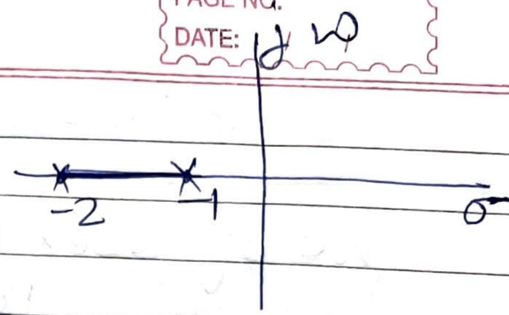
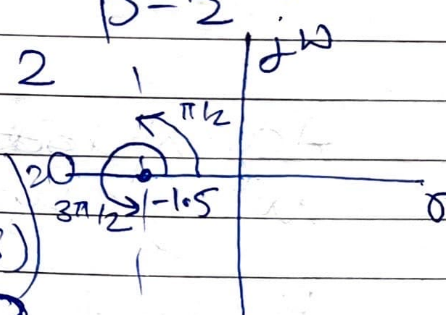
1 a) $KG(S)H(S) = \dfrac{K}{(S+1)(S+2)}$



No of open loop poles, = 2 (= P)   Poles -1, -2
No of open loop zeroes = 0 (= Z)
∴ No of branches of root locus = max(P, Z) = 2
Real axis segment → (-2, -1)
No of asymptotes = |P - Z| = 2
Real axis intercept, $\sigma_a = \dfrac{(-2)+(-1)}{2-0} = -1.5$

Asymptote angles, $\theta_a = \dfrac{(2r+1)\pi}{P-Z}$, r = 0, 1

$\Rightarrow \theta_a = \pi/2, 3\pi/2$



Breakaway point

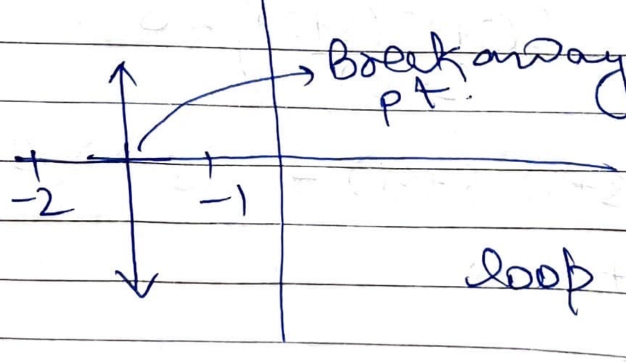
$\dfrac{dK}{ds} = 0 \Rightarrow \dfrac{d}{ds}\left(\dfrac{-1}{G(S)H(S)}\right) = 0$

$\dfrac{d}{ds}\left[(S+1)(S+2)\right] = 0$

$2S + 3 = 0 \Rightarrow S = -3/2$

$-1.5 \in (-2, -1)$. So -1.5 is a valid breakaway point. Breakaway angle $= \dfrac{180°}{2} = 90°$

Sketch



No jω-crossing for all values of K > 0, the closed loop system is stable

1 (b) $K\,G(S)\,H(S) = \dfrac{K(S+3)}{(S+1)(S+2)}$

no of open loop poles = 2 (= P)

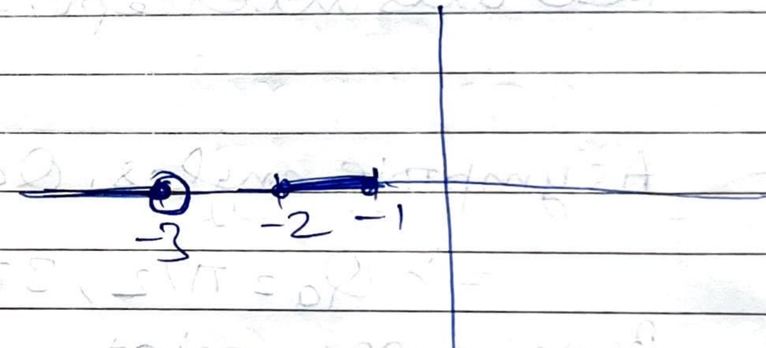
Poles = $-1, -2$

no of open loop zeroes = 1 (= Z)

zeroes = -3

∴ No of branches of root locus = max (P, Z) = 2

Real-axis segment = $(-\infty,-3) \cup (-2,-1)$

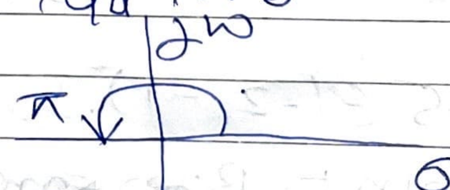


Asymptotes.

no of asymtotes = $|P-Z| = 1$

Real axis intercept, $\sigma_a = \dfrac{(-1)+(-2)-(-3)}{2-1} = 0$

Angle, $\theta_a = \dfrac{(2r+1)\pi}{2-1}, r=0 \Rightarrow \theta_a = \pi$ $\cancel{\text{or } 0}$



There will be a breakaway point b/w -2 & -1 & a breakin point b/w $-\infty$ & -3

$\dfrac{dK}{ds} = 0 \Rightarrow \dfrac{d}{ds}\left(\dfrac{-(s+1)(s+2)}{(s+3)}\right) = 0$

$(s+3)(s+2) + (s+1)(s+3) - (s+1)(s+2) =$

$\Rightarrow s^2 + 6s + 7 = 0$

$s = \dfrac{-3 \pm \sqrt{2}}{2}$

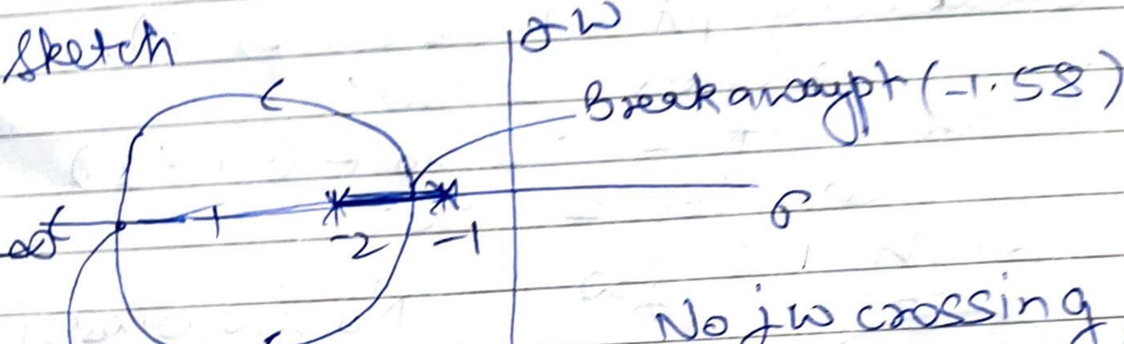
Break away points = ~~...~~ $-1.58 \in (-2,-1)$

Breakaway angle $= 180/2 = 90°$

Breakin point $= -4.41 \in (-\infty, -3)$

Breakin angle $= 180/2 = 90°$

Sketch



No jw crossing for all values of K>0, the closed Loop system is stable

1.C) $KG(s)H(s) = \dfrac{K(s+3)(s+4)}{(s+1)(s+2)}$

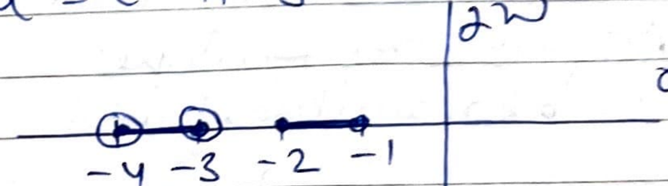
no of open loop poles = 2 (= P) Poles −1, −2

no of open loop zeroes = 2 (= Z) zeroes = −3, −4

∴ No of branches of root locus = max

max (P, Z) = 2

Real axis segment = $(-4,-3) \cup (-2,-1)$



Asymptotes

no of asymptotes = |P−Z| = 0 . no asymptotes

There will be a breakaway point b/w −2 & −1 & a breakin point between −4 & −3

Breakaway/in pt. calculation (another method)

$$\frac{1}{s+3} + \frac{1}{s+4} = \frac{1}{s+1} + \frac{1}{s+2}$$

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$\Rightarrow \frac{2s+7}{(s^2+7s+12)} = \frac{2s+3}{(s^2+3s+2)}$

$\Rightarrow 13s^2+25s+14 = 17s^2+45s+36$

$\Rightarrow \cancel{s^2+5s} - 4s^2+20s+22=0$

$\Rightarrow s = -1.63, -3.36$

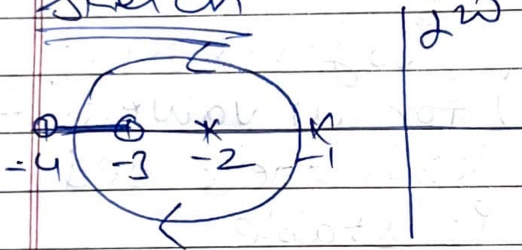
Breakaway pt $= 1.63 \in (-2,-1)$.

" angles $= 90'$

Breakin pt $= -3.36 \in (-4,-3)$

" angles $= 90°$

Sketch



No jω crossing for all $K>0$, the closed loop system is stable.

1.d. $KG(s)H(s) = \frac{K(s+2)(s+4)}{(s+1)(s+3)}$

No of open loop poles $= 2 (=P)$ Poles $= -1, -3$

no of open loop zeroes $= 2 (=Z)$ zeroes $-2, -4$

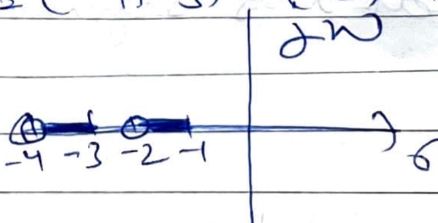
$\therefore$ No of branches of root locus $= \max(P, Z)$
$= 2$

Real axis segment $= (-4,-3) \cup (-2,-1)$

Asymptotes → None

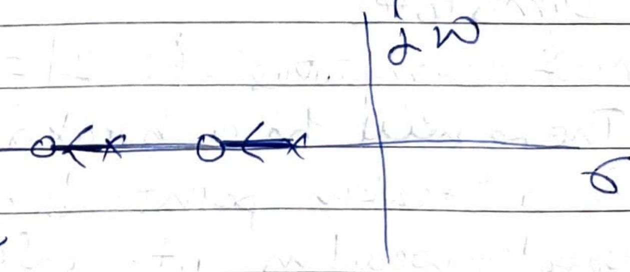
no breakaway/breakin pt.

No jω crossing



Sketch

for all $K>0$, the closed loop system is stable



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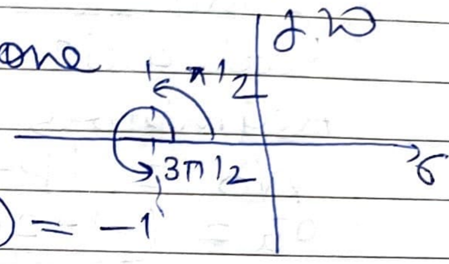
1.e $KG(s)H(s) = \frac{K}{(s^2+2s+5)} = \frac{K}{(s+1+2j)(s+1-2j)}$

open loop poles $= -1 \pm 2j$; $P = 2$

open loop zeroes – none ; $Z = 0$

∴ No of branches of root locus = max $(P, Z) = 2$

Real axis segment – none



Asymptotes

Number $= |P - Z| = 2$

$\sigma_a = \frac{(-1+2j) + (-1-2j)}{2} = -1$

$\theta_a = \frac{(2r+1)\pi}{2}, r = 0, 1 \Rightarrow \theta_a = \pi, 3\pi/2$

No breakaway/breakin pt

No jω crossing.

Angle of departure

for $-1+2j$

$-(\theta_{-1+2j} + \theta_{-1-2j}) = 180°$

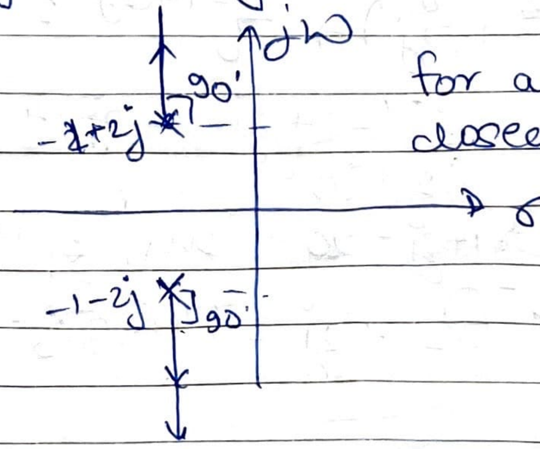
$\Rightarrow -\theta_{-1+2j} - \angle(-1+2j+1+2j) = 180°$

$\Rightarrow -\theta_{-1+2j} - 90 = 180 \Rightarrow \theta_{-1+2j} = -270° = 90°$

similarly angle of departure for

$\theta_{-1-2j} = -\theta_{-1+2j} = -90°$

Sketch



for all $K > 0$, the closed loop system is stable

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If $K G(s)H(s) = \frac{Ks}{s^2+2s+5} = \frac{Ks}{(s+1+2j)(s+1-2j)}$

Open loop poles $= -1 \pm 2j$, $P = 2$

open loop zeroes $= 0$ $Z = 1$

$\therefore$ No of branches of root locus
$= \max(P, Z) = 2$

Real axis segment $= (-\infty, -0)$

Asymptotes

No $= |P-Z| = 1$

$\sigma_a = \frac{(-1+2j)+(-1-2j)-0}{2-1} = -2$

$\theta_a = \frac{(2r+1)\pi}{2-1}, r=0 \Rightarrow \theta_a = \pi$

Breakin pt. b/w $-\infty$ & 0

$\frac{dK}{ds} = 0 \Rightarrow \frac{d}{ds}\left(\frac{-(s^2+2s+5)}{s}\right) = 0$

$(2s+2)s - (s^2+2s+5) = 0$

$s^2 = 5 \Rightarrow s = \pm\sqrt{5}$

Breakin pt $= -\sqrt{5} \in (-\infty, 0)$ Breakin angle
$= \frac{180}{2} = 90'$

Angle of departure
for pole at $-1+2j$

$-(\theta^{-1+2j}_{-1+2j} + \theta^{-1+2j}_{-1-2j}) + \theta^{-1+2j}_{0} = 180' \Rightarrow$

$-\theta_{-1+2j} - \angle(-1+2j-1-2j) + \angle(-1+2j+0)$
$= 180°$

$\Rightarrow -\theta_{-1+2j} - 90' + \tan^{-1}\left(\frac{2}{-1}\right) = 180°$

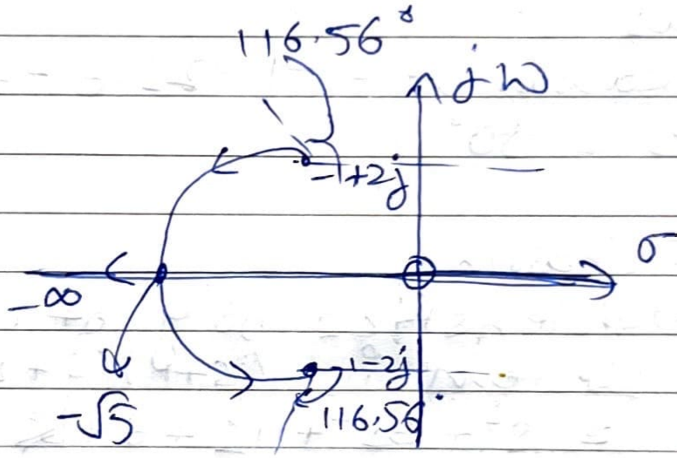
$\Rightarrow \theta_{-1+2j} = 116.56°$

for pole at $-1-2j$

$\theta_{-1-2j} = -\theta_{-1+2j} = -116.56^\circ$

No $j\omega$-crossing for all $k>0$ the closed loop system is stable

Sketch

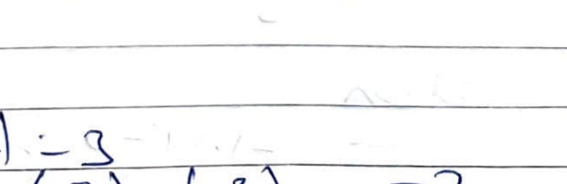


1.8) $KG(s)H(s) = \dfrac{K}{(s+1)(s+2)(s+3)}$

Open loop poles $= -1, -2, -3$. $P = 3$

″ ″ zeroes none $Z = 0$

∴ No of branches for root locus $= \max(P, Z) = 3$



Real axis segment $= (-\infty, -3) \cup (-2, -1)$

Asymptotes

No $= |P - Z| = 3$

$\sigma_a = (-1) + (-2) + (-3) = -2$

$\theta_a = \dfrac{(2r+1)\pi}{3}, \; r = 0, 1, 2 \Rightarrow \theta_a = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

Breakaway pt b/w -2 & -1

$\frac{dk}{ds} = 0 \Rightarrow \frac{d}{ds}\{-(s+1)(s+2)(s+3)\} = 0$

$\Rightarrow 3s^2+12s+11 = 0$

$\Rightarrow s = -1.423, -2.577$

$s = -2.577$ is discarded as it does not ~~be~~ lie on the root locus

Breakaway pt $= -1.423 \in (-2,-1)$ Breakaway angle $= 90°$

$j\omega$-crossing

characteristics poly of closed loop system is $(s+1)(s+2)(s+3)+k$

$= s^3+6s^2+11s+6+k$

let $s = j\omega$ be a root for some k

So $(j\omega)^3+6(j\omega)^2+11(j\omega)+6+k=0$

$\Rightarrow -j\omega^3 - 6\omega^2 + 11j\omega + 6 + k = 0$

$\Rightarrow 6+k = 6\omega^2$ & $11\omega - \omega^3 = 0$

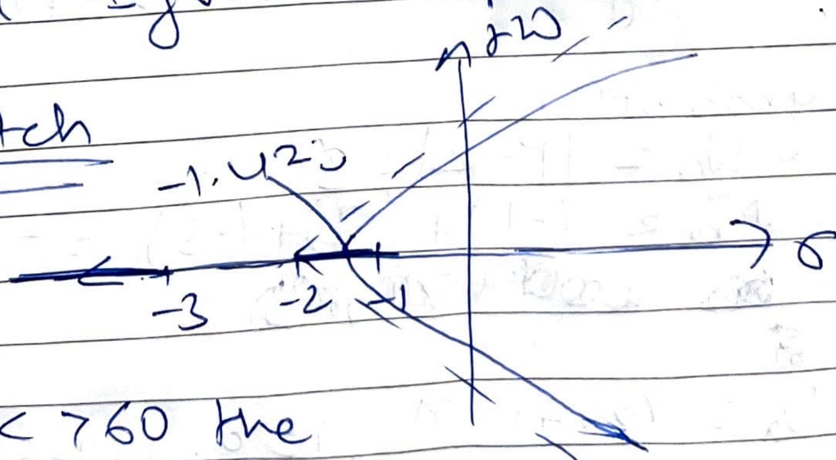
$\omega = 0$ or $\omega = \pm\sqrt{11}$

for $\omega = 0 \Rightarrow k = -6$ → discard

for $\omega = \sqrt{11}$ → $k = 60$ (feasible)

So $j\omega$ crossing happens for $k=60$ at $\pm j\sqrt{11}$

Sketch



for $k > 60$ the system becomes unstable.