Politecnico di Milano
Scuola di Ingegneria Industriale e dell'Informazione
Applied Statistics
Academic Year 2024/2025
Exam 2025-01-17 - Part B

Problem 1: Performance Metrics of Athletes

The file `athlete_stats.txt` contains information about the performance metrics of 100 athletes. Each column represents a unitless specific attribute of the athlete, indicating their level of performance in a particular skill or ability. Higher values correspond to better performance. The variables include:

- `sprint_speed`: Speed in short-distance sprints.
- `endurance`: Stamina for sustained activities.
- `vertical_jump`: Height achieved in vertical jumps.
- `agility`: Ability to change direction quickly and efficiently.
- `strength`: Muscular power and force.
- `reaction_time`: Speed of response to a stimulus.
- `accuracy`: Precision in tasks requiring targeting or control.
- `flexibility`: Range of motion and adaptability in movements.
- `throwing_power`: Strength and distance of throws.

a) Perform a Principal Component Analysis (PCA) on the dataset. Decide whether to use the original variables or the standardized ones, and justify your choice. Is there a *clear* number of principal components to consider? How many principal components are needed to explain at least $\approx 80\%$ of the total variability?

b) Report a plot of the loadings of the first three principal components. Provide an interpretation for each of these components based on the loadings.

c) Report the biplot of the data along the *second* and the *third* principal components. How would you qualify the athlete labeled as 78 (row index) based on the biplot?

d) Based on the dimensionality reduction suggested in part (a), project a new athlete with characteristics provided in Table 1 onto the reduced space, and compute its coordinates in the reference system of the first three principal components.

`sprint_speed`	1.85
`endurance`	1.74
`vertical_jump`	1.92
`agility`	1.89
`strength`	1.78
`reaction_time`	1.81
`accuracy`	1.69
`flexibility`	1.76
`throwing_power`	1.84

Table 1: Performance metrics of a new athlete.

Upload your results here: `https://forms.office.com/e/zAq8cM2i3a`