

Lecture 4: DC Optimal Power Flow

DTU Course 46770: Integrated Energy Grids

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Problem 4.1. This is a continuation of Problem 3.2 from Lecture 3.

Let us assume now that we are in an hour with an excess of wind generation in Denmark and a deficit in other countries so that the power injection p_i of the different countries is as follows:

Germany = - 2 MW, DK1=5 MW, DK2=6 MW, Norway = - 8 MW, Sweden = -1 MW,

Determine the voltage angles θ_i and the flows p_l in the lines of the network. Assume that $\theta_0=0$; i.e. the reference bus is at node 0 (Germany); and the reactance in all links is $x_l=1$.

Hint 1: Remember the relations between power injection p_i in every node and power flows p_l presented in Lecture 4.

$$p_i = \sum_j L_{ij} \theta_j$$

$$p_l = \frac{1}{x_l} \sum_j K_{lj} \theta_j$$

$$L_{ij} = \sum_l K_{il} \frac{1}{x_l} K_{lj}$$

Hint 2: you can use `numpy.linalg.solve` to solve the linear equation system.

Problem 4.2. This is a continuation of Problem 3.2 from Lecture 3.

- Assuming that the reactance in the links is $x_l=1$, calculate the Power Transfer Distribution Factor (PTDF) matrix.
- Assuming the power injection pattern described in Problem 4.1 determine the flows in the lines of the network.

Problem 4.3. This is a continuation of Problem 3.3 from Lecture 3. Using the Python package `networkX`.

- Assuming that the reactance in the links is $x_l=1$, calculate the Power Transfer Distribution Factor (PTDF) matrix.
- Assuming the power injection pattern described in Problem 4.1 determine the power flows in the lines of the network and plot them.
- Assume now that the links unitary reactance is $x_l=[1, 0.5, 0.5, 0.5, 1]$, calculate the weighted Laplacian (or susceptance matrix), the PTDF matrix, the power flows and plot them.

Problem 4.4. This is a continuation of Problem 3.4 from Lecture 3. For the synchronous zone corresponding to Scandinavia:

- a) Add to the network object the information on the links susceptance, calculate the weighted Laplacian (or susceptance matrix), and the Power Transfer Distribution Factor (PTDF) matrix.
- b) Assuming that power injection in the nodes increases linearly from -1 in the first node to +1 in the last node, calculate and plot the power flows in the network.