Linear Systems

Vectors , **Matrices**

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

3 D

v_1 v_2 v_3

(3) × (1) matrix

(3) × (3)
rows columns

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ → (3 × 1 matrix) → COLUMN VECTOR

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ → ROW VECTOR

1 × 3 matrix

[1] [2] [3]

multiplication

output → scalar → number

output → matrix → a vector n × 1 matrix

Row vector × column vector
(1×3)

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} (3\times1)$$

$$(1\times4) + (2\times5) + (3\times6)$$

$$= 4 + 10 + 18$$

$$= 32$$

[4 5 6]

[1 2 3]

4 × 1, 5 × 2, 6 × 3

32

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix}^T \quad \begin{bmatrix}1\\2\\3\end{bmatrix} \quad 3\times 1$$

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}^T \quad 1\times 3$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \quad 3\times 1$$

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

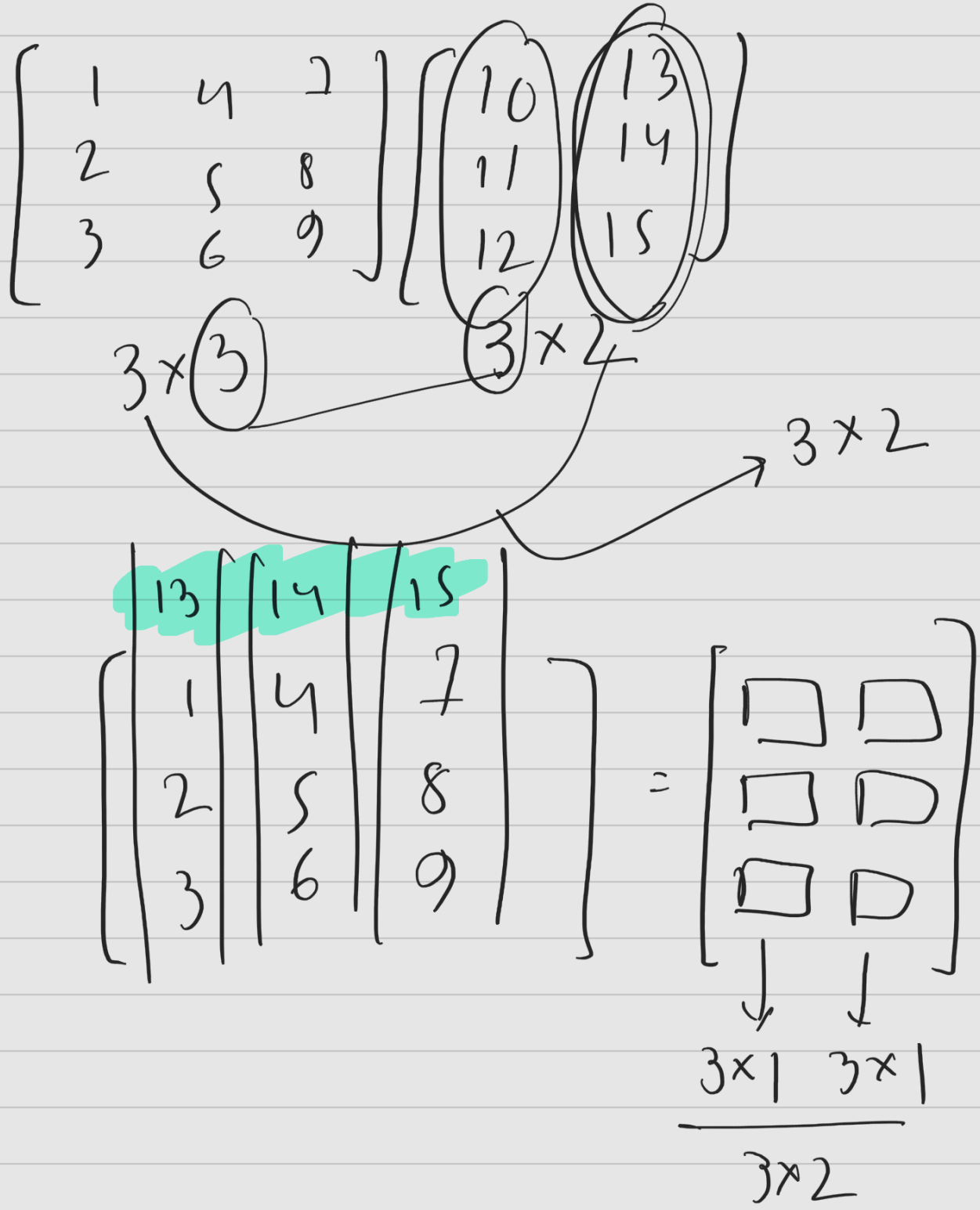
$$1+4+9$$

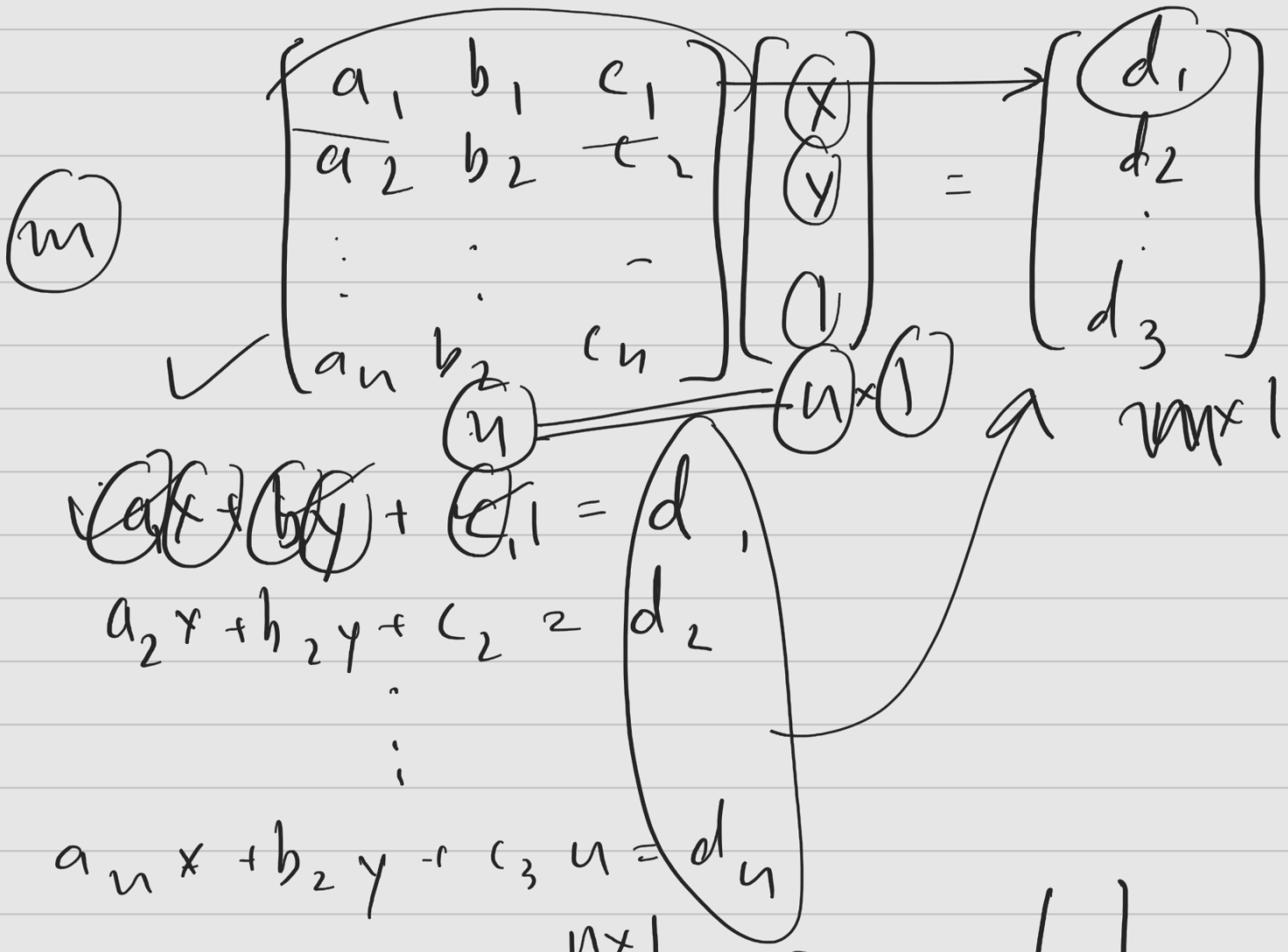
$$=14$$

$$3\times 3 \quad \begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\\3 & 6 & 9\end{bmatrix}\begin{bmatrix}10\\11\\12\end{bmatrix} \quad 3\times 1$$

$$\begin{bmatrix}10 & 11 & 12\end{bmatrix}\begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\\3 & 6 & 9\end{bmatrix} = \begin{bmatrix}10\times 1+11\times 4+12\times 7\\10\times 2+11\times 5+8\times 12\\3\times 10+6\times 11+9\times 12\end{bmatrix}$$

3×1


1 4 7
2 5 8
3 6 9
10 13
11 14
12 15
3×3
3×2
3×2
13 14 15
1 4 7
2 5 8
3 6 9
3×1 3×1
3×2


m
n
n×1
m×1
$a_1x + b_1y + c_1 = d_1$
$a_2x + b_2y + c_2 = d_2$
$a_nx + b_2y + c_3 = d_n$
n×1
A X = B
Dataoutput
input data
what companies collect
variables (columns)

Solving problems numerically (using data)

We always start with a SIMPLE model

$x + y + z \ldots = \bigcirc$

Linear combinations of variables / factors

$Ax = B$

$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$

parameters

↳ answer

$0.2 \quad 0.5 \quad 0.3$

$\bigcirc + \bigcirc + \bigcirc = \bigcirc$

Influences