Let us first consider the number of bullets in the modified game by b and number of chambers be n. Then, in this game, the probability of survival will be given by

$$P(surviving) = \frac{n-b}{n} \times \frac{n-b-1}{n-1}$$

Because, for b bullets, there are $n-b$ empty slots, also after one shot, the number of available empty slots will decrease by one and so is the number of chambers.

Here we assume that the gun is not reloaded after one shot, which seems the most rational as per the statement.

Hence, we want,

$$\frac{n-b}{n} \times \frac{n-b-1}{n-1} \geq \frac{1}{2}$$

By hit and trial, we get the solution for lowest b, n that is 1 bullet and 4 chambers.

Now for the second lowest solution, we find it by brute force using a program that checks for each value of b,n (where b less than equal to 9)

The combination that gives probability closest to 0.5 is with 3 bullets and 11 chambers.

Here the probability of surviving in 28/55 i.e. 0.50909.. $\approx$ 51%

Thus required t = 3, x = 11

Now for the mental hospital game, we have to consider 5 bullets and 11 chambers. And hence, the probability of surviving in this game is

$$P(surviving) = 6/11 \times 5/10 = \frac{3}{11} \approx 27\%$$

Let n be the number of patients = 100

Suppose when doctor has k number of patients.

We define the probability of doctor's winning recursively as

$$P(k) = P(surviving) \times P(k+1) + P(\overline{surviving}) \times P(k-1)$$

$$11P(k) = 3P(k+1) + 8P(k-1)$$

As base case, when there are zero patients, the doctor has already won, hence $P(0) = 1$, Also, when there is only one patient.

On the other hand, the doctor loses when the number of patients double,

hence $P(2n) = 0$
Solving for the general solution of the linear recurrence, we get the solution:

$$P(k) = c_1 \left(\frac{8}{3}\right)^k + c_2$$

Substituting the values, we get

$$P(0) = 1 = c_1 + c_2, P(2n) = 0 = c1 \left(\frac{8}{3}\right)^{2n} + c_2$$

Let $y = \left(\frac{8}{3}\right)^n$ Then we get 2 simple equations

$$c_1 + c_2 = 1, c_1 y^2 + c_2 = 0$$

Also we need $P(n) = c_1 y + c_2$
Solving the equations, we get

$$P(n) = \frac{y}{y+1} = \frac{8^n}{8^n + 3^n}$$

$$\mathbf{P(100)} = \frac{\mathbf{8^{100}}}{\mathbf{8^{100} + 3^{100}}}$$

Which is almost equal to 1
It also matches with our intuition that when there is only a 27% chance of a patient surviving ,then the probability that number of patient doubles is almost negligible, Hence the doctor almost always wins.