- Ejemplo Práctico?
  - Vida real o Deco

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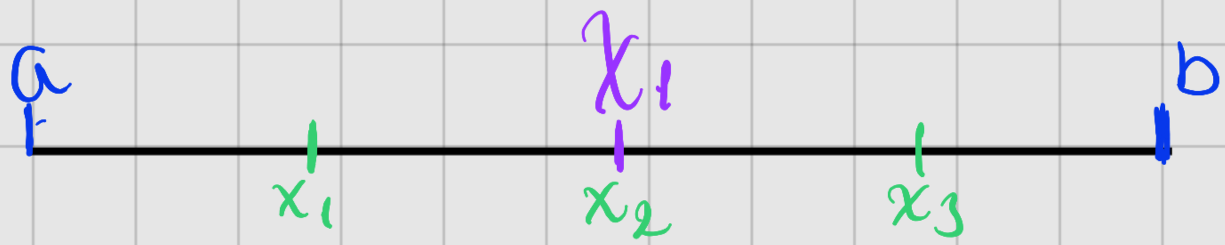
# Reglas recursivas de Integración

## Caso I: Trapecio




$$T_0 = \frac{h_0}{2}\left(f(a)+f(b)\right) \; ; \; h_0 = b-a$$

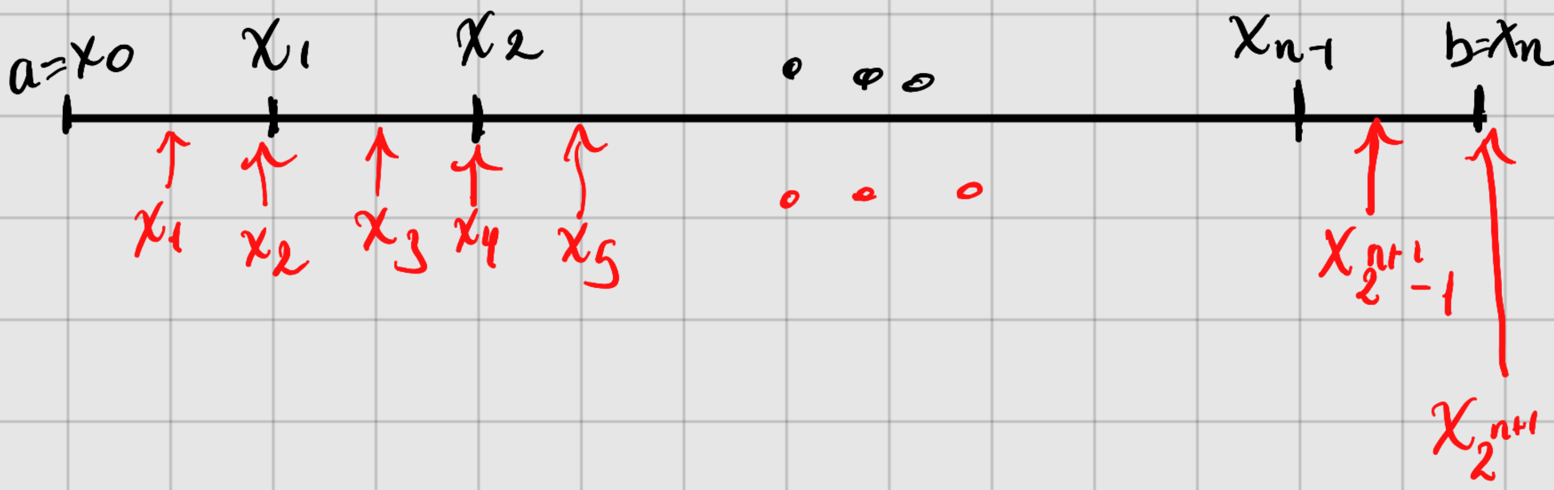
$$T_1 = \frac{h_1}{2}\left(f(a)+2f(x_1)+f(b)\right) ; \; h_1 = \frac{h_0}{2}$$

$$T_2 = \frac{h_2}{2}\left(f(a)+2\left(f(x_1)+f(x_2)+f(x_3)\right)+f(b)\right) ; \; h_2 = \frac{h_1}{2}$$

$$\vdots$$

$$\Rightarrow T_n = \frac{h_n}{2}\left[f(a)+2\sum^{n-1} f(x_i) + f(b)\right]$$

$i=1$




$$T_{n+1} = \frac{h_{n+1}}{2}\left[f(a) + 2\left(f(x_1) + f(x_2) + \dots + f(x_{2^{n+1}-1})\right) + f(b)\right]$$

$$= \frac{1}{2}\left(\frac{h_n}{2}\right)\cdot\left[f(a) + 2\left(f(x_2) + f(x_4) + \dots + f(x_{2^{n+1}-2})\right) + f(b) + 2\left(f(x_1) + f(x_3) + \dots + f(x_{2^{n+1}-1})\right)\right]$$

$$= \frac{1}{2}\left[\frac{h_n}{2}\left(f(a) + 2\left(f(x_2) + f(x_4) + \dots + f(x_{2^{n+1}-2})\right) + f(b)\right) + h_n\left(f(x_1) + f(x_3) + \dots + f(x_{2^{n+1}-1})\right)\right]$$

$$= \frac{1}{2} T_n + \frac{h_n}{2}\sum_{k=1}^{2^n} f(x_{2k-1}) \; ; \; n \geq 0$$

$$T_{n+1} = \frac{1}{2} T_n + h_{n+1} \cdot \sum^{2^n} f(x_{2k-1}) \; ; \; n \geq 0$$

$k=1$

* No es autocorrectivo

Analogamente:

Caso II: Regla recursiva de Simpson

$$S_{n+1} = \frac{4T_{n+1} - T_n}{3} \; ; \; n \geq 0$$

Caso III: Regla de Boole:

$$B_{n+1} = \frac{16\, S_{n+1} - S_n}{15}$$

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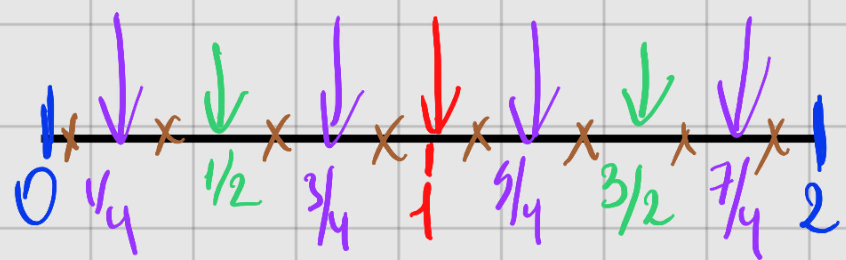
Ejm:

Dado $f(x) = 2^{x^2-1}$, $x \in [0;2]$

Aproxime el Area de la región R comprendida entre las rectas $x=0$; $x=2$; $y=0$ y la gráfica de $f$

$$A(R) = \int_0^2 2^{x^2-1}\, dx$$



$$n=0 : h_0 = 2-0 ; \quad T_0 = \frac{h_0}{2}\left(f(0)+f(2)\right) = 8{,}5$$

$$n=1 : h_1 = 1 ; \quad T_1 = \frac{T_0}{2} + h_1\left(f(1)\right) = 5{,}25$$

$$n=2 : h_2 = \frac{1}{2} ; \quad T_2 = \frac{T_1}{2} + h_2\left(f(1/2)+f(3/2)\right) = 4{,}11151$$

$$n=3 : h_3 = \frac{1}{4} ; \quad T_3 = \frac{T_2}{2} + h_3\left(f(1/4)+f(3/4)+f(5/4)+f(7/4)\right) = 3{,}78437$$

$$n=4 : h_4 = 1/8 ; \quad T_4 = \frac{T_3}{2} + h_4\left(f(1/8)+f(3/8)+\dots+f(13/8)+f(15/8)\right)$$

$$T_4 = 3{,}69903$$

| $n$ | $T_n$ | $S_n = \frac{4T_n - T_{n-1}}{3}$ | $B_n = \frac{16S_n - S_{n-1}}{15}$ |
|---|---|---|---|
| 0 | $T_0 = 8{,}5$ | | |
| 1 | $T_1 = 5{,}25$ | $S_1 = 4{,}16667$ | |
| 2 | $T_2 = 4{,}11151$ | $S_2 = 3{,}73201$ | $B_2 = 3{,}70302$ |
| 3 | $T_3 = 3{,}78437$ | $S_3 = 3{,}67532$ | $B_3 = 3{,}67150$ |
| 4 | $T_4 = 3{,}69903$ | $S_4 = 3{,}67058$ | $B_4 = 3{,}670264$ |

* Este esquema es conocido como la Regla de Romberg

En laboratorio:

1) Extrapolación de Richardson

Datos: $f(x)$, $x_0$, $n$: # iteraciones, $h$

$\Rightarrow$ Rsltd: $\dfrac{df(x_0)}{dx}$

2) Reglas Compuestas de Trapecio y Simpson

- Además de Romberg