Computational Methods and Modelling 3

Lecture 7 Supplement: Dr. Edward McCarthy

Topic: Linear Algebra, Simultaneous Equations

THE UNIVERSITY *of* EDINBURGH
School of Engineering

Gauss Elimination Applied to a Physics Problem

Exercise 1: Calculate Cord Tensions in a Tandem Team of Parachutists

- Three parachutists are connected by a weightless cord while free-falling at a velocity of 5 m/s.
- Calculate the tension in each cord, and the acceleration of the team given the following data

Parachutist	Mass, kg	Drag Coefficient, kg/s
1	70	10
2	60	14
3	40	17

- **Hint:** use free body diagrams to express the problem.

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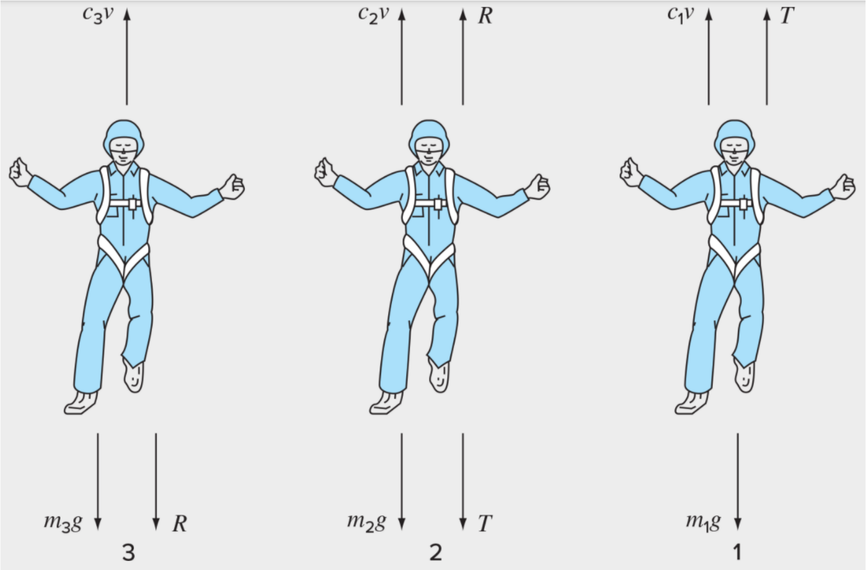
- We firstly write Newton's Second Law expressions for each of the three parachutists:



- Here, the accelerations of the three parachutists are each expressed as a balance of gravitational force (m_ig), and the tension force on each parachutist is R or/and T.

- The drag force exerted by air against direction of fall is cv, where c_i are the drag coefficients, v_i are the velocities.

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These three equations are linear and together comprise a linear equation system that can be solved using Gauss:

$$
\begin{aligned}
m_1 g - T - c_1 v \qquad &= m_1 a \\
m_2 g + T - c_2 v - R &= m_2 a \\
m_3 g \qquad - c_3 v + R &= m_3 a
\end{aligned}
$$

Filling in all the values we were provided with we get the following.

$$
\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ T \\ R \end{Bmatrix} = \begin{Bmatrix} 636.7 \\ 518.6 \\ 307.4 \end{Bmatrix}
$$

This is equivalent to a system **A.x = b**. We solve for x.

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```
import numpy as np

def linearsolver(A,b):
    n = len(A)

    #Initialise solution vector as an empty array
    x = np.zeros(n)

    #Join A and use concatenate to form an
augmented coefficient matrix
    M = np.concatenate((A,b.T), axis=1)

    for k in range(n):
        for i in range(k,n):
            if abs(M[i][k]) > abs(M[k][k]):
                M[[k,i]] = M[[i,k]]
            else:
                pass
                for j in range(k+1,n):
                    q = M[j][k] / M[k][k]
                    for m in range(n+1):
                        M[j][m] +=  -q * M[k][m]
    #Python starts indexing with 0, so the last
element is n-1
    x[n-1] =M[n-1][n]/M[n-1][n-1]
```

```
#We need to start at n-2, because of
Python indexing
    for i in range (n-2,-1,-1):
        z = M[i][n]
        for j in range(i+1,n):
            z = z  - M[i][j]*x[j]
        x[i] = z/M[i][i]

    return x

#Initialise the matrices to be solved.

A=np.array([[71., 1., 0],[60., -1., 1.],
[40, 0, -1]])
b=np.array([[636.7., 518.6, 307.4]])

print(linearsolver(A,b))
```

The answer received (as an array of three elements for x, is:

[8.55380117 29.38011696 34.75204678]

The first number is the acceleration of the team, while the second two numbers are the tensions.

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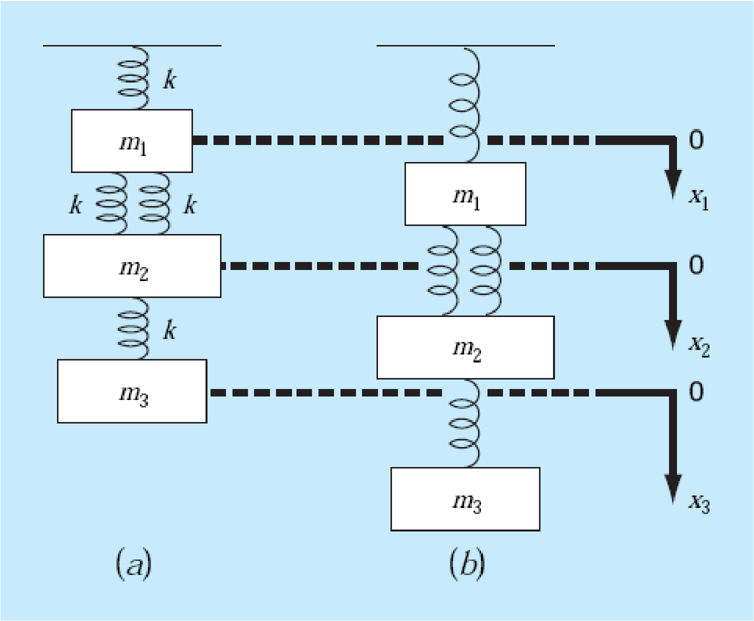
Exercise 2: Calculate the deflections of the three masses in a spring system



1. In this system, the masses for the three blocks are: m_1 = 2.0 kg, m_2 = 3.0 kg, m_3 = 2.5 kg, and all spring constant, k, values for the system are 10 kg/s^2.
2. **Draw a free body diagram** to express the forces in the system, and then **solve the system for the three deflections, x_i,** using coded Gauss Elimination.