1. Strong Form of the 1D Burgers' Equation

The 1D Burgers' equation with viscosity and a source term is given by:

$$\frac{\partial u(x,t)}{\partial t} + u(x,t)\frac{\partial u(x,t)}{\partial x} - \nu\frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t), \quad \forall x \in [0,L],\ \forall t \in [0,T] \tag{1}$$

where:

- $u(x,t)$ is the velocity field.
- ν is the kinematic viscosity.
- $f(x,t)$ is the source term.
- L is the length of the spatial domain.
- T is the final time.

1.1. Boundary and Initial Conditions

The equation is subject to the following boundary and initial conditions:

$$\begin{aligned} u(0,t) &= u_{\text{left}}(t) = \mu_1, \qquad \forall t \in [0,T], \\ u(x,0) &= u_{\text{initial}}(x) = 1, \quad \forall x \in [0,L]. \end{aligned} \tag{2}$$

1.2. Source Term

In the specific case we are considering, the source term $f(x,t)$ is given by:

$$f(x,t) = 0.02e^{\mu_2 x} \tag{3}$$

where μ_2 is a parameter that defines the spatial variation of the source term.