Problem 4: Elevation profile of the Passo dello Stelvio

The file `stelvio.txt` contains altitude measurements recorded at 200-meter intervals along the well-known 25-kilometer ascent of the *Passo dello Stelvio* in the Italian Alps. Our goal is to reconstruct the continuous altitude profile as a smooth function of horizontal distance, considering that these altitude measurements may be affected by noise.

a) Apply penalized smoothing to the altitude data using a basis of cubic B-splines, with knots placed at each horizontal distance point, penalizing the second-order derivative and using a smoothing parameter of $\lambda = 1$. Report the number of splines used and the generalized cross-validation (GCV) error.

b) Estimate the approximate dimension of the space in which the fitted curve lives. Provide a plot of the fitted curve along with its first derivative.

c) Determine the value of λ that minimizes the GCV error, and report the corresponding GCV error. Use a grid search with $\log_{10}(\lambda)$ values ranging from -1 to 3 in increments of 0.5. Refit the smoothed curve using this optimal λ value.

d) Calculate the slope[1] at the steepest point of the ascent.

Upload your results here: `https://forms.office.com/e/ibeeN59VhS`

[1] Defined as the tangent of the inclination angle