Q1

P	Q	R	$(P \to$	$(\neg R))$	$\vee$	$(\neg Q)$
T	T	T	F	F	F	F
T	T	F	T	T	T	F
T	F	T	F	F	T	T
T	F	F	T	T	T	T
F	T	T	T	F	T	F
F	T	F	T	T	T	F
F	F	T	T	F	T	T
F	F	F	T	T	T	T
			2	1	2	1

P	Q	R	$(P \wedge Q)$	$\leftrightarrow$	$(R \oplus Q)$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	F
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	T	F
			2	2	2

The models for the conjunction (the $\wedge$) are lines 2, 4, 5, 8 of the truth table.

Q2

(i) $Q(4) \vee (P(2) \leftrightarrow R(6))$

$\Leftrightarrow F \vee (F \leftrightarrow T)$ 1 per line

$\Leftrightarrow F \vee F$

$\Leftrightarrow F$

(ii) The $\exists$ is true since e.g. take $x = 4$, we note that $P(4) \wedge R(4)$ 2 is true & hence there exists 1 an x for which $P(x) \wedge R(x)$ is true.

(iii) Note the following

x	$P(x) \to (\neg Q(x))$
1	$F \to T$
2	$F \to T$
3	$F \to T$
4	$T \to T$
5	$F \to F$
6	$T \to T$

2

The truth values of all 1 of these is true & hence the $\forall x$ holds.

(iv) Truth value of $\forall x (\neg R(x))$ is false since e.g. $\neg R(1)$ is false. 1

Note that $P(1) \Leftrightarrow (Q(1) \Leftrightarrow R(1))$

$$\Leftrightarrow F \Leftrightarrow (F \Leftrightarrow T)$$
$$\Leftrightarrow F \Leftrightarrow F$$
$$\Leftrightarrow T$$

Hence $\exists x. (P(x) \Leftrightarrow (Q(x) \Leftrightarrow R(x))$ is true. 2

Hence the overall statement is of the form:

$$F \oplus T \Leftrightarrow T.$$ 1

Q3

1.	$P \to (\neg(Q \vee S))$	Premise
2.	$W \vee (\neg R)$	Premise
3.	$(\neg P) \Leftrightarrow R$	Premise
4.	$((\neg P) \to R) \wedge (R \to (\neg P))$	3, $\Leftrightarrow$, Equivalence
5.	$(\neg P) \to R$	4, Simplification
6.	Q	Assumption for Conditional Proof 1
7.	$Q \vee S$	6. Addition 1

3

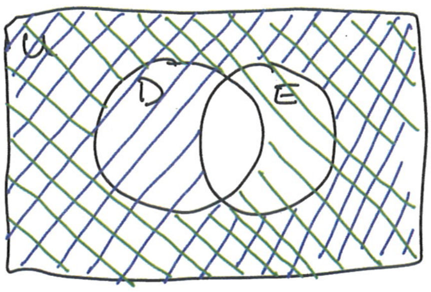
8.	$\neg P$	7, 1, Modus Tollens (2)
9.	R (1)	5, 8, Modus Ponens (1)
10.	W	2, 9, Disjunctive Syllogism (2)
11.	$R \wedge W$ (1)	9, 10, Conjunction
12.	$Q \to (R \wedge W)$ (1)	6, 11, Conditional Proof (1)

Q4 (a) $A \cap B = \{2, 3, 6\}$ (1h)

$(A \cap B) \times C = \{(2,0), (3,0), (6,0), (2,1), (3,1), (6,1)\}$ (1h)

(1) for correct bracketing.

b)



//// $\overline{E}$ (1)

\\\\\\ $\overline{D}$ (1)

Union is region covered by any or both colours. (1)
Alternatively: could recognise region as $(\overline{D \cap E})$

(c) (i) $F = \{-2, -1, 0, 1, 2, 3\}$ (2)

(ii). Every element of $\{0, 2\}$ is also in F.

- F does not contain a set as an element.
- F contains -2 as an element, $\mathbb{N}$ does not.

(1) per qn.

(iii) $|F| = 6$ (1)

Q5 (a)

(i) $g \circ f\,(4,6) = g(f(4,6))^{(1)} = g(-2)^{(1)}$

$= 2 + (-2)^{4\,(1)} = 18^{(1)}$

ii) Although $1 \in \text{codomain}(g)$ (1), g will not map any integer to 1 & hence g is not a surjection. (2)

b) $b^{-1}(x) = \dfrac{2x - 4}{3}$ (1) (1) (1)

c) We have $h(0) = \frac{5}{3} \notin \text{codomain}(h)$ (3)

Q6 (i) $S \circ R$ exists because right-set$(R) = B$ $=$ left-set(S) (1h)

but $R \circ S$ doesn't exist because right-set$(S) = C \neq A =$ left-set(R). (1h)

(ii) $S \circ R = \{(1,3),(1,5),(1,1),(1,2),(1,4),(2,3),(2,5),(3,3),(3,5),(3,1)\}$

p1 for any missing or incorrectly included.
ph for just one error

(b) (i) because, e.g. $(4,4) \notin W$ (1)

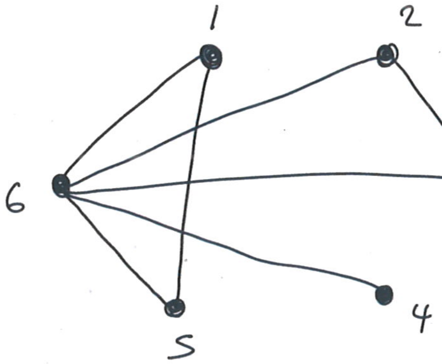
(ii) because $(3,3) \in W$ (1)

(iii) because $(4,2) \in W$ but $(2,4) \notin W$ (2)

(iv) because $(1,5) \in W$ & $(5,1) \in W$ but $(1,1) \notin W$. (2)

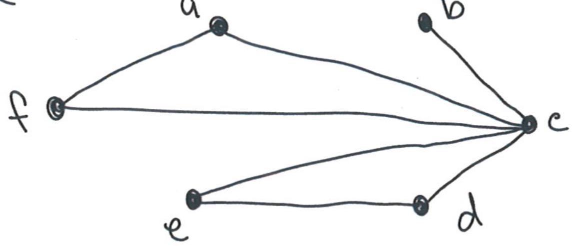
Q7

i) G1



(2)

G2



(2)

ii) The graphs are isomorphic. Relabelling G2 as follows: $c \to 6, b \to 4, d \to 2, e \to 3, a \to 1, f \to 5$ results in G1. 2

(b)

$$M_{G_3} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

2

ii) Looking for walks of length two and how many. 1

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 2 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 2 & 1 \end{pmatrix} \end{array}$$

2

iii) Convert any non-zero to a one: 1

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

1

Answers for section B

2 marks per correct answer for the answers below

Question B 8: c
Question B 9: d
Question B 10: d
Question B 11: c
Question B 12: d
Question B 13: b
Question B 14: a
Question B 15: b (1,3)
Question B 16: a
Question B 17: b
Question B 18: c
Question B 19: a
Question B 20: a
Question B 21: a
Question B 22: c
Question B 23: d
Question B 24: c
Question B 25: c
Question B 26: a
Question B 27: d
Question B 28: b
Question B 29: b (can reward 1 mark for c)
Question B 30: a
Question B 31: c