

Essential Maths for DTC DPhil Students

Michaelmas Term 2022

Problem Sheet 2: indices, logarithms, exponentials

Introductory problems

1. Simplify:

a) $\frac{a^{10}}{a^{-3}}$

b) $\frac{(\sqrt{x})^8}{x^4}$

c) $\frac{y^{\frac{1}{4}}}{y^{-\frac{2}{4}}}$

d) $\frac{10^{-2/3} \times 10^7 \times 10^{-16} \times x^{1/2} \times y^4 \times z^{-1/3}}{10^{-19} \times 10^{43} \times 10^{2/3} \times z^{-1/3} \times y^{1/4} \times x^{5/2}}$

2. Evaluate the following expressions without using a calculator:

a) 2^{-4}

b) $36^{\frac{1}{2}} + 64^{\frac{2}{3}}$

c) $\left(\frac{1}{3}\right)^{-2}$

d) $\left(\frac{81}{9}\right)^{\frac{3}{2}}$

3. Express the following in logarithmic form:

a) $5^3 = 125$

b) $8^{-\frac{1}{3}} = \frac{1}{2}$

c) $x^y = 4$

4. Evaluate the following expressions without using a calculator:

a) $\log_4(64)$

b) $\log_{\pi}(1)$

c) $\log_b(b^a)$

d) $6^{\log_6(\pi)}$

5. Solve the following equations for x :

a) $e^x = 4$

b) $e^{-x} = 2$

One of your solutions should be exactly -2 times the value of your other solution. Why?

6. Simplify:

a) $\ln(e^3)$

b) $\ln\left(\frac{1}{2e}\right)$

c) $e^{\ln x^4}$

d) $e^{3+\ln x}$

Main problems

1. In an experiment, the mass, m grams, of a reaction product is measured at various times, t minutes. It is believed that m and t are related by a law of the form:

$$m = At^n$$

The results are shown in the table below:

time (min)	1	2	3	4	5	6	7	8	12
mass (g)	0.5	0.63	0.72	0.8	0.85	0.9	0.95	0.99	1.0

- a) Confirm this postulate graphically. There is one result that does not conform to this law. Which?
- b) Find appropriate values of A and n , and in this context, explain the meaning of n . What are the units of A ?
- c) Explain, with reasons, whether you would use these results to predict the mass when $t = 15$.
2. These problems deal with pH:
- a) What is the pH of 130ml of a solution containing 4.7mg HCl, assuming that HCl is completely ionised in solution, and its molecular mass is 36.46?
- b) What would be the pH if the concentration of HCl were tripled?
3. Express in terms of $\log(a)$, $\log(b)$, $\log(c)$ and $\log(d)$:
- a) $\log\left(\frac{b}{ac}\right)$
- b) $\log(a^2bc^3d^4)$
- c) $\log\left(\sqrt{\frac{cd}{ab}}\right)$
4. Simplify:
- a) $\log(8) + \log(3) - \log(6)$
- b) $\log(x^2 - 1) - \log(x^2 + 1)$
- c) $3\log_a(4) + \log_a(5) - 2\log_a(9)$
- d) $\log(x^9) - \log(x^6)$

5. This problem gives you a chance to use the identity $\log_a(b) = \frac{1}{\log_b(a)}$.

Consider the equation $\log_3(x) + 4\log_x(3) = 5$:

- a) Verify that $x = 3$ satisfies this equation
 - b) There is one other value of x that also satisfies this equation. Find it.
6. Solve the following equations for x :
- a) $2(2^{2x}) - 5(2^x) + 2 = 0$
 - b) $3^{2x+1} - 28(3^x) + 9 = 0$
 - c) $16 = \log_2(x)$
 - d) $\left(2\sqrt{3}\log(x)\right)^2 - 7\log(x^2) + 2 = 0$
7. Write an expression for x or y without using logarithms:
- a) $\log(x) = \log(3r) - 5\log(s) + 3\log(t) - 3$
 - b) $\log(2y) = 5 + 5\log(4^3) - 15\log\left(\frac{2}{x}\right) - 6\log(y)$
8. Write x in terms of y for each of the following:
- a) $y = 2e^{4x}$
 - b) $\ln y = 3 + 2\ln x$
9. Express as a sum or difference of logarithms:
- a) $\ln \sqrt{\left(\frac{x-1}{x+1}\right)}$
10. Express as a single logarithm:
- a) $1 - \ln 4x$
 - b) $3\ln x - \frac{1}{2}\ln(5 - x^2)$