University of Dhaka

Department of Applied Mathematics

Fourth Year B.S. (Honors), Academic Session: 2023-2024

Course Title: Math Lab IV (Application Software), Course Code: AMTH 450

Assignment No.: 6 (Real-life Applications of Mathematical Finance)

Name:	Roll No.:	Group:

Instruction: *Write appropriate programming codes using* **Python** *to get the outputs of each of the following problems and hence visualize them properly.*

1. Which of the two investments described below is preferable? Assume the first payment will take place exactly one year from now and further payments are spaced one year apart. Assume the continually compounded annual interest rate is 4.33%.

Year	1	2	3	4	5
Investment A	225	215	250	225	205
Investment B	220	225	250	250	210

2. A 5-year bond with a yield of 11% (continuously compounded) pays an 8% coupon at the end of each year.
   (a) What is the bond's price?
   (b) What is the bond's duration?
   (c) Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.
   (d) Recalculate the bond's price on the basis of a 10.8% per annum yield and verify that the result is in agreement with your answer to (c)

3. Suppose an investor will receive payments at the end of the next 6 years in the amounts shown in the table below.

Year	1	2	3	4	5	6
Payment	460	235	640	370	330	250

If the interest rate is 4.5% compounded quarterly, what is the present value of the investments, given that the first payment will arrive one year from now?

4. A 1-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 10% per annum with continuous compounding.
    (a) What are the forward price and the initial value of the forward contract?
    (b) Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

5. Write a suitable Python code to sketch the Payoff and Profit diagrams of the following options:
    (a) Long call
    (b) Short call
    (c) Long put
    (d) Short put
    Consider $K = \$150$ & premium = $5 for all cases.

6. A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a suitable diagram showing the variation of the trader's profit with the asset price.

7. The price of a non-dividend-paying stock is $19 and the price of a 4-month European call option on the stock with a strike price of $20 is $1. The risk-free rate is 3% per annum. What is the price of a 4-month European put option with a strike price of $20?

8. Suppose that the price of a non-dividend-paying stock is $32, its volatility is 30%, and the risk-free rate for all maturities is 5% per annum. Use Python to calculate the cost of setting up the following positions:
    (a) A bull spread using European call options with strike prices of $25 and $30 and a maturity of 6 months.
    (b) A bear spread using European put options with strike prices of $25 and $30 and a maturity of 6 months.
    (c) A butterfly spread using European call options with strike prices of $25, $30, and $35 and a maturity of 1 year.

(d)   A butterfly spread using European put options with strike prices of $25, $30, and $35 and a maturity of 1 year.

(e)   A straddle using options with a strike price of $30 and a 6-month maturity.

(f)   A strangle using options with strike prices of $25 and $35 and a 6-month maturity.

In each case provide a table showing the relationship between profit and final stock price. Ignore the impact of discounting.

9.   Consider an option on a non-dividend-paying stock when the stock price is $30, the exercise price is $29, the risk-free interest rate is 5%, the volatility is 25% per annum, and the time to maturity is 4 months.

(a)   What is the price of the option if it is a European call?
(b)   What is the price of the option if it is an American call?
(c)   What is the price of the option if it is a European put?
(d)   Verify that put–call parity holds.

10.   A futures price is currently 60 and its volatility is 30%. The risk-free interest rate is 8% per annum. Use a two-step binomial tree to calculate the value of a six-month European call option on the futures with a strike price of 60. If the call were American, would it ever be worth exercising it early?

11.   A 2-month American put option on a stock index has an exercise price of 480. The current level of the index is 484, the risk-free interest rate is 10% per annum, the dividend yield on the index is 3% per annum, and the volatility of the index is 25% per annum. Divide the life of the option into four half-month periods and use the tree approach to estimate the value of the option.