

Matrices and Their Application in Automobiles

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CS 131 (Discrete Math)
Fall 2023





What are Matrices?

At the most basic level, a matrix is a rectangular arrangement of elements in rows and columns. These elements are often numbers but can also be symbols, expressions, or any significant mathematical symbol.

A matrix is written in the form $M \times N$.

M = the number of rows

N = the number of columns

Matrices are often related to linear systems, where we will find most of their applications.

Matrices in Math



$$\begin{array}{c} \text{Columns} \\ \begin{array}{cccc} 1 & 2 & \dots & n \end{array} \\ \begin{array}{c} \text{Rows} \\ \left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} \right\} \end{array} \end{array} \left[\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array} \right] = A_{m \times n}$$

Essential Concepts that Involve Matrices

Matrix Operations: Operations such as addition, subtraction, and multiplication can be done on a matrix. These operations work precisely how they would with regular integers, so the elements in the matrices being operated on will be altered as such.

Matrix Equations: Matrices can represent variables in an equation. This is called a Matrix equation. Matrix equations are foundational in solving linear equations with numerous real-life applications.

Matrix Addition

$$\begin{bmatrix} 5 & 4 \\ 8 & 6 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ 8 & 6 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 5+9 & 4+2 \\ 8+8 & 6+6 \\ 1+3 & 3+0 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 16 & 12 \\ 4 & 3 \end{bmatrix}$$

Matrix Subtraction

$$\begin{bmatrix} 7 & 6 & 9 \\ 4 & 8 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 9 & 1 \\ 2 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 7-6 & 6-9 & 9-1 \\ 4-2 & 8-5 & 2-7 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 8 \\ 2 & 3 & -5 \end{bmatrix}$$

Matrix Multiplication

$$\begin{bmatrix} 7 & 6 & 1 \\ 2 & 3 & 8 \end{bmatrix} * \begin{bmatrix} 4 & 9 \\ 1 & 7 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} 7*4+6*1+1*3 & 7*9+6*7+1*8 \\ 2*4+3*1+8*3 & 2*9+3*7+8*8 \end{bmatrix} = \begin{bmatrix} 37 & 113 \\ 35 & 103 \end{bmatrix}$$

Transpose Of A Matrix

$$\begin{bmatrix} 7 & 2 & 9 \\ 1 & 6 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 1 \\ 2 & 6 \\ 9 & 3 \end{bmatrix}$$

Essential Concepts cont.

Markov Chains: Markov Chains are a probability model composed of matrices representing the chances of something transitioning from one state to another based on the current time and state. These have several real-life applications in the automotive industry.

Kalman Filtering: The Kalman Filter is a recursive mathematical algorithm that operates on matrices to predict the state of a dynamic system. Dynamic systems are abundant in real life, and the Kalman Filter is famous for making accurate predictions.

$$\begin{array}{ccc} \begin{array}{cc} \text{Cadbury} & \text{Nestle} \\ \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \end{array} & \times & \begin{array}{cc} \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} \\ \text{Transition Matrix} \end{array} \\ \text{Initial Stage} & & \text{Next Stage} \end{array} = \begin{array}{cc} \begin{bmatrix} 0.45 & 0.55 \end{bmatrix} \end{array}$$

$$\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ y_k \\ z_k \\ \dot{x}_k \\ \dot{y}_k \\ \dot{z}_k \\ \ddot{x}_k \\ \ddot{y}_k \\ \ddot{z}_k \end{pmatrix} + \mathbf{v}_k$$

Real-Life Applications

Navigation (GPS): Error Models and Least Squares Estimation use matrix operations and inverses to find the best path for you. Matrix-dependent techniques such as Kalman Filter are used to predict your location on the navigation app. Markov Chains are also used to calculate estimated time of arrivals (ETA). These methods are essential when living in metropolitan cities with ever-changing traffic.

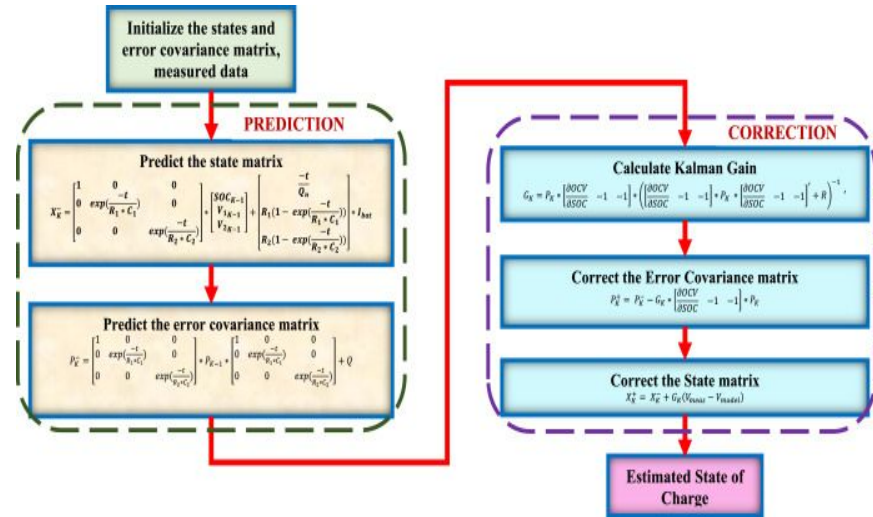
Detection Systems: Matrices are used to produce all the camera images vehicles display, such as the backup camera. Each element in a matrix is represented by a color pixel, and when you combine multiple matrices, an image is produced. Convolutional Neural Networks (CNN) directly involve matrix operations and are widely used for vehicle detection, such as blind spot detection on your car mirrors.



Real-Life Applications cont.

Electric Vehicle Design: A matrix can be used to represent an electric vehicle's battery storage system. State of Charge and State of Health can both be represented by matrices. The Kalman Filter can be used to estimate battery health and efficiency. Matrices also help optimize battery management, energy distribution, and overall vehicle efficiency.

Operations and Design: Matrices are used abundantly in representing inventory and production planning within the automotive industry. A matrix can be used to describe supply chain structures or a transportation schedule. Matrices can also be used to model production tasks, and they are used in many optimization algorithms that help improve the production process.





Conclusion

Matrices are essential in many areas of Mathematics and Computer Science, many of which are used in real-life applications. This means Matrices are vital for many real-life tools we use in our day to day. Their applications within automobiles are not outright, but they are essential.

Most cars produced today can only function with the use of matrices. They are used in production, design, event simulation, and navigation.