Error en la Integración Numérica

$$f(x) \approx P(x)$$

$$\int_a^b f(x)\,dx \approx \int_a^b P(x)\,dx$$

$$E = \int_a^b f(x)\,dx - \int_a^b P(x)\,dx$$

$$E = \int_a^b (f(x) - P(x))\,dx = \int_a^b \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \prod_{i=0}^{n} (x - x_i)\,dx$$

• Caso I: (Trapecio)

$n=1$ $$E_T = \int_{x_0}^{x_1} \frac{f^{(2)}(\xi)}{(1+1)!} \cdot \underbrace{(x-x_0).(x-x_1)}_{\text{Tomamos: } g(x) \neq 0}\,dx$$

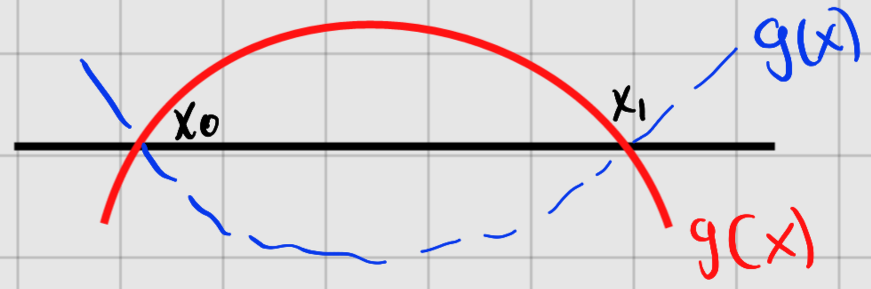
Recordar: $$\int_a^b f(x).g(x)\,dx = f(c)\int_a^b g(x)\,dx$$

suponemos: $g(x) \neq 0$

$x \in [a;b]$

$\Rightarrow \exists c \in \langle a;b \rangle$

Como $g(x) \neq 0 \; ; x \in \langle x_0 ; x_1 \rangle$



$$\exists \; \xi \in \langle x_0 ; x_1 \rangle$$

$$E_T = \frac{f^{(2)}(\xi)}{2} \int_{x_0}^{x_1} (x - x_0)(x - x_1)\, dx$$

Recordar: $x = x_0 + hs$

$x = x_1 + h(s-1)$

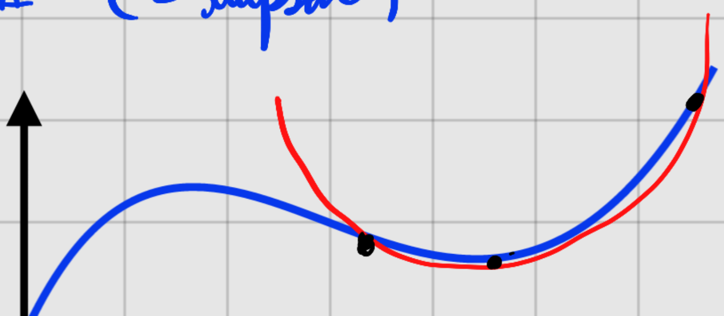
$$= \frac{f^{(2)}(\xi)}{2} \int_0^1 h.s.h(s-1).h\, ds$$

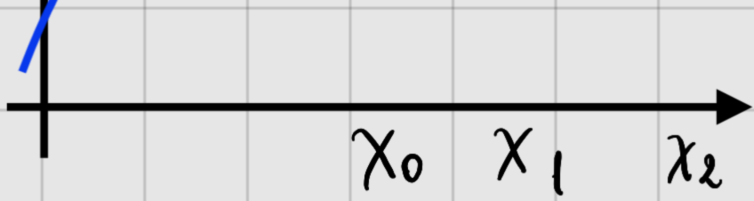
$$= \frac{h^3 f^{(2)}(\xi)}{2} \int_0^1 s(s-1)\, ds$$

$$|E_T| = \left| -\frac{h^3}{12} f^{(2)}(\xi) \right|$$

• Caso II: (Simpson)

$n = 2:$





$$E_S = -\frac{h^5}{90} f^{(4)}(\xi)$$

Además:

$$E_{CT} = -\frac{n}{12} h^3 f^{(2)}(\xi)$$

$$E_{CS} = -\frac{n}{180} h^5 f^{(4)}(\xi)$$

Ejemplo: Aproxime el valor del sólido de revolución que se obtiene, cuando la región plana:

$$D = \begin{cases} y = e^{-x^2} \\ y = 0;\ x = 0;\ x = 2 \end{cases}$$

gira alrededor del eje x empleando 7 nodos

$$V = \pi \int_0^2 \left(e^{-x^2}\right)^2 dx \qquad h = \frac{2-0}{6} = \frac{1}{3}$$

x	y

0	1
$1/3$	$0,80074$
$2/3$	$0,41111$
1	$0,13534$
$4/3$	$0,02857$
$5/3$	$0,00386$
2	$0,00034$

$$\Rightarrow V \approx \pi \left(\frac{1/3}{3}\right) \cdot \left(1 + 0,00034 + 4\sigma_1 + 2\sigma_2\right)$$

$$\approx 1,96854$$

Ejem:

Aproxime el valor de π empleando 7 nodos

$n+1=7 \rightarrow n=6$

$$\pi = 2\int_{-1}^{1} \sqrt{1-x^2}\, dx$$

$\Rightarrow x_0 = -1$

$x_n = 1$

$h = \frac{x_n - x_0}{n} = \frac{2}{6} = 0,3$

x	y
-1	0
$-\frac{2}{3}$	0,7454
$-\frac{1}{3}$	0,9428
0	1
$\frac{1}{3}$	0,9428
$\frac{2}{3}$	0,7454
1	0

$$\Rightarrow E_{CS} = \frac{0,3}{3}\left(0+0+4\left(0,7454+1+7454\right)+2\left(0,9428+0,9428\right)\right)$$