Matrix Addition and Multiplication

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What is a Matrix?

-A Matrix is a two dimensional array of numbers, expressions, or symbols.

-Matrix size is always denoted by “rows x columns.”

-The Matrix below would be considered a 2x4 Matrix.

-The defined sets of rows and columns can be used to identify specific elements within the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Identify Elements in Matrix

-To identify the element in the matrix we often say first the row that the element is in and then the column that the element is in.

-For example “2” would be 1st row, 2nd column.

-For example “8” would be 2nd row, 4nd column.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Matrix Addition

- One of the simplest operations performed on Matrices (plural of Matrix).

-Before adding two Matrices, check if they are of the same size (same amount of rows and columns).

-Matrix addition is undefined for adding two matrices of different sizes.

-The definition of matrix addition is…

Let $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ be $m \times n$ matrices. The *sum* of $\mathbf{A}$ and $\mathbf{B}$, denoted by $\mathbf{A} + \mathbf{B}$, is the $m \times n$ matrix that has $a_{ij} + b_{ij}$ as its (i, j)th element. In other words, $\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$.

(From *Discrete Math and Its Applications)*

Example of Addition

-Though the definition may sound wordy, Matrix addition is quite simple!

-Just add the two elements from each matrix that share the same spot.

-This is why only Matrices of equal size may be added together.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -3 \\ 3 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 4 & -1 \\ 1 & -3 & 0 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -2 \\ 3 & -1 & -3 \\ 2 & 5 & 2 \end{bmatrix}.$$

More Examples of Addition

$$\begin{bmatrix} 4 & 5 & 7 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 8 & 9 & 5 \\ 1 & 1 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 7 \\ 10 & 10 & 5 \\ 0 & -1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 2 \\ 11 & 15 & 23 \\ -12 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 9 \\ 1 & 6 & 10 \\ 11 & 7 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 11 \\ 12 & 21 & 33 \\ -1 & 11 & 15 \end{bmatrix}$$

Matrix Multiplication

- A bit more complex…

-The basic definition of matrix multiplication is…

Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix. The *product* of **A** and **B**, denoted by **AB**, is the $m \times n$ matrix with its (i, j)th entry equal to the sum of the products of the corresponding elements from the ith row of **A** and the jth column of **B**. In other words, if $\mathbf{AB} = [c_{ij}]$, then

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ik}b_{kj}.$$

(From *Discrete Math and Its Applications)*

Matrix Multiplication (cont.)

-That explanation is a bit complicated to understand so let's break it down.

-Matrices can only be multiplied if the number of columns of the first matrix matches the number of rows from the second matrix.

-One example would be that you could multiply a 2x3 matrix by a 3x5 matrix, but not a 3x2 matrix by a 3x5 matrix.

-This property makes matrix multiplication noncommutative unlike addition.

-This means that the order in which you multiply matrices matter.

-One example would be that you can not multiply a 3x5 matrix by a 2x3 matrix, although you could multiply them if your switch the order.

In general, for matrices A and B, $AB \neq BA$.

In fact, AB may be defined, while BA is undefined.

$$AB = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}}_{2 \times 2} \underbrace{\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}}_{2 \times 3} = \underbrace{\begin{bmatrix} 2 & 5 & 2 \\ 1 & 6 & 8 \end{bmatrix}}_{2 \times 3}$$

match

$$BA = \underbrace{\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}}_{2 \times 3} \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}}_{2 \times 2} \quad \text{undefined}$$

do not match

(From *MathBootCamps*)

Matrix Multiplication (cont.)

-The resultant matrix from multiplying two matrices has the number of rows from the first matrix and the number of columns from the second matrix.

-For example multiplying a 2x3 matrix by a 3x5 matrix will give a 2x5 matrix.

-Another example would be multiplying a 3x4 matrix by a 4x5 matrix will give a 3x5 matrix.

How to Multiply Matrices

Best shown in an example...

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 9 & 1 \\ 2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} (1)\cdot(7)+(2)\cdot(9)+(3)\cdot(2) & (1)\cdot(8)+(2)\cdot(1)+(3)\cdot(3) \\ (4)\cdot(7)+(5)\cdot(9)+(6)\cdot(2) & (4)\cdot(8)+(5)\cdot(1)+(6)\cdot(3) \end{bmatrix} =$$

$$\begin{bmatrix} 31 & 19 \\ 85 & 55 \end{bmatrix}$$

(From *eMathHelp*)

Application of Matrix Addition/Multiplication in the World

-Physics.

-In physics, matrices are used to represent vectors as individual components (x, y, and z).

-Vectors are things such as force, position, velocity, acceleration, etc.

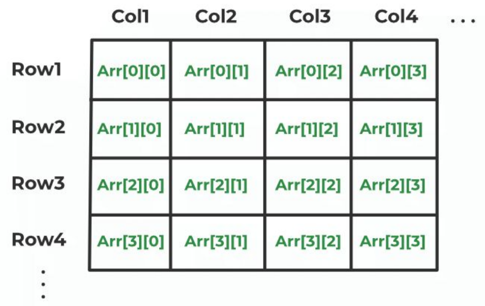
Application of Matrix Addition/Multiplication in the World

-Computer Science

-In Computer Science, matrices are often represented as a 2D array.

-2D arrays can be used to store data such as numbers or values.

-Can even be used to represent vectors from physics in order to simplify matrix addition and multiplication without doing the work by hand.



(From *Geeks4Geeks*)

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Thank You!