Question 1.2.1

a) There exists a student at UWE that takes this module. (True)

b) All students at UWE take this module. (False)

c) There exists a student at UWE that does not take this module. (True)

d) For all students at UWE they do not take this module. (False, since there are students that take this module) – d) is the negation of a) and means the same as e).

e) There does not exist a student at UWE that takes this module. (False, it is the negation of a)

f) Not all students at UWE take this module. (True)

g) There does not exist a student at UWE that does not take this module. (False – this is the negation of c) and we know c) is true.

h) Either there exists a student at UWE that takes this module exclusive-or all students at UWE take this module. (This has a truth value of true since it's the exclusive-or of part a) with part b), that is, $\mathrm{T} \oplus \mathrm{F} \Leftrightarrow \mathrm{T}$.)

Note: The following pairs of statements are equivalent in natural language and are logically equivalent in logic: (c,f), (d,e), (g,b).

Question 1.2.2

a) '0 $>$ 0' is False

b) '2 $>$ 1' is True

c) '1 $>$ 2' is False

d) 'There exists a number y in the domain such that $2 > y$'
True, for example, $2 > 0$.

e) 'There exists a number x in the domain such that $x > 2$'
False, remember that we limit our numbers to the domain { 0, 1, 2 } and in that domain there is no number that is greater than 2.

f) True, this is the negation of (e) and hence its truth value is the negation of False.

g) With the $P(x)$ in this question, $\exists x \cdot (\neg P(x,2))$ means the same as $\exists x \cdot (\neg(x > 2))$. This statement reads as, 'there exists an x in the domain such that it is not the case that x is greater than 2'.
Note that the truth value of $(\neg(0 > 2))$ is true and hence there does exist an x in the domain such that $(\neg(x > 2))$ is true.
Hence, $\exists x \cdot (\neg(x > 2))$ is true.

h) $\forall x \cdot (\neg P(x,x))$. In other words, for all numbers in {0, 1, 2}, it is not the case that x is greater than x.
Consider the following three statements, each of the form $\neg P(x,x)$ for different x's:

$\neg P(0,0)$ is True

$\neg P(1,1)$ is True

$\neg P(2,2)$ is True

and hence, $\neg P(x,x)$ is true for all x. Hence we deduce that:

$\forall x \cdot (\neg P(x,x))$ is true.

i) $\forall x \cdot (P(x,x) \to P(2,x))$:

Here, we consider, individually, the truth values of $P(x,x) \to P(2,x)$ for different x's:

We have:

$P(0,0) \to P(2,0)$	has a truth value of True (since of the form False → True).
$P(1,1) \to P(2,1)$	has a truth value of True (since of the form False → True).
$P(2,2) \to P(2,2)$	has a truth value of True (since of the form False → False).

and hence, since $P(x,x) \to P(2,x)$ is true for every x, we have $\forall x \cdot (P(x,x) \to P(2,x))$ being true.

Question 1.2.3

a)	"There exists someone who is an engineer"	E	$\exists x \cdot P(x)$
b)	"Everybody is employed"	G	$\forall z \cdot Q(z)$
c)	"There exists an engineer that is employed"	J	$\exists y \cdot (P(y) \wedge Q(y))$
d)	"All engineers are employed"	K	$\forall x \cdot (P(x) \to Q(x))$
e)	"There exists an engineer that is not employed"	A	$\exists y \cdot (P(y) \wedge (\neg Q(y)))$
f)	"Not all engineers are employed"	B	$\neg(\forall x \cdot (P(x) \to Q(x)))$
g)	"Every engineer is unemployed"	L	$\forall x \cdot (P(x) \to (\neg Q(x)))$
h)	"There does not exist an employed graduate"	H	$\neg(\exists x \cdot (P(x) \wedge Q(x)))$

Question 1.2.4

a) $P(7)$

b) $\neg(\exists x \cdot (P(x) \wedge Q(x)))$

c) $\exists x \cdot (\neg Q(x))$

d) $\forall x \cdot (P(x) \oplus Q(x))$

e) $\forall x \cdot (P(x) \to (\neg Q(x)))$

Question 1.2.5

Please note that, in this kind of question, it is the explanation that is of most importance.

(i) True.

To show this we investigate the truth values of $P(x) \vee (Q(x) \vee R(x))$ for the values in the domain. We have the following:

$$P(1) \vee (Q(1) \vee R(1)) \Leftrightarrow T \vee (T \vee T) \Leftrightarrow T$$
$$P(2) \vee (Q(2) \vee R(2)) \Leftrightarrow F \vee (T \vee T) \Leftrightarrow T$$
$$P(3) \vee (Q(3) \vee R(3)) \Leftrightarrow T \vee (T \vee F) \Leftrightarrow T$$
$$P(4) \vee (Q(4) \vee R(4)) \Leftrightarrow F \vee (F \vee T) \Leftrightarrow T$$
$$P(5) \vee (Q(5) \vee R(5)) \Leftrightarrow T \vee (F \vee F) \Leftrightarrow T$$
$$P(6) \vee (Q(6) \vee R(6)) \Leftrightarrow F \vee (F \vee T) \Leftrightarrow T$$

That is, $P(x) \vee (Q(x) \vee R(x))$ is true for all *x* and we can state:

$\forall x \cdot (P(x) \vee (Q(x) \vee R(x)))$ is true.

That is, every row (which correspond to an *x*) has at least one T

(ii) We have the following:

$\forall x \cdot P(x)$ is false (e.g. since *P*(2) is false)

$\forall x \cdot Q(x)$ is false (e.g. since *Q*(6) is false)

$\forall x \cdot Q(x)$ is false (since *R*(5) is false)

Hence, truth value of $((\forall x \cdot P(x)) \vee (\forall x \cdot Q(x))) \vee (\forall x \cdot R(x))$ is the same as the truth value of $(F \vee F) \vee F$ which is false.

(iii) $\exists x \cdot (P(x) \wedge (Q(x) \wedge R(x)))$

For this formula to be true then there must exist an *x* for which

$P(x) \wedge (Q(x) \wedge R(x))$

is true.

Consider the case when *x* is 1. We have

$$P(1) \wedge (Q(1) \wedge R(1)) \Leftrightarrow T \wedge (T \wedge T) \Leftrightarrow T$$

Hence there does exist an *x* for which $P(x) \wedge (Q(x) \wedge R(x))$ is true.

(iv) Note that $\forall x \cdot P(x)$ is false (see (ii)) and $\exists x \cdot P(x)$ is true (since, e.g. *P*(3) is true), hence the truth value of

$$(\forall x \cdot P(x)) \rightarrow (\exists x \cdot P(x))$$

is the same as that of $F \rightarrow T$, which is true.

(v) False. Note that the negation is the outermost operator so calculate the truth value of

$(\exists x \cdot Q(x)) \vee (\forall x \cdot P(x))$ first, and then calculate the truth value of the negation.

Since $(\exists x \cdot Q(x))$ is true (since, e.g. $Q(1)$ is true) and $\forall x \cdot P(x)$ is false (see (ii)) we have

$$(\exists x \cdot Q(x)) \vee (\forall x \cdot P(x)) \quad \Leftrightarrow \quad T \vee F \quad \Leftrightarrow \quad T$$

Since $(\exists x \cdot Q(x)) \vee (\forall x \cdot P(x))$ is true, then the negation of this (which is what the question asks for) is false.

(vi) We have $\exists x \cdot P(x)$ true since, e.g. $P(1)$ is true.

We have $\exists x \cdot (R(x) \wedge Q(x))$ true since, e.g. $R(2) \wedge Q(2) \quad \Leftrightarrow \quad T \wedge T \quad \Leftrightarrow \quad T$ is true.

Hence: $\neg(\exists x \cdot (R(x) \wedge Q(x)))$ is false.

Hence, $(\exists x \cdot P(x)) \rightarrow (\neg(\exists x \cdot (R(x) \wedge Q(x))))$ has the same truth value as True $\rightarrow$ False which is the value False.

(vii) We already know that $\forall x \cdot P(x)$ has a truth value of False (see (ii))

Further we note that $R(6) \wedge P(1) \quad \Leftrightarrow \quad T \wedge T \quad \Leftrightarrow \quad T$.

Hence the given formula is of the form True $\rightarrow$ False which is False.

Question 1.2.6

Note that we did not explicitly cover the topic of using transformational proof in <u>predicate logic</u> but we're giving you this solution anyway out of interest. Note that <u>all but one</u> of the steps involve logical equivalences in propositional logic. It's just the second step (marked with a *) that uses a law from predicate logic - the generalised DeMorgan's law that states $(\neg(\exists x \cdot \alpha)) \Leftrightarrow (\forall x \cdot (\neg\alpha))$.

$$\exists x \cdot (P(x) \wedge (\neg Q(x)))$$

$\Leftrightarrow$	$\neg(\neg(\exists x \cdot (P(x) \wedge (\neg Q(x)))))$	double negation law
$\Leftrightarrow$	$\neg(\forall x \cdot (\neg(P(x) \wedge (\neg Q(x)))))$	generalised DeMorgan's law*
$\Leftrightarrow$	$\neg(\forall x \cdot ((\neg P(x)) \vee (\neg(\neg Q(x)))))$	DeMorgan's law
$\Leftrightarrow$	$\neg(\forall x \cdot ((\neg P(x)) \vee Q(x)))$	double negation law
$\Leftrightarrow$	$\neg(\forall x \cdot (P(x) \rightarrow Q(x)))$	implication law