Computational Methods and Modelling

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Tutorial 7
Differential Equations

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Exercise 1: Euler and Runge-Kutta methods for the Logistic equation

- The Logistic Differential Equation is defined as:

$$\frac{dy}{dx} = y(1-y) \tag{1}$$

  It is relevant in a number of scientific and engineering contexts, including chemistry, psychology, machine learning. It is also one of the simplest models for disease spread as in the case of COVID-19.

- Write a Python code to solve the Logistic equation with the Euler method, with initial condition:

$$y_0 = \frac{e^{-4}}{e^{-4}+1} \approx 0.0179862 \quad \text{at} \quad x = 0 \tag{2}$$

  In particular, solve the equation in the interval $0 \leq x \leq 10$ with a step $h = 0.01$.

- For this initial condition, the Logistic equation has the analytical solution:

$$y_{exact} = \frac{e^{x-4}}{e^{x-4}+1} \tag{3}$$

  Verify your code by plotting the numerical and analytical solutions on the same graph.

- Solve the same equation with a Runge-Kutta method with order four (RK4).

Exercise 2: Order of accuracy of Euler and Runge-Kutta methods

- By performing computations with different values of h and computing an appropriate error, can you show that the Euler method is first order and the RK4 is order four? Hint: Plot the error vs h.
- A good definition of the error could be the Root Mean Square Error, defined as:

$$RMQE = \sqrt{\frac{1}{n_{steps}} \sum_{i=1}^{n_{steps}} (y_i - y_{exact}(x_i))^2} \tag{4}$$

where y_i is the numerical solution obtained at the location x_i and $y_{exact}(x_i)$ the analytical solution evaluated in x_i. n_{step} here is the number of steps performed for the Euler and RK methods to solve the equation from $x = 0$ to $x = 10$. Note that the number of steps will change for different h.