**Politecnico di Milano**
**Scuola di Ingegneria Industriale e dell'Informazione**
Applied Statistics
Academic Year 2024/2025
Exam 2025-01-17 - Part B

# Problem 3: Estimating Air Temperature in Alpine Regions

The file `temperature.txt` contains the yearly average air temperature ($y(s_i)$, in degrees Celsius) at 60 locations $s_i$ in the Alpine region surrounding the Aletsch Glacier. The dataset includes the UTM coordinates (`x`, `y`) of the locations $s_i$, whether the measurement site is north-oriented (`orient` = N), south-oriented (`orient` = S) or either east or west-oriented (`orient` = EW), and the elevation of the location $e(s_i)$ (in kilometers).

Consider the following model for air temperature estimation:

$$y(s_i) = \beta_{0,g} + \beta_{1,g} \cdot e(s_i) + \delta(s_i) \tag{2}$$

where $\delta(s_i)$ is a $2^{\text{nd}}$ order stationary residual with an exponential variogram *without* nugget effect, and $g = 0, 1, 2$ represents the grouping induced by the variable `orient` ($g = 0$ for `orient` = N, $g = 1$ for `orient` = EW and $g = 2$ for `orient` = S).

a) In Eq. 2, assume $\beta_{1,g} = \beta_1$ for $g = 0, 1, 2$ and fit the model. Report a plot of the fitted variogram, *initialising* the variogram fit with the model `vgm(0.2, "Exp", 1000)`. Indicate the estimate of the range and the sill.

b) Using the model fitted in part (a), estimate the yearly average air temperature $y^*(s_0)$ at the Jungfraujoch station which is located at $s_0 = (5140000, 427000)$, west-oriented, at an altitude of 3450m.

c) According to the model, what is the expected difference of temperature between a north and a south-exposed slope, all other things being equal?

d) Relaxing the assumption made in a) fit the model indicated by Eq. 2. Indicate the estimate of the sill, fitting the variogram with the same initialisation as in a). Should this model be preferred to the first one? Justify your answer.

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