

Problem 2: Impact of Sugar Consumption on Energy Levels

We are investigating how sugar intake affects individuals' perceived energy levels. The file `sugar_energy.txt` contains data from 2101 individuals following 200 different dietary patterns. Specifically, it reports:

- the average daily sugar intake in grams: `sugar` (this variable is centered)
- the dietary background of the individual, encoded as an integer variable: `diet`
- the self-reported average energy level on a scale from -10 to 10: `energy`

a) Implement the following linear regression model **M0**:

$$\text{energy} = \beta_{0,i} + \beta_1 \text{sugar} + \epsilon \quad (1)$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$, and $i \in \{1, \dots, 200\}$ represents the dietary background.

Report the estimates of $\beta_{0,2}$ and β_1 .

b) Consider now the model **M1**:

$$\text{energy}_i = \beta_0 + b_i \mathbf{1}_{n_i} + \beta_1 \text{sugar}_i + \epsilon_i \quad (2)$$

with $\epsilon_i \sim \mathcal{N}(0, \sigma^2 I_{n_i})$, $b_i \sim \mathcal{N}(0, \sigma_b^2)$ and n_i the number of individuals following dietary pattern i .

Fit the model and report the estimate of σ_b . Without performing any model comparison, what advantage does **M1** have over **M0**, in your opinion?

- c) A nutritionist suggests: "Sugar intake generally boosts energy levels, but the magnitude of this effect varies depending on the individual's dietary background." Propose and fit an updated model **M2** based on **M1** to account for this observation. Is there a dietary pattern for which sugar intake appears to have a negative effect on energy levels?
- d) Comment on whether **M1** or **M2** better explains the data, supporting your answer with an appropriate test.

Upload your results here: <https://forms.office.com/e/CMkAYpdrVn>