

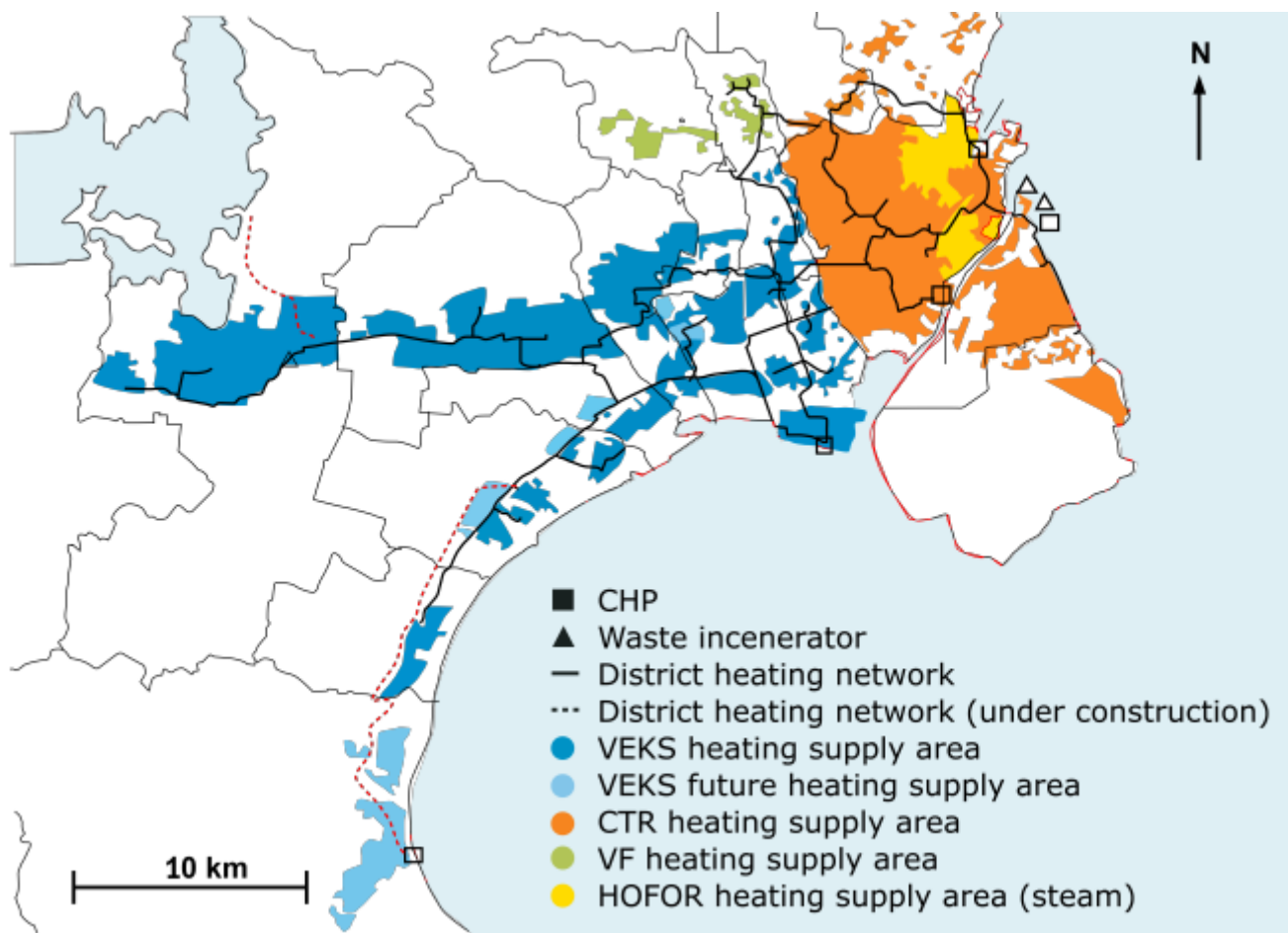
46770 Integrated energy grids

Lecture 7 – Heat Networks

Types of optimization problems and course structure

	One node	Network			
One time step	Economic dispatch or One-node dispatch optimization (Lecture 2)	Power		Gas flow (Lecture 6)	Heat flow (Lecture 7)
		Linearized AC power flow (Lecture 4)	AC power flow (Lecture 5)		
Multiple time steps	Multi-period optimization Join capacity and dispatch optimization in one node (Lecture 8)	Join capacity and dispatch optimization in a network (Lecture 10)			

District heating in Copenhagen



- serves 95% of population
- covers 18 municipalities
- compresses 4 integrated DH systems
- supplies 9,600 GWh/year of heat to around 500,000 end users

District heating in Aarhus

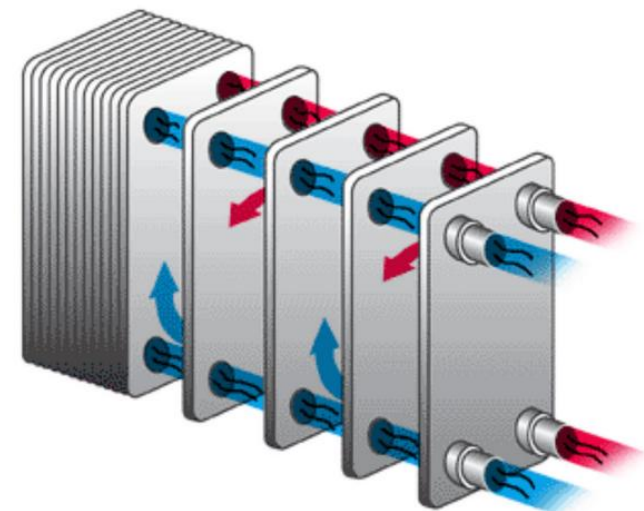


- covers Aarhus and 2 neighboring municipalities
- supplies 3,200 GWh/year of heat
- transmission system of 136 km, distribution system of 2100 km
- include CHP burning waste, straw, wood pellets and chips electric boiler, oil peak boiler



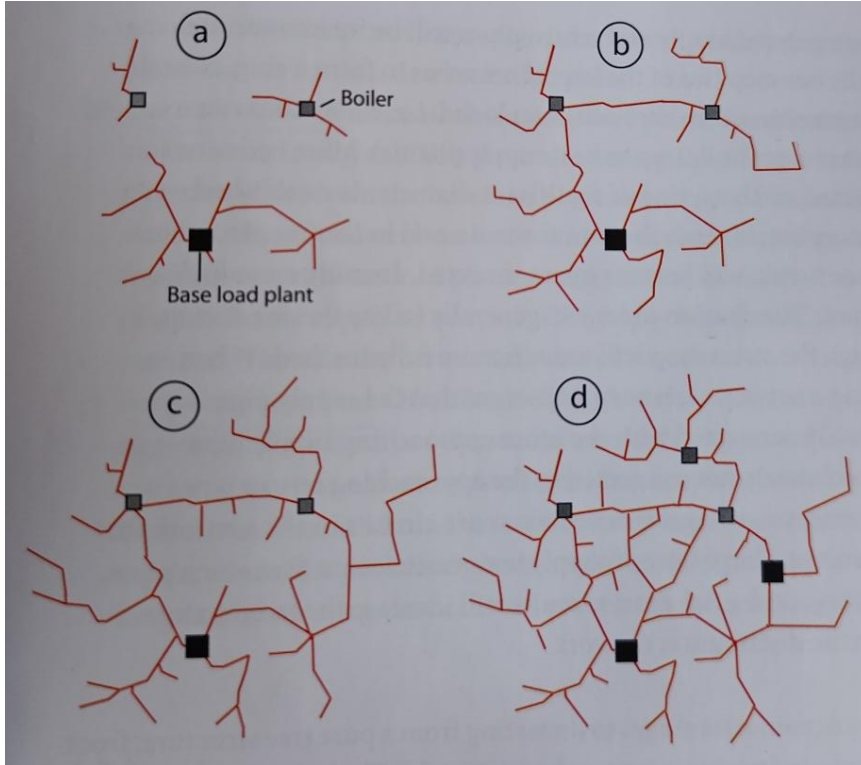
Supply and return networks are encapsulated and isolated in the same material

Plate heat exchanger are used in stations connecting the transmission network and distribution network

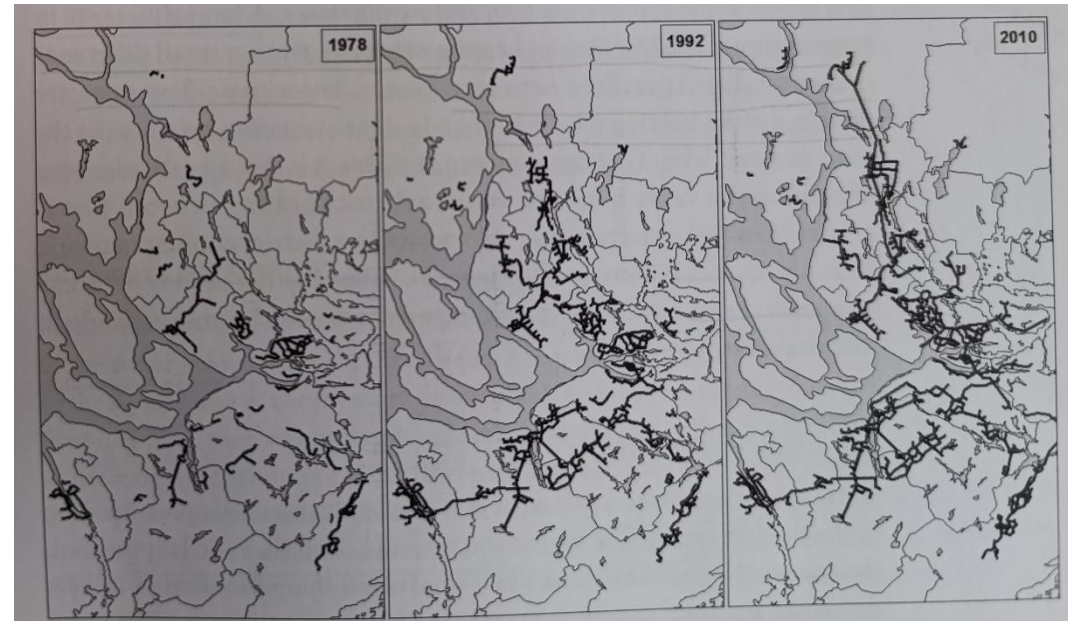


Deployment of district heating systems

Phases in district heating deployment



Major district heating pipelines in Stockholm



Frederick and Werner, District Heating and Cooling

Learning goals

- Describe the main characteristics of district heating systems
- Obtain the Darcy-Weisbach equation that relates pressure and mass flow in water pipelines
- Write the system cost minimization problem including optimal heat flow
- Describe the operation strategies for controlling heat flow in district heating systems
- Formulate the optimal heat flow problem on a computer.
- Describe the operation strategies for Combined Heat and Power (CHP) plants

Outline of the lecture

1. Previous knowledge
2. Heat networks and components
3. Nodal equations and heat flow in pipelines
4. Optimal heat flow formulation
5. Water flow in pipelines. Darcy-Weisbach equation that relates pressure and flow rates in water pipelines
6. Steady-state and dynamic model
7. Operating strategies: variable supply temperature and/or mass flow
8. Combined Heat and Power (CHP) plants

Previous knowledge

Previous knowledge

Convection (Newton's law of cooling): Heat transfer to the surrounding medium across the fluid/solid interface is

$$Q_{conv} = hA_{wet}(T_{solid} - T_{outside}) = U(T_{solid} - T_{outside})$$

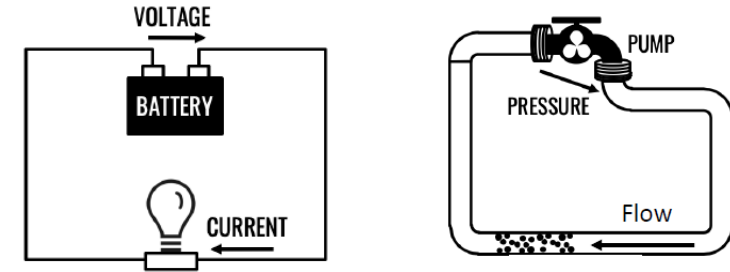
where h is the convection coefficient (W/m²K), T_{solid} is the temperature of the wall, $T_{outside}$ is the temperature of the surrounding fluid and A_{wet} is the surface area in contact with the fluid.

U is the heat transfer coefficient (W/k)

Thermal energy contained in water can be calculated as $c_p m T$

where m is the mass, T is the temperature and the water's specific heat capacity is $c_p = 4182$ J/kg °C.

Voltage and current in AC transmission lines are equivalent to pressure and water mass flow in heat networks

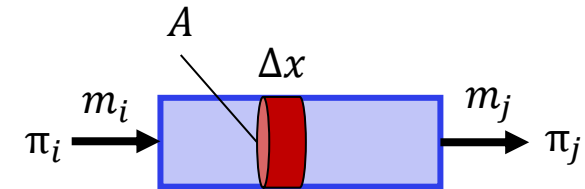


Mass flow in water pipelines

The mass-flow m (in kg/s) in a water pipeline can be calculated as

$$m = \rho A u$$

where ρ represents the density, u the velocity of the gas and A the cross-sectional area of the pipeline (water is an incompressible fluid with constant density ρ).

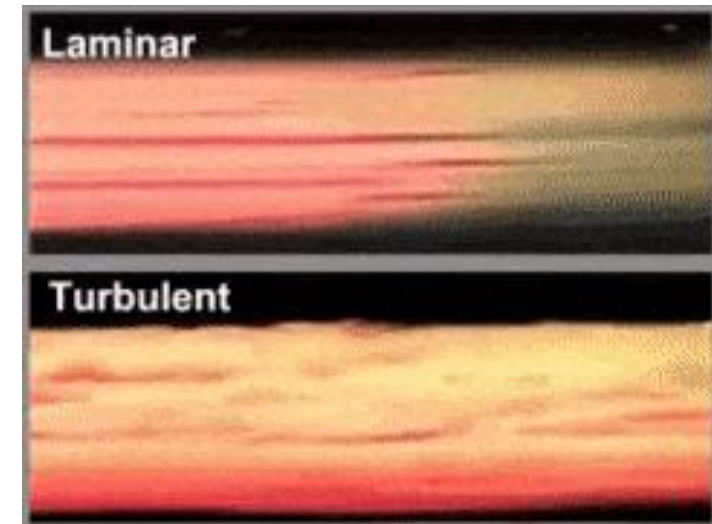


Previous knowledge

The **Reynolds number** determines if the flow is laminar ($Re < 2000$) or turbulent ($Re \gg 2000$)

Re provides an indication of whether inertial forces or viscous forces are more relevant

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho u D}{\mu}$$



Water mass flows has typically high Reynolds numbers (due to high density, velocity and pipe diameter), so **water flow is generally turbulent**

Previous knowledge

Darcy-Weisbach empirical formula is used to represent the pressure loss due to friction

$$\frac{f_D \rho u |u|}{2D}$$

where D is the diameter of the pipe and f_D is the Darcy friction coefficient.

Darcy friction coefficient f_D : can be estimated based on the characteristics of the pipeline (diameter D , roughness ε and Reynolds number Re)

For **laminar flow** ($Re < 2000$)

$$f_D = \frac{64}{Re}$$

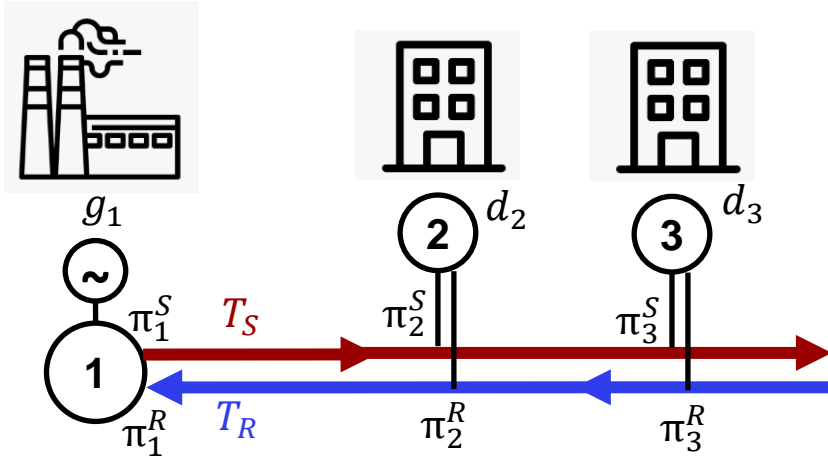
water flow is turbulent

For **fully-turbulent flow** ($Re \gg 2000$)

$$\frac{1}{\sqrt{f_D}} = -2 \log_{10} \left(\frac{\varepsilon}{3.7D} \right)$$

where ε is the roughness and D the diameter of the pipe

Heat networks and components



Similarly to power network, heat networks comprise transmission networks and distribution networks.

Heat is transported by a flow of **water (incompressible fluid) at different temperatures**. A **supply network** at high temperature T^S transports hot water to demand nodes and a **return network** at lower temperature T^R transports cold water back to heat generation nodes.

In some nodes, heat is produced, e.g. Combined Heat and Power (CHP) units, heat pumps, solar thermal collectors.

In some nodes, heat is extracted, e.g. in heat exchanger that transport heat from the transmission network to the distribution network or in a building that uses heat to warm up the building.

Nodal balance and pipeline equations (I)


In every node, there should be a balance between the nodal supply, demand, and the water mass flowing in and out.

$$g_i - d_i = \sum_j m_{i \rightarrow j} - \sum_j m_{j \rightarrow i} \quad \text{mass nodal balance}$$

Every pipeline has a certain capacity

$$|m_{i \rightarrow j}| \leq M_{i \rightarrow j} \quad \text{Pipeline capacity}$$

The mass flow in a pipeline is related to the pressure difference. Pressure drops due to friction can be estimated with the Darcy-Weisbach equation



$$b_{ij} m_{i \rightarrow j}^2 = \pi_i - \pi_j \quad \rightarrow \quad \text{The Darcy-Weisbach equation is not linear (but contrary to the Weymouth equation that we used for gases is linear on the pressure)}$$

Note: We use π to represents pressure (because we use p for power flows).

Nodal balance and pipeline equations (II)

The heat flow $g_{s,i}$ injected by a heat producing technology s in node i is

$$g_{s,i} = c_P m_s (T_i^S - T_i^R)$$

where m_s is the mass flow injected in that node by technology s , T_i^S is the supply temperature and T_i^R is the return temperature.

The heat flow d_i extracted by heat exchanger in node i is

$$d_i = -c_P m_s (T_i^R - T_i^S)$$

Nodal balance and pipeline equations (II)

The electricity consumed by the pump in every heat producing technology to keep the mass flow of water is related to the pressure different between the supply network and the return network

$$\text{electricity demand}_s^{pump} = -m_s(\pi_i^S - \pi_i^R)$$

Electricity demand depends on the pressure drop and the mass flow, so it is proportional to m^3

The supply temperature T_i^S at node i is the weighted sum of all the temperatures arriving at that node ([temperature mixing equation](#))

$$T_i^S = \frac{\sum_l m_l T_l^{out}}{\sum_l m_l}$$

Economic dispatch with heat flow

Determine the optimal economic dispatch to supply the heat demand d_n in a certain hour and the optimal heat flows while minimizing the total system cost.

Economic dispatch with heat flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} + \text{electricity demand}_s^{pump}$$

subject to:

$$\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} \cdot e \cdot m_l$$

Nodal mass (or energy rate) balance

$$g_{s,i} = c_P m_s (T_i^S - T_i^R)$$

Heat generation in node i

$$d_i = -c_P m_s (T_i^R - T_i^S)$$

Heat demand in node i

$$T_i^S = \frac{\sum_l m_l T_l^{out}}{\sum_l m_l}$$

Temperature mixing

$$|m_l| \leq M_l$$

Pipelines capacities (in mass flow or energy)

$$b_{ij} m_{ij}^2 = \pi_i - \pi_j \quad b_{ij} = \frac{f_D L}{2 \rho A^2 D}$$

Physical relations in the links

$$\text{electricity demand}_s^{pump} = -m_s (\pi_i^S - \pi_i^R)$$

Electricity demand pumps

Unknown variables are energy production in every heat generation unit $g_{s,i}$ and either temperatures T_i^S, T_i^R or mass flows in the links m_l

Heat flow in pipelines

Darcy-Weisbach equation relating pressure and mass flow

momentum
conservation
(steady-state)

$$\underbrace{\frac{\partial(\rho u)}{\partial t}}_{\text{inertia}} + \underbrace{\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} - \underbrace{\frac{f_D \rho u |u|}{2D}}_{\text{Friction force}} = 0$$

$$m = \rho u A \quad \rho u = \frac{m}{A}$$

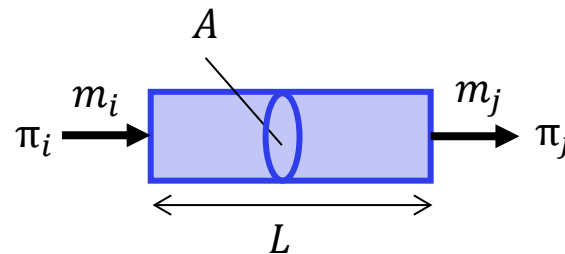
$$\frac{\partial \pi}{\partial x} = \frac{f_D m |m|}{2 \rho A^2 D}$$

$$\pi_i - \pi_j = \underbrace{\frac{f_D L}{2 \rho A^2 D}}_{b_{ij}} m^2$$

Darcy-Weisbach equation

For incompressible flows, equation of state does not exist. In practice, this means that the energy equation is decoupled from the other two equations.

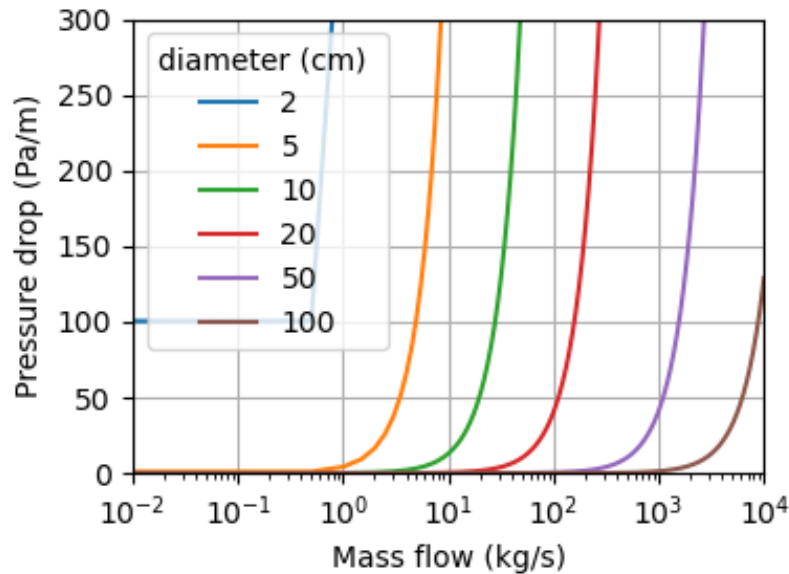
We can assume constant density and solve mass and momentum conservation without taking into account energy conservation.



Pressure in heat pipelines

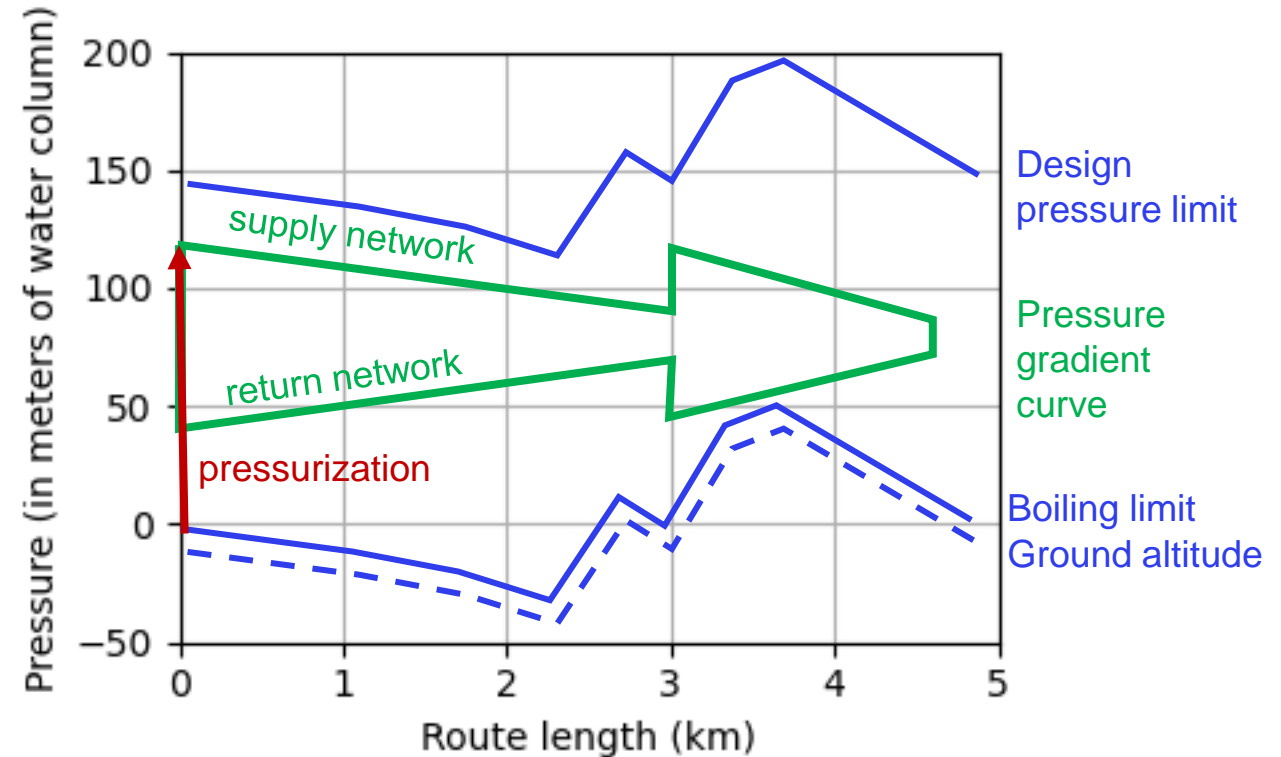
Once full flow is reached in a pipe it is nearly impossible to increase it by greater pump capacity and pipes with larger diameters should be used.

$$\pi_i - \pi_j = \frac{f_D L}{2\rho A^2 D} m^2$$



Assuming $f_D=0.025$

Pressure in pipelines should be below the design pressure limit, above the water boiling limit and is affected by the round altitude.



DTU Energy conservation equation

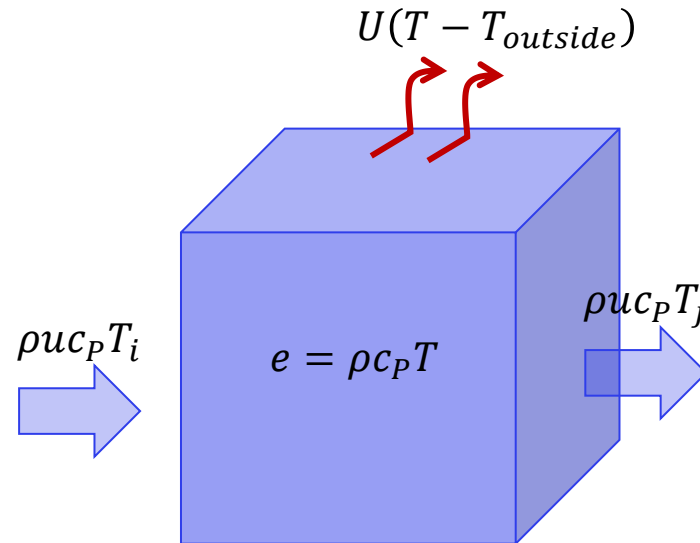
Energy conservation

$$\underbrace{\rho c_P \frac{\partial T}{\partial t}} + \underbrace{\rho u c_P \frac{\partial T}{\partial x}} + \underbrace{U(T - T_{outside})} = 0$$

Rate of
change of
thermal
energy
stored
within the
volume

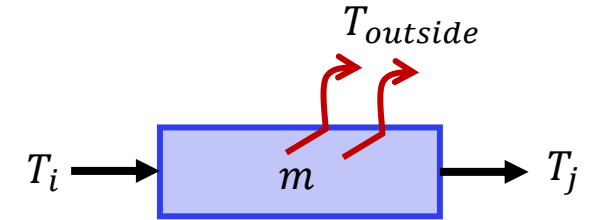
Energy
transfer
by mass
flow

Heat exchange
with the outside



Energy conservation for well-insulated pipes and steady-state

Energy conservation in the pipeline



$$\rho c_P \frac{\partial T}{\partial t} + \rho u c_P \frac{\partial T}{\partial x} + \underbrace{U(T - T_{outside})}_{\text{Heat exchange with the outside}} = 0$$

Heat exchange with the outside

where $T_{outside,i}$ is the temperature of the medium surrounding the pipe and the heat transfer coefficient U_i depends on the characteristics of the water and the pipeline.

For **well-insulated pipelines**, we can neglect the heat exchange with the outside: $\rho c_P \frac{\partial T}{\partial t} + \rho u c_P \frac{\partial T}{\partial x} = 0$

For **steady-state conditions**: $\rho c_P \frac{\partial T}{\partial x} = 0 \rightarrow T_i = T_j$

mass conservation in steady-state $m_i = m_j$

Propagation of changes in mass flow and supply temperature

Let's assume a 10 km pipe:

- A pressure change travels with the speed of sound in water (~ 1000 m/s) so the new flow situation will be established in ~ 10 seconds.
- Assuming a mass flow velocity of 2 m/s, a change in supply in temperature will reach the end of the pipe in ~ 1.5 hours

Changes in demand/supply that induce **changes in mass flow propagates quickly** while **changes affecting the difference between the supply and return temperature propagate slowly**

How can we express the temperature at the outlet of the pipe as a function of time?

Let us assume now a variable mass flow $m(t) = \rho A u(t)$

We can define a time delay $t - t_0$ (time that it takes the water to reach the outlet of a pipeline with length L)

Because the mass flow depends on time $m(t)$, so does the time delay $t - t_0(t)$

$$\int_{t_0(t)}^t u dt = \int_{t_0(t)}^t \frac{m(t)}{\rho A} dt = L$$

We write again the energy conservation equation

$$\rho c_P \frac{\partial T}{\partial t} + \rho u c_P \frac{\partial T}{\partial x} + U(T - T_{outside}) = 0$$

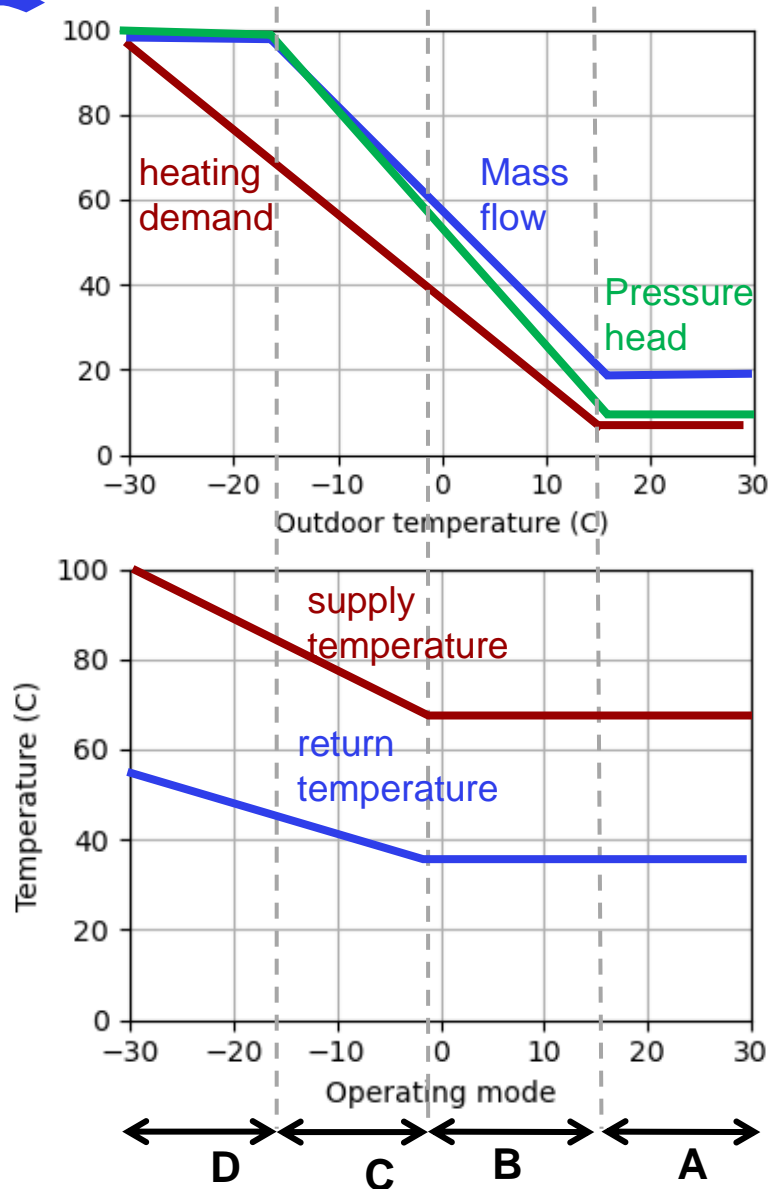
We reorganized the energy conservation equation and make explicit the space and time dependence

$$\frac{\partial T}{\partial t}(x, t) + u(t) \frac{\partial T}{\partial x}(x, t) + \frac{U}{\rho c_P}(T(x, t) - T_{outside}) = 0$$

This equation has an analytical solution for the temperature at the outlet $T_j(t)$ of a pipeline

$$T_j(t) = T_{outside} + (T_i(t - t_0(t)) - T_{outside}) e^{-\frac{U}{\rho c_P}(t - t_0(t))}$$

Control strategies for district heating networks



- We can keep fixed/vary mass flow and/or supply temperature
- When both vary, and the energy injected in a pipeline is higher than the energy extracted, the network is storing energy and vice versa

Some heuristics for operating modes

A: low heating demand (only for domestic hot water), constant mass flow and supply temperature

B: constant supply temperature and variable mass flow

C: variable supply temperature and mass flow (trade-off between keeping supply temperature low to reduce thermal losses and keep mass flow low to reduce pumps' consumption)

D: constant mass flow (the maximum allowed pressure is reached) and variable supply temperature

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Modelling approaches for heat flows

Non-discretized pipelines	Fixed mass flow Variable temperature	Variable mass flow Fixed temperature	Variable mass flow Variable temperature
No time delays	Fixed time delays	Variable time delays	Variable time delays and energy storage within the pipelines can be modelled



 Increase computational complexity

4th Generation District Heating system

1st Generation: steam as heat carrier

2nd Generation: pressurized water as heat carrier with temperature $> 100^{\circ}\text{C}$

3rd Generation: pressurized water as heat carrier with temperature $< 100^{\circ}\text{C}$, prefabricated and pre-insulated pipes buried directly into the ground

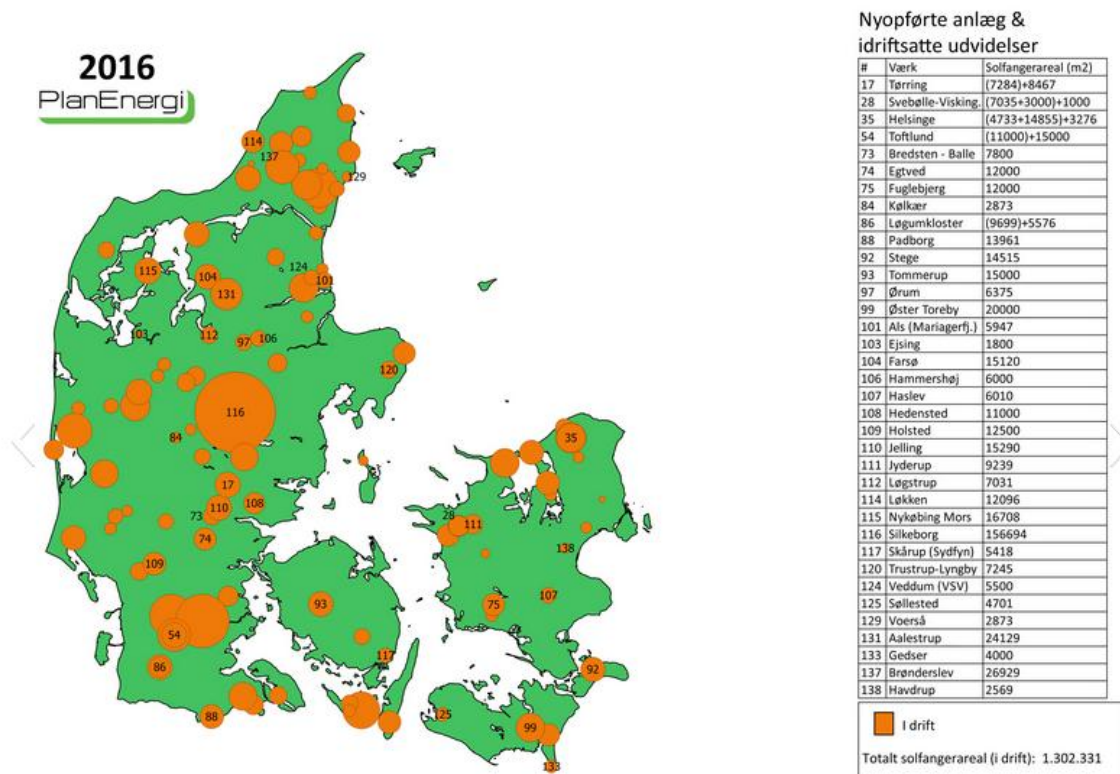
4th Generation: pressurized water as heat carrier with **lower temperature**, which enable higher efficiency for CHP plants, higher COP for heat pumps, lower losses in solar thermal collectors, higher availability for geothermal and waste heat from industry, increased capacity in water-based thermal energy storage

As main drawback, lower supply temperature requires larger-area radiators to heat up buildings

Technologies used in district heating systems

Technologies used in district heating systems

Thermal solar collectors



See the installation in different years in

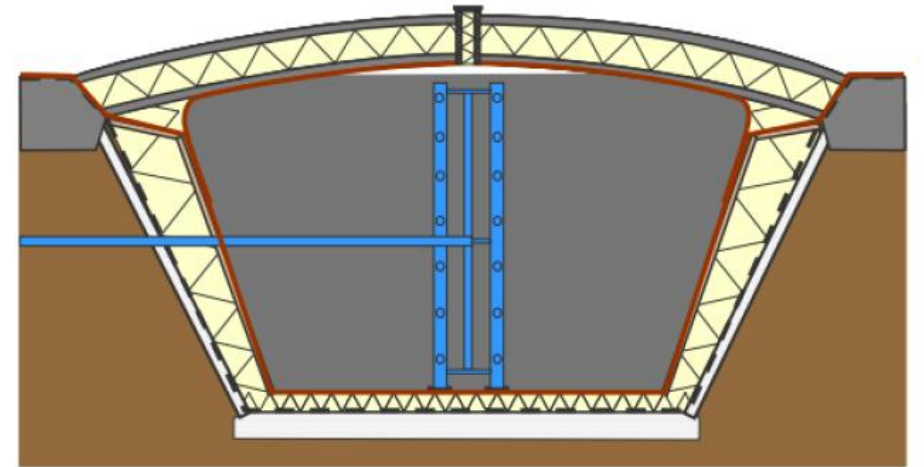
<https://planenergi.dk/arbejdsmraader/fjernvarme/solvarme/solvarme-i-danmark-1988-2018/>

Technologies used in district heating systems

Long-term thermal energy storage



Long-term thermal energy storage in Voens (South Jutland)

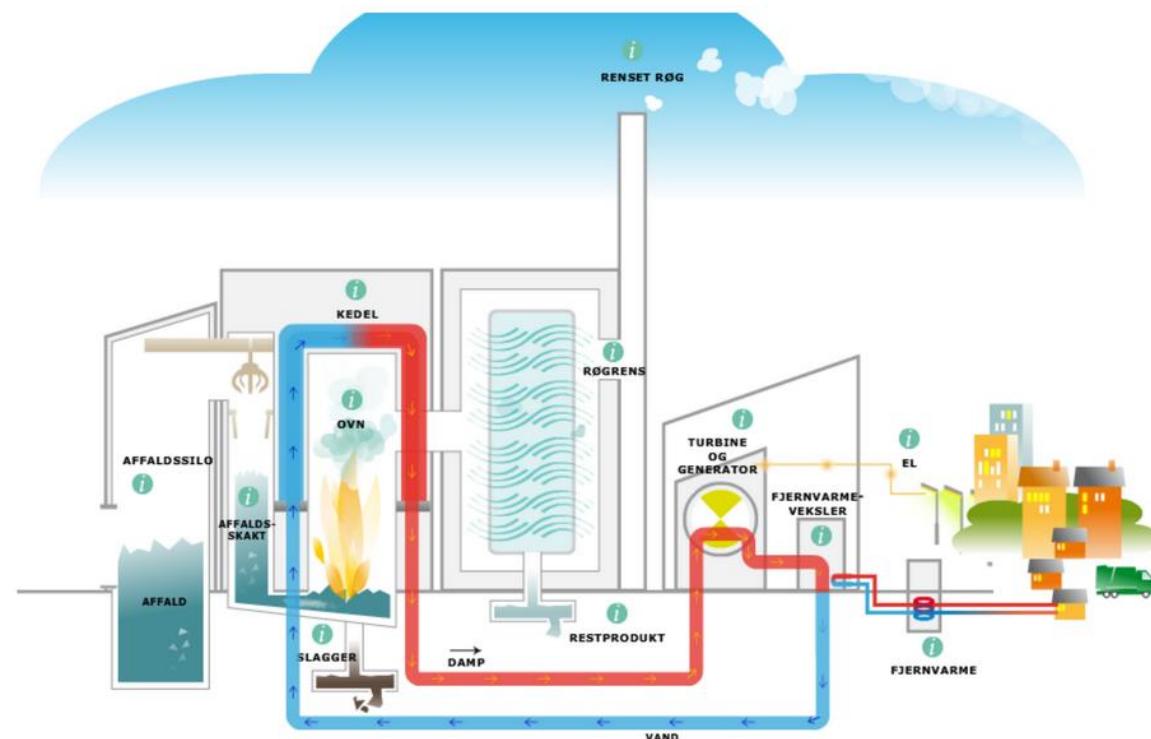


Technologies used in district heating systems

Waste-to-heat plants

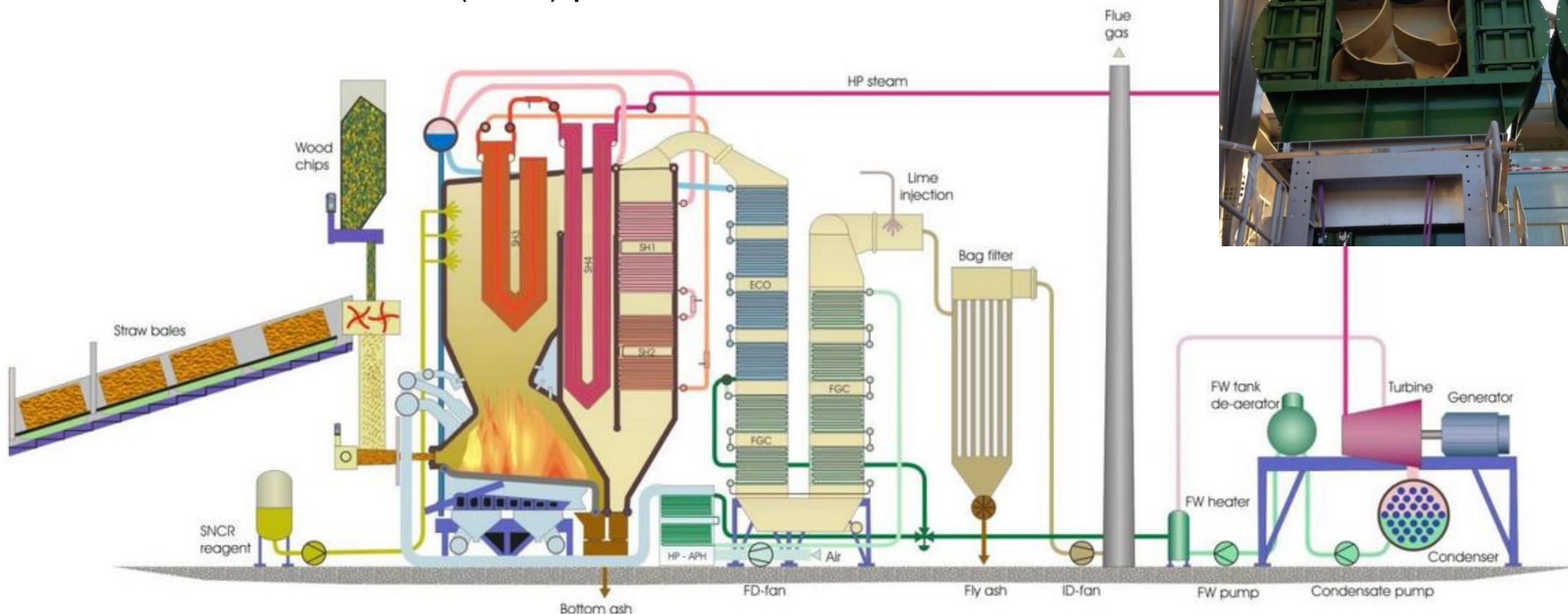


Waste-to-heat, biomass CHP in Lisbjerg (Aarhus)



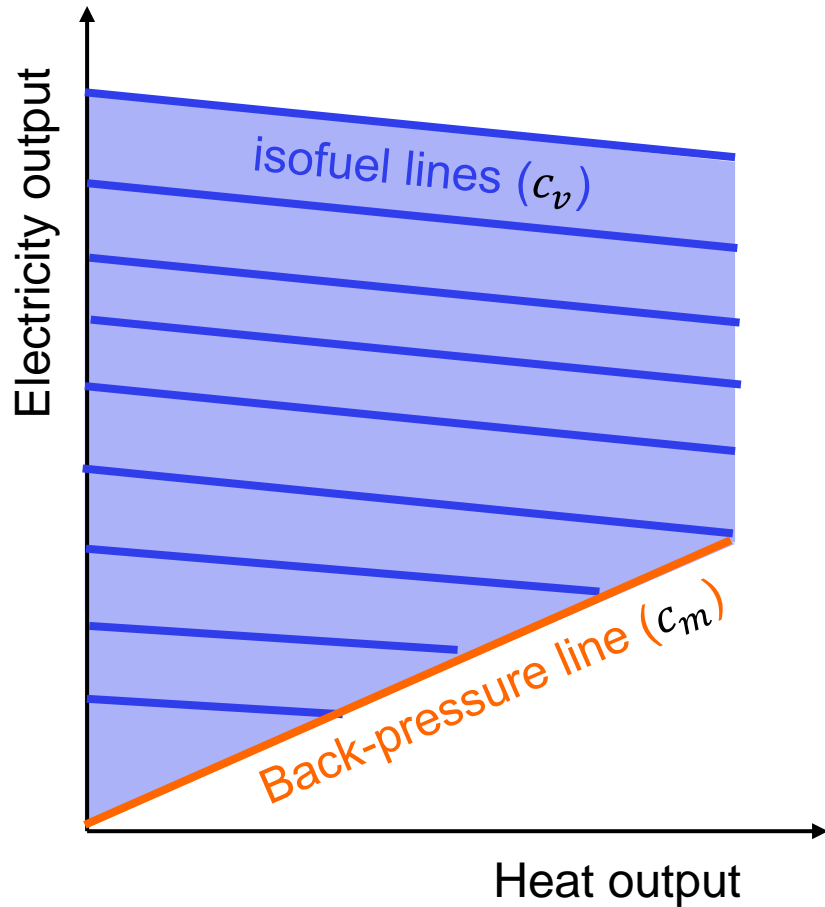
Technologies used in district heating systems

Combined Heat and Power (CPH) plants



Waste-to-heat, biomass CHP in Lisbjerg (Aarhus)

Combined Heat and Power (CHP) units



Combined Heat and Power (units) are also called cogeneration plants.

They can be operated in:

- Condensing mode
(turbine exhausts steam at very low pressure into a condenser, where it turns back into water. Maximizes electricity generation)
- Back-pressure mode (turbine exhausts steam at a higher pressure which is utilized to provide heat)



Problems for this lecture

Problems 7.1, 7.2 (**Group 14**)

Problems 7.3 and 7.4 (**Group 15**)

We are right in the middle of the course, please provide feedback so that we can improve

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