Problems 17: Probability, Permutations and Combinations

1 Permutations and Combinations

Exercise 1. In how many ways can the letters of the word STATISTICS be arranged?

Exercise 2. There are 10 balls in a bag numbered from 1 to 10. Three balls are selected at random. How many different ways are there of selecting the three balls (if order doesn't matter and balls are never replaced)?

Exercise 3. How many permutations of 3 digits are there, chosen from the ten digits 0 to 9 inclusive (i.e. with replacement)?

Exercise 4. How many different committees of 5 people can be chosen from 10 people?

Exercise 5. Jones is the Chairman of a committee. In how many ways can a committee of 5 be chosen from 10 people given that Jones must be one of them?

Exercise 6. Bob makes passwords by choosing four *different* letters of the alphabet. How many different possible passwords are there?

Exercise 7. (Assessed) Alice makes passwords by choosing two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?

Exercise 8. An encyclopedia has eight volumes. In how many ways can the eight volumes be replaced on the shelf?

Exercise 9. Assuming that any arrangement of letters forms a 'word', how many 'words' of any length can be formed from the letters of the word SQUARE (without repeating letters)?

Exercise 10. A restaurant offers 5 choices of appetizer, 10 choices of main meal and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming all choices are available, how many different possible meals does the restaurant offer?

Exercise 11. There are 10 students in a class, and they take an exam. It is observed that the three highest scores in the class go to the students whose names are earliest in the alphabet. How likely is that to happen purely by chance?

Exercise 12. There are 5 men and 5 women in a class, and they take an exam. It is observed that the three highest scores in the class go to women. How likely is that to happen purely by chance?

2 Probability

Exercise 13. Consider the experiment of drawing a single card from a standard pack of 52 playing cards. Are the events 'the card is between 5 and 7 inclusive' and 'the card is a face card' (i.e. jack, queen or king) (a) exhaustive, (b) independent, (c) exclusive? What about the events 'the card is red' and 'the card is an ace'?

Exercise 14. A box contains 5 green balls and 1 black one. Five balls are chosen randomly and removed. What is the probability that the black ball was removed?

Exercise 15. (Extension) Consider a series of hands dealt at bridge. Let A be the event that in a given deal each player has exactly one ace. Show that the probability that A occurs at least once in 7 deals is approximately $1/2$.

Exercise 16. A species of plant has 4 different types: yellow flowering (40%), red (32%), blue (18%) and white.

(a) What is the probability that the flowers on a randomly selected plant will be blue or red?

(b) What is the probability that a coloured (non-white) flower is blue?

Exercise 17. (Assessed) A male chimpanzee makes sexual advances to 12 females. If he has an (independent) 10% chance of mating on each occasion, what is the probability that he mates (a) at least once, (b) at least twice?