Problem :- 1

Solution of Matrix eqn with Gaussian Elimination

The given Linear System is:

$$\begin{pmatrix} 4 & 1 & 2 \\ 2 & 4 & 7 \\ 1 & 1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ -5 \\ -9 \end{pmatrix}$$

In ~~a~~ a two bit floating point decimal computer when the machine stores data in mantissa and exponent form; the same eq reads:

$$\begin{pmatrix} 0.4\times10^1 & 0.1\times10^1 & 0.2\times10^1 \\ 0.2\times10^1 & 0.4\times10^1 & -0.1\times10^1 \\ 0.1\times10^1 & 0.1\times10^1 & -0.3\times10^1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.9\times10^1 \\ -0.5\times10^1 \\ -0.9\times10^1 \end{pmatrix}$$

(There is '0' at 2nd decimal for each mantissa which I've not written down)

Now start Gaussian elimination.

Step : 1 :$\longrightarrow$ $R_1 \longrightarrow \dfrac{R_1}{0.4\times10^1}$

$$\begin{pmatrix} 0.10\times10^1 & 0.25\times10^0 & 0.50\times10^0 \\ 0.20\times10^1 & 0.40\times10^1 & 0.10\times10^1 \\ 0.10\times10^1 & 0.10\times10^1 & -0.30\times10^1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.23\times10^1 \\ -0.50\times10^1 \\ -0.90\times10^1 \end{pmatrix}$$

Step 2 : $R_2 \longrightarrow R_2 - 0.2\times10^1\times R_1$ ; $R_3 \longrightarrow R_3 - R_1$

(In machine the steps successively are

i) $R_2 \rightarrow R_2 - 0.2\times10^1\times R_1$

ii) $R_3 \rightarrow R_3 - 0.1\times10^1\times R_1$

I've ~~at~~ written them in one step)

$$\begin{pmatrix} 0.10\times10^1 & 0.25\times10^0 & 0.50\times10^0 \\ 0 & 0.35\times10^1 & -0.2\times10^1 \\ 0 & 0.75\times10^0 & -0.35\times10^1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.23\times10^1 \\ -0.96\times10^1 \\ -0.11\times10^2 \end{pmatrix}$$

Step 3: $R_2 \to \frac{R_2}{A_{22}}$ i.e. $R_2 \to \frac{R_2}{0.35\times10^1}$

$$\begin{pmatrix} 0.10\times10^1 & 0.25\times10^0 & 0.50\times10^0 \\ 0 & 0.10\times10^1 & -0.57\times10^0 \\ 0 & 0.75\times10^0 & -0.35\times10^1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.23\times10^1 \\ -0.27\times10^1 \\ -0.11\times10^2 \end{pmatrix}$$

Step : 4 : $R_3 \to R_3 - 0.75\times10^0 \times R_2$.

$$\begin{pmatrix} 0.10\times10^1 & 0.25\times10^0 & 0.50\times10^0 \\ 0 & 0.10\times10^1 & -0.57\times10^0 \\ 0 & 0 & -0.31\times10^1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.23\times10^1 \\ -0.27\times10^1 \\ -0.9\times10^1 \end{pmatrix}$$

Back Substitution:

$$x_3 = -\frac{0.9\times10^1}{-0.31\times10^1} = 0.29\times10^1 = 2.9$$

$$x_2 - 0.57\times0.29\times10^1 = -0.27\times10^1$$

$$\Rightarrow x_2 = -0.10\times10^1 = -1.0.$$

$$x_1 + 0.25\times(-0.10\times10^1) + (0.5\times10^0\times0.29\times10^1) = 0.23\times10^1$$

$$\Rightarrow x_1 = 0.11\times10^1$$

So the solution is: $(0.11\times10^1, -0.10\times10^1, 0.29\times10^1)$

i.e. $(1.1, -1.0, 2.9)$ Ans

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Problem: 3

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For the operator eq $Ax = b$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} ; \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

If A is of tridiagonal form i.e only the diagonal, superdiagonal & subdiagonal elements are non zero.

$$\text{i.e } A = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \cdots & 0 \\ 0 & a_{32} & a_{33} & a_{34} & \cdots & & 0 \\ 0 & & \ddots & \ddots & \ddots & & \vdots \\ 0 & & & \ddots & \ddots & & a_{m-1\,m} \\ \vdots & & & & & & \\ 0 & 0 & \cdots & 0 & 0 \cdots & a_{m,m-1} & a_{mm} \end{pmatrix}$$

To solve eq $Ax = b$ the Augmented matrix be

$$\tilde{A} = \left(\begin{array}{cccccc|c} a_{11} & a_{12} & 0 & 0 & 0 \cdots & 0 & b_1 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \cdots & b_2 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \cdots & \vdots \\ \vdots & & \ddots & \ddots & & a_{m,m-1} & \vdots \\ 0 & \cdots & & & a_{m,m-1} & a_{mm} & b_m \end{array}\right)$$

Now we apply Gaussian elimination:

Step 1: $R_1 \longrightarrow \dfrac{R_1}{a_{11}}$ ; $R_2 \longrightarrow R_2 - a_{21} R_1$

$$\tilde{A} = \left(\begin{array}{cccccc|c} 1 & \frac{a_{12}}{a_{11}} & 0 & 0 & \cdots & 0 & b_1/a_{11} \\ 0 & a_{22} - \frac{a_{21}}{a_{11}}.a_{12} & a_{23} - \frac{a_{21}}{a_{11}}.a_{12} & 0 & 0 \cdots & 0 & b_2 - \frac{b_1}{a_{11}} \\ & \cdots & & \cdots & & & \vdots \\ & & & & & & b_m \end{array}\right)$$

Now the $(m-1) \times (m-1)$ lower block is tridiagonal form

Applying the same rule $m$ times for which in $j$th step $R_j \to \frac{R_j}{a_{jj}}$ ; $R_{j+1} \to R_{j+1} - a_{j,j-1} \times R_{j-1}$ gives the ultimate Augmented matrix to be upper triangular form

$$\tilde{A}_{m\cdot itn} = \left(\begin{array}{cccccc|c} 1 & a'_{12} & 0 & 0 & 0 \cdots & 0 & b'_1 \\ 0 & 1 & a'_{23} & 0 & 0 \cdots & 0 & b'_2 \\ 0 & 0 & 1 & a'_{34} & 0 \cdots & 0 & \vdots \\ \vdots & & \ddots & & & & \vdots \\ & & & & & a'_{m-1\,m} & b'_m \\ 0 & 0 & \cdots & & 0 & 1 & \end{array}\right)$$

in A part

Now, to do this job we need 3 multiplication & 3 subtraction in any one of $m$ steps.

(except $j = 1, m$ needs 2). All other terms are zero.

So at $j$th step total arithmatic operations are $3 + 3 = 6$.

As there are $m$ steps so total operations

$= 6m$.

but for $j = 1, m$ there ~~are~~ are two multiplications & ~~2 sub~~ 2 subtractions.

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$\therefore$ no of steps till now $N_1 = 6m - 2$.

for the right side there are (in b part) $(m-1)$ divisions & $(m-1)$ subtractions.

$\therefore$ $N_1 \longrightarrow N_1 + 2(m-1) = 8m - 4$.

For the back substitution (Now with the form we can do that) we need at

$j = 1$ th step $\longrightarrow$ no operation

$\underbrace{\text{all other step}}_{m-1 \text{ steps}} \longrightarrow \underbrace{1 \text{ multiplication} + 1 \text{ subtraction}}_{2 \text{ operation}}$

So total no of ~~step~~ operations:

$N_1 \longrightarrow N = N_1 + 2(m-1)$

$= 10m - 6$

i.e at $m \to \infty$ limit the complixity asymptotically goes to $O(m)$ limit.

$\therefore$ For tridiagonal operator eq; the computational complexity varies as $\sim O(m)$. Proved