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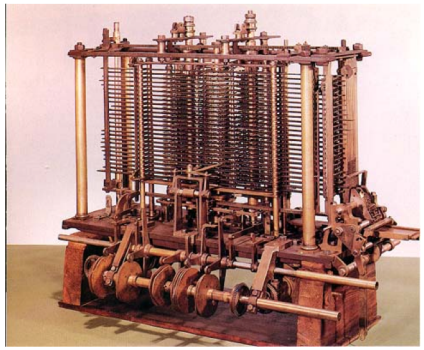
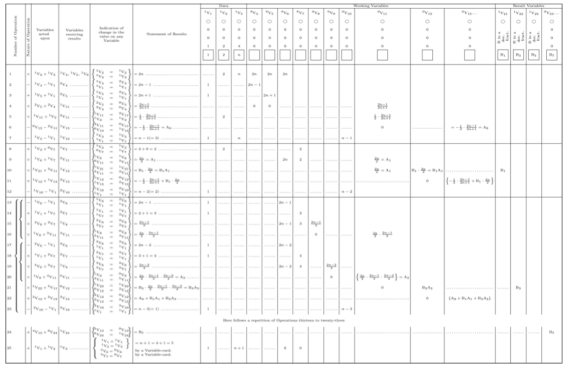
# Efficiency

## From the beginning

*"As soon as an Analytic Engine exists, it will necessarily guide the future course of science. Whenever any result is sought by its aid, the question will arise – By what course of calculation can these results be arrived at by the machine in the shortest time?" – Charles Babbage (1864)*

*"In almost every computation a great variety of arrangements for the succession of the processes is possible, and various considerations must influence the selections amongst them for the purposes of a calculating engine. One essential object is to choose that arrangement which shall tend to reduce to a minimum the time necessary for completing the calculation." – Ada Lovelace (1843)*





*Computing the Numbers of Bernoulli on the Analytical Engine, by Ada Lovelace.*

## Caveat discipulus

*"More computing sins are committed in the name of efficiency (without necessarily achieving it) than for any other single reason — including blind stupidity."*

**William A. Wulf**

*"We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil."*

**Donald E. Knuth**

*"We follow two rules in the matter of optimization: Rule 1: Don't do it. Rule 2 (for experts only): Don't do it yet - that is, not until you have a perfectly clear and unoptimized solution."*

**Michael A. Jackson**

## Efficiency

*"We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil.* ***Yet we should not pass up our opportunities in that critical 3%****."*

**Donald E. Knuth**

**efficiency** (noun) Skillfulness in avoiding wasted time and effort.

*WordNet 3.1 http://wordnetweb.princeton.edu/perl/webwn?s=efficiency*

We always want to be skillful in avoiding wasted time and effort, but particularly so when the difference in efficiency can make a critical difference.

## Linear Search: Avoiding wasted time and effort

## Efficiency

A boolean version of list search: return true if found, false if not.

```
public boolean search(List<T> a, T target) {
   boolean found = false;
   for (T element : a) {
      if (element.equals(target))
         found = true;
   }
   return found;
}
```

**Version A**

Which is more efficient?

```
public boolean search(List<T> a, T target) {
   for (T element : a) {
      if (element.equals(target))
         return true;
   }
   return false;
}
```

**Version B**

**"early exit"**

# Efficiency

List search has a best, average, and worst case running time. If we timed both versions, what would you expect the data to look like for **worst** and **average** cases?

## Version A (no early exit)

```
boolean search(List<T> a, T target){
   boolean found = false;
   for (T element : a) {
      if (element.equals(target))
         found = true;
   }
   return found;
}
```

| No early exit | |
|---|---|
| Worst case | 4.75 ms |
| Average case | 4.00 ms |

*Average time over four runs. List size = 100,000*

## Version B (early exit)

```
boolean search(List<T> a, T target) {
   for (T element : a) {
      if (element.equals(target))
         return true;
   }
   return false;
}
```

| Early exit | |
|---|---|
| Worst case | 4.70 ms |
| Average case | **3.25 ms** |

*Average time over four runs. List size = 100,000*


| ?? | |
|---|---|
| Worst case | **0.064 ms** |
| Average case | **0.065 ms** |

*Average time over four runs. List size = 100,000*

## Improving efficiency

**Tune/tweak the algorithm** (e.g., early exit)

| | No early exit | Early exit |
|---|---|---|
| Worst case | 4.75 ms | 4.70 ms |
| Average case | 4.00 ms | **3.25 ms** |

**Apply a heuristic**

*Example*: On each successful search, move the target to the first position in the list.

```
public boolean search(List<T> a, T target) {
   for (T element : a) {
      if (element.equals(target)){
         a.remove(element);
         a.add(0, element);
         return true;
      }
   }
   return false;
}
```

Assume there is a "working set" of search targets that follows the 90-10 rule. (90% of the searches are for only 10% of the elements.)

**Average** time of 10,000 runs with N = 100,000: **0.085ms**

**Neither of these techniques improve worst case.**

## Binary Search: Improving the worst case

## Improving worst case

Improving worst case efficiency often involves a more fundamental change to the algorithm or to the assumptions/constraints on the problem.

Current list search constraints: A list of elements that can be compared to each other.

| 20 | 12 | 35 | 15 | 10 | 80 | 30 | 17 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

New list search constraints: A list of elements that can be compared to each other, **arranged in non-decreasing order**.

| 1 | 2 | 10 | 12 | 15 | 17 | 20 | 30 | 35 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

This new constraint leads to a new algorithm that will be much more efficient than our current linear search.

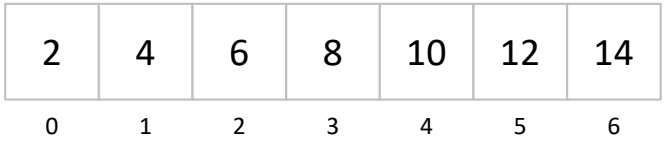
## Binary search

Solves the list search problem when the search space is ordered.

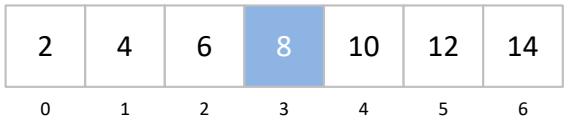
Assumes the input has been sorted into a monotonically increasing sequence: $[a_1 \leq a_2 \leq \ldots a_n]$

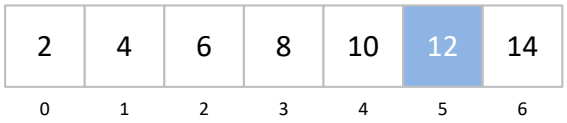
**Strategy:**

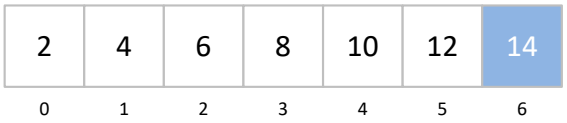
Look at the middle element.
If it equals target, return.
If it is less than target, repeat on right half.
If it is greater than target, repeat on left half.




target = 15



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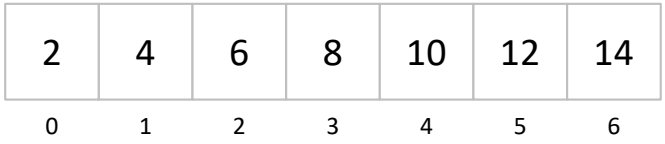
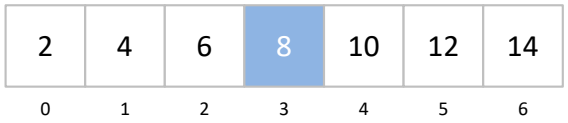
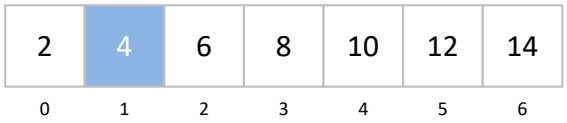



target = 4



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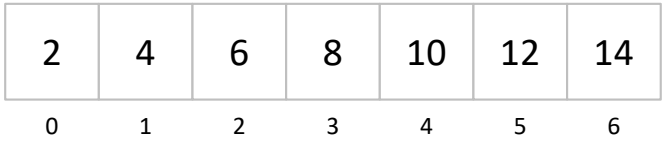
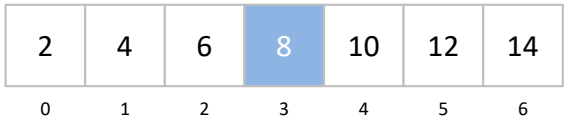
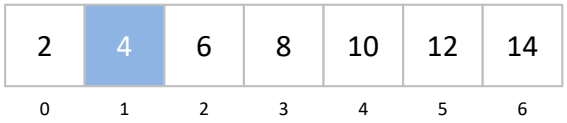
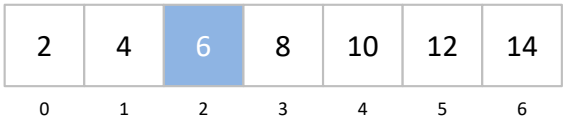



target = 5

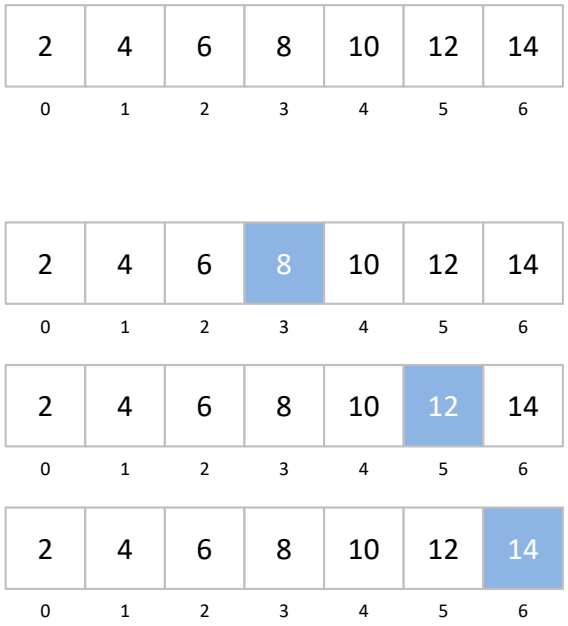


# Binary search

```
public int search(int[] a, int target) {
   int left = 0, right = a.length-1;
   while (left <= right) {
      int middle = (left + right) / 2;
      if      (target < a[middle])        right = middle - 1;
      else if (target > a[middle])        left = middle + 1;
      else                                return middle;
   }
   return -1;
}
```

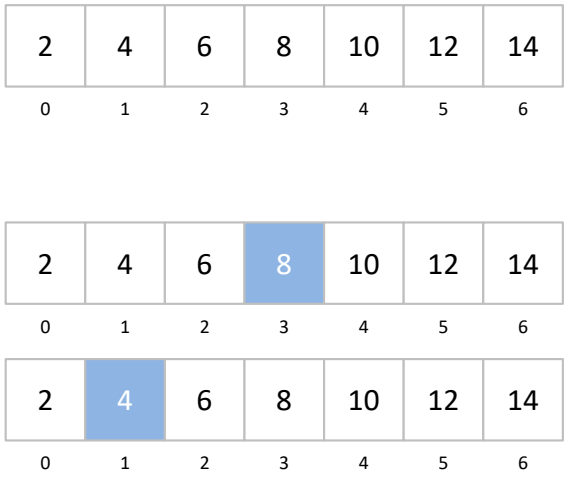



**Trace for target = 15**

| left | right | middle | a[middle] |
|---|---|---|---|
| 0 | 6 | 3 | 8 |
| 4 | 6 | 5 | 12 |
| 6 | 6 | 6 | 14 |
| 7 | 6 | / | / |

# Binary search

```
public int search(int[] a, int target) {
   int left = 0, right = a.length-1;
   while (left <= right) {
      int middle = (left + right) / 2;
      if      (target < a[middle])        right = middle - 1;
      else if (target > a[middle])        left = middle + 1;
      else                                return middle;
   }
   return -1;
}
```

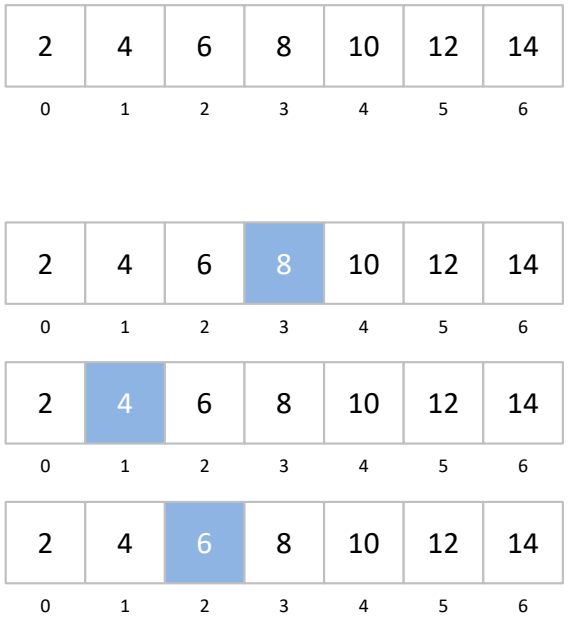



**Trace for target = 4**

| left | right | middle | a[middle] |
|---|---|---|---|
| 0 | 6 | 3 | 8 |
| 0 | 2 | 1 | 4 |

# Binary search

```
public int search(int[] a, int target) {
   int left = 0, right = a.length-1;
   while (left <= right) {
      int middle = (left + right) / 2;
      if      (target < a[middle])        right = middle - 1;
      else if (target > a[middle])        left = middle + 1;
      else                                return middle;
   }
   return -1;
}
```




## Trace for target = 5

| left | right | middle | a[middle] |
|---|---|---|---|
| 0 | 6 | 3 | 8 |
| 0 | 2 | 1 | 4 |
| 2 | 2 | 2 | 6 |
| 3 | 2 | / | / |

## Quick question:

**Q:** How many array elements will binary search examine when searching for the value 42 in the array below?

**A. 0**
**B. 3**
**C. 5**
**D. 7**
**E. 20**

```
l:r:m
0:19:9
10:19:14
15:19:17
18:19:18
19:19:19
```

| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 23 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

# Time Complexity: Characterizing efficiency

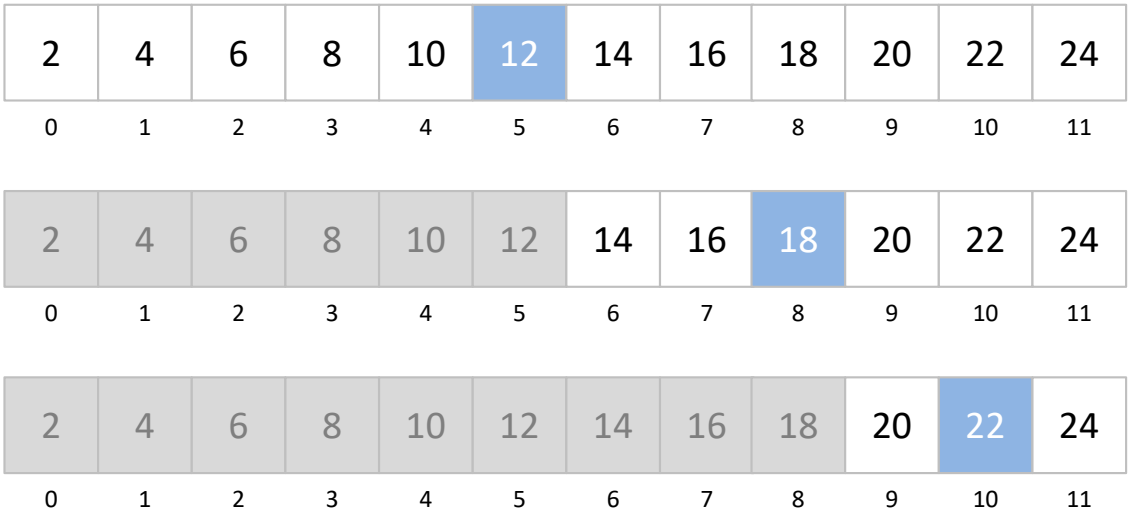
# Comparing efficiency

## Linear Search


| | |
|---|---|
| How many elements are eliminated by one comparison? | **1** |
| How many comparisons would be made in an array of size N? | **N** |
| How many *more* comparisons if the array doubled in size? | **N** |

# Comparing efficiency

## Binary Search

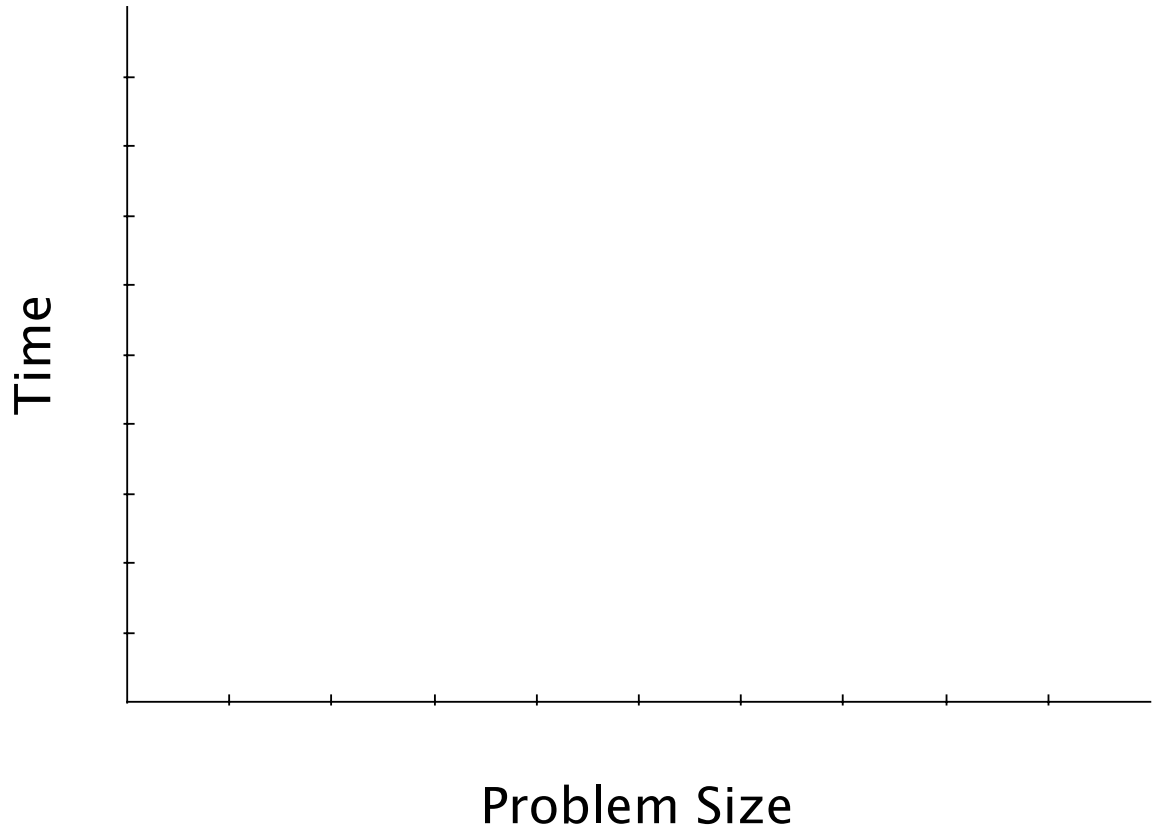



How many elements are eliminated by one comparison? **half**

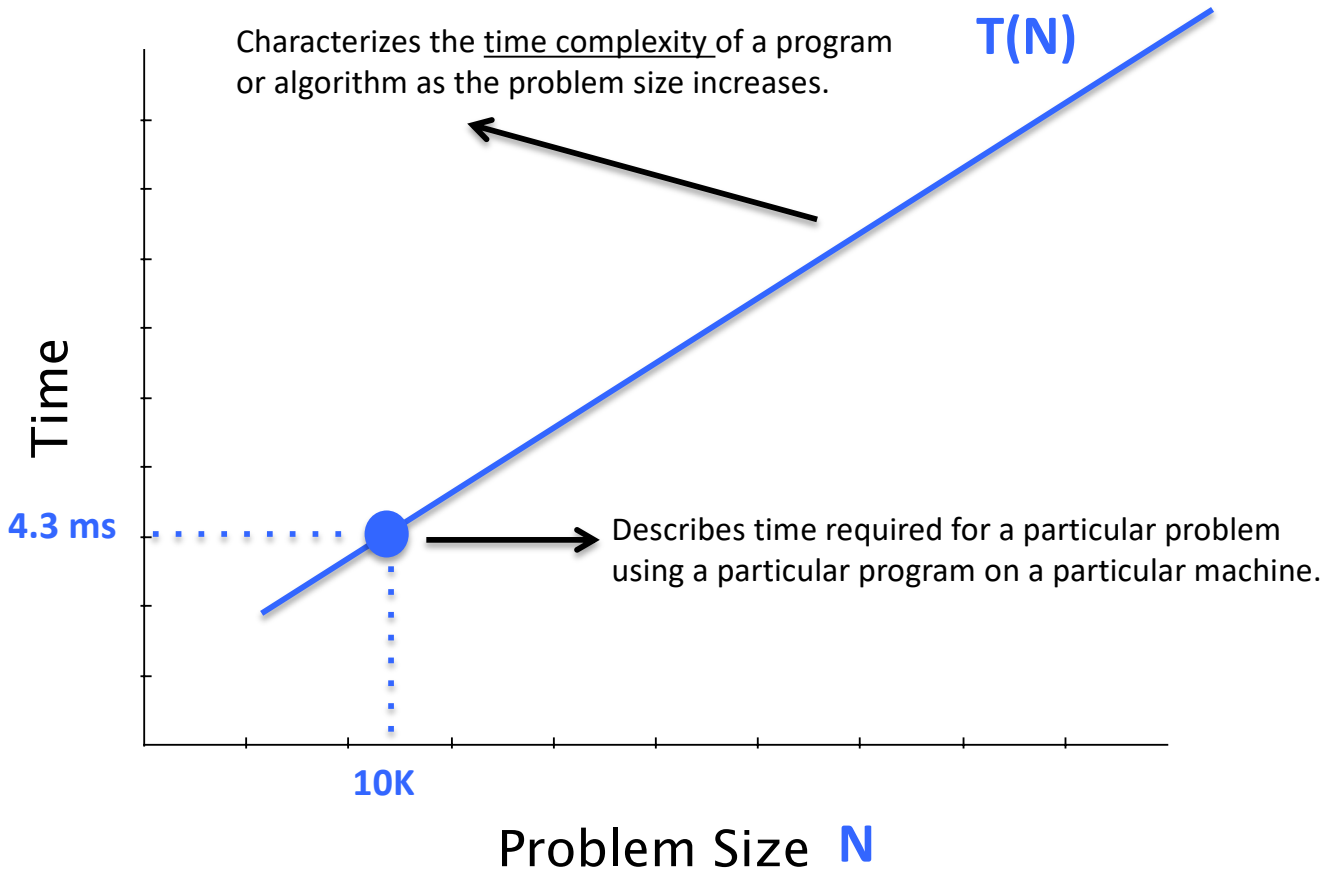
How many comparisons would be made in an array of size N? **~$\log_2 N$**

How many *more* comparisons if the array doubled in size? **1**

## Categorizing running time



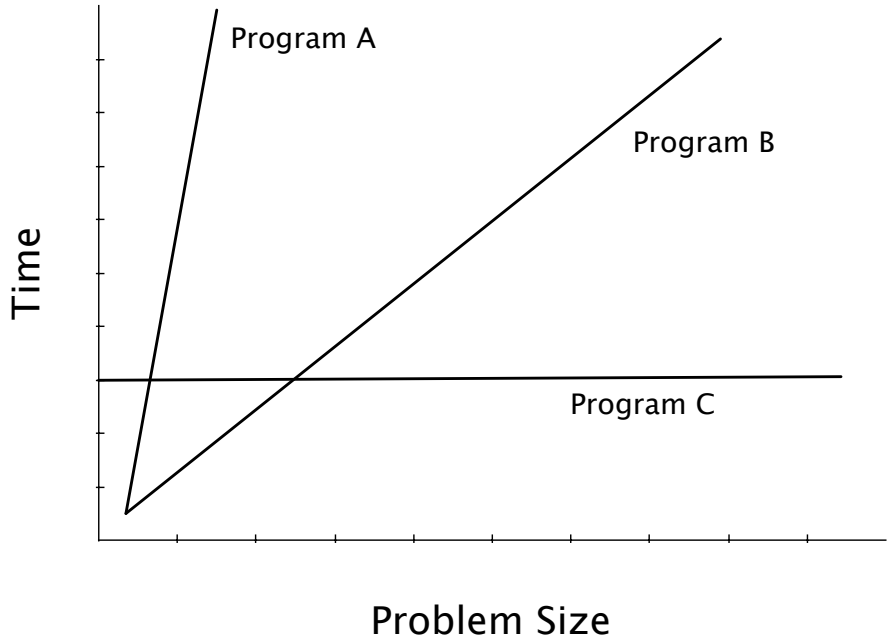
## Categorizing running time



## Quick question:

**Q:** Which program offers the best performance as the problem size scales up?

**A. Program A**
**B. Program B**
**C. Program C**

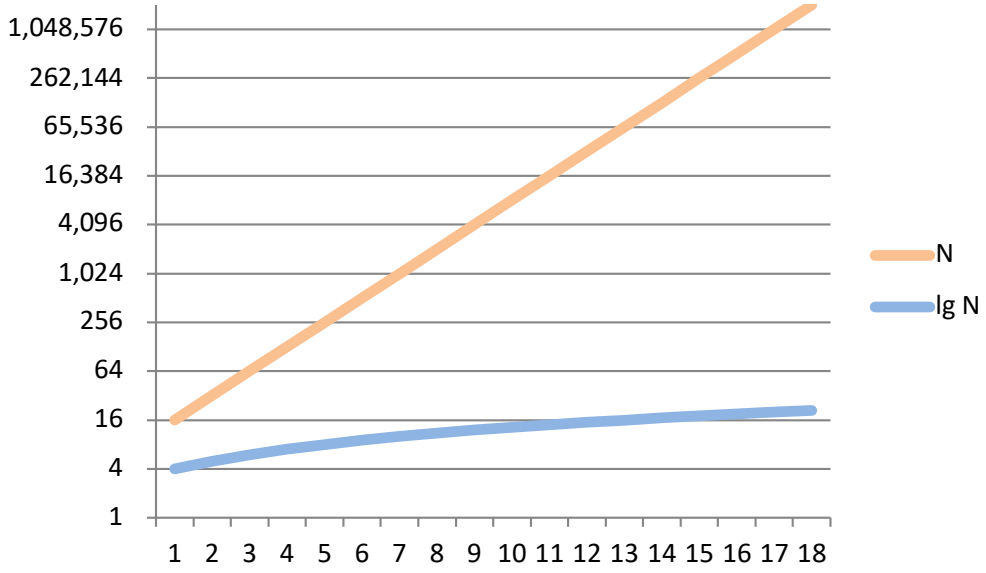


# Linear search v. binary search

**Number of middle comparisons for a list of size N**

Linear search: ~N

Binary search: ~lg N




| N | lg N |
|---|---|
| 16 | 4 |
| 32 | 5 |
| 64 | 6 |
| 128 | 7 |
| 256 | 8 |
| 512 | 9 |
| 1,024 | 10 |
| 2,048 | 11 |
| 4,096 | 12 |
| 8,192 | 13 |
| 16,384 | 14 |
| 32,768 | 15 |
| 65,536 | 16 |
| 131,072 | 17 |
| 262,144 | 18 |
| 524,288 | 19 |
| 1,048,576 | 20 |
| 2,097,152 | 21 |

# Comparison