Computational Methods and Modelling

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Tutorial 8
Solution of a system of differential equations

Exercise 1: Numerical simulation of chaos, the Lorenz system

- Edward Lorenz developed the following system for atmospheric fluid dynamics

$$\frac{dy_1}{dt} = \sigma(y_2 - y_1)$$
$$\frac{dy_2}{dt} = \rho y_1 - y_2 - y_1 y_3$$
$$\frac{dy_3}{dt} = -\beta y_3 + y_1 y_2$$

  which can show chaotic dynamics, depending on the value of σ, β, and ρ. The system has wide applicability, being relevant also for convection in toroidal pipes (nuclear fusions) and in single-mode lasers.

Exercise 1: Numerical simulation of chaos, the Lorenz system

- **Write a Python code to solve numerically the Lorenz equations** in the interval $0 \le t \le t_f$ $(t_f = 30)$, assuming $\sigma = 10$; $\beta = 8/3$; $\rho = 28$ and with initial conditions:

$$y_1^0 = y_2^0 = y_3^0 = 5 \quad \text{at} \quad t = 0$$

- To this end, use the python function solve_ivp included in the module scipy[1].
- Plot the solution in the interval $0 \le t \le t_f$ for the 3 variables.
- Plot the solution in the planes $y_1 y_2$ and $y_1 y_3$: one plot with y_1 in the abscissa and y_2 in the ordinate and one plot with y_1 in the abscissa and y_3 in the ordinate.
- Repeat the tasks above for $\sigma = 10$; $\beta = 8/3$; $\rho = 10$. In this case the solution is not chaotic and the plots will look different.
- Tip: in order to obtain nice plots, specify explicitly the times at which to store the computed solution. This can be done by using the optional argument `t_eval` of the python function solve_ivp. A good number of elements for the array `t_eval` is 5000. The array `t_eval` can be defined easily with the python code line:
  ```
  t_eval = np.linspace(0, t_final, num=5000)
  ```
  where `t_final` is the final time $t_f = 30$ mentioned above.
  The function `np.linspace` is used to generate an array of 5000 elements equispaced in the range `0, t_final`.

[1] See also the reference manual online
`docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.solve_ivp.html`