University of the West of England
BRISTOL

Faculty of Environment and Technology

Academic Year: **20/21**
Examination Period: **May-21**

Module Leader: **Ibrahim Mimrah**
Module Code: **UFCFA3-30-1 Principles**
Module Title: **of Computing EX1-**
Work Item Code: **Component A**

Duration: **Online**

THIS IS YOUR QUESTION PAPER AND ANSWER BOOK

Please complete your Student Number on the line below:

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Standard materials required for this examination:

Examination Answer Booklet		Yes
Multiple Choice Answer Sheet		No
Graph Paper	Type of paper e.g. G3, G14	No
	Number of sheets per student	0

Additional materials required for this examination:

Details of additional material **supplied by UWE**: UFCFA3-30-1 Exam Formula Booklet To be collected with Answer Booklet	**No**
Details of approved material **supplied by Student**: **None**	
University approved Calculator	Yes
Candidates permitted to keep Examination Question Paper	**No**

Candidates are NOT permitted to turn the page over until the exam starts

Instructions to Candidates

This paper is partitioned into two sections, namely Section A and Section B. **Section A** is worth **52** marks and **Section B** is worth **48** marks.

Candidates must answer **four** (out of seven) questions in **Section A**.
Candidates must answer **all** 24 questions in **Section B**.
Questions in Section A are labelled **A1 - A7** and those in Section B are labelled **B8 – B31**.

For **Section A**, candidates must show all workings for all attempted questions. If more than **four** questions are answered in **Section A**, then all answered questions will be marked and the candidate will be awarded the highest four marks.

For **Section B**, candidates need only provide their selected answers to each of the multiple choice questions.

For Questions A1 and A3 in Section A and all the questions in Section B (multiple choice), your answers must be written on **this** Examination Question Paper. For Questions A1 and A3, pages 9 and 10 on **this** Examination Question Paper are allocated for you to write your answer. For the questions in Section B you should make each of your multiple choice selections by circling your chosen selection.

At the end of the examination, you are required to attach all your Examination Booklets and this Examination Question paper together.

SECTION A [52 Marks]
Answer any 4 questions (out of 7) from this section.

Question A1

Construct the truth tables for the following two propositional formulae:

i) $\big(P \to (\neg R)\big) \vee (\neg Q)$ [**6 marks**]

ii) $(P \wedge Q) \leftrightarrow (R \oplus Q)$ [**6 marks**]

Hence, state all the models for the formula:

$\Big(\big(P \to (\neg R)\big) \vee (\neg Q)\Big) \wedge \big((P \wedge Q) \leftrightarrow (R \oplus Q)\big)$ [**1 mark**]

Note: Your answer to this question should be made on the *Question A1 Answer Sheet* which is on page 9 of this examination paper.

Question A2

Consider the following interpretation for propositional functions P, Q and R with domain $D = \{1,2,3,4,5,6\}$. In parts (i) – (iv) you are given formulae in predicate logic, together with their truth values under this interpretation. You are required to explain why these formulae, have the given truth values.

x	$P(x)$	$Q(x)$	$R(x)$
1	F	F	T
2	F	F	T
3	F	F	F
4	T	F	T
5	F	T	F
6	T	F	T

Explain why, under the above interpretation:

(i) $Q(4) \vee (P(2) \leftrightarrow R(6))$ is False [**3 marks**]

(ii) $\exists x \cdot (P(x) \wedge R(x))$ is True [**3 marks**]

(iii) $\forall x \cdot \big(P(x) \to (\neg Q(x))\big)$ is True [**3 marks**]

(iv) $\Big(\forall x \cdot \big(\neg R(x)\big)\Big) \oplus \Big(\exists x \cdot \big(P(x) \leftrightarrow (Q(x) \leftrightarrow R(x))\big)\Big)$ is True [**4 marks**]

Question A3

Below is a partially complete proof of the following argument:

$$\{ P \to (\neg(Q \vee S)),\ W \vee (\neg R),\ (\neg P) \leftrightarrow R \} \ \vdash\ Q \to (R \wedge W)$$

You are required to fill in the gaps in the following proof using only the standard inference and logical equivalence rules of Propositional Logic supplied in Sections 1.2 and 1.3 of the Formula Booklet.

Note: Your answer to this question should be made on the *Question A3 Answer Sheet* which is on page 10 of this examination paper.

1.	$P \to (\neg(Q \vee S))$	Premise
2.	$W \vee (\neg R)$	Premise
3.	$(\neg P) \leftrightarrow R$	Premise
4.	$((\neg P) \to R) \wedge (R \to (\neg P))$	……
5.	$(\neg P) \to R$	……
6.	Q	Assumption for ……
7.	$Q \vee S$	6, ……
8.	$\neg P$	……
9.	……	5,8, ……
10.	W	……
11.	……	9,10, Conjunction
12.	……	……

[**13 marks**]

Question A4

(a) Let A, B and C be sets given as follows:

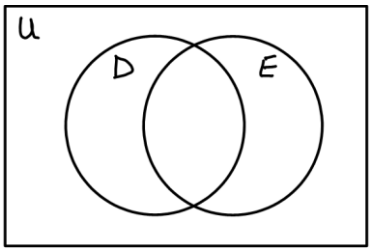
$$A = \{1, 2, 3, 5, 6, 8\},\ B = \{2, 3, 4, 6, 9\} \text{ and } C = \{0,1\}.$$

Write down the following set by explicitly stating each individual element:

$$(A \cap B) \times \mathrm{C}.$$

[**4 marks**]

(b) Use a Venn Diagram of the following form,



to display the region represented by the following formula in set-theory, where D and E represent sets:

$$\overline{D} \cup \overline{E}$$

[**3 marks**]

(c) Consider the set $F = \{x \in \mathbf{Z} \mid -2 \leq x < 4\}$.

(i) Write down the set F by explicitly stating each individual element.

[**2 marks**]

(ii) Explain <u>why</u> each of the following three statements are true:

- $\{0,2\} \subseteq F$
- $\{0\} \notin F$
- $F \not\subseteq \mathbf{N}$

[**3 marks**]

(iii) State $|F|$. [**1 mark**]

Question A5

a) Consider the following two functions:

$f: \boldsymbol{N} \times \boldsymbol{N} \to \boldsymbol{Z}$, given by $f(m,n) = m - n$

$g: \boldsymbol{Z} \to \boldsymbol{N}$, given by $g(p) = 2 + p^4$

(i) Calculate $(g \circ f)(4,6)$. **[4 marks]**

(ii) Explain why g is not a surjection. **[3 marks]**

(b) It is given that the following function b is a bijection:

$b: \boldsymbol{R} \to \boldsymbol{R}$, given by $b(x) = \frac{1}{2}(3x + 4)$.

State its inverse function $b^{-1}: \boldsymbol{R} \to \boldsymbol{R}$. **[3 marks]**

(c) The following attempted definition at a function is incorrect. Explain, by means of an example, what is wrong with the following definition:

$h: \boldsymbol{Z} \to \boldsymbol{Z}$ given by $h(x) = \frac{1}{3}(5 - x)$ **[3 marks]**

Question A6

(a) Let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$ and $C = \{1, 2, 3, 4, 5\}$. Consider the following two relations R and S given as follows:

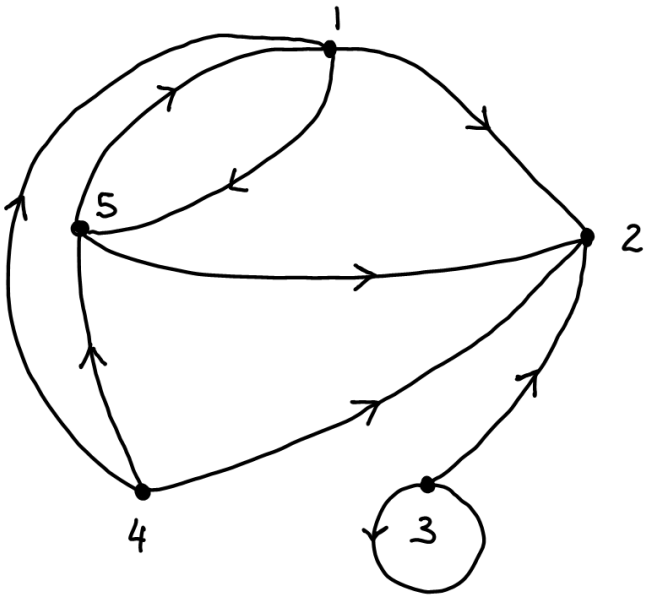
$R: A \times B$ given by $R = \{ (1, a), (1, b), (2, a), (3, a), (3, c) \}$

$S: B \times C$ given by $S = \{ (a, 3), (a, 5), (b, 1), (b, 2), (b, 4), (c, 1), (c, 5) \}$

(i) Give an explanation as to why the composition $S \circ R$ exists but the composition $R \circ S$ does not exist. **[3 marks]**

(ii) Write down the composition $S \circ R$, stating the result as a set containing ordered pairs. **[4 marks]**

(b) Consider the relation W on the set $D = \{ 1, 2, 3, 4, 5 \}$, given by the following representation in the form of a directed graph:



The following statements are all correct. Give an explanation as to the reasons for them to be correct:

(i) W is not reflexive.

(ii) W is not irreflexive.

(iii) W is not symmetric.

(iv) W is not transitive.

[6 marks]

Question A7

(a) Consider two graphs, G_1 and G_1 which are defined as follows:

G_1 has vertex set $V_1 = \{1, 2, 3, 4, 5, 6\}$ and has the following adjacency matrix:

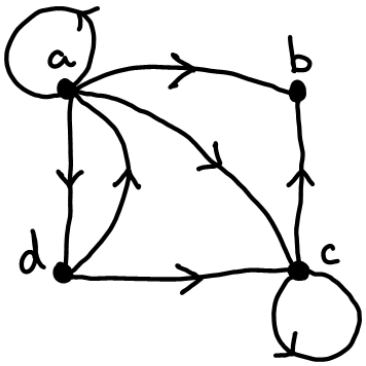
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

G_2 has vertex set $V_2 = \{a, b, c, d, e, f\}$ and edge set $E_2 = \{\{a, c\}, \{a, f\}, \{b, c\}, \{c, d\}, \{c, e\}, \{c, f\}, \{d, e\}\}$.

(i) Sketch both graphs G_1 and G_2. **[4 marks]**

(ii) Discuss whether or not G_1 and G_2 are isomorphic. **[2 marks]**

(b) Consider the directed graph G_3 as shown:



The directed graph G_3.

Let M_{G_3} be the adjacency matrix of graph G_3, using the alphabetic ordering of the vertices.

(i) Write down the adjacency matrix M_{G_3}. **[2 marks]**

(ii) By inspection of the graph G_3, or otherwise, write down the product $M_{G_3} \times M_{G_3}$. **[3 marks]**

(iii) Using your answer to part (ii), or otherwise, write down the Boolean product $M_{G_3} \odot M_{G_3}$.

[2 mark]

Question A1 Answer Sheet

Enter your answer to Question A1 on this page:

Complete this truth table:

P	Q	R	$\big(P \to (\neg R)\big) \vee (\neg Q)$	$(P \wedge Q) \leftrightarrow (R \oplus Q)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Hence, state all the models for the formula:

$$\Big(\big(P \to (\neg R)\big) \vee (\neg Q)\Big) \wedge \big((P \wedge Q) \leftrightarrow (R \oplus Q)\big)$$

Question A3 Answer Sheet

Enter your answer to Question A3 on this page:

1.	$P \rightarrow \big(\neg(Q \vee S)\big)$	Premise
2.	$W \vee (\neg R)$	Premise
3.	$(\neg P) \leftrightarrow R$	Premise
4.	$\big((\neg P) \rightarrow R\big) \wedge \big(R \rightarrow (\neg P)\big)$	……
5.	$(\neg P) \rightarrow R$	……
6.	Q	Assumption for ……
7.	$Q \vee S$	6, ……
8.	$\neg P$	……
9.	……	5,8, ……
10.	W	……
11.	……	9,10, Conjunction
12.	……	……

SECTION B (48 Marks)

Answer all 24 multiple-choice questions from this section.

Each question in this section is worth 2 marks.

Question B8)

Which conversion of a binary number 110001, to decimal and then to a hex number is correct?

a) 110001 -- to decimal -- 49 -- to hex -- 61
b) 110001 -- to decimal -- 98 -- to hex -- 81
c) 110001 -- to decimal -- 49 -- to hex -- 31
d) 110001 -- to decimal -- 98 -- to hex -- f1

Question B9)

A computer program which tells you the time returns different results at different times. Does this mean that it is non-deterministic?

a) Yes, it is non-deterministic.
b) Yes or no, it is non-deterministic because the results is different; but it is deterministic because it returns the correct time.
c) No, it is deterministic because the program performs the exact same operation.
d) No, it is deterministic because for any given time the result is same.

Question B10)

Within computer science, what do we mean by a language?

a) A language is a sequence of symbols.
b) A language is a set of symbols.
c) A language is a sequence of strings.
d) A language is a set of strings.

Question B11)

If a language A is {a} and another language B is {Λ,1}, what is $A.B^3$?

a) $A.B^3$ = {Λ, a, a1, a2, a3}
b) $A.B^3$ = {a, 1, 11, 111, 2, 3}
c) $A.B^3$ = {a, a1, a11, a111}
d) $A.B^3$ = {a, Λ, 1, 11, 111}

Question B12)

If A = {a,b} and B = {b,c}, what is A – B and what is B – A?

("–" means difference, ref Formula Booklet) A – B = {a,b} and B – A = {b,c}

b) A – B = {b} and = B – A = {b}

c) A – B = {c} and = B – A = {a}

d) A – B = {a} and = B – A = {c}

Question B13)

If A = {Λ, a} and B = {a ,b}, what is $(B^2) \cap (A.B)$? ("∩" means intersection)

a) {ab, ba}

b) {aa, ab}

c) {Λ, a, aa, ab}

d) {Λ, a, b, aa, ab, bb, ba}

Question B14)

If A and B are languages with m and n strings respectively does it follow that A.B has m×n strings? ("m×n" means "m times n")

a) No, it is not true when A and B both have Λ, and share some common strings.

b) No, it is not true when A and B both have Λ.

c) No, it is not true when A = B.

d) Yes.

Question B15)

How many of the following languages can be defined by Finite Automata?

{ab, ababab, abababababab, abababababababab,....}

{abb, aabbbb, aaabbbbbb, aaaabbbbbbbb, ...}

{a, aa, aba, abba, abbba, abbbba, abbbbba, ...}

{a, bab, bbabb, bbbabbb, bbbbabbbb, ...}

a) 1

b) 2

c) 3

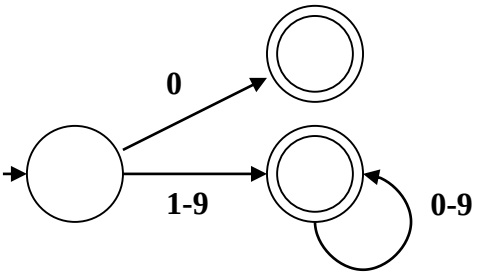
d) 4

Question B16)

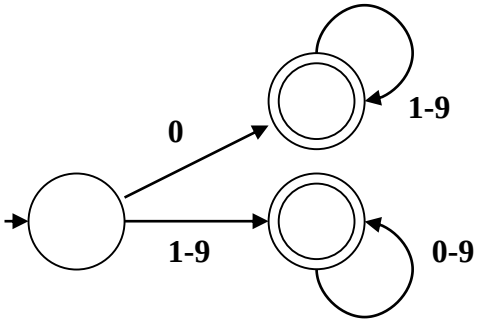
Which of the following Finite Automata **recognise** non-negative integers without leading zeros? (e.g. 0010 has two leading zeros, and 10 has none). Note that, when we say '**recognise**', we mean 'accept and only accept'.

You can only choose one answer.

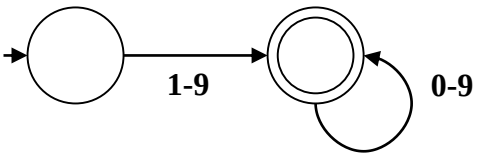
a)



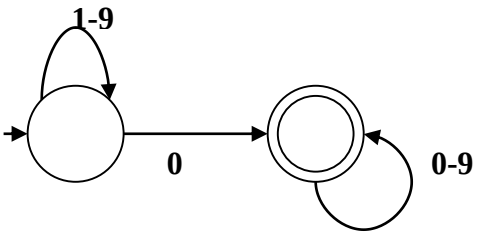
b)



c)



d)



Question B17)

One of the following deterministic Finite Automata accepts the same language as this non-deterministic Finite Automata shown in Figure 1. Which one is it?

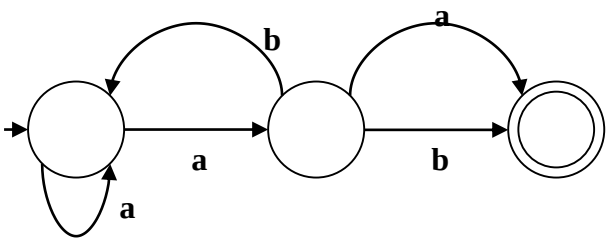
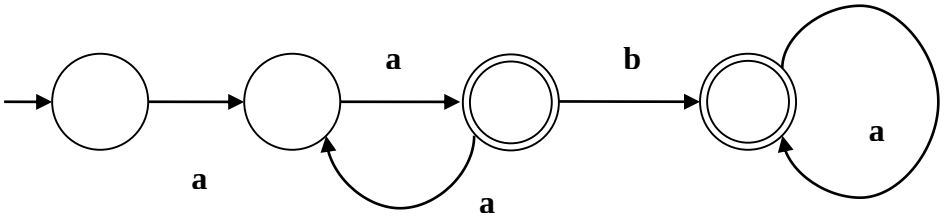
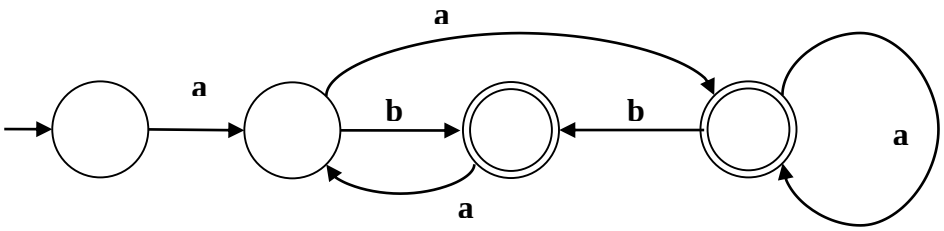


Figure 1: A non-deterministic Finite Automata.

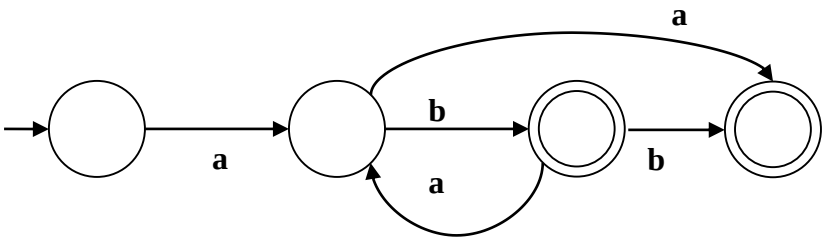
a)



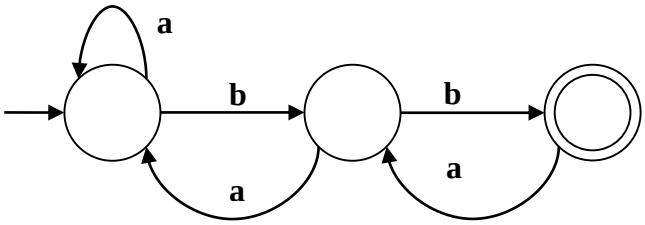
b)



c)

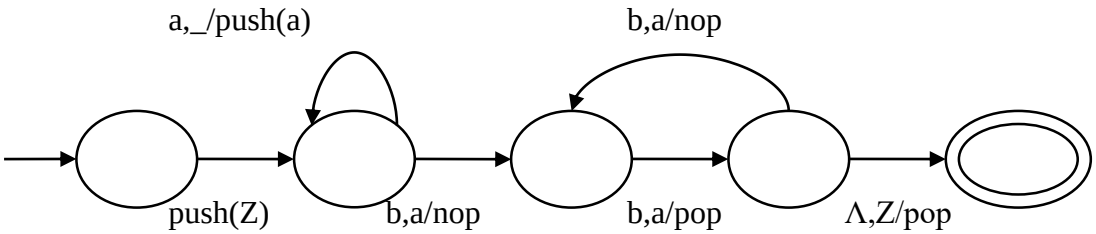


d)



Question B18)

Which of the following languages is accepted by this Pushdown Automaton?



a) $\{ a^n b^n \mid n > 0 \}$

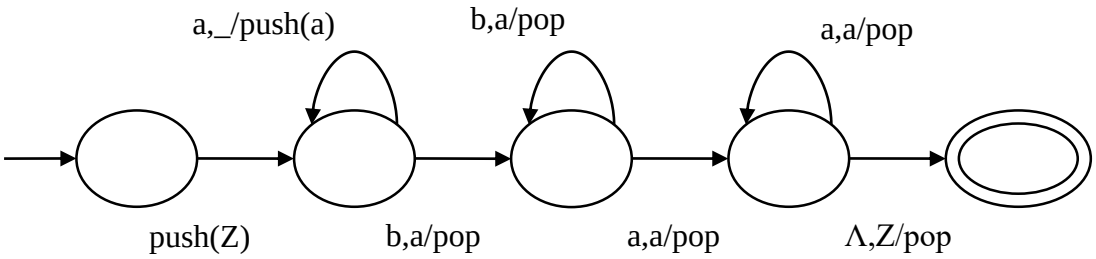
b) $\{ a^n b^m \mid n > 0, m > 0 \}$

c) $\{ a^n b^{2n} \mid n > 0 \}$

d) $\{ (a^n b^n)^m \mid n > 0, m > 0 \}$

Question B19)

Which of the following languages is accepted by this Pushdown Automaton?



a) $\{ a^n b^p a^q \mid n > 1, q > 0, p > 0, n = p+q \}$

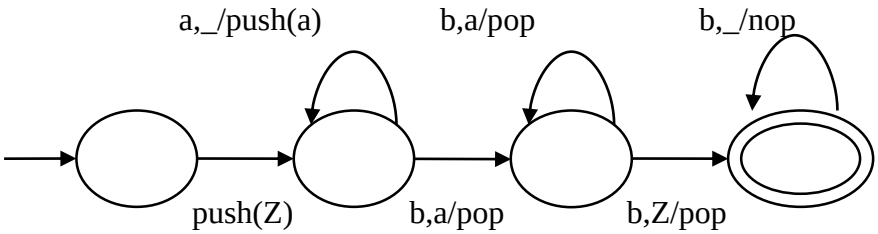
b) $\{ a^n b^p a^q \mid n > 1, q > 0, p > 0, p = n+q \}$

c) $\{ a^n b^p a^n \mid n > 1, \ p > 0 \}$

d) $\{ a^n b^n a^q \mid n > 1, q > 0 \}$

Question B20)

Which of the following languages is accepted by this Pushdown Automaton?



a) $\{ a^n b^m \mid n > 0, m > n \}$

b) $\{ a^n b^m \mid n > 0, m > n+1 \}$

c) $\{ a^n b^m \mid n > 0, m > 0 \}$

d) $\{ a^n b^m \mid n > 0, m > 1 \}$

Question B21)

Which one of statements below is true?

a) LIFO is another name for stack

b) Stack in Pushdown Automata has a limited size

c) Stack in Pushdown Automata can be replaced by queue.

d) Stack in Pushdown Automata is always initialised by “$” or “Z”

Question B22)

The following is a recursive program. Assume that it only takes the positive integers as the input:

```
int compute(float x, int n) {
  if(n==0) return 1;
  else
    if(n is even) return (compute(x, n/2) * compute(x, n/2));
    else return (x * compute(x, n-1);
}
```

Can you tell what the above program really does?

a) It always returns 1.

b) It returns the value x.

c) It returns the value x^n.

d) It returns the value x^2.

Question B23)

Which one of the following statements gives us a correct inductive definition of a palindrome?

a) A palindrome is a string that reads the same left to right as right to left.

b) A palindrome is a string whose left half is a mirror image of its right half.

c) A single character is a palindrome and if a string ***s*** is a palindrome, then ***xsx*** is also a palindrome.

d) Both single character and an empty string are a palindrome. If a string ***s*** is a palindrome, then ***xsx*** is also a palindrome.

Question B24)

Which of the following statements is the inductive rule for an ascending list?

a) An empty list is an ascending list.

b) A list with single element is also an ascending list.

c) If you are given an ascending list, then adding to the start of the list an element which is smaller than or equal to the first element in the list makes a new ascending list.

d) An ascending list means that reading its elements from left to right the values get bigger.

Question B25)

Which of the following pseudo code returns the `length` of a given list?

a)
```
length(L) {
      if(L has a single number x) return x;
      else return (head(L) + length(tail(L)));
}
```

b)
```
length(L) {
      if(L has a single number x) return 1;
      else return (1 + length(tail(L)));
}
```

c)
```
length(L) {
      if(L is an empty list) return 0;
      else return (length(tail(L)) + 1);
}
```

d)
```
length(L) {
      if(L is an empty list) return 0;
      else return (head(L) + length(tail(L)));
}
```

Question B26)

Read the following Java program. What output does it produce when n is “-9”?

```
void print(int n) {
        if(n > 0) {
                System.out.print(n + ", ");
                print(n-1);
        }
        else if (n < 0) {
                print(n+1);
                System.out.print(n + ", ");
        }
        else if(n==0) System.out.printnl(n + ", ");
}
```

a) 0, -1, -2, -3, -4, -5, -6, -7, -8, -9,

b) -9, -8, -7, -6, -5, -4, -3, -2, -1, 0,

c) 9, -8, 7, -6, 5, -4, 3, -2, 1, 0,

d) 0, 1, -2, 3, -4, 5, -6, 7, -8, 9,

Question B27)

The following four statements describe the term recursion. Which one of them is INCORRECT?

a) Recursion is induction applied to programming.

b) When a program calls itself, we say it is a recursive program.

c) Recursion in programming needs to use an invisible stack

d) Recursion is a programming style which is similar to iteration.

Question B28)

Why was an extra tape added to the original Turing Machine?

a) to make the machine work faster

b) to store the transition table

c) to increase memory space

d) to allow the head write and read separately

Question B29)

Suppose n > 2 is an odd number. To decide if n is prime you can try dividing n by all odd numbers up to $\sqrt{n}$. If any of them divide n exactly, it is not a prime. What is the average cost of deciding in whether n is prime? Use big-O notation to express it.

a) $O(n)$

b) $O(\sqrt{n})$

c) $O\left(\frac{1}{2}\sqrt{n}\right)$

d) $O\left(\frac{1}{4}\sqrt{n}\right)$

Question B30)

For the bubble sort algorithm, what is the time complexity of the best/worst case? (assume that the computation stops as soon as there are no more swaps in one pass)

a) best case: $O(n)$ worst case: $O(n^2)$

b) best case: $O(n)$ worst case: $O(n \log(n))$

c) best case: $O(n \log(n))$ worst case: $O(n \log(n))$

d) best case: $O(n \log(n))$ worst case: $O(n^2)$

Question B31)

Work out what language the following grammar generates.

S → Λ

S → aSb

S → aaS

a) {Λ, ab, aa}

b) {Λ, ab, aa, aabb, aaab, aaaa, aaabb, aaaab, aaaaa, aabbbb, aaaabb, aaaaaa, …}

c) {Λ, ab, aa, aabb, aaab, aaaa, aaabbb, aaaabb, aaaaab, aaaaaa, ...}

d) {Λ, ab, aa, abab, aaaa, ababab, aaaaaa, ….}

END OF QUESTION PAPER