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# Quantum computer hardware based on rare-earth-ion-doped inorganic crystals

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## Abstract

We present a scheme for generating multiple, strongly interacting qubits in rare-earth-ion-doped inorganic crystals at cryogenic temperatures. Two ground state hyperfine levels, with hour long lifetimes and ms decoherence times are chosen as qubit states. Controlled logic between the qubits is accomplished using the change in permanent dipole moment induced by an optical transition between the ground and excited state of these ions. The scheme is based on existing material data and established measurement techniques and should therefore be straightforward to realise in practice. The procedure used for creating the qubits can be generalised also to other solid state systems. © 2002 Published by Elsevier Science B.V.

There has been a very rapid development in the field of quantum information and quantum computing over the last decade. Path breaking discoveries by Shor and others in developing quantum algorithms among other things have spurred the quest for suitable quantum hardware for their implementation. The stringent requirements, imposed on the hardware in terms of entanglement and decoherence, have spawned several ingenious schemes that range from linear ionic traps [1] to nuclear spins in molecules [2]. Systems of stationary atoms or molecules, where the shift due to their interaction exceed the rele-

vant optical linewidths, have been investigated. All solid state quantum computing involving systems such as quantum dots in semiconductors [3], nuclear spins in doped silicon wafers [4] and impurities in spectral hole burning materials [5] have been projected as very promising, since scalability to a large number of quantum bits (qubits) can be envisaged.

In this paper we present a scheme for controlled logic and quantum computations in liquid helium cooled rare-earth-ion-doped inorganic crystals [6]. This is achieved because we devise a procedure by which subgroups of ions with specific interactions tailored after our needs, can be picked out from a macroscopic number of randomly interacting ions. In this paper the discussion is limited to rare-earth crystals, but the procedure can be generalised to other systems where there is a need to select a

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subgroup of interacting entities (ions, atoms, molecules, etc.) within a larger group of randomly distributed interacting and non-interacting entities.

The two-level systems that we use as the qubits correspond to two hyperfine levels in the electronic ground state of the rare-earth ions. The different qubits are distinguishable since they have different absorption frequencies within an inhomogeneously broadened optical transition. Interactions between qubits are accomplished using the change in the permanent dipole moments of the ions induced by an optical transition. In our opinion this system has several features making it very favourable for quantum computations: It gives the possibility for controlled interactions between any subset of qubits, allowing implementation of  $n$ -bit quantum gates. Decoherence times for the hyperfine levels have not been measured thoroughly, but are a few ms or longer, which should be sufficient to allow a very large number of consecutive quantum operations without loss of coherence. Scaling to several qubits is readily within reach using existing technology.

We will use the  ${}^7\text{F}_0$ – ${}^5\text{D}_0$  transition in Eu-doped  $\text{YAlO}_3$  as a model system, but many other combinations of rare-earth ion dopants and inorganic crystal hosts could be considered. The rare-earths have a number of unique features making them highly interesting as systems for quantum computing. The optical transitions considered here involve electrons situated in an inner shell, the f-shell, which is shielded from perturbations induced by the environment. The consequence of this shielding is that the rare-earth ions have very narrow homogeneous transition linewidths, even when doped into crystals. On the other hand, the exact frequencies of these transitions depend strongly on the surrounding electric fields. Because of imperfections in the crystals, the local field is different for different ions, thus making the optical transitions inhomogeneously broadened. The remarkable combination of narrow homogeneous linewidths and large inhomogeneous broadening provides possibilities to address narrow individual frequency channels within a large frequency interval. For Eu-doped  $\text{YAlO}_3$  the homogeneous linewidth is about 1 kHz [7] and the inhomoge-

neously broadened profile is more than 10 GHz [8], which yields about  $10^7$  individual frequency channels.

The ground state of  ${}^{151}\text{Eu}$  has three hyperfine levels, each doubly degenerate. (The excited state also has three hyperfine levels, but for simplicity, this splitting will be disregarded here.) The splitting between the ground state hyperfine levels are 46 and 23 MHz, respectively, [9] and the lifetimes of these levels are several hours [8]. We propose to use two of these levels as qubit states. At cryogenic temperatures the population is essentially uniformly spread between the three levels. In order to get a pure quantum state, optical pumping to one of the levels is therefore necessary. This can be achieved by continuously pumping from e.g. the two higher lying levels to an excited state using an optical transition. If this is done for several excited state lifetimes, the whole population will end up in the third hyperfine level (Fig 1(a)). As the excited  ${}^5\text{D}_0$  state lifetime is 2 ms [8] and the  ${}^7\text{F}_0$  ground state hyperfine level lifetime several hours it should be straightforward to perform the pumping very efficiently. Once this optical pumping is done, the ions will stay in the same state for a very long time due to the extremely long lifetimes of the hyperfine levels. One-qubit operations, such as population transfer between the qubit states, can be performed using resonant Raman transitions between hyperfine levels via the excited state. Such population transfer has recently been demonstrated experimentally in this type of systems, e.g. [10,11].

Depending on the doping concentration and the position in frequency under the inhomogeneously broadened profile, each frequency channel will be occupied by different number of ions (at low doping possibly less than one). In the case of many ions in each frequency channel it is actually the complete ensemble of ions in the frequency channel and not the individual ions that act as a qubit, whereas in the low doping case, where each frequency channel has at most one ion, there is a one to one correspondence between ions and qubits. We would like to stress that it is not the energy splittings of the two level systems constituting the qubits that are different for different qubits, but instead the energy of the optical transition to the excited state.

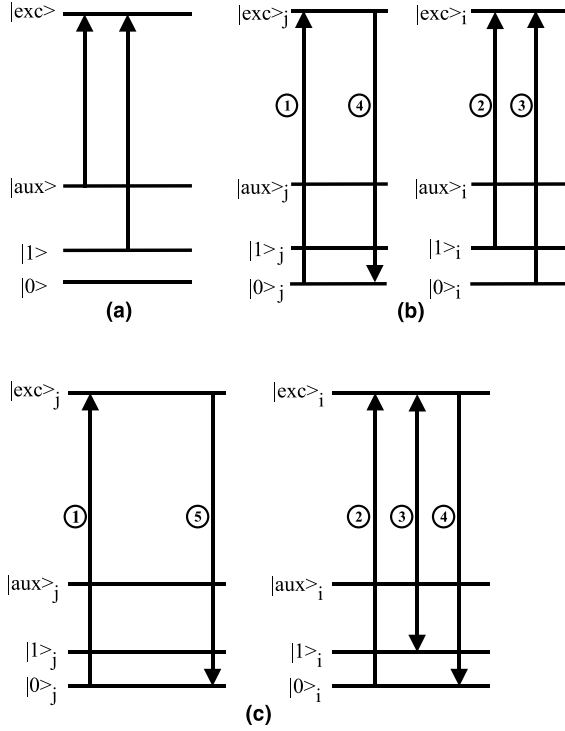


Fig. 1. Pulse sequences showing the most critical steps of the scheme. The arrows represent optical transitions and the numbering indicates the time ordering of the light pulses to be applied. (a) Level diagram for the optical pumping from the  $|aux>$  and the  $|1>$  state via the excited state, preparing the qubit in the  $|0>$  state. (b) Optical excitation of the ions in qubit  $j$  followed by optical pumping of the ions in qubit  $i$  that were not shifted due to interaction with ions from qubit  $j$ . After the deexcitation of the ions in qubit  $j$ , the shifted ions in qubit  $i$  will return to their original absorption frequency. It is now possible for qubit  $j$  to act as a control for qubit  $i$ . (c) Pulse sequence performing a controlled-NOT operation with qubit  $j$  used as the control bit and qubit  $i$  as the target. If qubit  $j$  is in the  $|0>$  state, it will be transferred to the excited state by the first pulse. Pulses 2, 3 and 4 will in this case have no effect on the ions in qubit  $i$ , since they will all be shifted out of resonance by the ions in qubit  $j$ . The fifth pulse will then transfer the  $j$  ions back to the  $|0>$  state, leaving everything unchanged. If on the other hand the  $j$  ions are in the  $|1>$  state, pulses 1 and 5 will have no effect, whereas pulses 2, 3 and 4 will swap the population between the qubit states for the  $i$  ions.

In order to perform controlled logic between different qubits, coupling between qubits is necessary. In our scheme this coupling is possible because the rare-earth ions have permanent electric dipole moments both in the ground (qubit) states ( $^7F_0$ ) and in the excited state ( $^5D_0$ ), denoted  $\mu_g$  and

$\mu_e$  [12]. Because of the difference in dipole moments, the electric field from each ion will change if it undergoes a transition between the ground and excited state. The dipole moments, however, are the same for all the hyperfine levels of a given electronic state. As stated above, the absorption frequencies of the ions in the crystal strongly depend on the surrounding electric field. When one ion is excited, exhibiting a change in its dipole moment, ions located close (in space) to the excited ion experience this change in electric field and consequently their absorption frequencies change. The change in transition frequency for ion  $i$ , due to interaction with ion  $j$ ,  $\Delta v_{ij}$ , is given by

$$\Delta v_{ij} = \frac{(\Delta\mu_i)(\Delta\mu_j)}{4\pi h \epsilon \epsilon_0 r_{ij}^3} [(\hat{\mu}_i \cdot \hat{\mu}_j) - 3(\hat{\mu}_i \cdot \hat{r})(\hat{\mu}_j \cdot \hat{r})], \quad (1)$$

where  $\Delta\mu = |\mu_e - \mu_g|$ ,  $h$  is Planck's constant,  $\epsilon_0$  is the permittivity of vacuum and  $r_{ij}$  is the distance between the ions.  $\hat{\mu}$  and  $\hat{r}$  are unit vectors along  $(\mu_e - \mu_g)$  and  $\mathbf{r}_{ij}$ . For Eu in  $\text{YAlO}_3$  where  $\Delta\mu \approx 3 \times 10^{-31}$  Cm [12], this implies that one excited ion could shift the absorption frequency of the neighbouring ions by about 1 GHz, 1 MHz and 1 kHz for inter-ionic distances of 1, 10 and 100 nm, respectively.

An important aspect in order to do controlled logic using the dipole–dipole interaction is that the induced frequency shift of the qubit ions is large enough, so that a laser pulse resonant with the absorption frequency of the unperturbed ions, after the change in the local electric field, has a negligible effect on these ions (i.e. the shift must be of the order of a few linewidths or more). With 0.5% doping of Eu in  $\text{YAlO}_3$ , the average distance between Eu-ions would be about 2 nm.

Since we have chosen two hyperfine levels in the ground state to be the qubit states, the qubits do not interact per se. The absence of interaction between the qubits is advantageous since single-qubit operations can be defined and performed independent of the state of all other qubits. On the other hand, we can create interactions between qubits by selectively exciting from one of the qubit states to the excited state via an optical transition. If the qubit was originally in the chosen qubit state, it will shift the absorption frequencies of the

other qubits. After the controlled logic has been performed, the population in the excited state can be transferred back to its original qubit state. For Eu-doped  $\text{YAlO}_3$ , the excited state that is used for the control has a decoherence time of about 250  $\mu\text{s}$  [7], which is the maximum time for completing any quantum gates involving more than one qubit. Typically a  $\pi$ -pulse in these systems using a gated continuous wave dye laser would be a few  $\mu\text{s}$ . Thus, there is sufficient time to perform even complicated operations.

When discussing how to perform controlled logic in the rare-earth-ion-doped inorganic crystals, we distinguish between two different situations. First we will discuss controlled logic when each qubit consists of a single ion. In this case (denoted case A) we assume that any ion can be used for controlling any other ion (qubit). Experimentally, this situation corresponds to some 10–100 ions doped into a small volume,  $\sim (10 \text{ nm})^3$ , where any individual ion is close enough to the other ions to shift them out of resonance while the probability for two ions to occupy the same frequency is low. We will then discuss controlled logic in a conventionally doped crystal (case B). In such a crystal, each frequency channel is occupied by several ions and most ions are too far apart to be able to control each other (by means of the dipole–dipole interaction). In the description that follows, we will assume, in both situations mentioned above, that the qubits have been prepared in the hyperfine qubit states using optical pumping.

In case A, excitation of any ion to the  $^5\text{D}_0$  state will shift the absorption frequencies of all the other ions by more than a homogeneous linewidth. This is accomplished because of the above mentioned difference in dipole moments between the ground and excited state. A controlled-NOT operation between two arbitrary ions  $i$  and  $j$ , with ion  $j$  as the control qubit, would for example be done in the following way: First a  $\pi$ -pulse resonant with the transition between the qubit state  $|0\rangle$  and the excited state is applied to ion  $j$ . If this ion was in the  $|0\rangle$  state, the  $\pi$ -pulse will cause the ion to be transferred to the excited state with certainty, thus shifting all other ions (and specifically ion  $i$ ) out of their original absorption frequency. Now the idea is to exchange the population between the  $|0\rangle$  and

$|1\rangle$  state of ion  $i$ , if and only if ion  $j$  is not excited. This can be done by applying appropriate pulses on the original absorption frequency for ion  $i$ . These pulses will only affect ion  $i$  if ion  $j$  is not excited, since the excitation of ion  $j$  causes a shift of ion  $i$ 's absorption frequency, thus making it insensitive to pulses at its original frequency. After the population transfer on ion  $i$  has been completed, another  $\pi$ -pulse (with appropriate phase) is applied to ion  $j$ , transferring any population back from the excited state to the  $|0\rangle$  state (Fig. 1(c)).

For a crystal with a doping concentration of e.g. 0.5% in the whole crystal (case B), the above described scheme does not immediately work. Since the volume that can be addressed by a laser contains a very large number of ions, each frequency channel in the material will be occupied by many ions. Further, the ions are spread out over a larger volume, making mutual control using the above described change in dipole moments impossible because the interaction between ions at large distances does not cause the absorption frequencies to shift by a sufficient amount. Controlled logic in these crystals therefore requires further preparation of the qubits. For any pair of frequency channels,  $C_i$  and  $C_j$ , there will be a number of ions in  $C_j$  that are sufficiently close to some ion in  $C_i$  such that they can control each other. The aim of the preparation is to select only these ions for the qubits. We will first describe how to select ions for two qubits such that they can control each other mutually, and then generalise this scheme so that it works for an arbitrary number of qubits. The first step is to, more or less arbitrarily, choose the two frequency channels,  $C_i$  and  $C_j$ , that are going to be used for the qubits. The next step is to choose the ions that will constitute the qubits (Fig. 1(b)). This is done by exciting the ions in channel  $C_j$  to the excited state using a  $\pi$ -pulse. The ions in channel  $C_i$  situated close enough to  $C_j$  ions will then shift out of their original absorption frequency because of the dipole–dipole interaction. However,  $C_i$  ions with no  $C_j$  ions close by are not affected by the  $C_j$  excitation and they will still absorb at their original frequency. These  $C_i$  ions can therefore be excluded from any further interaction by means of optical pumping to an auxiliary state, in this case the third hyperfine level. The next step is to return the  $C_j$

ions to the ground state by means of a  $\pi$ -pulse. The result of this is that the fraction of  $C_i$  ions that were originally shifted out of resonance return to their original position (in frequency). We now have a situation where the ions in the first frequency channel can be used as a control for the second one. If the procedure is repeated with the ions remaining in the second channel ( $C_i$ ) as a control, the two channels can control each other mutually. The procedure can easily be generalised to an arbitrary number of qubits. The procedure above can then be applied to all the qubits simultaneously, i.e. the control bit is first excited and the remaining population for all the other bits is sequentially transferred to the auxiliary state. When the selection of ions for the qubits has been done, controlled logic can be performed in exactly the same way as for the single-ion case described above.

We will now describe the complete scheme for performing a controlled-NOT operation between two qubits,  $i$  and  $j$ , where bit  $j$  is used as the control bit (Fig. 1(c)).

1. Prepare the qubits in the two qubit states using optical pumping.
2. Select the ions that are going to be used for the qubits. (Not needed in the case of low doping in small volume.)
3. Apply the following pulse sequence:
  - (I)  $\pi$ -pulse on  $|0\rangle_j - |\text{exc}\rangle_j$ ,
  - (II)  $\pi$ -pulse on  $|0\rangle_i - |\text{exc}\rangle_i$ ,
  - (III)  $\pi$ -pulse on  $|1\rangle_i - |\text{exc}\rangle_i$ ,
  - (IV)  $\pi$ -pulse on  $|0\rangle_i - |\text{exc}\rangle_i$ ,
  - (V)  $\pi$ -pulse on  $|0\rangle_j - |\text{exc}\rangle_j$ .

In order to generalise the above given pulse sequence to a (controlled) <sup>$n$</sup> -NOT gate, using  $n$  control bits, it is sufficient to repeat the first and the last step under point 3 in the scheme for all control bits. Since the excitation of any qubit shifts the absorption frequency of all the other qubits, the pulse sequence for the gate will only lead to the excitation of one qubit. This is similar to earlier proposals based on Rydberg atoms [13]. This means that we will not have to keep track of the more complicated shifts that might occur if several qubits were excited.

Even though the single and multi-ion qubit cases described above are equivalent when it comes to the pulse sequences needed to perform con-

trolled logic, the experimental difficulties that arise are different for the two situations. Since the scheme using crystals with a relatively high doping seems easier to implement experimentally, we will focus on that. In this situation it is important, for the coherent population transfer to work, that all the ions constituting one qubit experience the same pulse area. If the frequency channel selected for the qubit is broader in frequency than the homogeneous linewidth for the ions, it is important that the applied laser pulses have a uniform intensity for the different frequencies within the qubit. This can be accomplished using short laser pulses, covering a broader frequency range. In this case, ions absorbing at frequencies close to the qubit ions need to be removed by optical pumping to prevent them from being excited by the pulses controlling the qubit. The ions in the qubits are also located at different spatial positions in the crystal. Therefore it is also important to make sure that the laser pulses have a homogeneous intensity profile over the whole volume of the qubit.

In a system with several qubits, where each qubit should be able to control all the other qubits, the preparation sequence will have selected only the ions being close to ions from all frequency intervals corresponding to all the other qubits. This means that it is especially interesting to consider materials where the dopants are sitting relatively close to each other.

If, for a given ion, the number of ions situated close enough to control this ion is small compared to the total number of spectral channels in the material, the preparation sequence will create “islands” of controlling ions with one ion from each qubit. In this case, the preparation sequence should be an efficient way of ensuring controllable interactions between the ions. If on the other hand, several ions from each qubit are situated in a volume small enough for them to control each other, the situation will be more complicated since e.g. the frequency shifts due to two excited ions could have different signs and therefore cancel each other. We believe that the magnitude of the dipole interaction in combination with the spacing between the ions in the rare-earth-ion-doped crystals will lead to the former situation where the preparation scheme is an efficient way of selecting

the strongly interacting ions. A study of the latter case, where many ions in the same frequency channel can control another ion, is beyond the scope of the present work, but we think that it would be interesting to identify such systems and to study the effects of multiple quantum gates and creation of entanglement.

The read-out of the qubits can plainly be done by using conventional absorption spectroscopy or other standard techniques used in spectral hole burning, e.g. [14]. Here it is a major advantage that the qubits consist of many ions, since the low oscillator strength of the optical transition would otherwise make the read-out a difficult task.

We now summarise some essential features of the approach proposed. Using a cryogenically cooled crystal with appropriate doping concentration, scaling to several qubits should readily be possible with existing technology. The mechanism which distinguishes between different qubits is not the same as the mechanism by which the controlled interaction is mediated. This is different from e.g. ion traps, where interactions are mediated using coulomb repulsion, which gets stronger with smaller distance, while qubit selection relies on the condition that the distance between ions is sufficiently large. It is a strength of the rare-earth crystals that it is possible to have a strong coupling between the qubits without losing the control of the individual qubits. Timing between the different steps (pumping, preparation and computing) is not critical in our scheme because of the very long lifetime (hours) of the hyperfine levels. This means that the pumping and preparation steps can be performed for one qubit at a time, which simplifies the experimental work.

As previously mentioned quantum computing in rare-earth-ion-doped inorganic crystals and related systems has already been discussed in the literature [5]. It may therefore be motivated to briefly comment on the differences between the approach here and that in [5].

First, in [5] the interaction mechanism between the qubits is suggested to be the induced transition dipole moment and *not* the permanent dipole moment as in this paper. To explain why this matters we note that in our approach an ion can act as a control bit for all other ions that reside

within a, let us say, 10 nm radius, irrespective of their transition frequencies within the several GHz inhomogeneous linewidth. In [5] the same ion can essentially only control ions in a 1 MHz subgroup within the inhomogeneous line. Unless ions spatially close are correlated in frequency this will have the consequence that the number of ions that could act as qubits are a factor 1000 or 10,000 higher in the approach presented in this paper and the probability for  $n$ -qubit entanglement by the same reasoning would be of the order of  $(1000)^n$  times as probable in our approach.

Second, in this paper we also describe how to generate multi-ion qubits (e.g. a billion ions/qubit). With present technology the signal level in most materials would be too low to detect single-ion qubits. However, multi-ion qubits would give strong signals in basically any material in our opinion.

To conclude a scheme for constructing simple solid state quantum gates in rare-earth-doped crystals is described. The scheme is based on well established experimental techniques and well documented material data and should be straightforward to realise experimentally. The scheme for selecting ions with sufficiently strong interaction to control each other should be possible to apply also to other solid state systems.

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