Student ID: 1805106

(1) (a) here, $a = 8$, $b = 20$, $n = 4$

$$h = \frac{b-a}{n} = \frac{12}{4} = 3$$

$f(8) = 177.2667$

$f(8+6) = f(14) = 339.295$

$f(8+3) = f(11) = 252.847$

$f(8+9) = f(17) = 422.142$

$f(20) = 517.35$

Using simpson's ⅓ rule,

$$\int_8^{20} f(t)\,dt = \frac{h}{3}\left[f(8) + 4f(11) + 4f(17) + 2f(14) + f(20)\right]$$

$$= 4063.0627 \text{ m}$$

(b) here, $a = 20$, $b = 30$, $n = 2$

$$h = \frac{b-a}{2} = \frac{10}{2} = 5$$

$f(20) = 517.35$

$f(30) = 901.674$

$f(25) = 695.007$

$$\therefore \int_{20}^{30} f(t)\,dt = \frac{h}{2}\left[f(20) + 2f(25) + f(30)\right] = 7022.595 \text{ m}$$

(c)

$$\therefore \int_8^{30} f(t)\,dt = \int_8^{20} f(t)\,dt + \int_{20}^{30} f(t)\,dt = 4063.0627 + 7022.595$$

$$= 11085.6577 \text{ m}$$

Student ID: 1805106

(2) here, $f(0) = \sin(0) = 0$ $\quad f(\pi/2) = \sin(\frac{\pi}{2}) = 1$

$f(\pi/4) = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$x_1 = 0,\ y_1 = 0,\ x_2 = \frac{\pi}{4},\ y_2 = \frac{1}{\sqrt{2}},\ x_3 = \frac{\pi}{2},\ y_3 = 1$

$f_1[x_1] = 0,\ f_1[x_2] = \frac{1}{\sqrt{2}},\ f_1[x_3] = 1$

$$f_1[x_2, x_1] = \frac{f_1[x_2] - f_1[x_1]}{x_2 - x_1} = \frac{\frac{1}{\sqrt{2}}}{\frac{\pi}{4}} = 0.9$$

$$f_1[x_3 x_2] = \frac{f_1[x_3] - f_1[x_2]}{x_3 - x_2} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{\pi}{4}} = 0.373$$

$$f_1[x_3, x_2, x_1] = \frac{f_1[x_3, x_2] - f_1[x_2, x_1]}{x_3 - x_1} = \frac{0.373 - 0.9}{\frac{\pi}{2}}$$

$$= -0.3355$$

$$\therefore f(x) = b_0 + b_1(x - x_1) + b_2(x - x_1)(x - x_2)$$

$$= f_1[x_1] + f_1[x_2, x_1](x - 0) + f_1[x_3, x_2, x_1](x - 0)(x - \frac{\pi}{4})$$

$$= 0 + 0.9x - 0.3355\,x(x - \frac{\pi}{4})$$

$$= -0.3355x^2 + 1.1635x$$

(Ans.)

Student ID: 1805106

(4) As we are considering polynomial of order 3, we can select 4 closest point of value 4, (2, 12), (3, 19), (5, 33), (7, 51) can be picked

$$L_1(4) = \prod_{\substack{i=1 \\ j \neq 1}}^{4} \frac{4 - x_j}{x_1 - x_j} = \frac{4-3}{2-3} \times \frac{4-5}{2-5} \times \frac{4-7}{2-7} = \frac{1}{-1} \times \frac{-1}{-3} \times \frac{-3}{-5}$$
$$= -\frac{1}{5}$$

$$L_2(4) = \prod_{\substack{i=1 \\ j \neq 2}}^{4} \frac{4 - x_j}{x_2 - x_j} = \frac{4-2}{3-2} \times \frac{4-5}{3-5} \times \frac{4-7}{3-7} = \frac{2}{1} \times \frac{-1}{-2} \times \frac{-3}{-4}$$
$$= \frac{3}{4}$$

$$L_3(4) = \prod_{\substack{i=1 \\ j \neq 3}}^{4} \frac{4 - x_j}{x_3 - x_j} = \frac{4-2}{5-2} \times \frac{4-3}{5-3} \times \frac{4-7}{5-7} = \frac{2}{3} \times \frac{1}{2} \times \frac{-3}{-2}$$
$$= \frac{1}{2}$$

$$L_4(4) = \prod_{\substack{i=1 \\ j \neq 4}}^{4} \frac{4 - x_j}{x_4 - x_j} = \frac{4-2}{7-2} \times \frac{4-3}{7-3} \times \frac{4-5}{7-5} = \frac{2}{5} \times \frac{1}{4} \times \frac{-1}{2}$$
$$= -\frac{1}{20}$$

$$\therefore f(4) = \sum_{i=1}^{4} L_i(4) \times f(x_i)$$
$$= -\frac{1}{5} \times 12 + \frac{3}{4} \times 19 + \frac{1}{2} \times 33 - \frac{1}{20} \times 51$$
$$= 25.8$$

(Ans.)

(6) here, $v = \sqrt{2gH} \tanh\left(\frac{\sqrt{2gH}}{2L} t\right)$

$v = 6.5$, $t = 2$, $L = 4.5$,

$h_l = 0$, $h_u = 3$ (max height)

$0 = \left\{\sqrt{19.6H} \tanh\left(\frac{\sqrt{19.6H}}{4.5}\right)\right\} - 6.5$

$\Rightarrow f(H) = 0$

here, $f(h_l) \times f(h_u) < 0$

$f(h_l) = -6.5$

$f(h_u) = 0.67$

$f(h_l) \times f(h_u) < 0$

Iteration 1:

$h_m = \frac{0+3}{2} = 1.5$

$f(h_m) = 5.42 \times \tanh(1.2) - 6.5$

$= -1.97972$

As $f(h_m) < 0$, $h_l = h_m = 1.5$

Error = N/A

Iteration 2:

$h_m = \frac{h_l + h_u}{2} = \frac{1.5 + 3}{2} = 2.25$

$f(h_m) = -0.523$

As $f(h_m) < 0$, $h_l = h_m = 2.25$

$E = \frac{2.25 - 1.5}{2.25} \times 100 = 33\%$

Iteration 3:

$h_m = \frac{2.25+3}{2} = 2.625$

$f(h_m) = 6.6$, As $f(h_m) > 0$, $h_u = 2.625$

$E = \frac{2.625 - 2.25}{2.625} = 14.286\%$

Ans: 2.625 m

Student ID : 1805106

7

$$y = axe^{bx}$$

~~$\partial y \quad E = y$~~

$$E_s = \sum E_i^2 = \sum \left(y_i - a x_i e^{bx_i}\right)^2$$

$$\frac{\partial E_s}{\partial a} = \sum 2 \left(y_i - a x_i \cdot e^{bx_i}\right)$$