Computational Methods and Modelling

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Solution tutorial 4
Error in root finding methods

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Exercise 1: Analyse the convergence of root finding methods

error_root_finding.py

```
#Note that this is not a complete code!!!
import math
import numpy as np
import matplotlib.pyplot as plt

# Newton
def newton2(f,Df,x0,N):
    ...

# define array to collect the solution results
n_array_N   = np.zeros(n_max-1)
sol_array_N = np.zeros(n_max-1)
fun_array_N = np.zeros(n_max-1)

f = lambda x: x**2 + 4*x - 12
df= lambda x: 2*x + 4

# Initial guess for Newton
x0=1

for i in range(1,n_max):
    solution = newton2(f,df,x0,i)
    n_array_N[i-1] = i
    sol_array_N[i-1] = solution
    fun_array_N[i-1] = np.absolute(f(solution))

plt.figure()
plt.plot(n_array_N,sol_array_N, '-o',n_array_S,sol_array_S, '-o')
plt.xlabel("Number of iterations")
plt.ylabel("Solution")
plt.xlim(0,n_max)
```

Solution strategy:

- The root finding method (Newton here) is applied many times for different numbers of iteration.
- The results are collected in arrays that are predefined
- Results collected are plotted with the matplotlib library

- **This is not a complete code!!!**
- Complete python files available on Learn

Exercise 2: Pencil and paper solution

SYNTHETIC DIVISION

- Consider the example:

$$\frac{x^3-12x^2-42}{x-3}$$

- numerator $p(x) = x^3 - 12x^2 + 0x - 42$
- denominator $g(x) = x - 3$. The zero of $g(x)$ is 3

① Arrange the coefficients of $p(x)$ and the zero in the following way:

$$\begin{array}{r|rrrr} & 1 & -12 & 0 & -42 \\ 3 & & & & \\ \hline \end{array}$$

② The first coefficient is dropped:

$$\begin{array}{r|rrrr} & 1 & -12 & 0 & -42 \\ 3 & & & & \\ \hline & 1 & & & \end{array}$$

③ The dropped number is multiplied by the number before the bar and placed in the next column

$$\begin{array}{r|rrrr} & 1 & -12 & 0 & -42 \\ 3 & & 3 & & \\ \hline & 1 & & & \end{array}$$

④ Perform addition and repeat:

$$\begin{array}{r|rrrr} & 1 & -12 & 0 & -42 \\ 3 & & 3 & -27 & -81 \\ \hline & 1 & -9 & -27 & -123 \end{array}$$

⑤ $q(x) = \frac{p(x)}{g(x)} = x^2 - 9x - 27$

with reminder

$r(x) = -123$

Exercise 2: Python code for the solution

”

```
import numpy as np

def poly_iter(A, t):
    # compute q(x) = p(x)/(x-t) and residual r
    # array A contains coefficients of p(x)
    n = len(A)-1
    # q: array of integers to store coefficients of q(x)
    q=np.zeros(n,dtype=np.int8)
    r = A[n]
    for a in reversed(range(n)):
        s=A[a]
        q[a]=r
        r = s + r * t
    print('----------------------------------------')
    print('Coefficients a0, a1, a2, ..., an')
    print('of quotient a0+a1*x+a2*x^2+...an*x^n:')
    print(q)
    print('----------------------------------------')
    print('Residual:')
    print(r)
    print('----------------------------------------')
    return []

#A = np.array([ -24, 2, 1])
#t = 4

A = np.array([ -42, 0, -12 ,1])
t=3

poly_iter(A,t)
```