

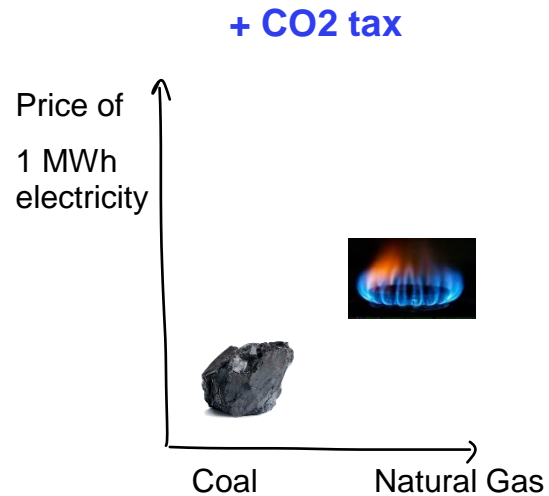
46770 Integrated energy grids

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Lecture 9 – Limiting CO₂ emissions

Introduction

How can we ensure that the cost optimization also leads to CO₂ emissions reductions?



A CO₂ tax can be imposed to alter the merit order of generators

How is that related to our optimization problem?

Learning goals for this lecture

- Demonstrate cost recovery in electricity markets
- Explain how the CO₂ price affects the cost recovery in electricity markets
- Interpret the meaning of the Lagrange/KKT multiplier associated with the CO₂ emissions constraints and analyze its value.
- Formulate joint capacity and dispatch optimization with CO₂ emissions constraints on a computer.
- Define financial discount rate and social discount rate
- Calculate the cost annualization factor
- Describe the different approach that classic economy has taken to include climate change in economic optimization

Cost recovery in electricity markets

Cost recovery in optimized markets (“non-profit rule”)

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$\left\{ \begin{array}{l} \min_{g_{s,t}, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s g_{s,t} \right] \\ \text{subject to:} \\ \sum_s g_{s,t} - d_t = 0 \leftrightarrow \lambda_t \\ -g_{s,t} + G_s \geq 0 \leftrightarrow \overline{\mu_{s,t}} \end{array} \right.$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda_t^* + \overline{\mu_{s,t}^*} = 0 \quad \rightarrow \quad o_s = \lambda_t^* - \overline{\mu_{s,t}^*}$$

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = \frac{\partial f}{\partial G_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial G_s} - \sum_j \mu_j \frac{\partial g_j}{\partial G_s} = c_s - \sum_t \overline{\mu_{s,t}^*} \cdot (1) = 0 \quad \rightarrow \quad c_s = \sum_t \overline{\mu_{s,t}^*}$$

Cost recovery in optimized markets (“non-profit rule”)

Total cost for generator s

$$\begin{aligned}
 c_s G_s^* + \sum_t o_s g_{s,t}^* &= c_s G_s^* + \sum_t (\lambda_{s,t}^* - \overline{\mu_{s,t}^*}) g_{s,t}^* \\
 c_s G_s^* + \sum_t o_s g_{s,t}^* &= c_s G_s^* + \sum_t \lambda_t^* g_{s,t}^* - \underbrace{\sum_t \overline{\mu_{s,t}^*}}_{c_s} \underbrace{g_{s,t}^*}_{G_s^*} \\
 c_s G_s^* + \sum_t o_s g_{s,t}^* &= \cancel{c_s G_s^*} + \sum_t \lambda_t^* g_{s,t}^* - \cancel{c_s G_s^*} \\
 c_s G_s^* + \sum_t o_s g_{s,t}^* &= \sum_t \lambda_t^* g_{s,t}^*
 \end{aligned}$$

This is only true if the constraint is binding, but otherwise $\overline{\mu_{s,t}^*} = 0$

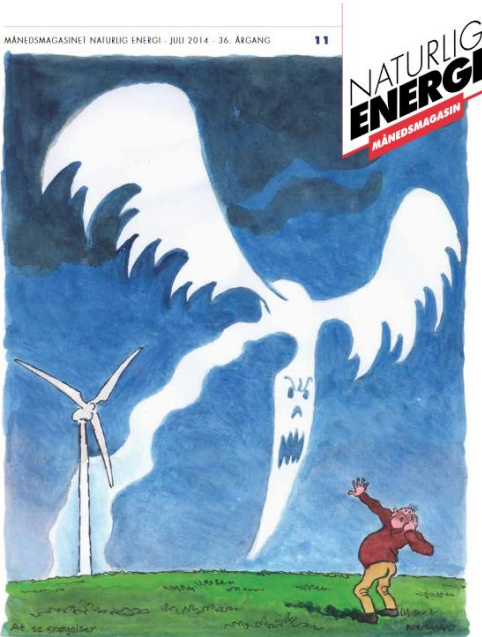
The generator costs (fixed and variable) are **exactly fully-recovered** from market revenues (generation times market price). This is known as **long-term market equilibrium** or **non-profit rule**.

If a price cap is set in the market, this is not true anymore. Some countries set a price cap and use capacity markets to compensate for the unsuccessful recovery of capital cost.

Assumptions included in the model / Limitations

The optimization problem makes three assumptions:

- Perfect competition: no generator has the power to impose artificially high electricity prices.
- Long-term market equilibrium: thanks to market revenues every technology recovers exactly its cost (fixed & variable) throughout its lifetime. This requires that, for some hours, electricity prices are higher than the marginal cost of the most expensive technology.
- Perfect foresight through the year: This might not be true (e.g., the model assumes that reservoir hydropower can be used at the optimal time).



We can include additional constraints to represent social acceptance issues (e.g., limit transmission expansion or the maximum capacity for onshore wind).

$$\sum_l Capacity_l \cdot Length_l < Transmission\ Cap$$

We can search for alternative solutions with higher public acceptance and quantify the economic impacts.

Global CO₂ constraint

Global CO₂ constraint

We can add a constraint to limit the total CO₂ emissions in the system

$$\sum_{s,t} \frac{\epsilon_s}{\eta_s} g_{s,t} \leq CAP_{CO_2} \leftrightarrow \mu_{CO_2}$$

where ϵ_s are the specific emissions per technology (in kgCO₂/MWh_{thermal}), and η_s is the efficiency of the generator (in MWh_{electricity} / MWh_{thermal})

μ_{CO_2} is the Lagrange or Karush-Kuhn-Tucker (KKT) multiplier associated with the CO₂ constraint. It represents the change in the objective function at the optimal solution, with respect to a small change in the constraint.

Small change in constraint : $CO_2 \text{ limit}^* = CO_2 \text{ limit} + 1 \text{ tonne}$

Change in objective function : $System \text{ cost}^* = System \text{ cost} + \Delta System \text{ cost}$

Hence μ_{CO_2} represents the cost of 1 tone of CO₂

- If $\mu_{CO_2} = 0$ the constraint is not binding.
- If $\mu_{CO_2} \neq 0$ we would obtain the same solution if we remove the constraint and add a CO₂ price equal to μ_{CO_2}

μ_{CO_2} is also known as **CO₂ price, CO₂ tax, shadow price or marginal abatement cost**

Setting a global CO₂ constraint is equivalent to adding a CO₂ tax

If we repeat the demonstration of cost recovery in optimized markets (“non-profit rule), including the CO₂ constraint

$$\left\{ \begin{array}{l} \min_{g_s, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s g_{s,t} \right] \\ \text{subject to:} \\ \sum_s g_{s,t} - d_t = 0 \leftrightarrow \lambda_t \\ -g_s + G_s \geq 0 \leftrightarrow \overline{\mu_{s,t}} \\ -\sum_{s,t} \frac{\epsilon_s}{\eta_s} g_{s,t} + CAP_{CO_2} \geq 0 \leftrightarrow \mu_{CO_2} \end{array} \right.$$

The derivative of the Lagrangian with respect to the generation gets an extra factor

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda_t^* + \overline{\mu_{s,t}^*} + \mu_{CO_2} \frac{\epsilon_s}{\eta_s} = 0 \quad \rightarrow \quad o_s = \lambda_t^* - \overline{\mu_{s,t}^*} - \mu_{CO_2} \frac{\epsilon_s}{\eta_s}$$

Setting a global CO₂ constraint is equivalent to adding a CO₂ tax

We repeat the demonstration of cost recovery in optimized markets (“non-profit rule)

$$c_s G_s^* + \sum_t o_s g_{s,t}^* = c_s G_s^* + \sum_t (\lambda_{s,t}^* - \overline{\mu_{s,t}^*} - \mu_{CO2} \frac{\epsilon_s}{\eta_s}) g_{s,t}^*$$

$$c_s G_s^* + \sum_t o_s g_{s,t}^* = c_s G_s^* + \sum_t (\lambda_t^* - \mu_{CO2} \frac{\epsilon_s}{\eta_s}) g_{s,t}^* - \underbrace{\sum_t \overline{\mu_{s,t}^*}}_{c_s} \underbrace{g_{s,t}^*}_{G_s^*}$$

This is only true if the constraint is binding, but otherwise $\overline{\mu_{s,t}^*} = 0$

$$c_s G_s^* + \sum_t o_s g_{s,t}^* = \cancel{c_s G_s^*}_0 + \sum_t (\lambda_t^* - \mu_{CO2} \frac{\epsilon_s}{\eta_s}) g_{s,t}^* - \cancel{c_s G_s^*}_0$$

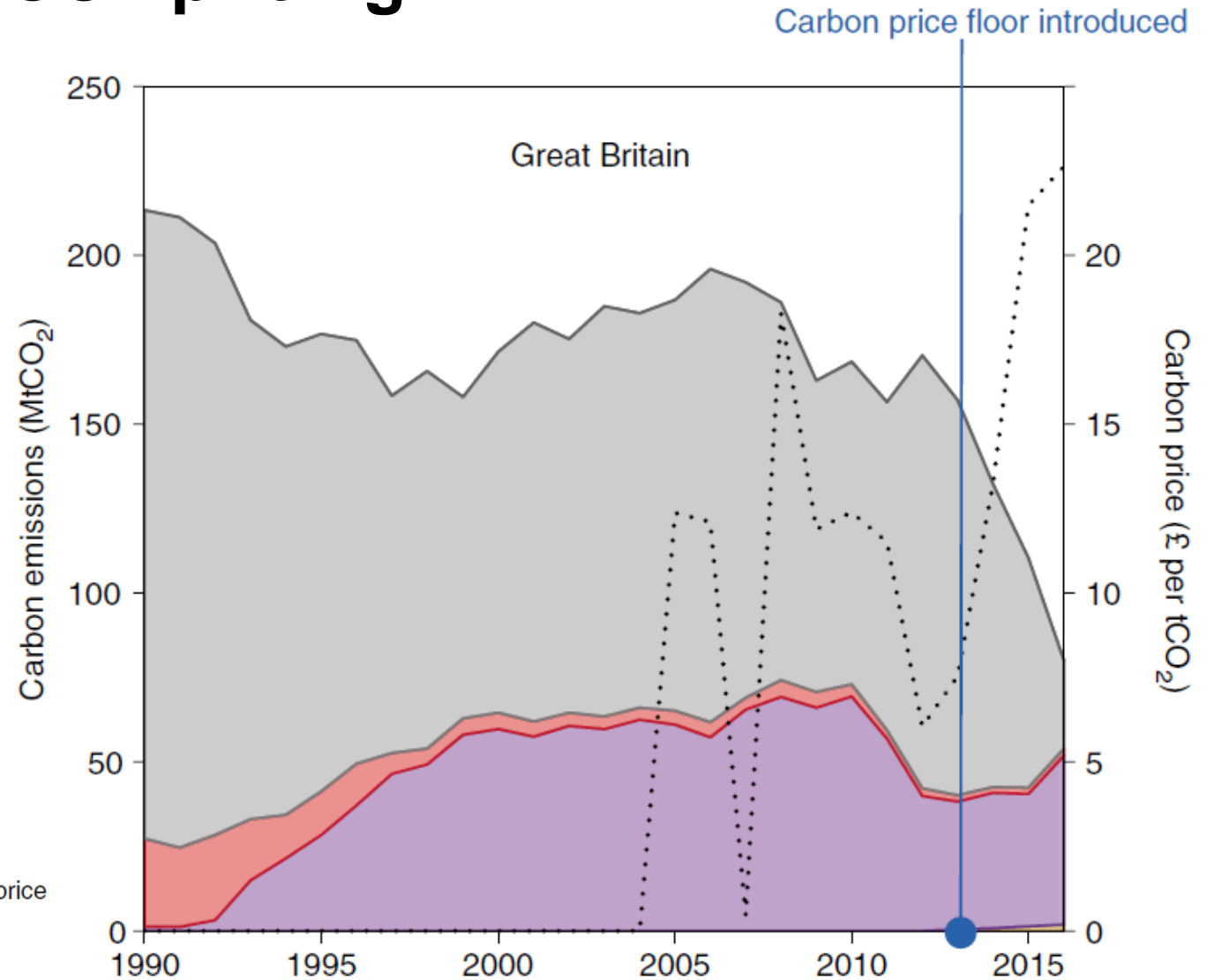
$$c_s G_s^* + \sum_t o_s g_{s,t}^* = \sum_t \underbrace{(\lambda_t^* - \mu_{CO2} \frac{\epsilon_s}{\eta_s}) g_{s,t}^*}_{\text{Market revenues minus cost of CO}_2 \text{ price times generation}}$$

This shows that **setting a global CO₂ constraint is equivalent to adding a CO₂ price**
(i.e. substituting o_s by $o_s + \mu_{CO2} \frac{\epsilon_s}{\eta_s}$)

An example of effective CO₂ pricing

“Great Britain carbon emissions fell drastically and quickly not due to a surge in low-carbon nuclear or renewable sources. Instead, it was the impact of fuel switching from coal to natural gas generation incentivized through a stable and strong carbon price.”

■ Lignite
■ Coal
■ Oil
■ Gas
■ Biomass
..... Carbon price



Source: Wilson and Staffell, [Rapid fuel switching from coal to natural gas through effective carbon pricing](#), Nature (2018)

Current CO₂ price in Europe

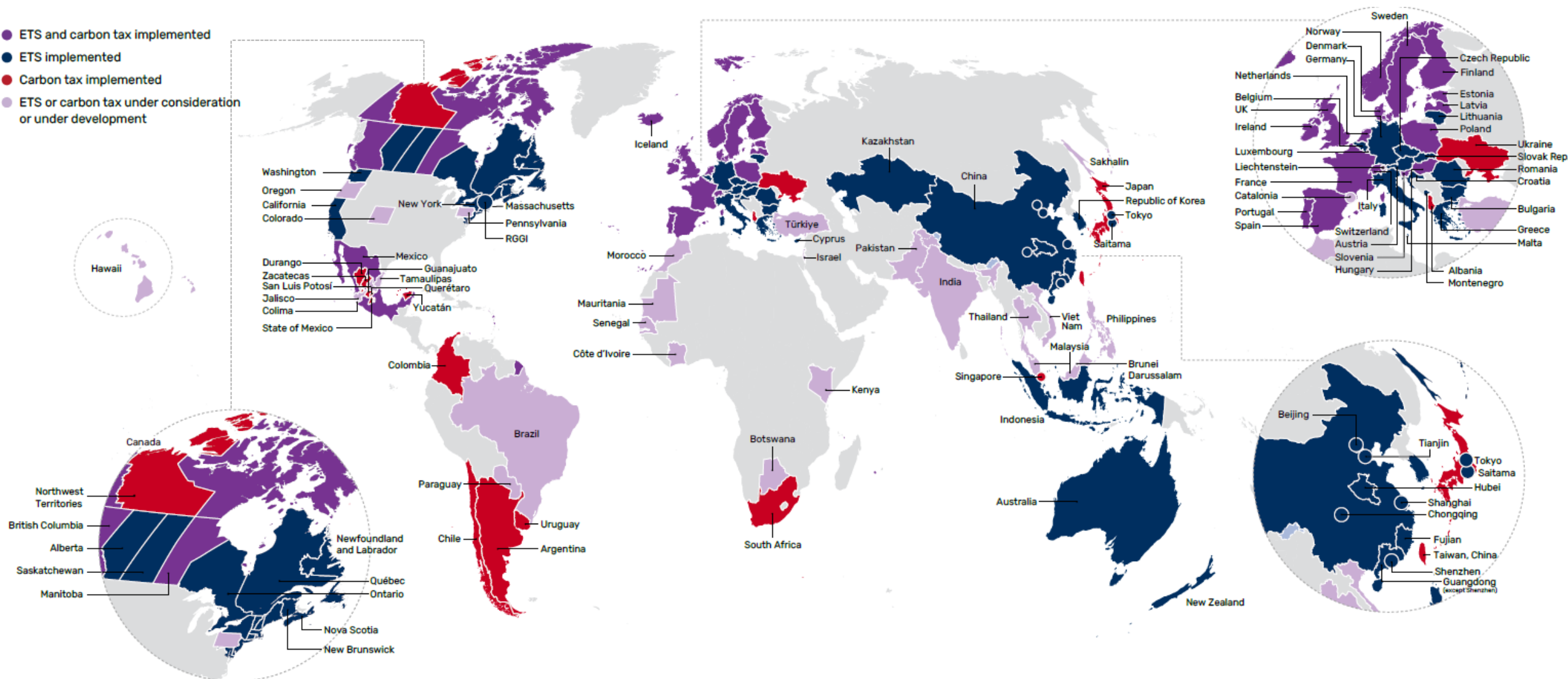
In Europe, electricity generators need to buy their rights to emit CO₂ in the Emissions Trading System (ETS). Currently ETS includes power and heat generation, energy-intensive industry sectors, commercial aviation.



[EU carbon price tracker](#)

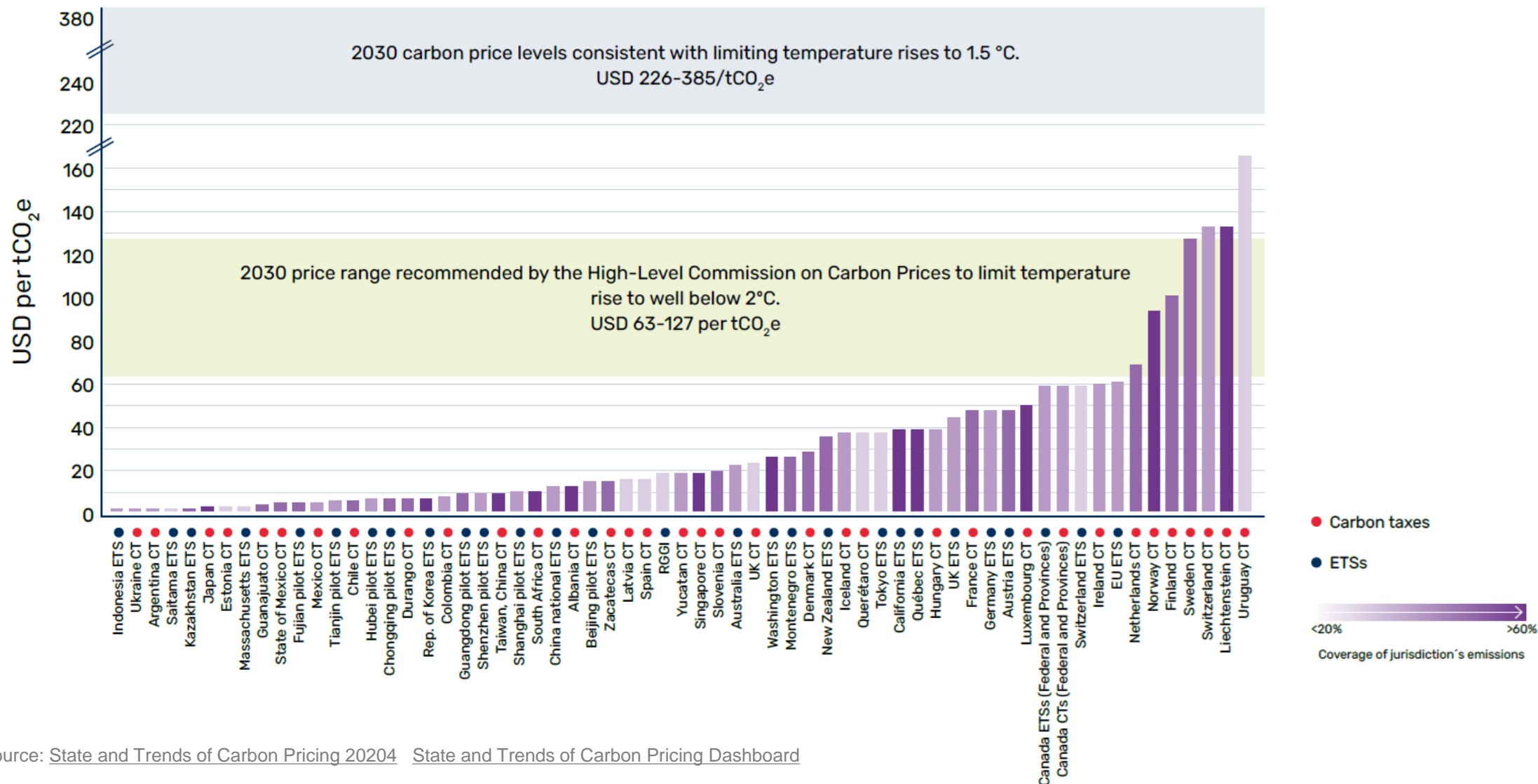
Existing CO₂ tax in different countries

- ETS and carbon tax implemented
- ETS implemented
- Carbon tax implemented
- ETS or carbon tax under consideration or under development



Source: [State and Trends of Carbon Pricing 2020](#) [State and Trends of Carbon Pricing Dashboard](#)

CO₂ price levels continue to fall short of the ambition needed to achieve the Paris Agreement goals



Source: [State and Trends of Carbon Pricing 2020/4](#) [State and Trends of Carbon Pricing Dashboard](#)

Cost annualization and financial discount rate

Cost annualization and financial discount rate

The investment cost I_0 needs to be recovered by payments in every year, which can be expressed by the *Eq. A*.

$$I_0 = \frac{I_A}{(1+r)^1} + \frac{I_A}{(1+r)^2} + \frac{I_A}{(1+r)^3} + \dots + \frac{I_A}{(1+r)^N} \quad (\text{Eq. A})$$

If we multiply *Eq. A* by $\frac{1}{1+r}$, compare it to *Eq. A* and subtract both expressions

$$I_0 = \frac{I_A}{(1+r)^1} + \frac{I_A}{(1+r)^2} + \frac{I_A}{(1+r)^3} + \dots + \frac{I_A}{(1+r)^N}$$

minus

$$\frac{I_0}{1+r} = \frac{I_A}{(1+r)^2} + \frac{I_A}{(1+r)^3} + \frac{I_A}{(1+r)^4} + \dots + \frac{I_A}{(1+r)^{N+1}}$$

We get *Eq. B*.

$$I_0 \left(1 - \frac{1}{1+r} \right) = \frac{I_A}{(1+r)^1} - \frac{I_A}{(1+r)^{N+1}} \quad (\text{Eq. B})$$

Cost annualization and financial discount rate

We now multiply Eq. B by $(1 + r)^{N+1}$ and get:

$$I_0((1 + r)^{N+1} - (1 + r)^N) = I_A((1 + r)^N - 1)$$

Finally, by reordering the equation we get:

$$I_A = I_0 \frac{(1 + r)^N \cdot r}{(1 + r)^N - 1}$$

Cost annualization

r = financial discount rate

N = number of years the asset is in operation

I_A = annualized cost

I_0 = capital cost, investment on the first year.

For investment in the energy sector, a financial discount rate of 7% is typically assumed.

Including climate change in economic optimization and social discount rate

Including climate change in economic optimization

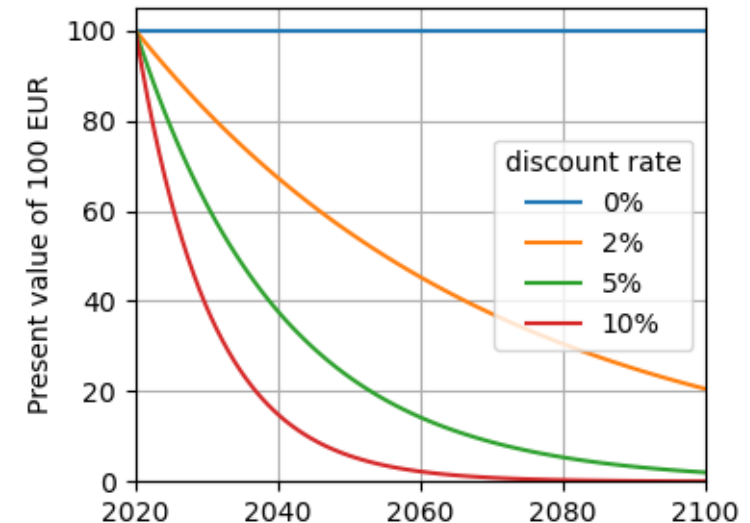
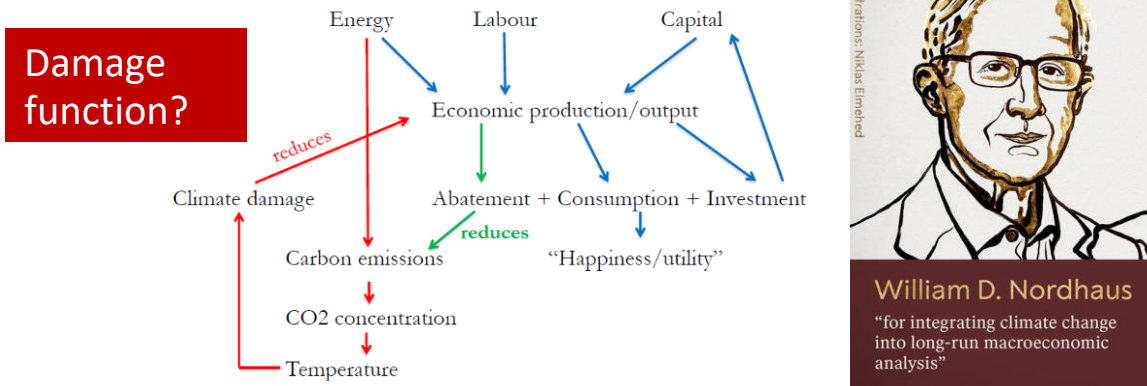
Initially, **cost-benefit analyses** were developed, aiming at evaluating to what extent the cost of climate change mitigation compensates for the avoided climate change impacts.

Which temperature increase is cost-optimal (from an economic point of view)?

The main challenges were:

1. How to quantify impacts of climate change through damage functions or using the social cost of carbon (SCC)
2. The assumed social discount rate determines the results

Figure 1: Schematic illustration of the DICE model



Stern/Nordhaus controversy:

- Nordhaus found optimal temperature increase 3.5°C ($r = 4.5\%$), SSC of 10 USD/tCO₂ ($r = 3\%$)
- Stern: SSC of 85 USD/tCO₂ ($r = 1.4\%$)

D. Roberts, [Discount rates: A boring thing you should know about](#), 2012

Including climate change in economic optimization

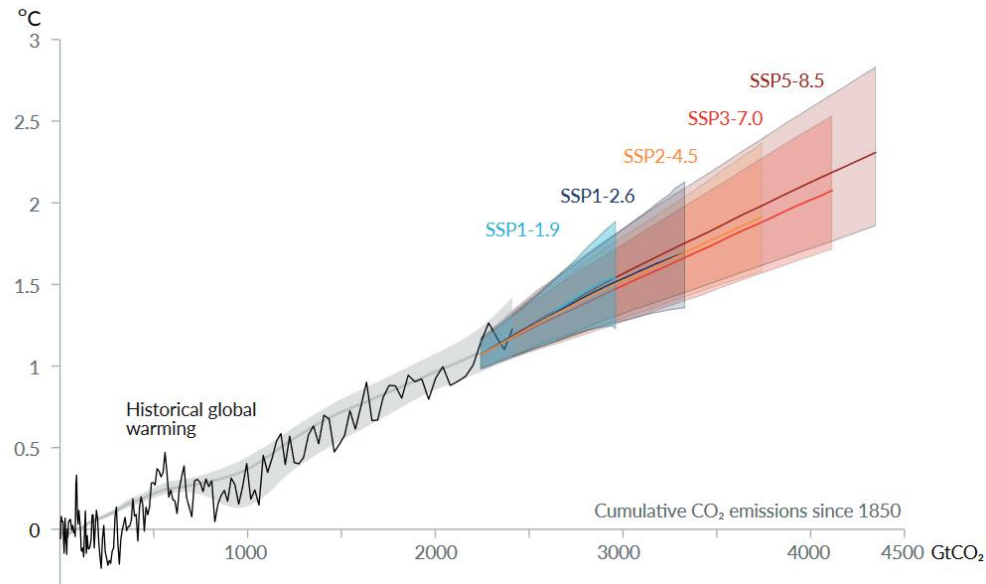
Later, **cost-effectiveness analyses** were developed, aim at estimating the most cost-effective strategy to attain a climate target.

What is the best way of transforming the system while keeping emissions below CO₂ budget?

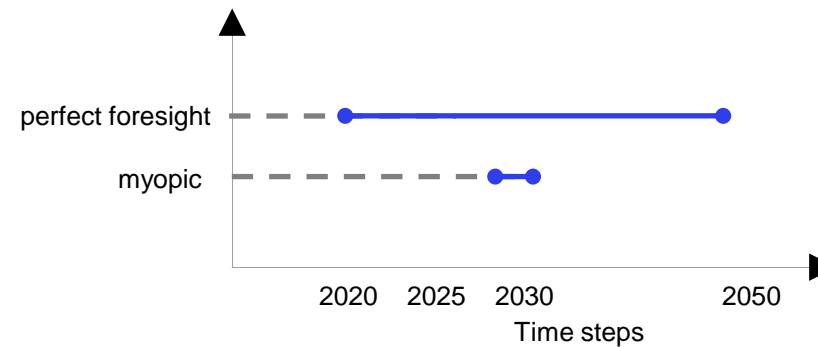
We can impose the CO₂ constraint throughout a transition path to limit cumulative emissions

$$\sum_{s,t} CO_2 \text{ emissions} \leq CO_2 \text{ limit} \leftrightarrow \mu_{CO_2}$$

$$\int_{2020}^{2050} \sum_{s,t} CO_2 \text{ emissions} \leq CO_2 \text{ budget}$$



We can assume perfect-foresight or myopic transition.



The perfect-foresight approach obtains the optimal transformation path and is influenced by the assumed social discount rate. The myopic approach captures the short-sighted behaviour of policy-makers and the sunk costs of bad decisions.



Problems for this lecture

Problems 9.1, 9.2 (**Group 18**)

Problems 9.2 (**Group 19**)

DTU

