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To proof that the clique problem is np-complete we have to proof to statement—

(i) The clique problem is NP.

(ii) The clique problem is NP-hard which means ~~Th~~ we can reduce another NP-complete problem into this clique problem.

(i) To belong to the NP class, we have to verify ~~the~~ a given solution to the problem in polynomial time.

It is always possible to verify in polynomial time. If we provide a subset of vertices to verify that if it is the solution to the ~~cli~~ clique problem then we first check if it's size is at least K. Then we iterate over all pair of the given verdices set and ~~che~~ check if there is an edge between

them. If there exists all the edges then we can say that the solution is verified. It take $O(n^2)$ time.
Thus the problem is in NP-class.

(ii) We know that Independent set problem is a NP-complete problem. In this problem we have to say that if there exists ~~at least one~~ a subset of vertices where any pair of vertices in the subset does not have an edge and the size of subset is at least K.

Now, if we change the edge set of graph, then we can solve the problem with clique problem. We ~~define an~~ create a new graph with same vertices and define the edgeset ~~such that~~ with all pair except the edges that belongs to the main graph. Now if the new graph has a clique with size at least K then we can say that there is an ~~vertices set~~ independent set with size at least k. Because if the new graph has a clique of size k then all the nodes in that set does not have an edge with another node in that set in the original graph.

Thus,

$$A \leq_p B$$

where, A = independent set problem
B = clique problem.

Thus, clique problem is NP-hard and it belongs to NP class. hence, clique problem is NP complete.