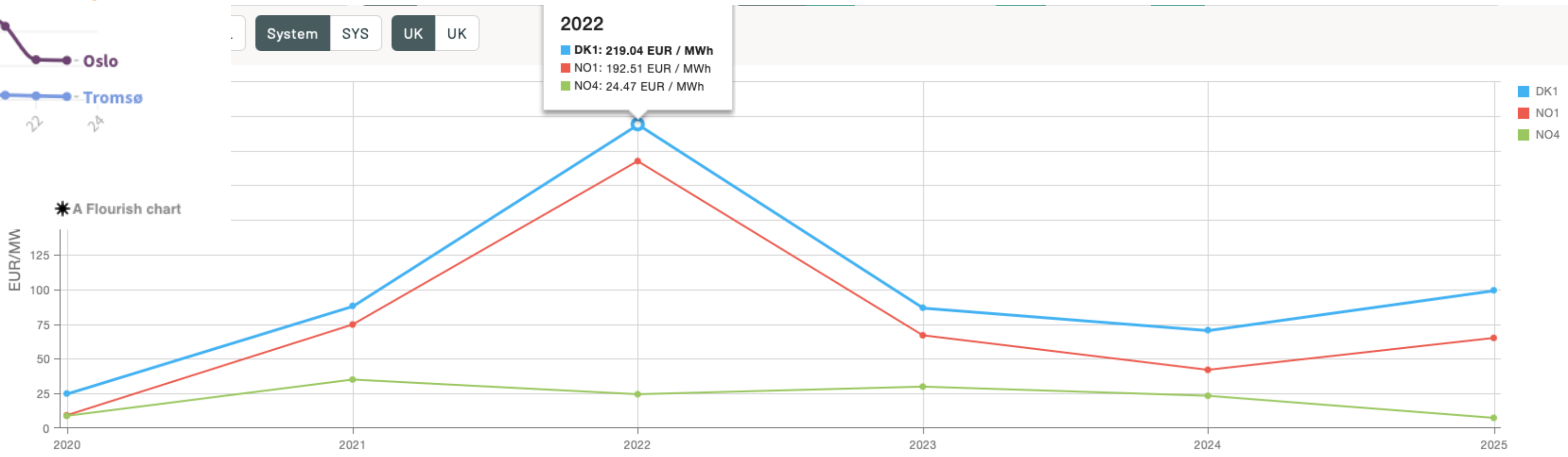
- Other countries are split up in different market zones
  - Norway, Sweden, Denmark, Italy
- Often justified by network infrastructure, particularly lack of transmission
  - Can lead to large price differences, and as we have seen congestion rent
  - Investment in transmission expansion can be financed by congestion rent
- Efficient from a market perspective, as price level determines profitability of investments or spurs investment

Market failure or functioning markets?



Source: NRK

- Integrated markets reduce manoeuvring space of policymakers
  - National policy might be short- or mid-term hard to achieve (price stability or similar national prices), example during gas/energy crisis in Norway
- Appeal of fixed prices: predictability
  - Drawbacks: lack of incentives for efficiency or investments; cost recovery



Source: Nordpool

Capacity and dispatch optimisation in a network with storage

$$\min \underbrace{\sum_{s,i} c_{s,i} \cdot G_{s,i} + \sum_{l} c_l \cdot F_l + \boxed{\sum_{i,r} c_{r,i} \cdot G_{i,r,\text{charge}} + \sum_{i,r} c_{r,i} \cdot G_{i,r,\text{discharge}} + \sum_{r,i} c_{r,i} \cdot E_{r,i}}}_{\text{investment costs}}$$

$$+ \underbrace{\sum_{s,t,i} o_{s,i} \cdot g_{s,i,t} + \boxed{\sum_{r,i,t} o_{i,r,\text{charge}} \cdot g_{i,r,\text{charge}} + \sum_{r,i,t} o_{i,r,\text{discharge}} \cdot g_{i,r,\text{discharge}}}}_{\text{operational costs}} \quad (3)$$

Minimising system cost or maximising social welfare

such that:

$\sum_s g_{s,t,i} + \sum_r (g_{r,t,i,\text{discharge}} - g_{r,t,i,\text{charge}}) - d_{t,i} - \sum_l K_{il} \cdot f_{l,t}$	$= 0$	$\leftrightarrow \lambda_{i,t}$	for all i, t	Balance constraint (gen., demand, im-/export) must be fulfilled at all times
$g_{s,t,i} - G_{s,i}$	≤ 0	$\leftrightarrow \mu_{i,s,t}$	for all $i, s, t,$	Electricity generation limited by installed capacity
$f_{l,t}$	$\leq F_l$	$\leftrightarrow \overline{\mu_{l,t}}$	for all $l, t,$	Flow through links limited by maximum capacity (recall discussions about net-transfer-capacities/linearized AC power flow)
$-f_{l,t}$	$\leq F_l$	$\leftrightarrow \underline{\mu_{l,t}}$	for all $l, t,$	
$\sum_c C_{lc} \cdot x_l \cdot f_l$	$= 0$	$\leftrightarrow \lambda_c$	for all cycles $c,$	Kirchhoff's voltage law for every cycle
$g_{r,t,i,\text{charge}} - G_{r,i,\text{charge}}$	≤ 0	$\leftrightarrow \mu_{r,t,i,\text{charge}}$	for all $i, r, t,$	Charging and discharging limited by capacity
$g_{r,t,i,\text{discharge}} - G_{r,i,\text{discharge}}$	≤ 0	$\leftrightarrow \mu_{r,t,i,\text{discharge}}$	for all $i, r, t,$	
$0 \leq e_{i,r,t}$	$\leq E_{r,i}$	$\leftrightarrow \overline{\mu_{r,t,i}}, \underline{\mu_{r,t,i}}$	for all $i, r, t,$	Storage level limited by capacity
$\boxed{\eta_0 \cdot e_{i,r,t-1} + \eta_{\text{charge}} \cdot g_{i,r,t,\text{charge}} - \frac{1}{\eta_{\text{discharge}}} \cdot g_{i,r,t,\text{discharge}}}$	$= e_{i,r,t}$	$\leftrightarrow \mu_{i,r,t}$	for all $i, r, t.$	Consistency of storage levels across time steps (+efficiencies)

(4)

Capacity and dispatch optimisation in a network with storage

$$\min \underbrace{\sum_{s,i} c_{s,i} \cdot G_{s,i} + \sum_{l} c_l \cdot F_l + \sum_{i,r} c_{r,i} \cdot G_{i,r,\text{charge}} + \sum_{i,r} c_{r,i} \cdot G_{i,r,\text{discharge}} + \sum_{r,i} c_{r,i} \cdot E_{r,i}}_{\text{investment costs}}$$
$$+ \underbrace{\sum_{s,t,i} o_{s,i} \cdot g_{s,i,t} + \sum_{r,i,t} o_{i,r,\text{charge}} \cdot g_{i,r,\text{charge}} + \sum_{r,i,t} o_{i,r,\text{discharge}} \cdot g_{i,r,\text{discharge}}}_{\text{operational costs}} \quad (3)$$

Minimizing system cost or maximizing social welfare

such that:

$\sum_s g_{s,t,i} + \sum_r (g_{r,t,i,\text{discharge}} - g_{r,t,i,\text{charge}}) - d_{t,i} - \sum_l K_{il} \cdot f_{l,t}$	$= 0$	$\leftrightarrow \lambda_{i,t}$	for all i, t	Balance constraint (gen., demand, im-/export) must be fulfilled at all times
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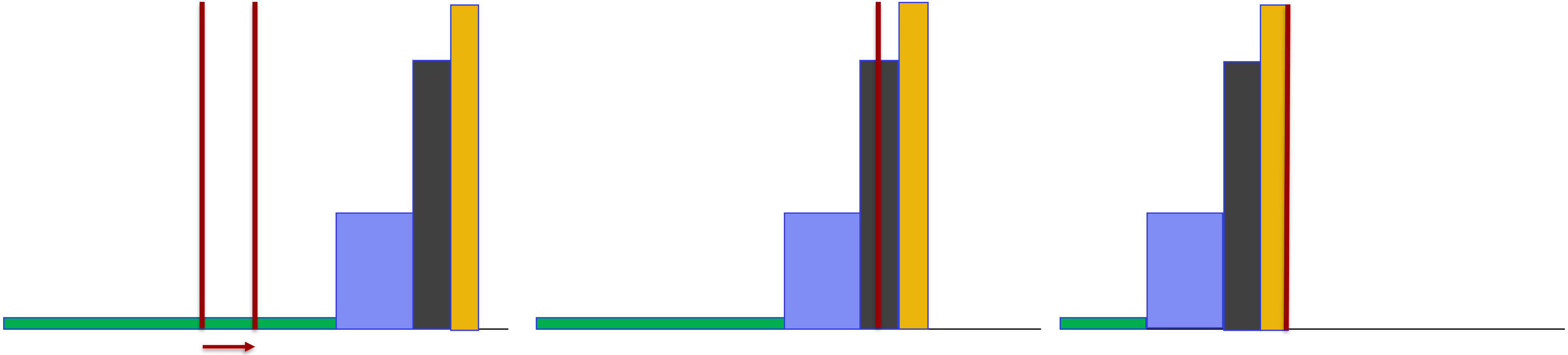
$f_{l,t}$	$\leq F_l$	$\leftrightarrow \overline{\mu_{l,t}}$	for all $l, t,$	Flow through links are limited by maximum capacity (recall discussions about net-transfer-capacities/linearized AC power flow)
$-f_{l,t}$	$\leq F_l$	$\leftrightarrow \underline{\mu_{l,t}}$	for all $l, t,$	
$\sum_c C_{lc} \cdot x_l \cdot f_l$	$= 0$	$\leftrightarrow \lambda_c$	for all cycles $c,$	Kirchhoff's voltage law for every cycle
$g_{r,t,i,\text{charge}} - G_{r,i,\text{charge}}$	≤ 0	$\leftrightarrow \mu_{r,t,i,\text{charge}}$	for all $i, r, t,$	Charging and discharging limited by capacity
$g_{r,t,i,\text{discharge}} - G_{r,i,\text{discharge}}$	≤ 0	$\leftrightarrow \mu_{r,t,i,\text{discharge}}$	for all $i, r, t,$	
$0 \leq e_{i,r,t}$	$\leq E_{r,i}$	$\leftrightarrow \overline{\mu_{r,t,i}}, \underline{\mu_{r,t,i}}$	for all $i, r, t,$	Storage level limited by capacity
$\eta_0 \cdot e_{i,r,t-1} + \eta_{\text{charge}} \cdot g_{i,r,t,\text{charge}} - \frac{1}{\eta_{\text{discharge}}} \cdot g_{i,r,t,\text{discharge}}$	$= e_{i,r,t}$	$\leftrightarrow \mu_{i,r,t}$	for all $i, r, t.$	Consistency of storage levels across time steps (+efficiencies)

(4)

Symbols	Description
i, t, s, r, l	Indices for node, time step, generator, storage, line
$G_{s,i}, F_l, G_{i,r,\text{charge}}, E_{i,r}$	Investment variable of generator s at node i, line l, charger and energy storage
$g_{s,i,t}, f_{l,t}, g_{i,r,\text{discharge}}, e_{i,r,t}$	Operational variable of generator g at node i, line l at time t, discharge and storage level
$c_{s,i}, c_l, c_{r,i,\text{charge}}, c_{r,i,\text{discharge}}, c_{r,i}$	Investment costs of generator g at node i, line l, (dis-)charger, storage
$o_{s,i}, o_{r,i,\text{charge}}, o_{r,i,\text{discharge}}$	Operational costs of generator g at node i and (dis-)charger
$d_{i,t}$	Demand at node i at time step t
$f_{l,t}$	Transmission flow through line l at time t
$\lambda_{i,t}$	Electricity price at node i at time t
$\mu_{i,r,t}$	Value of stored energy in r at node i at time t
$\mu_{i,s,t}, \mu_{l,t}, \lambda_c, \mu_{r,t,i,\text{discharge}}$	other KKT multipliers
$\eta_0, \eta_{\text{charge}}, \eta_{\text{discharge}}$	Efficiency of storage, charging and discharging
K_{il}	Incidence matrix
C_{lc}	Cycle matrix

Figure 2: Formulation of an energy system optimisation model in terms of a LP.

Cost recovery of storage



Charging bid raises electricity demand

Marginal price/value of storage higher than electricity price

Storage discharge in generation stack, sets price in merit order

Low prices: storage charged

Medium prices: no storage bids

High prices: storage discharged

(efficiencies and expectations influence value of storage)

See this paper for a proof and more detail.

Storage and transmission smooth production and demand over space and time

Cost recovery is similar to transmission; storage operators need to recover their costs

$$c_{r,\text{discharge}} G^*_{i,r,\text{discharge}} + c_{r,\text{charge}} G^*_{i,r,\text{charge}} + c_{r,i} E^*_{i,r} = \sum_t \lambda^*_t g^*_{i,t,\text{discharge}} - \sum_t \lambda^*_t g^*_{i,t,\text{charge}}$$

- Need to recover capital cost for storage, charging and discharging capacities

- Challenges of renewable production (and weather-dependent demand) can be alleviated by smoothing both
  - In space via transmission: imports can supply additional power, exports can raise demand during times of overproduction to avoid curtailment (congestion rent)
  - In time via storage: charging during overproduction/low price hours, discharging during tight supply (market participation as generator/consumer, also trading/hedging)
- However, both transmission and storage do not come for free and we need to find cost-efficient designs with finite resources.

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Revisiting today’s learning goals

- Discuss benefits and risks of interconnection and market integration
- Calculate how transmission infrastructure recovers its cost
- Implement previous learnings about networks, power flow and capacity and dispatch optimisation
- Illustrate how energy storage recovers its cost
- Operate energy storage short-term and model it in long-term planning
- Build a mathematical model of a renewable-based power system

Recap on what we can now optimise

- Short-term market equilibrium: dispatch optimisation and balancing of supply and demand, also over multiple time steps:
  – Operation of different generators
  – Operation of transmission, i.e. network with different nodes
  – Operation of storage, i.e. balancing across time steps
- Long-term market equilibrium: capacity expansion and dispatch optimisation; system planning over longer time horizons while ensuring short-term operations
  – Investment in different generators and their operation
  – Investment in transmission network and its operation
  – Investment in storage technologies and their operation

- Next: multiple sectors beyond the electricity sectors, but similar ideas