46770 Integrated energy grids

Lecture 6 – Gas Networks

Types of optimization problems and course structure

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Join capacity and dispatch optimization in one node
(Lecture 8)</td><td colspan="4">Join capacity and dispatch optimization in a network
(Lecture 10)</td></tr>
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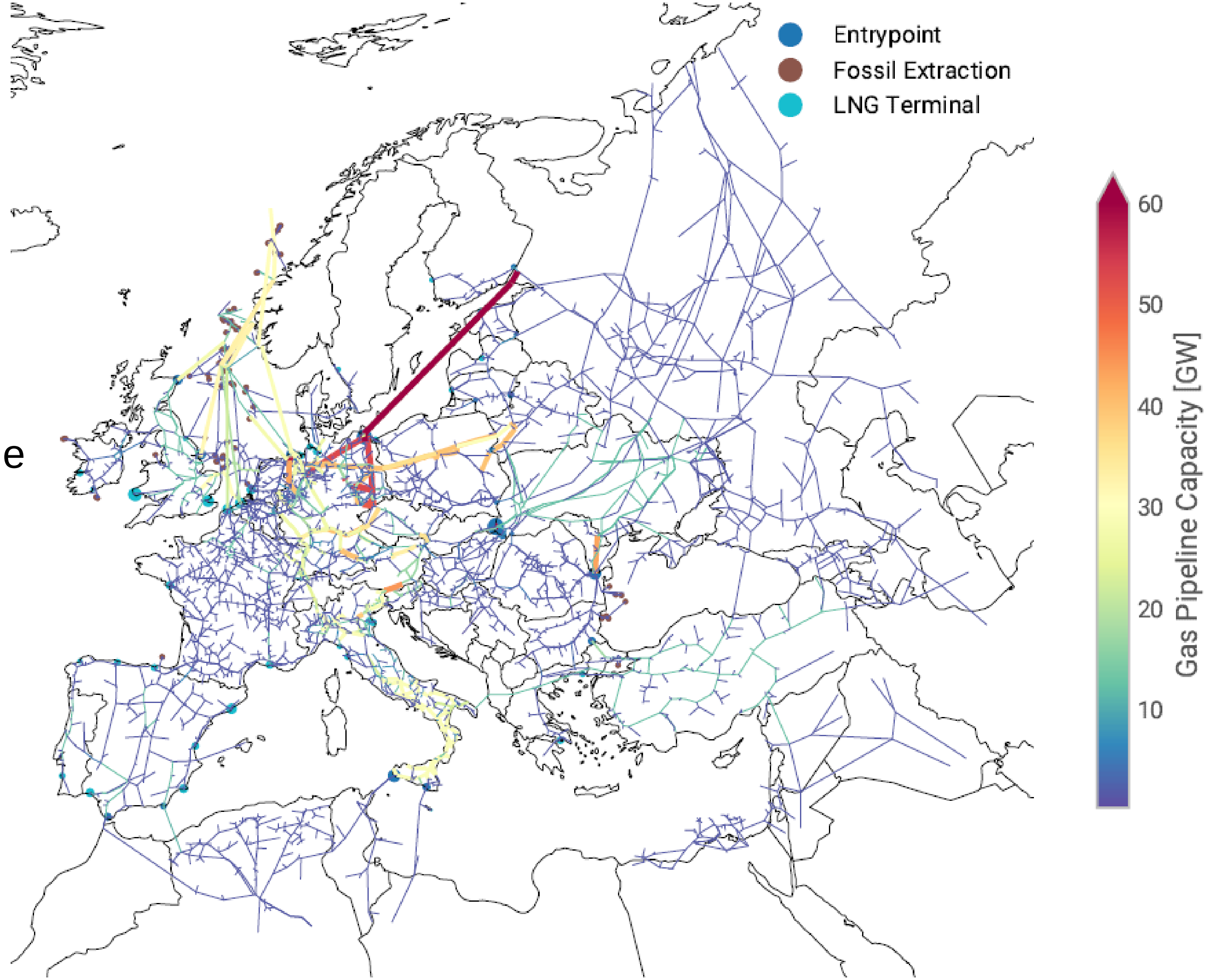
European gas network

There is a methane gas network in Europe which transports methane from fossil extraction points and LNG (liquified natural gas) terminals to consumption points.

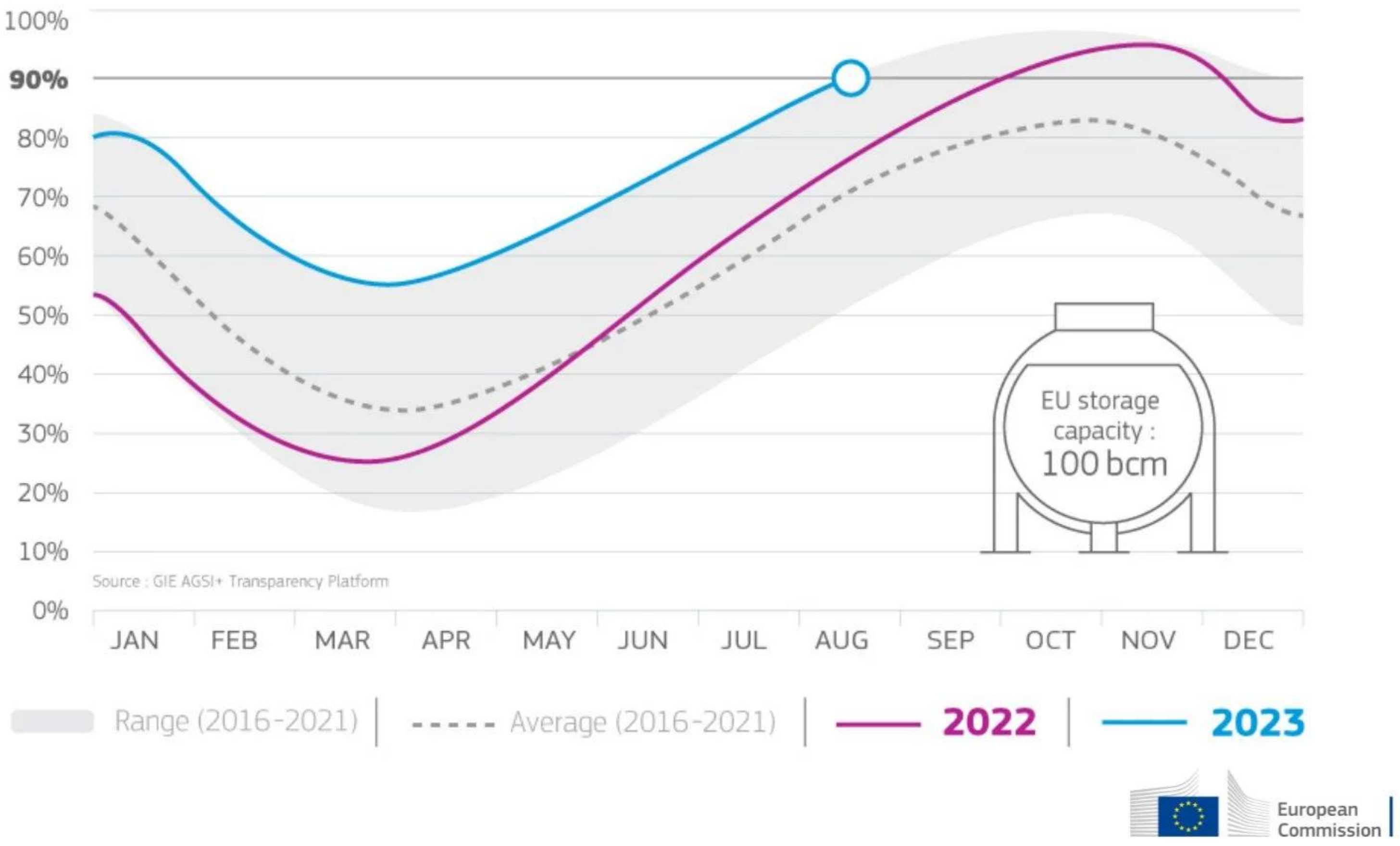
As with power networks, gas networks include transmission and distribution networks.



Neuman, Zeyen, Victoria, Brown, Joule, 2023

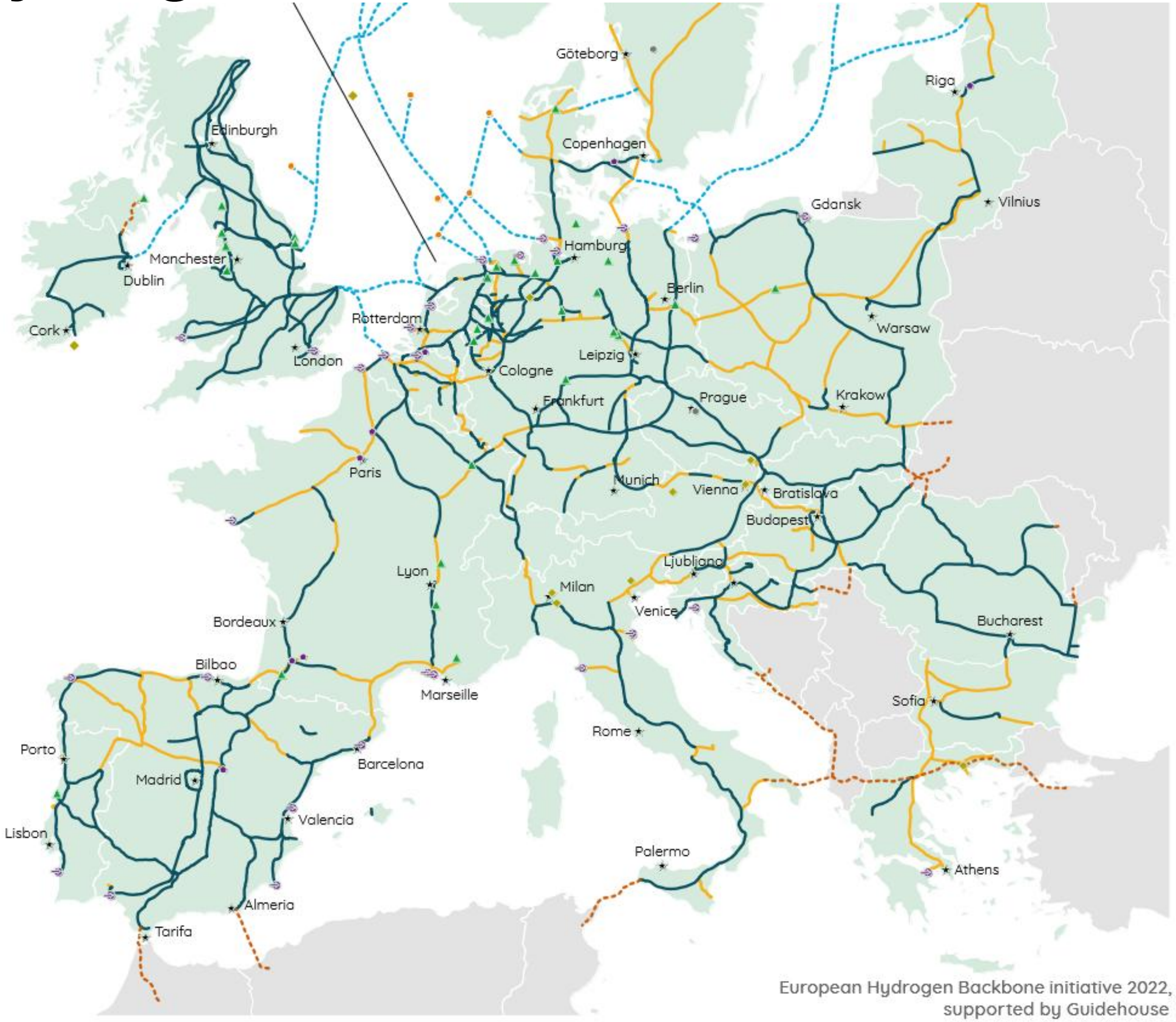
Gas storage in Europe

Depleting gas storage in Europe is the main strategy to deal with high energy demand in winter.



A potential European hydrogen network

The possibility of building a hydrogen network (from scratch or by repurposing existing methane pipelines) is being discussed as a strategy to move electrolytic hydrogen from production to consumption locations.



European Hydrogen Backbone initiative 2022, supported by Guidehouse

Dispatch optimization in a network

How can we include gas flows between nodes in our optimization problem? Or more specifically, what are our nodal energy balance equations and links equations?

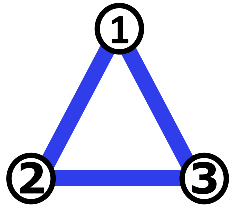
$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} g_{s,i} - d_i = \boxed{?}$$

$$0 \leq g_{s,i} \leq G_{s,i}$$

$$\boxed{?}$$



Learning goals

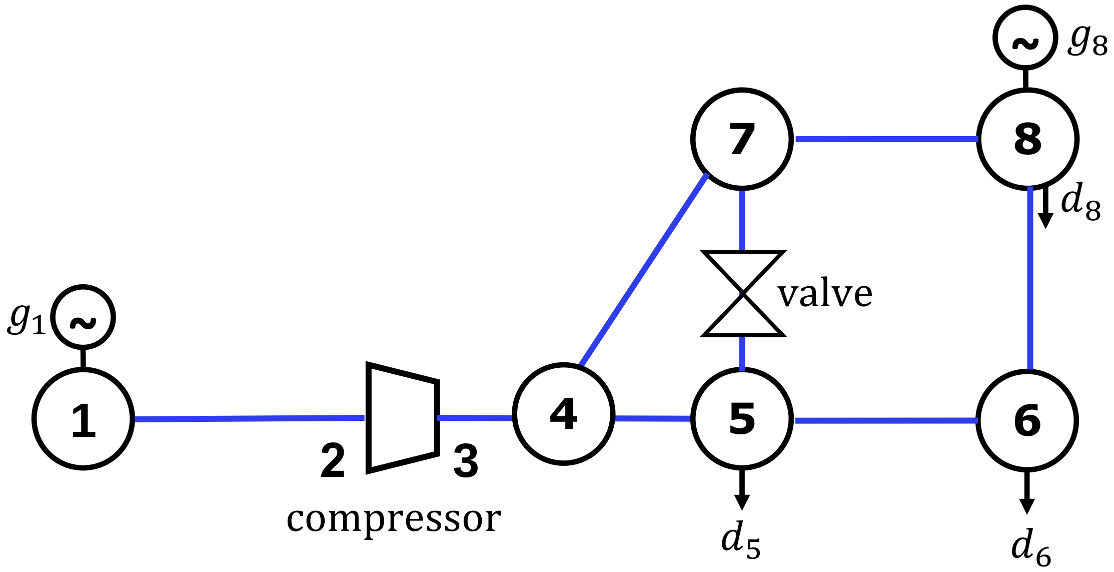
- Calculate the mass flow capacity and energy rate capacity of a gas pipeline
- Estimate the energy that can be stored as linepack in a pipeline
- Obtain the Weymouth equation that relates pressure and mass flow in gas pipelines
- Write the system cost minimization problem including optimal gas flow
- Describe approaches the discretize the gas flow equations
- Formulate the optimal gas flow problem on a computer.

Outline of the lecture

1. Gas networks and components
2. Gas flow in pipelines.
3. Dynamic, quasi-dynamic, steady-state models
4. Weymouth equation that relates pressure and flow rates in gas pipelines
5. Linepack
6. Discretization
7. Optimal gas flow formulation

Gas networks and components

Gas network and components



A gas network is directed (because gas flow has a direction) and weighted (because pipelines have different capacities). The network contains links (pipelines) and nodes.

In the nodes, we can have gas demand (for example from gas power plants), gas supply, compressors (increase pressure), regulators (decrease pressure), and valves (reconfigure the network topology by opening or closing).

The gas flows can be **balanced (steady-state conditions)** or **dynamic (non-stationary)**

Linepack is the volume of gas that can be "stored" in a gas pipeline by increasing the pressure. Compare to the fast dynamic of power systems the slow dynamics of gas pipelines function as short-term storage.

Nodal balance and pipeline equations

In every node, there should be a balance between the nodal supply, demand, and the mass flowing in and out.

$$g_i - d_i = \sum_j m_{i \to j} - \sum_j m_{j \to i}$$

mass nodal balance

Every pipeline has a certain capacity

$$\left| m_{i \to j} \right| \le M_{i \to j}$$

Pipeline capacity

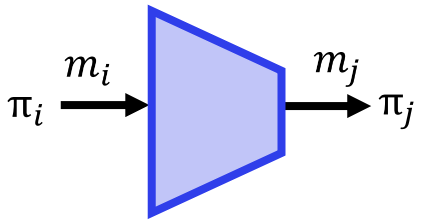
The mass flow in a pipeline is related to the pressure difference

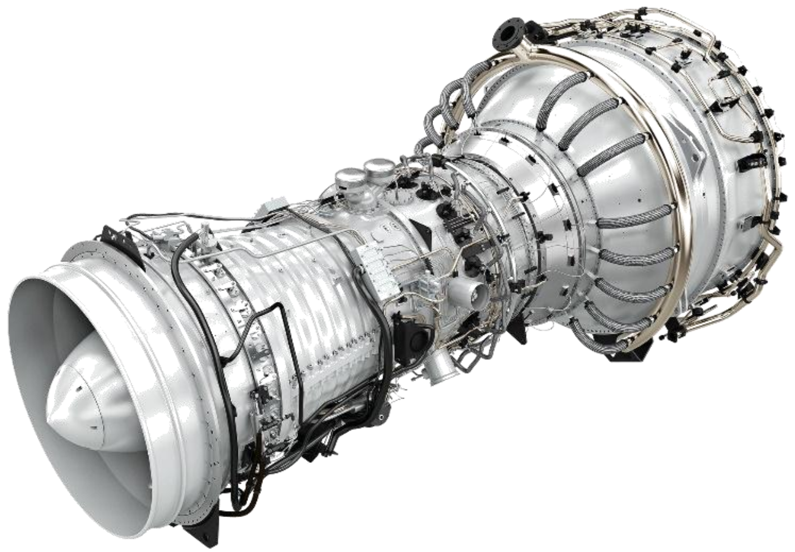
$$a_{ij} m_{i \to j}^2 = \pi_i^2 - \pi_j^2$$

In the next slides we will derive the equation that relates mass flows and pressure in a gas network (momentum conservation)

Note: We use π to represents pressure (because we use p for power flows).

Compressors





A compressors increases pressure by $c_{i\to j}$

$$\pi_j = c_{i\to j}\pi_i$$

To represent the compressor energy demand, we can follow different approaches

a) Assume that the compressors consumes a certain gas mass flow

$$m_i = m_{compressor} + m_j$$

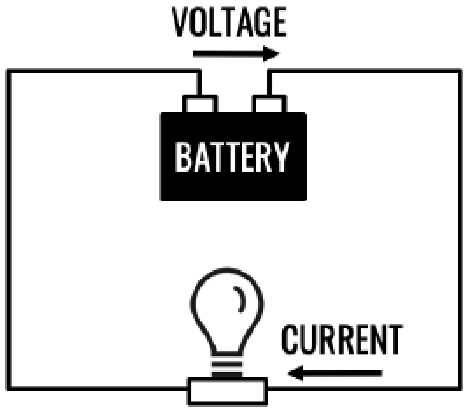
b) Assume that the compressors consumes electricity

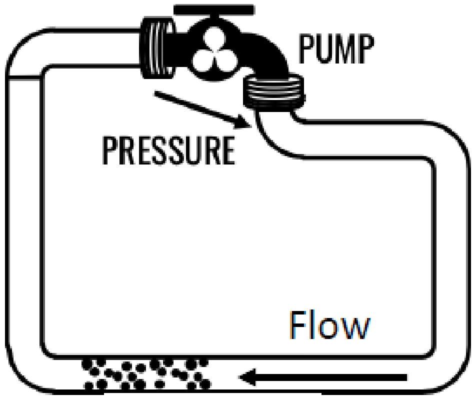
c) Assume that a percentage of the energy transported by the pipeline needs to be consumed to maintain the high pressure, typically 2%/1000 km

Gas flow in pipelines

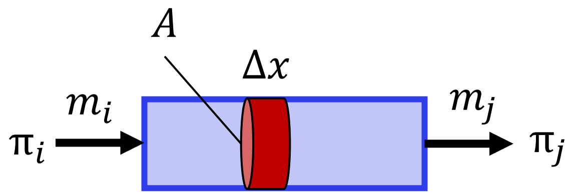
Gas flow in pipelines

Voltage and current in AC transmission lines are equivalent to pressure and mass flow in gas networks





Gas flow in pipelines



The mass-flow m (in kg/s) in a gas pipeline can be calculated as

$$m = \rho A u$$

where ρ represents the density, u the velocity of the gas and A the cross-sectional area of the pipeline.

Using the ideal gas equation, density can be expressed as function of the pressure in the pipeline and the speed of sound in gas c

$$\frac{\pi}{\rho} = \frac{ZRT}{M} = c^2 \qquad\qquad m = \rho A u = \frac{\pi}{c^2} A u$$

where M is the molar mass, R =8.314 J/mol·K is the universal gas constant and Z the compressibility factor.

The energy rate that transported by a gas pipeline q (in MWh/h or MW) can be calculated as

$$q = me = \frac{\pi}{c^2} Aue$$

where e is the energy content in GJ/tonnes or in MWh/kg

Note: m represents a mass flow and q represents an energy flow

How to model gas flow?

Navier-Stokes equations are partial differential equations which describe the motion of fluids.

Navier-Stokes equations in one dimension

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\text{Density change}} + \underbrace{\frac{\partial (\rho u)}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\frac{\partial (\rho u)}{\partial t}}_{\text{inertia}} + \underbrace{\frac{\partial (\rho u^2)}{\partial x}}_{\text{momentum net inflow}} + \underbrace{\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} - \underbrace{\frac{\partial \tau}{\partial x}}_{\text{Friction force}} - \underbrace{\cancel{f(x,t)}}_{\text{External force}} = 0$$

0

where $u(x,t)$ is the velocity of the fluid particle in location x and time step t, $\rho(x,t)$ is the density of the fluid, $\pi(x,t)$ is the pressure, and $f(x,t)$ is some external force per unit area (e.g. gravity)

We use **Euler's equations** (a particularization of Navier-Stokes with zero viscosity and zero thermal conductivity) to impose conservation of mass and momentum

mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$$

momentum conservation

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + \pi)}{\partial x} + \frac{f_D \rho u |u|}{2D} = 0$$

Main hypotheses

1. We model a single pipe not tilted (gravity neglected) $f(x,t) = 0$

2. Darcy-Weisbach empirical formula is used to represent the pressure loss due to friction

$$-\frac{\partial \tau}{\partial x} = \frac{f_D \rho u |u|}{2D}$$

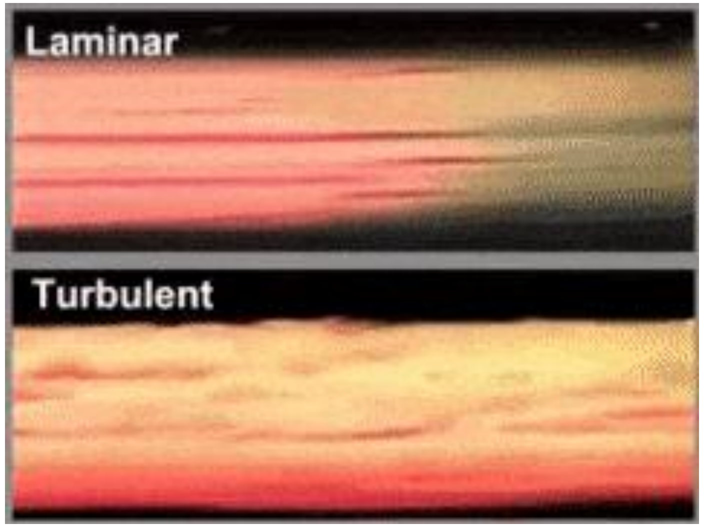
where D is the diameter of the pipe and f_D **is the Darcy friction** coefficient (also named λ) that depends on the characteristics of the pipe (diameter, roughness) and the fluid (Reynolds number)

Reynolds number

The Reynolds number determines if the flow is laminar ($Re < 2000$) or turbulent ($Re \gg 2000$)

Re provides an indication of whether inertial forces or viscous forces are more relevant

$$Re = \frac{inertial\ forces}{viscous\ forces} = \frac{\rho u D}{\mu}$$



Gas flows has typically very large Reynolds numbers (due to very low viscosity, high velocity and large diameter gas pipes), so **gas flow is turbulent**

Darcy friction coefficient

Darcy-Weisbach empirical formula is used to represent the pressure loss due to friction

$$\frac{f_D \rho u|u|}{2D}$$

where D is the diameter of the pipe and f_D is the Darcy friction coefficient.

Darcy friction coefficient $\boldsymbol{f_D}$: can be estimated based on the characteristics of the pipeline (diameter D, roughness ε and Reynolds number Re)

gas flow is turbulent

For **laminar flow** $(Re < 2000)$

$$f_D = \frac{64}{Re}$$

For **fully-turbulent flow** $(Re >> 2000)$

$$\frac{1}{\sqrt{f_D}} = -2log_{10}(\frac{\varepsilon}{3.7D})$$

where ε is the roughness and D the diameter of the pipe

How to model gas flow?

We use **Euler's equations** to impose conservation of mass and momentum

mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \qquad \frac{1}{c^2}\frac{\partial \pi}{\partial t} + \frac{1}{A}\frac{\partial m}{\partial x} = 0 \qquad \frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$

momentum conservation

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + \pi)}{\partial x} + \frac{f_D \rho u|u|}{2D} = 0 \qquad \frac{1}{A}\frac{\partial m}{\partial t} + \frac{\partial(\cancelto{0}{\frac{\pi}{c^2}u^2} + \pi)}{\partial x} + \frac{f_D m|m|}{2DA^2\frac{\pi}{c^2}} = 0 \qquad \frac{\partial m}{\partial t} + A\frac{\partial \pi}{\partial x} + \frac{f_D c^2}{2DA}\frac{m|m|}{\pi} = 0$$

Ideal gas equation

$$\frac{\pi}{\rho} = \frac{ZRT}{M} = c^2$$

Main hypotheses

3. We consider an ideal gas, and the gas flow is **isothermal**. Hence, pressure and density in linear relation

4. Negligible expansion or contraction of the pipe wall and constant cross-sectional area A, then

mass flow $\quad m = \rho u A \qquad \rho u = \frac{m}{A}$

5. We ignore fast transients: the gas flow velocity is much lower than the speed of sound in gas $c \quad (u \ll c)$

Dynamic, quasi-dynamic and steady-state model

How to model gas flow? Dynamic model

In the **dynamic model** we keep all the elements in the mass and momentum conservation equations

$$\underbrace{\frac{\partial \pi}{\partial t}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\frac{\partial m}{\partial t}}_{\text{inertia}} + \underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f_D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

How to model gas flow? Quasi-dynamic model

In the **quasi-dynamic model,** we consider the inertia term negligible.

$$\underbrace{\frac{\partial p}{\partial t}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\cancel{\frac{\partial m}{\partial t}}}_{\text{inertia}} \xrightarrow{} 0 \quad + \underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f_D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

Generally, the inertia term is much smaller than the friction force.

The inertia term is only relevant when fast dynamics occur, for example, when there is a sudden shut-down of a gas-fired power plant.

How to model gas flow? Steady-state model

In the **steady-state model**, we consider steady-state conditions (the derivative with respect to time is equal to zero)

$$\underbrace{\cancel{\frac{\partial p}{\partial t}}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0 \qquad \frac{\partial m}{\partial x} = 0 \qquad m_{in} = m_{out}$$

In this case, mass conservation equation states that outflow is equal to inflow.

This means that **linepack** is neglected in this model

$$\underbrace{\cancel{\frac{\partial m}{\partial t}}}_{\text{inertia}} + \underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f_D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

Weymouth equation and linepack

How to model gas flow? Weymouth equation

We can apply the momentum conservation to the flow in a pipeline to obtain the equation that relates gas flows and pressure in the optimal gas flow problem. This is known as **Weymouth equation**.

$$\underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f_D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

$$\pi_i \xrightarrow{m_i} \boxed{} \xrightarrow{m_j} \pi_j$$

We approach mass flow and pressure in the pipeline by average values and the pressure derivative as the in-out difference

$$m_{ij} = \frac{m_{ij}^{in} + m_{ij}^{out}}{2} \qquad \pi_{ij} = \frac{\pi_i + \pi_j}{2} \qquad \frac{\partial \pi}{\partial x} = \frac{\pi_i - \pi_j}{\Delta x}$$

$$\frac{f_D c^2}{2DA^2} m|m| = \pi \frac{\partial \pi}{\partial x}$$

$$\frac{f_D c^2}{2DA^2} m_{ij}|m_{ij}| = \frac{\pi_i + \pi_j}{2}\frac{\pi_i - \pi_j}{\Delta x}$$

$$\boxed{\frac{f_D c^2}{DA^2} m_{ij}|m_{ij}| = \frac{(\pi_i^2 - \pi_j^2)}{L}}$$

Weymouth equation

$$a_{ij} m_{ij}^2 = \pi_i^2 - \pi_j^2 \qquad a_{ij} = \frac{f_D c^2 L}{DA^2}$$

a_{ij} depends on the physical properties of the pipeline (length L, diameter D, cross-sectional area A), friction coefficient f_D and the speed of sound in gas c

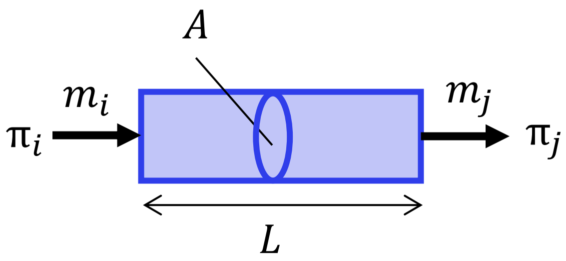
Linepack

Linepack is the volume of gas that can be stored/discharged in a gas pipeline by increasing/decreasing the pressure. This is why changes in average pressure also are referred to as changes in linepack

Linepack is the amount of gas (in kg) in pipeline at step t

where π is the average pressure

$$Linepack(kg) = \rho AL = \frac{\pi}{c^2} AL$$



We can also express linepack in MWh by multiplying by the energy content e (in MWh/kg or GJ/tonnes)

$$Linepack(MWh) = \rho ALe = \frac{\pi}{c^2} ALe$$

The mass-conservation equation can be modified to include an explicit representation of linepack.

We get an equation similar to a storage constraint

$$\frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$

$$\underbrace{\frac{A}{c^2}\frac{\partial \pi}{\partial t}}_{\text{Change in linepack}} = \frac{\partial m}{\partial x}$$

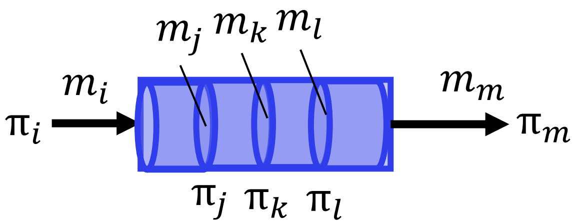
$$Linepack_t = Linepack_{t-1} + \Delta t(m_t^{in} - m_t^{out})$$

Discretization

Discretization

The obtained system of partial differential equations has no analytical solution. We can use numerical methods to solve them. We can discretize the spatial domain, the temporal domain or both.

We can **discretize the pipes by dividing them into segments of equal length** Δx and use the finite different methods (cell-centered method). We can **discretize temporal derivatives using a uniform time step** $\boldsymbol{\Delta t}$



Average pressure and mass flow for **subpipe** $i \rightarrow j$ at time step t

$$\pi_{ij,t} = \frac{\pi_{i,t}+\pi_{j,t}}{2} \qquad m_{ij,t} = \frac{m_{i,t}+m_{j,t}}{2}$$

Partial Differential Equations

$$\frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$

$$\frac{\partial m}{\partial t} + A\frac{\partial \pi}{\partial x} + \frac{f^D c^2}{2DA}\frac{m|m|}{\pi} = 0$$

Discrete versions

$$\frac{\pi_{ij,t} - \pi_{ij,t-1}}{\Delta t} + \frac{c^2}{A}\frac{m_{j,t} - m_{i,t}}{\Delta x} = 0$$

$$\frac{m_{ij,t} - m_{ij,t-1}}{\Delta t} + A\frac{\pi_{j,t} - \pi_{i,t}}{\Delta x} + \frac{f_D c^2}{2DA}\frac{m_{ij,t}|m_{ij,t}|}{\pi_{ij,t}} = 0$$

Discretization

In the **steady-state model**, we consider steady-state conditions (the derivative with respect to time zero)

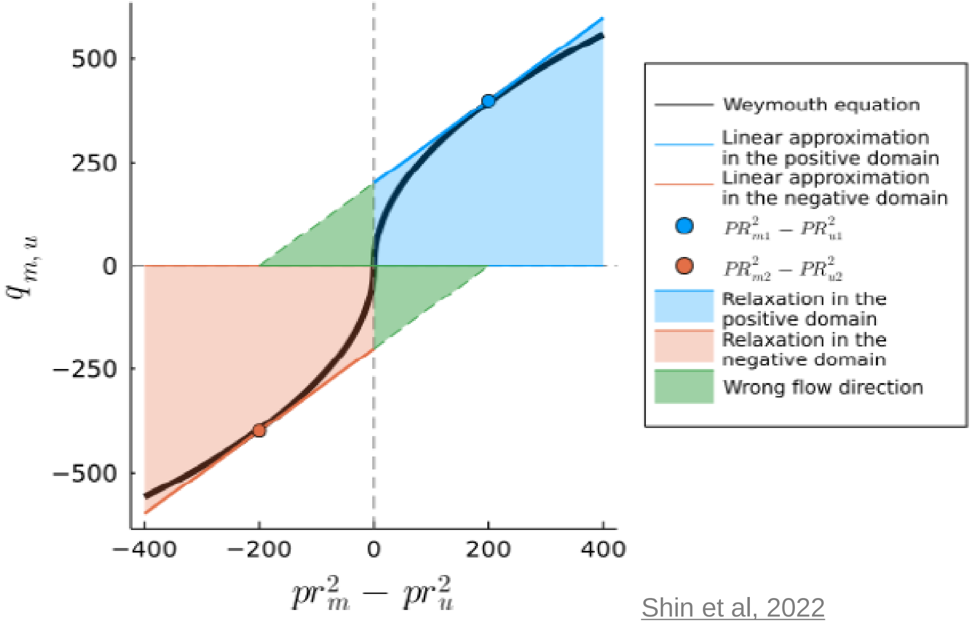
$$\frac{\pi_{ij,t} - \cancelto{0}{\pi_{ij,t-1}}}{\Delta t} + \frac{c^2}{A}\frac{m_{j,t} - m_{i,t}}{\Delta x} = 0$$

$$\frac{m_{ij,t} - \cancelto{0}{m_{i,jt-1}}}{\Delta t} + A\frac{\pi_{j,t} - \pi_{i,t}}{\Delta x} + \frac{f_D c^2}{2DA}\frac{m_{ij,t}\left|m_{ij,t}\right|}{\pi_{ij,t}} = 0$$

The discrete equations are still not linear.

1. We can use Newton-Raphson algorithm

2. We can use linear approximations or relaxation



Shin et al, 2022

3. If the problem is solved sequentially, we can approximate the non-linear term with values from the previous time step

$$\frac{m_{ij,t}\left|m_{ij,t}\right|}{\pi_{ij,t}} \approx \frac{m_{ij,t-1}\left|m_{ij,t-1}\right|}{\pi_{ij,t-1}}$$

Modelling approaches for gas flows

Non-discretized pipelines	Discretized pipelines		
	Steady-state	Quasi-dynamic	Dynamic
$\lvert m_l \rvert \leq M_l$	$\frac{\partial m}{\partial x} = 0$ $A\frac{\partial \pi}{\partial x} + \frac{f_D c^2}{2DA}\frac{m\lvert m\rvert}{\pi} = 0$	$\frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$ $A\frac{\partial \pi}{\partial x} + \frac{f_D c^2}{2DA}\frac{m\lvert m\rvert}{\pi} = 0$	$\frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$ $\frac{\partial m}{\partial t} + A\frac{\partial \pi}{\partial x} + \frac{f_D c^2}{2DA}\frac{m\lvert m\rvert}{\pi} = 0$
• Linear • Compressors demand modelled as link efficiency • Linepack modelled as energy storage	• No representation of Linepack • Can be linearized or not	• Not good enough to model fast dynamics, e.g shut-down of a gas-fired power plant	

Increase computational complexity

Optimal gas flow

Nodal balance and pipeline equations

We recall here the nodal balance and pipeline equations.

In every node, there should be a balance between the nodal supply, demand, and the mass flowing in and out.

$$g_{i,t} - d_{i,t} = \sum_j m_{i\to j} - \sum_j m_{j\to i}$$

mass nodal balance

We can use the incidence matrix to compute all the pipelines connected to a node.

$$g_{i,t} - d_{i,t} = \sum_l K_{il} m_l$$

Every pipeline has a certain capacity

$$|m_{i\to j}| \leq M_{i\to j}$$

Pipeline capacity

The mass flow in a pipeline is related to the pressure difference

$$a_{ij} m_{i\to j}^2 = \pi_i^2 - \pi_j^2 \qquad a_{ij} = \frac{L f_D c^2}{D A^2}$$

Momentum conservation in pipelines

where m_i represents the injection in node i and a_{ij} depends on the physical properties of the pipeline (length L, diameter D, cross-sectional area A), friction coefficient $f^D$and the speed of sound in gas c

Economic dispatch with gas optimal power flow

Determine the optimal economic dispatch to supply the demand d_n in a certain hour and the optimal gas flows while minimizing the total system cost.

Economic dispatch with AC power flow

$$\begin{cases} \min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ \text{subject to:} \\ \sum_{s,i} g_{s,i} - d_i = \sum_{l} K_{il} m_l \quad \leftrightarrow \quad \lambda_i \\ |m_l| \leq M_l \\ (\text{ or } |q_l| \leq Q_l) \\ a_{ij} m_{ij}^2 = \pi_i^2 - \pi_j^2 \qquad a_{ij} = \frac{L f_D c^2}{D A^2} \end{cases}$$

Nodal mass (or energy rate) balance

Pipelines capacities
(in mass flow or energy)

Physical relations in the links

m represents a mass flow and q represents an energy flow (i.e. $q = m \cdot u$ where u is the energy content in MWh/kg or GJ/tonne

Problems for this lecture

Problems 6.1, 6.2 (**Group 12**)

Review tutorial on gurobipy

https://martavp.github.io/integrated-energy-grids/intro-gurobipy.html

Problems 6.3 (**Group 13**)

DTU