Incorporating Relativistic Features into FLASH

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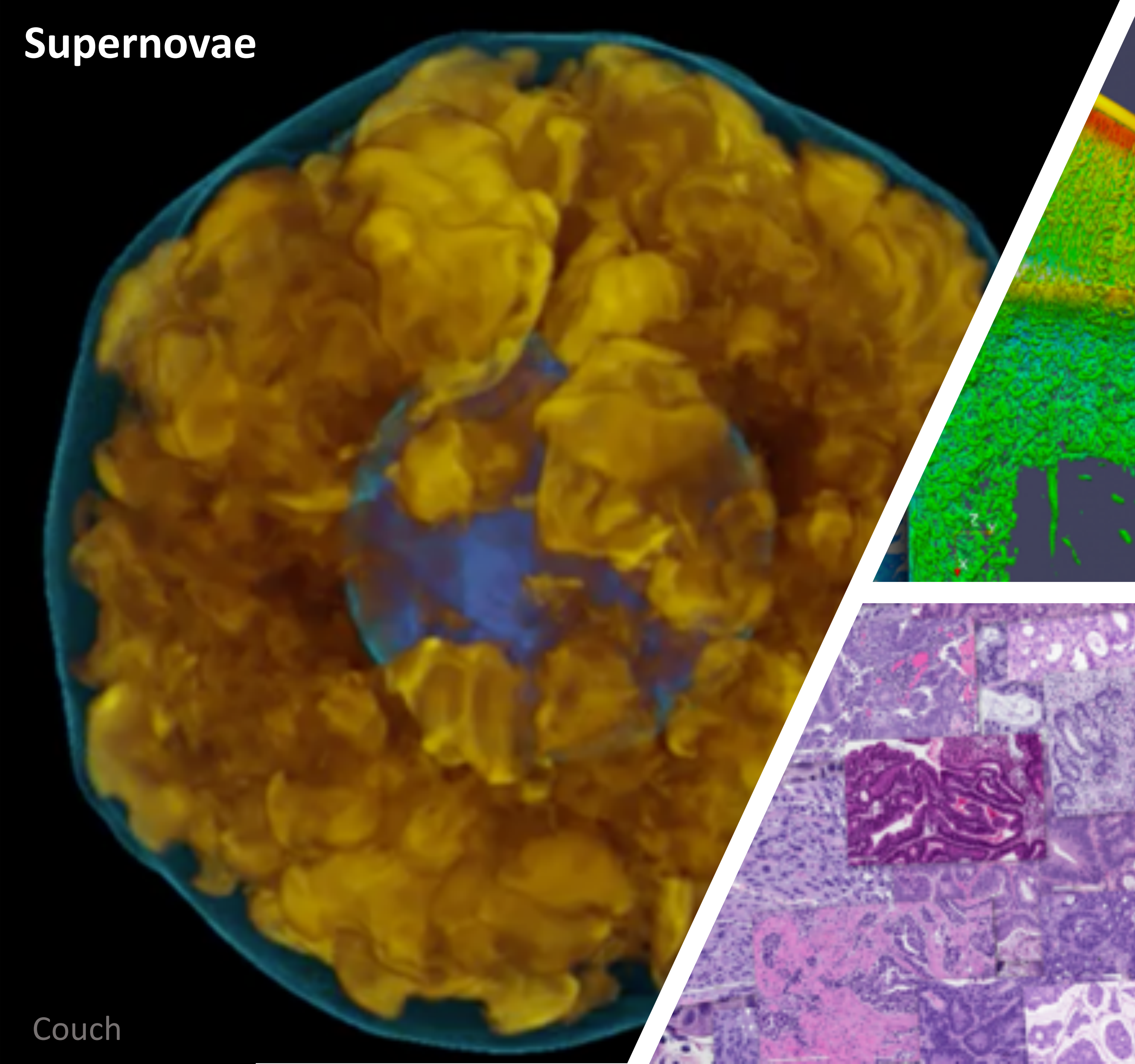
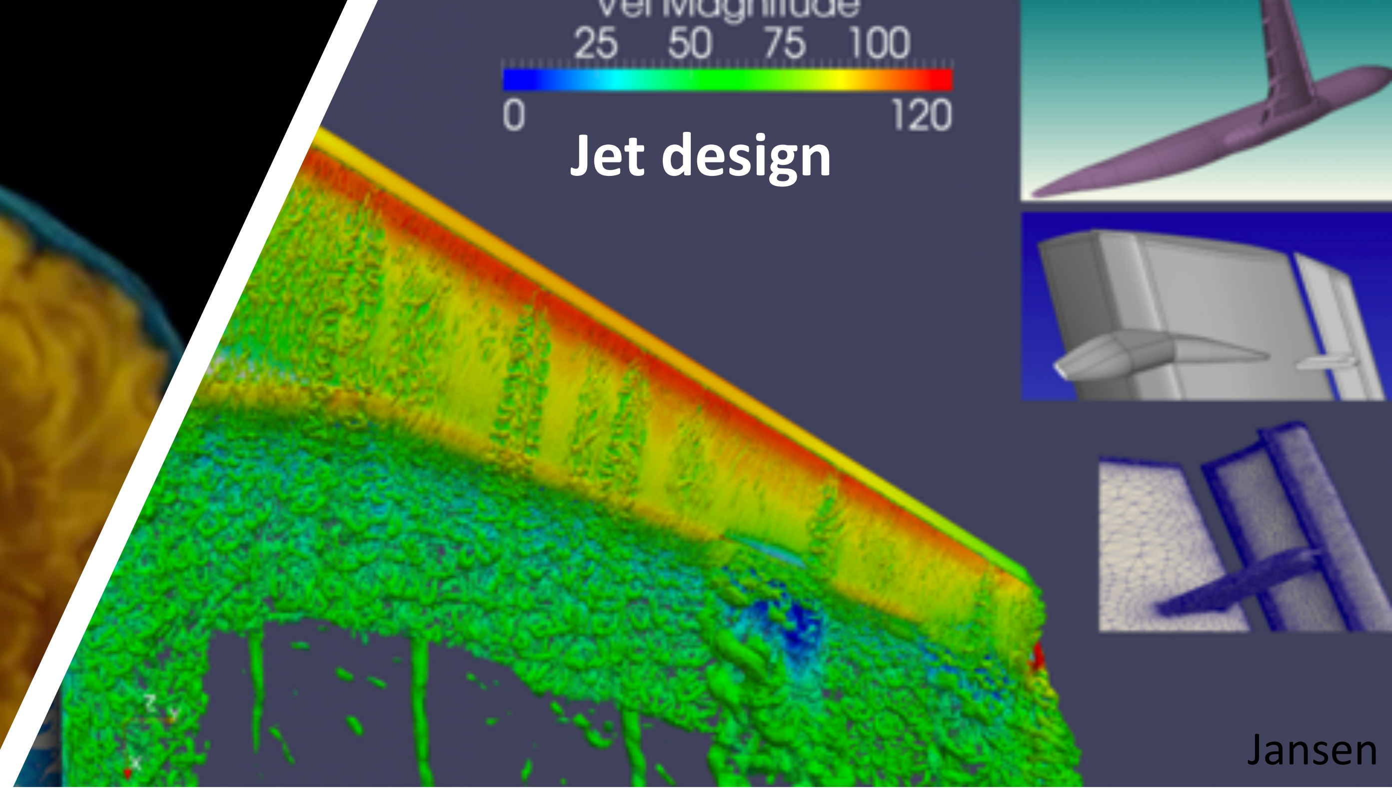
EXASCALE COMPUTING PROJECT

Argonne NATIONAL LABORATORY

SNAPhU


Supernovae
Couch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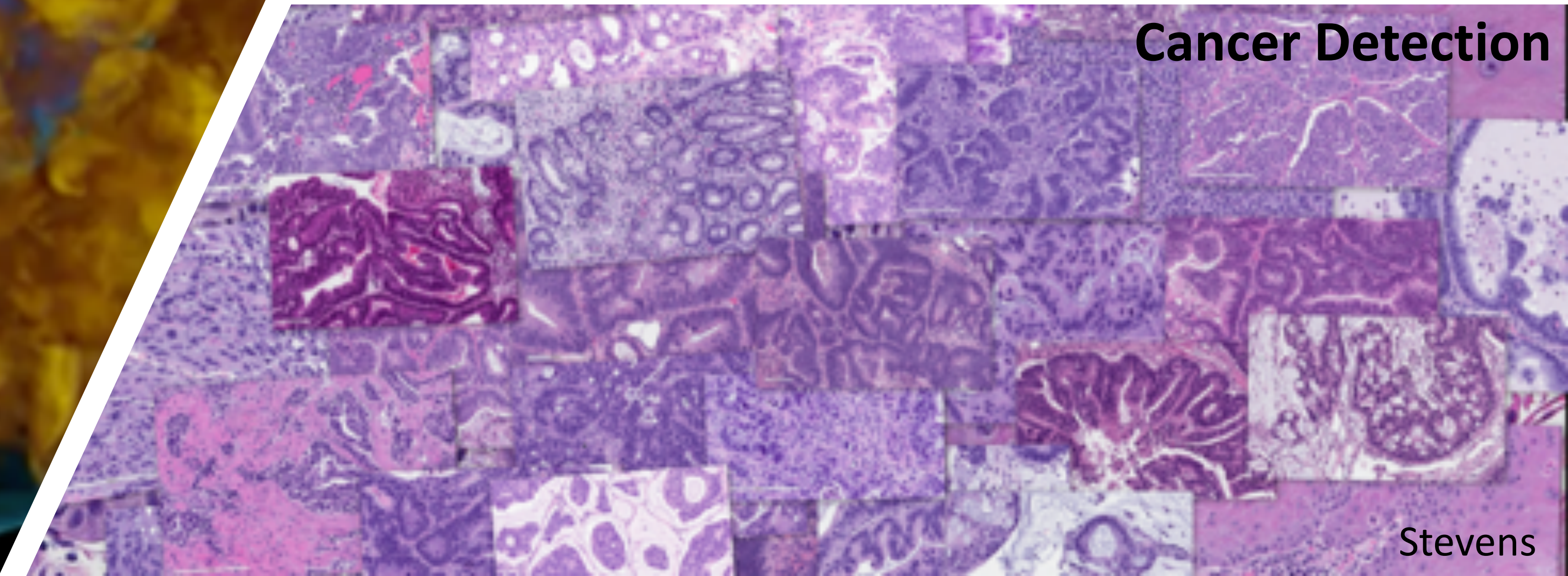
Jet design
Jansen

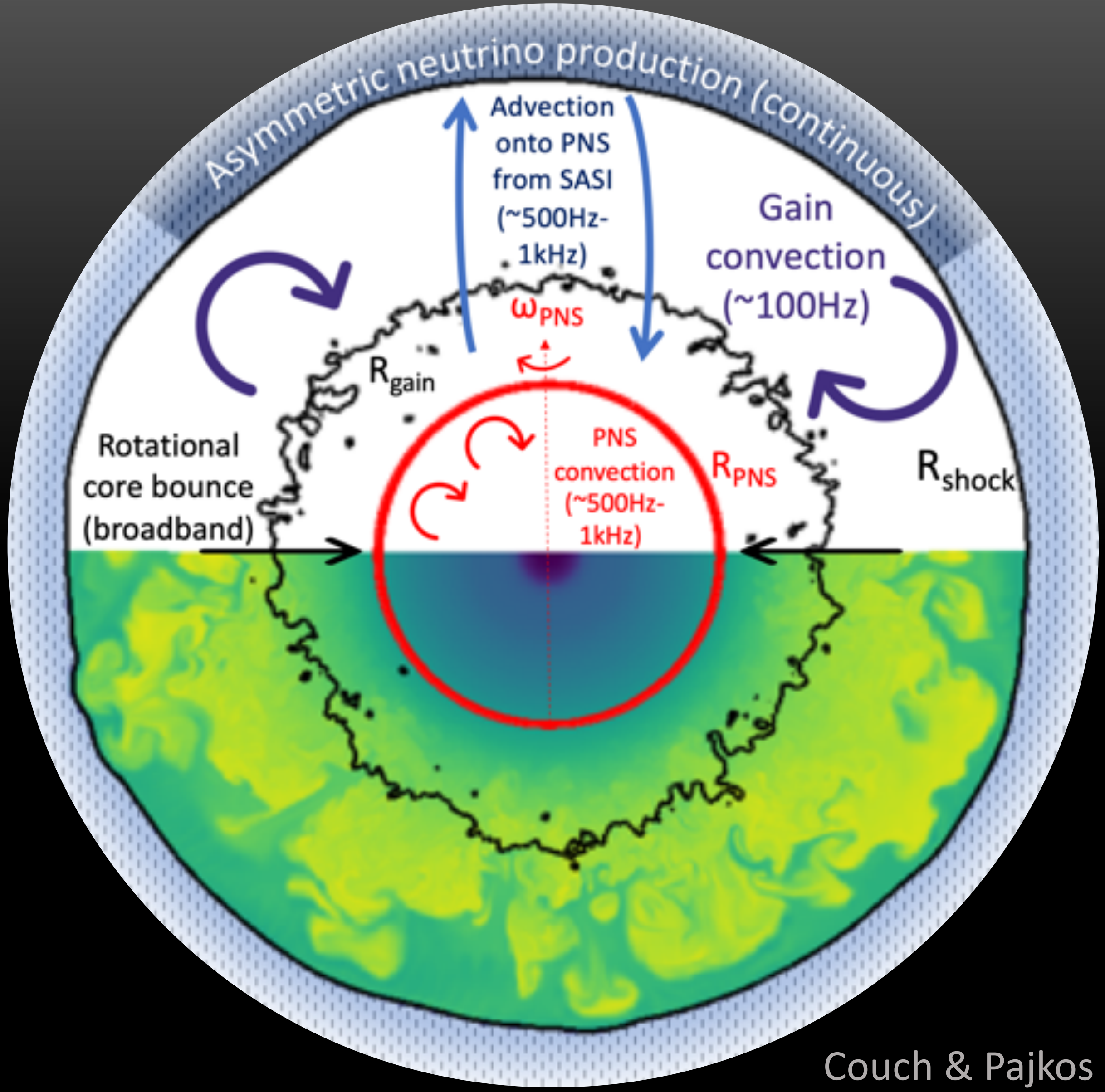

Cancer Detection
Stevens

Review

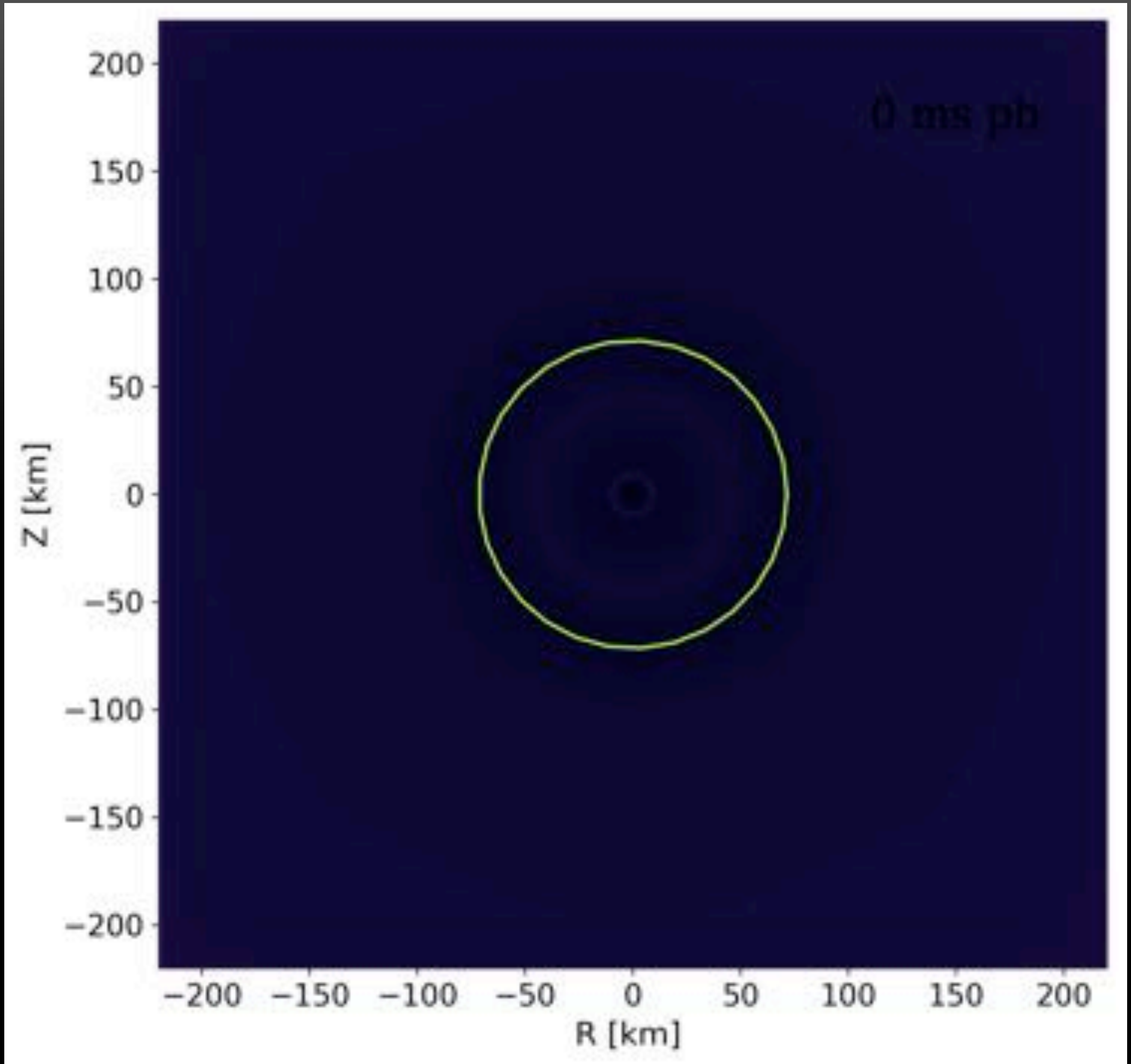
- Gravitational wave (GW) predictions depend on robust hydrodynamics
- Recent features included in FLASH5
- Current developments and future FLASH

GWs from CCSNe

- PNS convection
- Post shock turbulence
- Can cause PNS oscillations



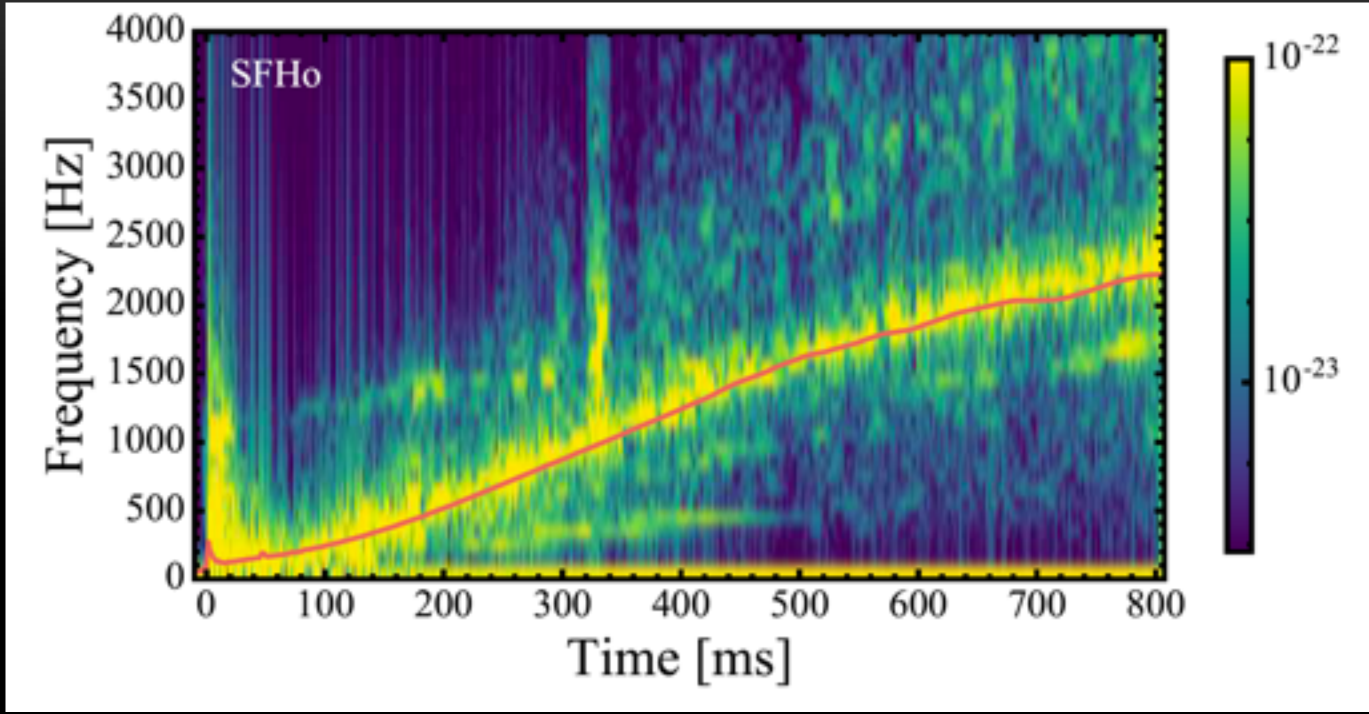
Couch & Pajkos


0 ms pb
Z [km]
R [km]
200
150
100
50
0
−50
−100
−150
−200
−200 −150 −100 −50 0 50 100 150 200

R_{PNS}

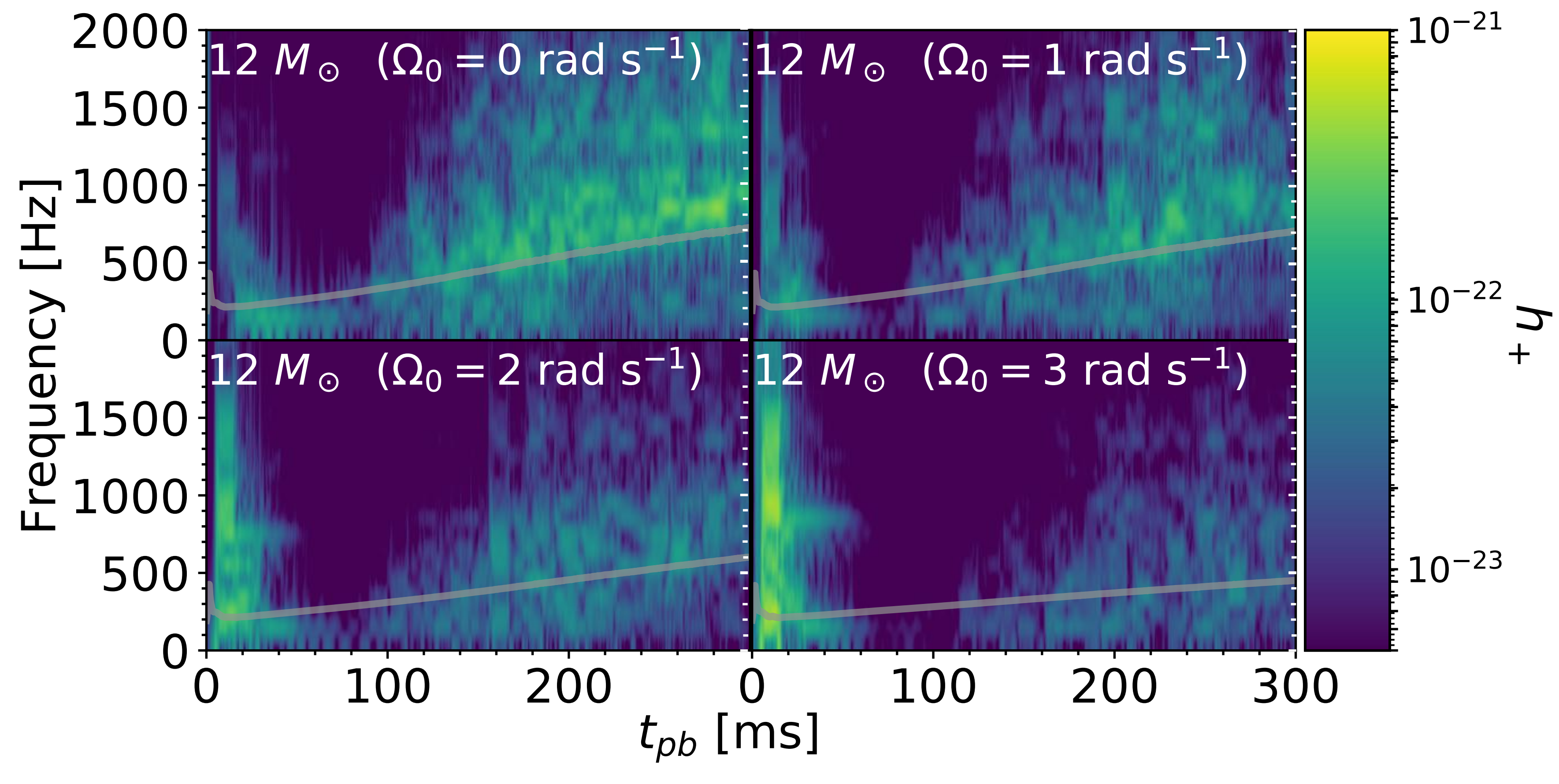
Features of GW Signal

- Mass accretion & neutrino cooling
- PNS radius decreases
- PNS "ramp up"



Pan+ 2018

Influence of Rotation

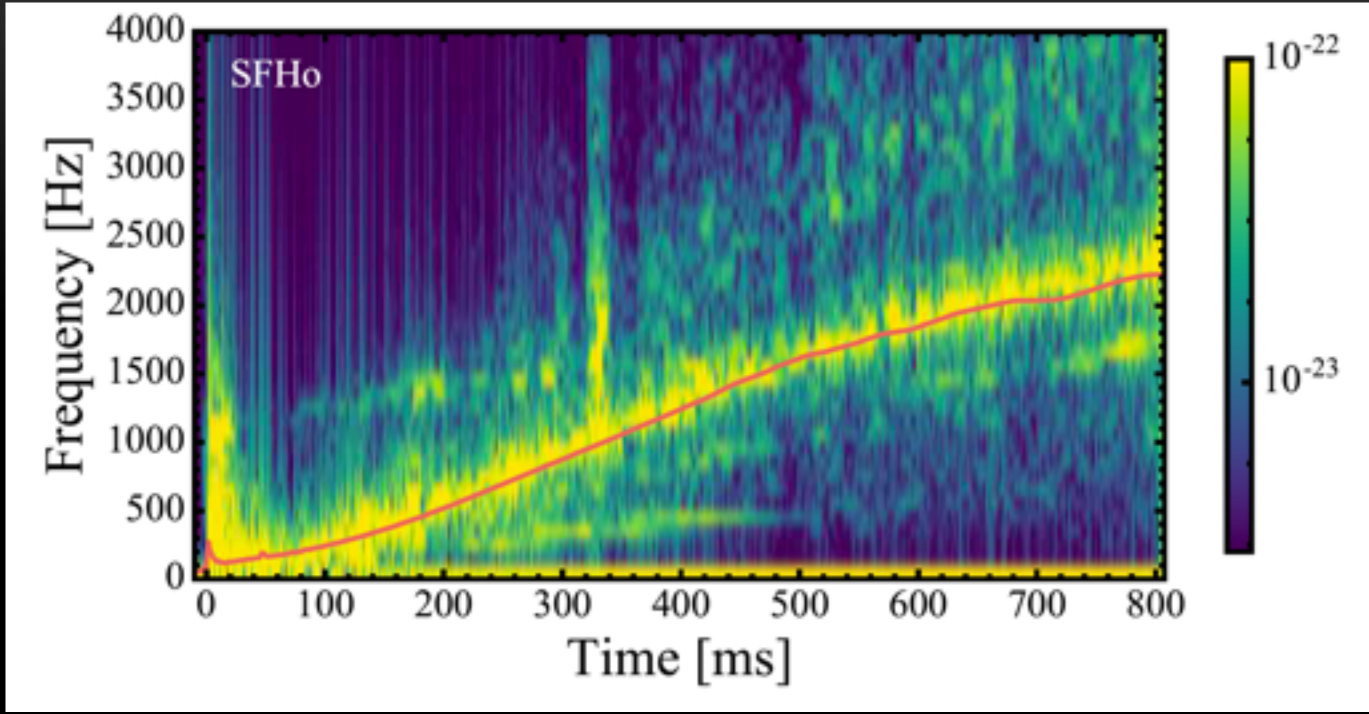


Pajkos+ 2019

arXiv:1901.09055

PNS Characteristics from GWs

- Contains mass & radius information
- Depends on $\bar{\nu}_e$ energy (Muller+ 2013)
- Places constraints on nuclear EOS



Pan+ 2018

Importance of GR Hydrodynamics

- PNS compactness ↑ (Kuroda+ 2012)
  - Mean neutrino energy ↑
- GW peak frequencies ↑ (Muller+ 2013)
- Need GR Hydrodynamics to properly translate GWs to physical information

An Intermediate Solution

- Modified Euler equations (Zha+ 2020)
- Account for time dilation (lapse function)
- Additional momentum source term, relativistic pressure support

Euler Equations

Mass conservation

Momentum conservation

Energy conservation

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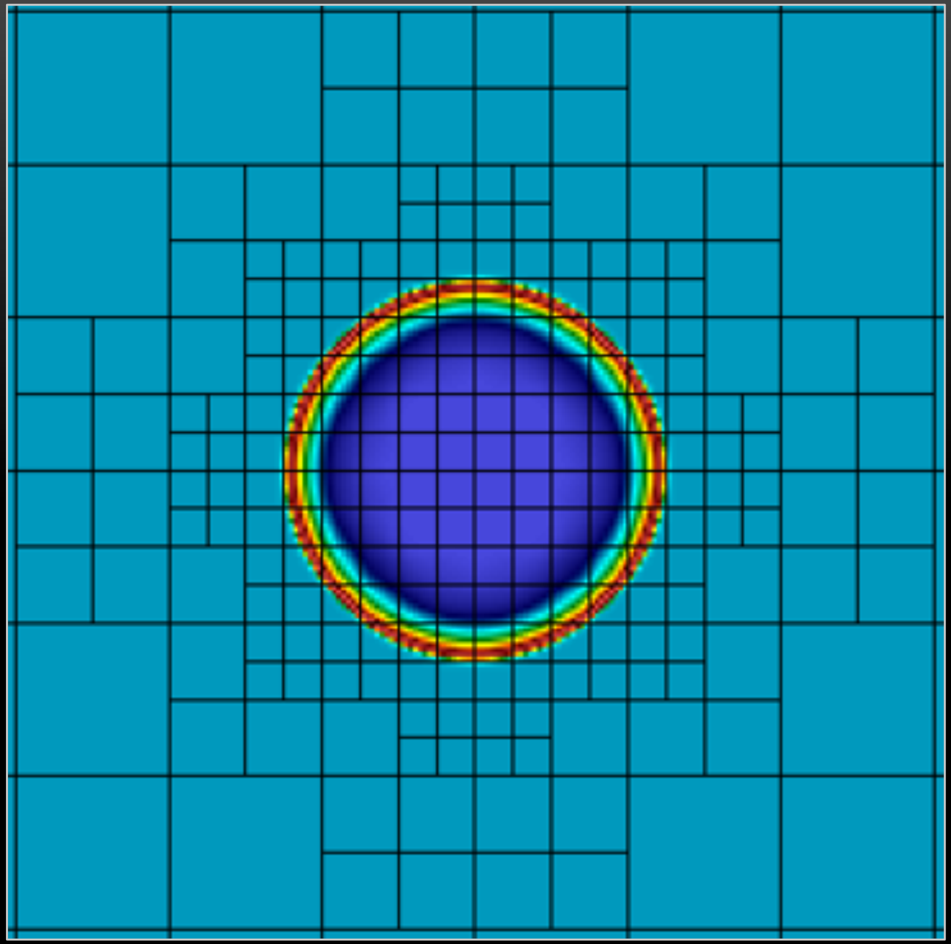
Increasing Extensibility

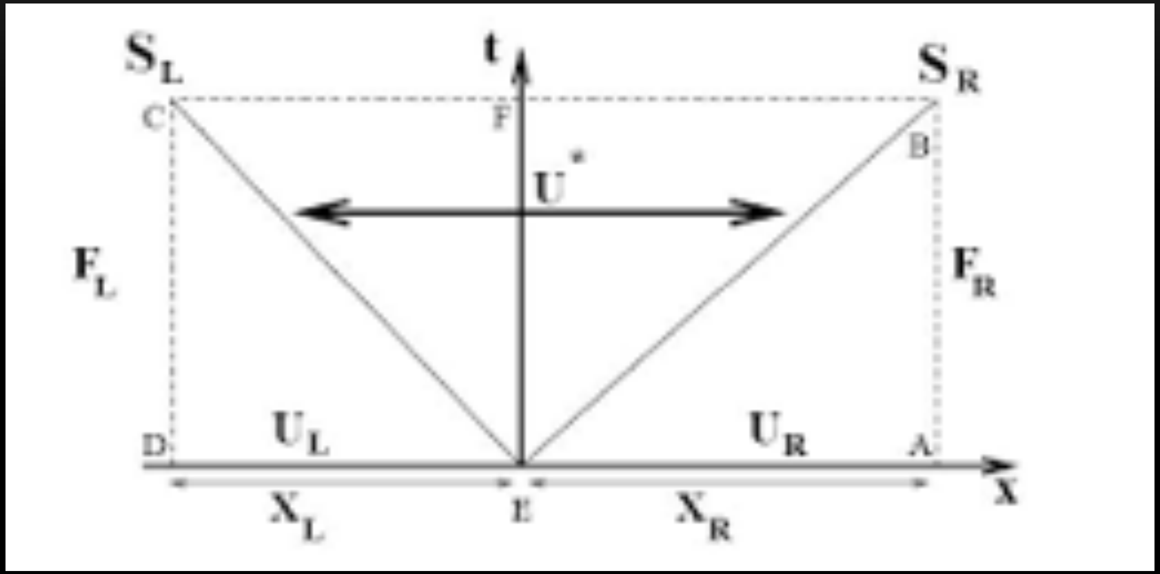
- Compatibility with Paramesh & AMReX
- Fixed & varied block size
- Octree AMR structure



Sedov test problem using AMReX

Relativistic Corrections to Riemann Solver

- Sound speeds depends on thermodynamic derivatives (Rezzolla & Zanotti 2013)
- Divergence cleaning:
  $v_{hyperbolic}=c$ (Mosta+ 2013)
- Characteristics nonlinearly depend on sound speeds (Gammie+ 2003)



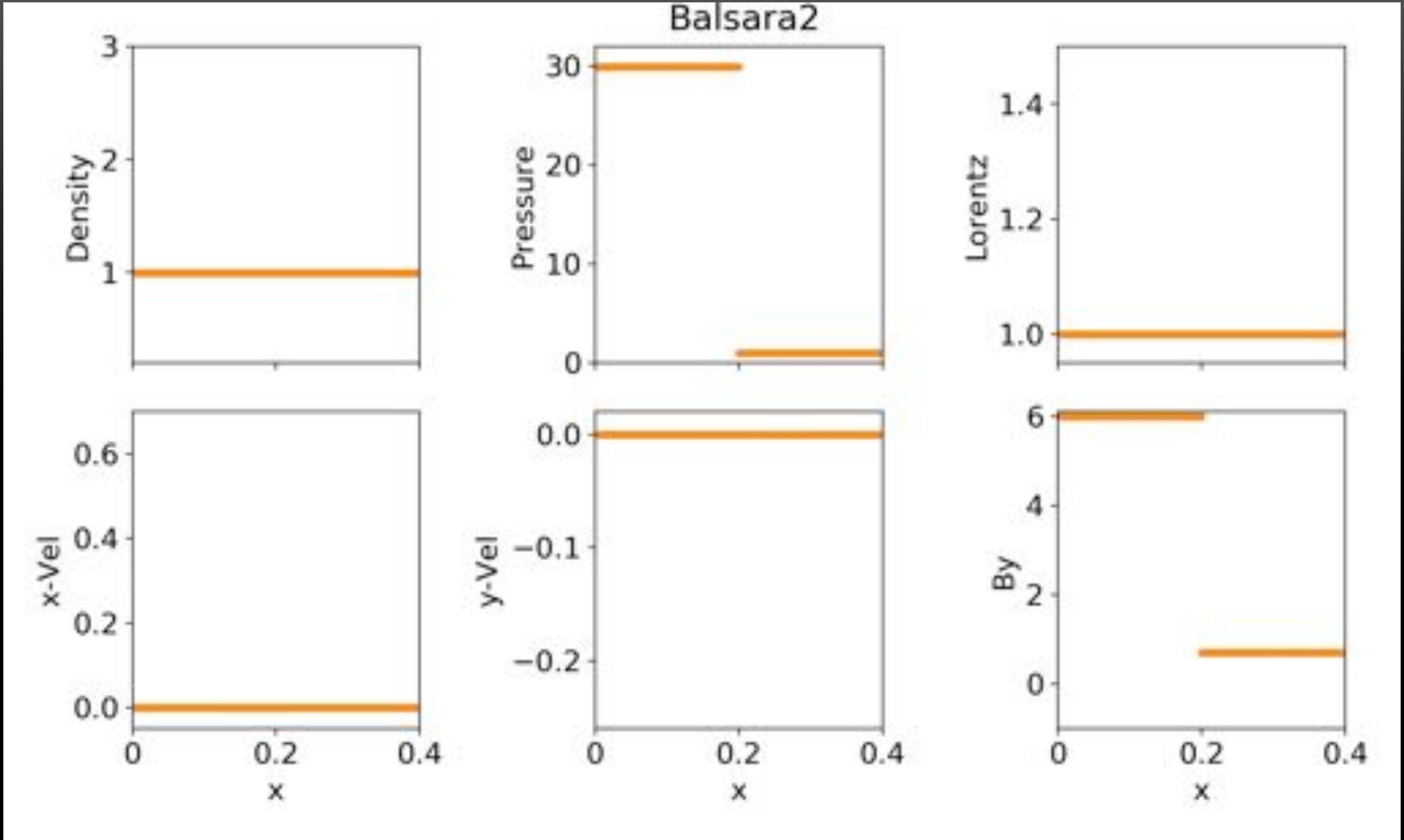
Tramel + 2009

Redefining Conserved Quantities & Fluxes

- Relativistic analogues to density, momentum, & energy
- Dependence on *total* pressure, Lorentz factor, & lapse function
- For curved spacetime, new source terms depend on metric & matter

Cost of Conservative to Primitive (con2prim)

- Relativistic con2prim relies on root finding (eg. *1D Brent*, 3D Newton-Raphson) (Siegel+ 2018)
- Con2prim scheme must balance robustness & efficiency
- Compounds cost with each (tabular) EOS call


Balsara2
Density
Pressure
Lorentz
x-Vel
y-Vel
By
x
x
x
0
0.2
0.4
3
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1
30
20
10
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1.4
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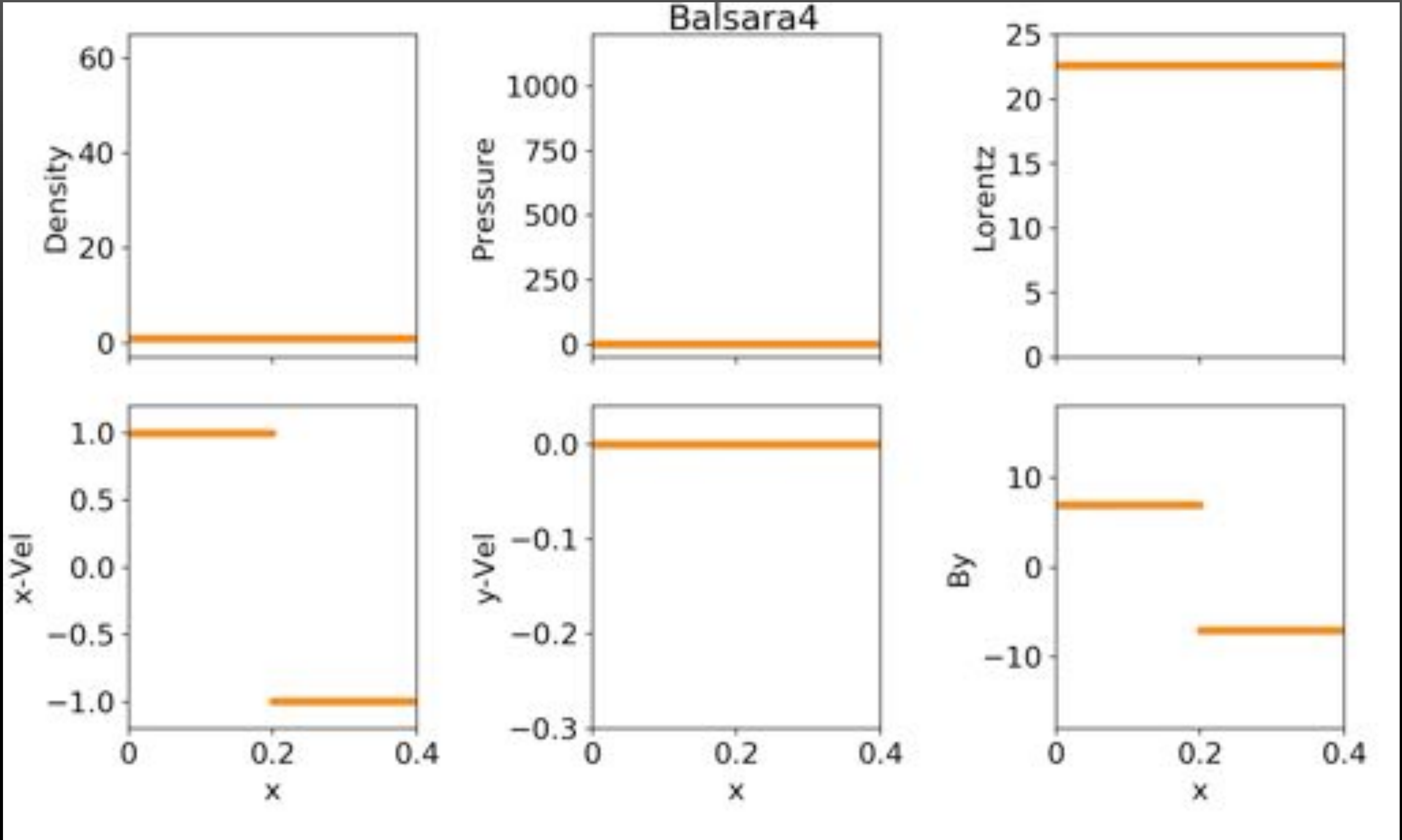
Balsara4
Density
Pressure
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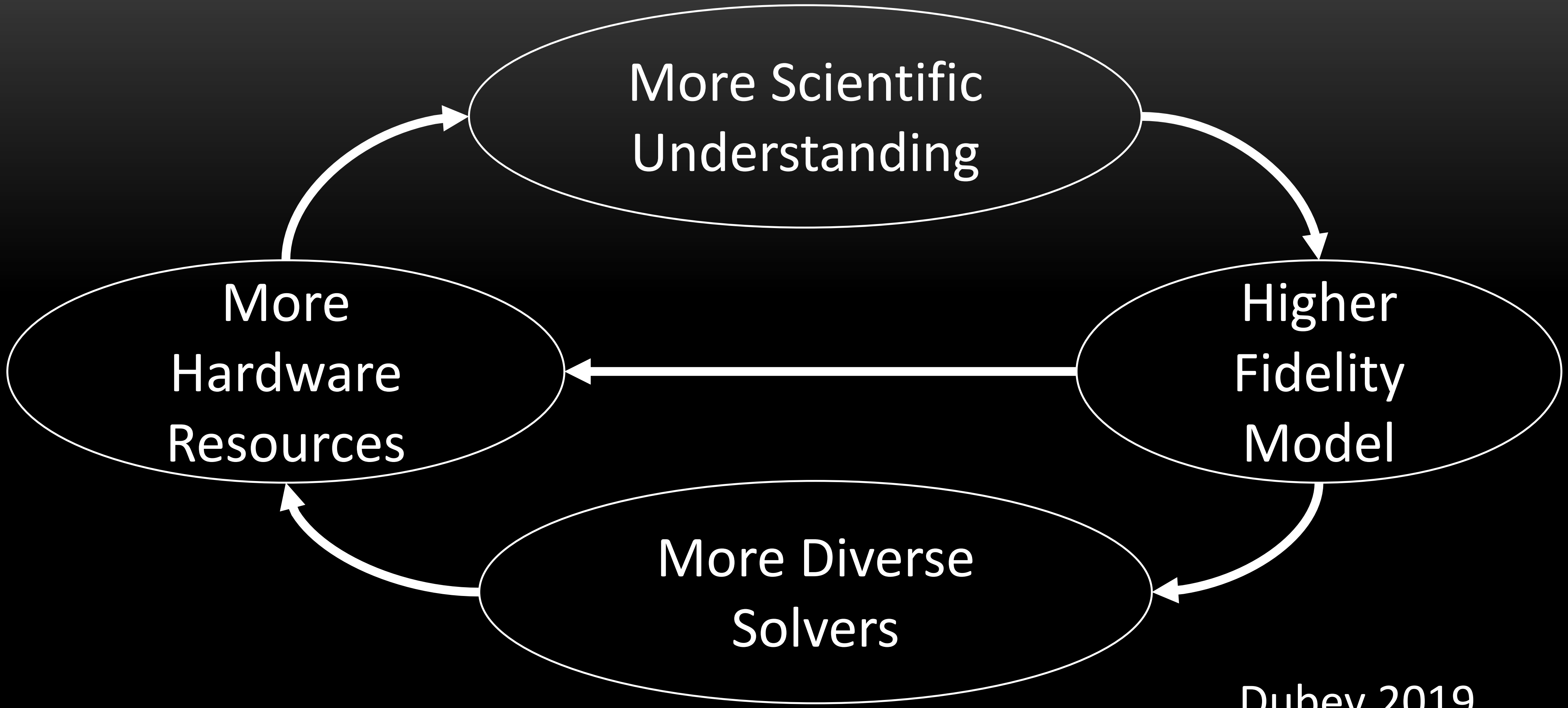
−0.2
−0.3
10
−10

Science-Computation Cycle



Dubey 2019

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Increasing Portability

- Future FLASH—distribute intelligence
- FLASH is composed before runtime
- User provides a ‘recipe’, orchestrator produces executable code

Current Developments

- Incorporating curved (fixed) metric into data structure
- Construction of TOV verification test
- Will examine convergence of central density oscillations

Future Developments

- Dynamically evolving spacetime
- Conformal flatness as possible intermediary
- BSSN or CCZ4 for final spacetime evolution

FLASH5 Overview

- Open-source

- ECP-Astro/FLASH5

Github

Conclusion

- High fidelity GW predictions rely on relativistic terms in MHD
- FLASH5 has recently improved the extensibility & robustness of MHD
- Next steps for FLASH include curved, evolving space-times

Modified Euler Equations

$$\partial_t \rho + \nabla \cdot (\alpha \rho \vec{v}) = 0,$$

$$\partial_t (\rho \vec{v}) + \nabla \cdot [\alpha(\rho \vec{v}\vec{v} + P)] = \alpha(\rho - P/c^2)\nabla\Phi,$$

$$\partial_t \tau + \nabla \cdot [\alpha(\tau + P)\vec{v}] = \alpha \rho \vec{v} \cdot \nabla \Phi,$$

Zha+ 2020

Relativistic Conservative Vars & Fluxes

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}^i}{\partial x^i} = \mathbf{S},$$

$$\mathbf{U} = [D, S_j, \tau, \mathcal{B}^k],$$

$$\mathbf{F}^i = \alpha \times \begin{bmatrix} D\tilde{v}^i \\ S_j\tilde{v}^i + \sqrt{\gamma}P^*\delta^i_j - b_j\mathcal{B}^i/W \\ \tau\tilde{v}^i + \sqrt{\gamma}P^*v^i - \alpha b^0\mathcal{B}^i/W \\ \mathcal{B}^k\tilde{v}^i - \mathcal{B}^i\tilde{v}^k \end{bmatrix},$$

$$\mathbf{S} = \alpha\sqrt{\gamma} \times \begin{bmatrix} 0 \\ T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^\lambda_{\mu\nu}g_{\lambda j}\right) \\ \alpha\left(T^{\mu 0}\frac{\partial \ln \alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\mu\nu}\right) \\ \vec{0} \end{bmatrix},$$

$$S_j = \sqrt{\gamma}\left(\rho h^* W^2 v_j - \alpha b^0 b_j\right),$$

$$\tau = \sqrt{\gamma}\left(\rho h^* W^2 - P^* - (\alpha b^0)^2\right) - D$$

$$\mathcal{B}^k = \sqrt{\gamma}B^k,$$

Mosta+ 2013

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Mosta+ 2013