ED: 2; ASSIGNMENT 1; PART: 2
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Problem: 1.10

Maxwell's eqn in space (in the given unit system) in the source free ($\rho = 0$; $\vec{J} = 0$) form is given by:

$$\vec{\nabla}\cdot\vec{E} = 0; \quad \vec{\nabla}\cdot\vec{B} = 0; \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}; \quad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t}.$$

The given transformation is $\begin{pmatrix} E' \\ B' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} E \\ B \end{pmatrix}$

for the transformed system; we get:

$$\vec{\nabla}\cdot\vec{E}' = \cos\theta(\vec{\nabla}\cdot\vec{E}) + \sin\theta(\vec{\nabla}\cdot\vec{B})$$

$$= 0 + 0$$

i.e $\vec{\nabla}\cdot\vec{E}' = 0$

Similarly $\vec{\nabla}\cdot\vec{B}' = \cos\theta(\vec{\nabla}\cdot\vec{B}) - \sin\theta(\vec{\nabla}\cdot\vec{E}) = 0.$

$$\vec{\nabla}\times\vec{E}' = \cos\theta(\vec{\nabla}\times\vec{E}) + \sin\theta(\vec{\nabla}\times\vec{B})$$

$$= -\cos\theta\frac{\partial\vec{B}}{\partial t} + \sin\theta\frac{\partial\vec{E}}{\partial t}$$

$$= -\frac{\partial}{\partial t}(\cos\theta.\vec{B} - \sin\theta\vec{E})$$

i.e $\vec{\nabla}\times\vec{E}' = -\frac{\partial\vec{B}'}{\partial t}.$

And $\vec{\nabla}\times\vec{B}' = -\sin\theta(\vec{\nabla}\times\vec{E}) + \cos\theta(\vec{\nabla}\times\vec{B})$

$$= -\sin\theta.\left(-\frac{\partial\vec{B}}{\partial t}\right) + \cos\theta\frac{\partial\vec{E}}{\partial t}$$

$$= \frac{\partial}{\partial t}(\cos\theta\vec{E} + \sin\theta.\vec{B})$$

i.e $\vec{\nabla}\times\vec{B}' = \frac{\partial\vec{E}'}{\partial t}.$

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i.e $\vec{E}'$; $\vec{B}'$ satisfies all 4 maxwell's eq'n in source free form ($\rho = 0$; $\vec{j} = 0$)

→ proved

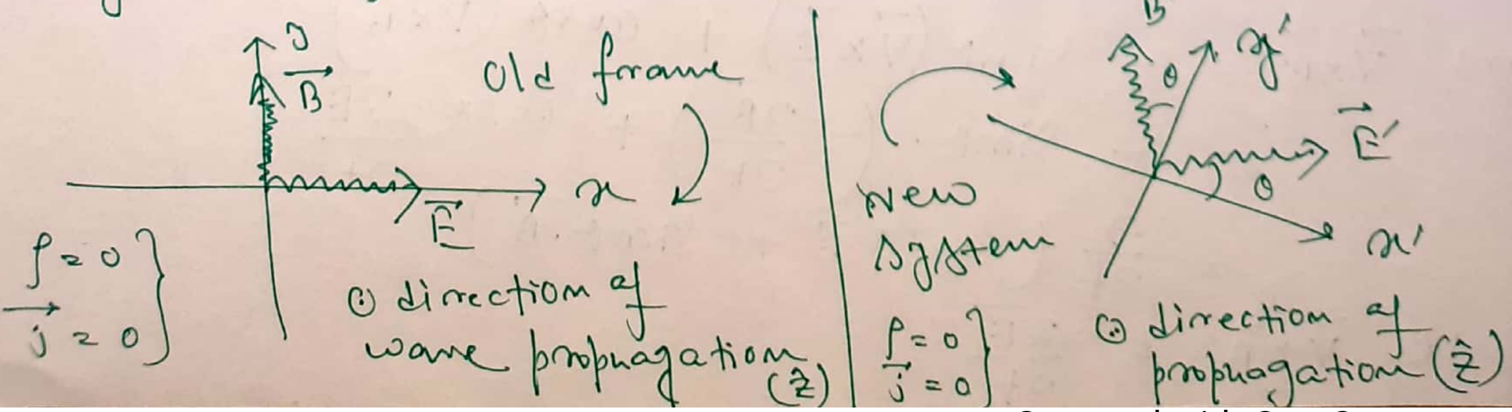
[iv] In source free region; the condition $\vec{E} \perp \vec{B}$ is the consequence of wave eq from Maxwell's eq'n.

i.e If we take our coordinate system so that $\vec{E}$ is along $\hat{x}$: $\vec{B}$ is along $\hat{y}$; then it produces a propagation of E.M wave along $\hat{z}$.

i.e
$$\left.\begin{aligned} \vec{E} &= E_0 \hat{x}\, e^{i(\cancel{kx}\; kz - \omega t)} \\ \vec{B} &= B_0 \hat{y}\, e^{i(kz - \omega t)} \end{aligned}\right\}$$

So in that frame the previous result means that we have just rotated the coordinate system about z axis (by clockwise) at an angle of θ.

The ~~Pay~~ Poynting vector is similar. Only the polarization of light in new frame is tilted by θ angle from x' axis.



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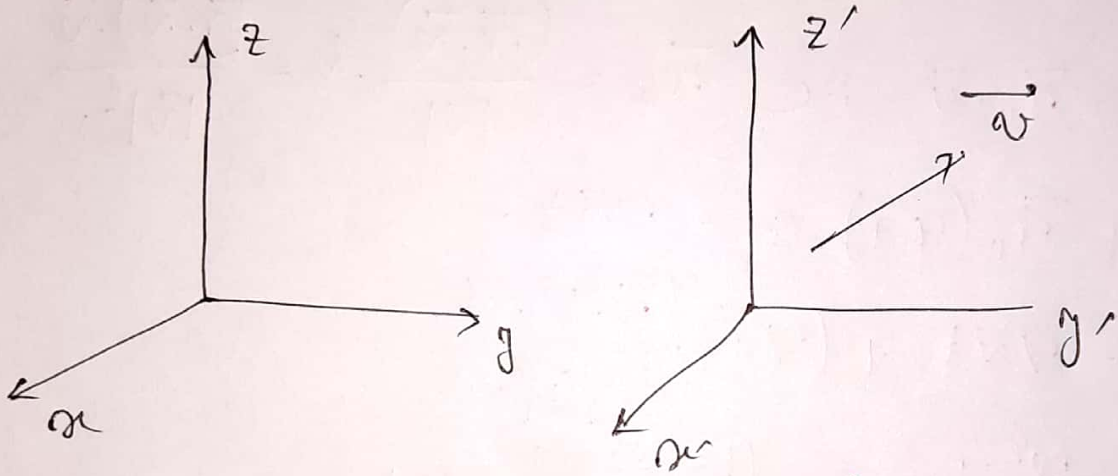
Problem: 1.11

Let in two frames (inertial) the same spacetime events be given by $(ct, x, y, z)^T$ & $(ct', x', y', z')^T$.
The frames were co-incided / overlapped at $t = t' = 0$.
The velocity of primed frame S' be $\vec{v}$ w.r.t the unprimed frame S.

Ans $\vec{r} = (x, y, z)^T$; $\vec{r}' = (x', y', z')^T$.



clearly $\vec{v} = (v_x, v_y, v_z)^T$ (in S)

Now let the parallel & perpendicular part of $\vec{r}$ w.r.t $\vec{v}$ be given by: $\vec{r}_{\parallel}$ & $\vec{r}_{\perp}$

$\therefore \quad \vec{r} = \vec{r}_{\parallel} + \vec{r}_{\perp}$; $(\vec{r}_{\parallel} \parallel \vec{v}$ & $\vec{r}_{\perp} \perp \vec{v})$

clearly in S' frame we get: (from L.T in a particular direction)

$$\vec{r}\,' = \vec{r}_{\perp} + \gamma(\vec{r}_{\parallel} - \vec{v}t)$$

Now, $\vec{r} \cdot \vec{v} = \vec{r}_{\parallel} \cdot \vec{v} + \vec{r}_{\perp} \cdot \vec{v} = \vec{r}_{\parallel} \cdot \vec{v} = |\vec{r}_{\parallel}| \cdot |\vec{v}|$

And in standard L.T we get: $ct' = \gamma\left(ct - \frac{xv}{c}\right)$

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Here in general case we generize it by:

$$ct' = \gamma\left(ct - \frac{x v_x}{c} - \frac{y v_y}{c} - \frac{z v_z}{c}\right)$$

i.e $$t' = \gamma\left(t - \frac{\vec{v}\cdot\vec{r}}{c^2}\right)$$

Now; $$\vec{r}\,' = \vec{r}_\perp + \gamma(\vec{r}_\parallel - \vec{v}t)$$

but $$\vec{r}_\perp = \vec{r} - \vec{r}_\parallel$$

i.e $$\vec{r}\,' = \vec{r} - \vec{r}_\parallel + \gamma(\vec{r}_\parallel - \vec{v}t)$$

As $\vec{r}\cdot\vec{v} = \vec{r}_\parallel\cdot\vec{v}$ i.e $\vec{r}_\parallel = \frac{(\vec{r}\cdot\vec{v})}{|\vec{v}|}\cdot\hat{v} = \frac{(\vec{r}\cdot\vec{v})\cdot\vec{v}}{|\vec{v}|^2}$.

$$\therefore \vec{r}\,' = \vec{r} + \vec{r}_\parallel(\gamma - 1) - \gamma\vec{v}t.$$

$$= \vec{r} - \gamma\vec{v}t + (\gamma-1)\cdot\frac{(\vec{r}\cdot\vec{v})\cdot\vec{v}}{|\vec{v}|^2}$$

but $$\frac{\gamma-1}{v^2} = \frac{\gamma^2-1}{(\gamma+1)v^2} = \frac{\left(1-\frac{v^2}{c^2}\right)^{-1} - 1}{v^2(\gamma+1)}$$

$$= \frac{1 - 1 + \frac{v^2}{c^2}}{(1-\beta^2)\cdot v^2(\gamma+1)} = \frac{\gamma^2}{c^2(\gamma+1)}$$

i.e $$\vec{r}\,' = \vec{r} + \frac{\gamma^2}{c^2(\gamma+1)}\frac{(\vec{r}\cdot\vec{v})\cdot\vec{v}}{1} - \gamma\vec{v}t.$$

i.e the L.T is given by:

$$\vec{r}\,' = \vec{r} + \frac{\gamma^2}{(\gamma+1)}\frac{(\vec{r}\cdot\vec{v})\cdot\vec{v}}{c^2} - \gamma\vec{v}t$$

$$t' = \gamma\left(t - \frac{\vec{v}\cdot\vec{r}}{c^2}\right)$$

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So by inverse L.T we get:
(This problem ~~woul~~ would not arise if from beginning I would take S' as S & S as S' ; i.e $\vec{v} \to -\vec{v}$ yeilding the required formula in problem.)

$$\left.\begin{aligned}\vec{r} &= \vec{r}' + \frac{\gamma^2}{(\gamma+1)} \frac{(\vec{r}'\cdot\vec{v})\cdot\vec{v}}{c^2} + \gamma\vec{v}t \\ t' &= \gamma\left(t + \frac{\vec{v}\cdot\vec{r}'}{c^2}\right)\end{aligned}\right\}$$

interchanging primes & unprimes (which is just a matter of notation):

$$\left.\begin{aligned}\vec{r}' &= \vec{r} + \frac{\gamma^2}{(\gamma+1)} \frac{(\vec{r}\cdot\vec{v})\cdot\vec{v}}{c^2} + \gamma\vec{v}t \\ t' &= \gamma\left(t + \frac{\vec{v}\cdot\vec{r}}{c^2}\right)\end{aligned}\right\}$$

proves

By breaking the formula we get separately:

$$\cancel{x_i' = + \gamma v_{x_i} t + x_i + \frac{\gamma^2}{(\gamma^2+1)c^2}\left(x_i v_{x_i} v_{x_i} + 2 v_y v_x + 2 v_z v_x\right)}$$

$$x_i' = x_i + \gamma v_i t + \frac{\gamma^2 v_{x_i}}{c^2(\gamma+1)} \cdot \sum_j x_j v_j$$

i.e in matrix form the L.T can be written as,

$$\left(\text{using } \frac{\gamma^2}{(\gamma+1)} = \frac{\gamma-1}{\beta^2}\right)$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ & 1+(\gamma-1)\frac{\beta_x^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_x}{\beta^2} & (\gamma-1)\frac{\beta_z\beta_x}{\beta^2} \\ & & 1+(\gamma-1)\frac{\beta_y^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ & & & 1+(\gamma-1)\frac{\beta_z^2}{\beta^2} \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Symmetric

$$\parallel$$
$$\Lambda.$$

Clearly $c^2t'^2 - \Sigma_i x_i'^2 = (ct', \vec{r}')\cdot\begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}\begin{pmatrix} ct' \\ \vec{r}' \end{pmatrix}$

$= (ct, \vec{r})\, \Lambda^T \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \Lambda \begin{pmatrix} ct \\ \vec{r} \end{pmatrix}$

$= c^2t^2 - x_j x_j = (ct, \vec{r}) \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}\begin{pmatrix} ct \\ \vec{r} \end{pmatrix}$

only if $\Lambda^T g \Lambda = g$.

$\therefore$ we have to proof : $\Lambda^T g \Lambda = g$

Let $\Lambda^T g \Lambda = A. = \Lambda g \Lambda$ $\quad (\because \Lambda^T = \Lambda)$

i.e $\boxed{A_{\mu\nu} = \Lambda_{\mu\sigma}\, g_{\sigma\rho}\, \Lambda_{\rho\nu}.}$

~~i.e A_{00} = A~~ but $g_{\sigma\rho} = \delta_{\sigma\rho}$ for $\sigma = 0$
$\qquad = -\delta_{\sigma\rho}$ for $\sigma \neq 0$

i.e $A_{\mu\nu} = \Lambda_{\mu\sigma}\,\delta_{\sigma\rho}\,\Lambda_{\rho\nu}\Big|_{\sigma=0} - \Lambda_{\mu\sigma}\,\delta_{\sigma\rho}\,\Lambda_{\rho\nu}\Big|_{\sigma\neq0}$

$= \Lambda_{\mu0}\Lambda_{0\nu} - \Lambda_{\mu1}\Lambda_{1\nu} - \Lambda_{\mu2}\Lambda_{2\nu} - \Lambda_{\mu3}\Lambda_{3\nu}.$

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i.e $A_{00} = \Lambda_{00}\Lambda_{00} - \Lambda_{01}\Lambda_{10} - \Lambda_{02}\Lambda_{20} - \Lambda_{03}\Lambda_{30}$

$$= \Lambda_{00}^2 - \Sigma_i \Lambda_{0i}^2 \quad (\because \Lambda_{0i} = \Lambda_{i0})$$

$$= \gamma^2 - \Sigma \gamma^2 \beta_i^2 = \gamma^2 (1 - \Sigma \beta_i^2)$$

$$= \gamma^2 (1-\beta^2) = 1 \quad \left(\because \gamma^2 = \frac{1}{1-\beta^2}\right)$$

for $i \neq 0$:

$$A_{ii} = \Lambda_{0i}\Lambda_{0i} - \Lambda_{1i}\Lambda_{1i} - \Lambda_{2i}\Lambda_{2i} - \Lambda_{3i}\Lambda_{3i} \quad \left(\begin{array}{c}\text{using} \\ \Lambda_{ij} = \Lambda_{ji}\end{array}\right)$$

$$= \gamma^2\beta_i^2 - \left(1 + \frac{(\gamma-1)\beta_i^2}{\beta^2}\right)^2 - \frac{(\gamma-1)^2\beta_i^2}{\beta^2}(\beta_j^2 + \beta_k^2) \quad (i \neq j \neq k \neq 0)$$

$$= \gamma^2\beta_i^2 - 1 - \frac{(\gamma-1)^2\beta_i^4}{\beta^4} - \frac{(\gamma-1)^2\beta_i^2(\beta_j^2+\beta_k^2)}{\beta^2}$$

$$- \frac{2(\gamma-1)\beta_i^2}{\beta^2}.$$

$$= \gamma^2\beta_i^2 - \frac{(\gamma-1)^2\beta_i^2}{\beta^2 \times \cancel{\beta^2}}\cancel{(\beta_i^2+\beta_j^2+\beta_k^2)} - \frac{2(\gamma-1)\beta_i^2}{\beta^2} - 1$$

$$= -\frac{\beta_i^2(\gamma-1)^2}{\beta^2} - \frac{2(\gamma-1)\beta_i^2}{\beta^2} + \gamma^2\beta_i^2 - 1$$

$$= -\frac{(\gamma-1)\beta_i^2}{\beta^2}(\gamma - 1 + 2) + \gamma^2\beta_i^2 - 1$$

$$= \gamma^2\beta_i^2 - \frac{(\gamma^2-1)}{\beta^2}\beta_i^2 - 1 \quad \left(\begin{array}{l}\because \gamma^2 = \frac{1}{1-\beta^2} \\ \text{i.e } \gamma^2 - 1 = \frac{\beta^2}{1-\beta^2} \\ \therefore \frac{\gamma^2-1}{\beta^2} = \frac{1}{1-\beta^2} = \gamma^2\end{array}\right)$$

$$= \gamma^2\beta_i^2 - \gamma^2\beta_i^2 - 1$$

i.e $A_{ii} = -1$ (for $i \neq 0$)

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Again for $i \neq 0$

$$A_{0i} = \Lambda_{00}\Lambda_{0i} - \Lambda_{01}\Lambda_{1i} - \Lambda_{02}\Lambda_{2i} - \Lambda_{03}\Lambda_{3i}$$

Let's take for $i = 1$:

$$A_{01} = \Lambda_{00}\Lambda_{01} - \Lambda_{01}\Lambda_{11} - \Lambda_{02}\Lambda_{21} - \Lambda_{03}\Lambda_{31}$$

$$= -\gamma^2\beta_1 + \gamma\beta_1\left(1 + \frac{(\gamma-1)\beta_1^2}{\beta^2}\right) + \frac{\gamma\beta_2(\gamma-1)\beta_1\beta_2}{\beta^2} + \frac{\gamma\beta_3(\gamma-1)\beta_1\beta_3}{\beta^2}.$$

$$= -\gamma^2\beta_1 + \gamma\beta_1\left(1 + \frac{(\gamma-1)}{\cancel{\beta^2}}\cancel{(\beta_1^2+\beta_2^2+\beta_3^2)}\right)$$

$$= -\gamma^2\beta_1 + \gamma\beta_1(1+\gamma-1) = 0;$$

Similarly $A_{02} = 0 = A_{03}$

i.e $A_{0i} = 0 \quad (\text{for } i \neq 0)$

And lastly:

$$A_{ij} = \Lambda_{i0}\Lambda_{0j} - \Lambda_{i1}\Lambda_{1j} - \Lambda_{i2}\Lambda_{2j} - \Lambda_{i3}\Lambda_{3j}$$

for $(i,j) = (1,2)$ we get:

$$A_{12} = (-\gamma\beta_1)(-\gamma\beta_2) - \left(1 + \frac{(\gamma-1)\beta_1^2}{\beta^2}\right)\frac{(\gamma-1)\beta_1\beta_2}{\beta^2} - \frac{(\gamma-1)\beta_1\beta_2}{\beta^2}\left(1 + \frac{(\gamma-1)\beta_2^2}{\beta^2}\right) - \frac{(\gamma-1)\beta_1\beta_3}{\beta^2}\cdot\frac{(\gamma-1)\beta_2\beta_3}{\beta^2}.$$

$$= \gamma^2\beta_1\beta_2 - \frac{(\gamma-1)}{\beta^2}\left\{\beta_1\beta_2\left(1 + \frac{(\gamma-1)\beta_1^2}{\beta^2}\right) + \beta_1\beta_2\left(1 + \frac{(\gamma-1)\beta_2^2}{\beta^2}\right) + \frac{(\gamma-1)\beta_3^2\beta_1\beta_2}{\beta^2}\right\}$$

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$$= \gamma^2\beta_1\beta_2 - \frac{(\gamma-1)}{\beta^2}\left\{2\beta_1\beta_2 + \frac{(\gamma-1)\beta_1\beta_2}{\cancel{\beta^2}}\cancel{(\beta_1^2+\beta_2^2+\beta_3^2)}\right\}$$

$$= \gamma^2\beta_1\beta_2 - \frac{(\gamma-1)(\gamma+1)}{\beta^2}\beta_1\beta_2$$

$$= \gamma^2\beta_1\beta_2 - \frac{(\gamma^2-1)}{\beta^2}\beta_1\beta_2 \qquad \left(\because \frac{\gamma^2-1}{\beta^2} = \gamma^2 \text{ previously shown}\right)$$

$$= \gamma^2\beta_1\beta_2 - \gamma^2\beta_1\beta_2$$

$$= 0$$

Similarly for all $i \neq j\ (\neq 0)$ we get, $A_{ij} = 0$.

i-e $A = \Lambda^T g \Lambda = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} = g.$

i-e the transformation preserves the norm.

i-e $\boxed{c^2t'^2 - \sum_i x_i'^2 = c^2t^2 - \sum_i x_i^2}$

$\longrightarrow$ proved

Problem: 1.14

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$A^\mu = \left(\phi/c , \vec{A}\right)$$

(here I've used the traditional notation I always use i.e $A^\mu = (\phi/c, \vec{A})$; $A_\mu = (\phi/c, -\vec{A})$;

$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$; unlike the given in problem.

However this is just a fact of notation & physics does not change anything. As I'm familier more with this notation, I use this. Please let me know if there is any problem.)

clearly $F^{\nu\mu} = -F^{\mu\nu}$; i.e F is antisymmetric.

$\therefore F^{\mu\mu} = 0$.

~~Ans; $F_{0i} = \partial^0 A^i - \partial^i A^0 = \frac{1}{c}\frac{\partial\phi}{\partial t} + \frac{1}{c}\frac{\partial A^i}{\partial x_i}$ $\left(\because \partial^i = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)\right)$ $= \frac{1}{c}\left\{\frac{\partial A^i}{\partial x_i}\right.$~~

Ans $F^{0i} = \partial^0 A^i - \partial^i A^0 = \frac{1}{c}\frac{\partial \phi}{\partial x_i} + \frac{1}{c}\frac{\partial A_i}{\partial t}$

$$= \frac{1}{c}\left(\frac{\partial A_i}{\partial t} + \frac{\partial \phi}{\partial x_i}\right) \qquad \left(\because \partial^i = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)\right)$$

$$= -\frac{1}{c}\left(-\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}\right)_i = -\frac{E_i}{c}$$

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$$F^{io} = -F^{oi} = \frac{E_i}{c}$$

And $F^{12} = \partial^1 A^2 - \partial^2 A^1 = -\frac{\partial A^2}{\partial x_1} + \frac{\partial A^1}{\partial x^2} = -(\vec{\nabla}\times\vec{A})_3$
$= -B_3.$

$$F^{23} = \partial^2 A^3 - \partial^3 A^2 = \frac{\partial A^2}{\partial x_3} - \frac{\partial A^3}{\partial x^2} = -(\vec{\nabla}\times\vec{A})_1 = -B_1$$

$$F^{13} = \partial^1 A^3 - \partial^3 A^1 = \frac{\partial A^1}{\partial x^3} - \frac{\partial A^3}{\partial x^1} = (\vec{\nabla}\times\vec{A})_2 = B_2.$$

$\therefore$ The $F^{\mu\nu}$ matrix be given by (using $F^{21} = -F^{12}$; ... etc)

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

And the field tensor with covariant indices is,

$$F_{\mu\nu} = g_{\mu\alpha} F^{\alpha\beta} g_{\beta\nu}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix} \begin{pmatrix} 1 & & 0 \\ & 7 & \\ 0 & & 7 \;\; 7 \end{pmatrix}$$

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$$= \begin{pmatrix} 0 & E_1/c & E_2/c & E_3/c \\ -E_1/c & 0 & -B_3 & B_2 \\ -E_2/c & B_3 & 0 & -B_1 \\ -E_3/c & -B_2 & B_1 & 0 \end{pmatrix}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\left(\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\right) \;;\quad \tilde{F}^{\nu\mu} = -\tilde{F}^{\mu\nu}$$

$$\text{i.e. } \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}.$$

$$\text{clearly } \tilde{F}_{ij} = \frac{1}{2}\left(F^{k0}\epsilon_{ijk0} + \epsilon_{ij0k} F^{0k}\right) = \epsilon_{ijk} F^{k0}$$

$$= + \frac{\epsilon_{ijk} E_k}{c}$$

$$\text{i.e. } \tilde{F}_{12} = + \epsilon_{123}\frac{E_3}{c} = + \frac{E_3}{c}$$

$$\tilde{F}_{23} = + \epsilon_{231}\frac{E_1}{c} = + \frac{E_1}{c}$$

$$\tilde{F}_{13} = + \epsilon_{132}\frac{E_2}{c} = - \frac{E_2}{c}$$

$$\text{And: } \tilde{F}_{0i} = \frac{1}{2}\epsilon_{0ijk} F^{jk} = -\frac{1}{2}\epsilon_{ijk0} F^{jk}$$

$$\therefore \tilde{F}_{01} = -\frac{1}{2}\left(\epsilon_{1230} F^{23} + \epsilon_{1320} F^{32}\right) = \frac{2\epsilon_{1230} F^{23}}{2}$$

$$= B_1$$

$$\text{Similarly: } \tilde{F}_{02} = B_2 \;;\; \tilde{F}_{03} = B_3.$$

$$\text{i.e. } \tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & +E_z/c & -E_y/c \\ -B_y & -E_z/c & 0 & +E_x/c \\ -B_z & +E_y/c & -E_x/c & 0 \end{pmatrix}$$

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And the contravariant duel field tensor:

$$\tilde{F}^{\mu\nu} = g^{\mu\alpha}\tilde{F}_{\alpha\beta}g^{\beta\nu}$$

$$= \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & \cdots & & \\ & \cdots & & \end{pmatrix} \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ & 0 & +E_z/c & -E_y/c \\ & & 0 & +E_x/c \\ & & & 0 \end{pmatrix} \text{ antisymmetric.}$$

[14] As $A^\mu = (\phi, \vec{A})$ forms a 4 vector; so here the transformation of A^μ be given by:

$$\phi' = \gamma(\phi + \beta A^3)$$

$$A'^1 = A^1$$

$$A'^2 = A^2$$

$$A'^3 = \gamma(\beta\phi + A^3)$$

i.e

$$\begin{pmatrix} \phi' \\ A'^1 \\ A'^2 \\ A'^3 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \phi \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}$$

Transformation matrix for 4 vector A^μ.

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The transformation of F is:

$$F' = \Lambda^T F \Lambda$$

$$= \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \times \Lambda$$

(c = 1 in Gaussian unit in this part)

$$\cancel{= \begin{pmatrix} \gamma\beta E_z & (-\gamma E_x - \gamma v B_y) & (-\gamma E_y + \gamma v B_x) & -\gamma E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ \gamma E_z & -(\gamma B_y + \gamma v B_x) & (\gamma B_x + \gamma & \end{pmatrix}}$$

$$= \begin{pmatrix} \gamma\beta E_z & (-\gamma E_x - \gamma\beta B_y) & (-\gamma E_y + \gamma\beta B_x) & -\gamma E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ \gamma E_z & (-\gamma\beta E_x - \gamma B_y) & (-\gamma\beta E_y + \gamma B_x) & -\gamma\beta E_z \end{pmatrix}$$

$$\times \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

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$$= \begin{pmatrix} 0 & (-\gamma E_x - \gamma\beta B_y) & (-\gamma E_y + \gamma\beta B_x) & \gamma^2 E_z(1-\beta^2) \\ & 0 & -B_z & \gamma\beta E_x + \gamma B_y \\ & & 0 & \gamma\beta E_y - \gamma B_x \\ & & & 0 \end{pmatrix}$$

Antisymmetric

comparing we get:

$$\left.\begin{aligned} E_x' &= \gamma E_x + \gamma\beta B_y \\ E_y' &= \gamma E_y - \gamma\beta B_x \\ E_z' &= E_z \end{aligned}\right\}$$ Transformations of $\vec{E}, \vec{B}$ for the given L.T.

$$\left.\begin{aligned} B_x' &= \gamma B_x - \gamma\beta E_y \\ B_y' &= \gamma B_y + \gamma\beta E_x \\ B_z' &= B_z \end{aligned}\right\}$$ Answer.

(iii) In terms of F, $\tilde{F}$ & $j^\mu = (c\rho, \vec{j})$ the Maxwell's eqn's can be written as:

1) for Homogeneous M.E:

$\nabla \cdot \vec{B} = 0$; $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$:

we get: $\partial_\alpha \tilde{F}^{\alpha\beta} = 0$

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for $\beta = 0$ gives:

$$\partial_0 \tilde{F}^{00} + \partial_1 \tilde{F}^{10} + \partial_2 \tilde{F}^{20} + \partial_3 \tilde{F}^{30} = 0$$

i.e $\quad 0 + \partial_x B_x + \partial_y B_y + \partial_z B_z = 0$

i.e $\quad (\vec{\nabla} \cdot \vec{B}) = 0 \longrightarrow$ M.E (1).

$\beta = 1$ gives,

$$\partial_0 \tilde{F}^{01} + \partial_1 \tilde{F}^{11} + \partial_2 \tilde{F}^{21} + \partial_3 \tilde{F}^{31} = 0$$

i.e $-\frac{1}{c} \partial_t B_x + 0 - \frac{\partial_y E_3}{c} + \frac{\partial_3 E_2}{c} = 0$

i.e $\left(\vec{\nabla} \times \vec{E}\right)_x = -\left(\frac{\partial B}{\partial t}\right)_x$

for $\beta = 2, 3$ gives (combing with $\beta = 1$) in vector form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \text{M.E(2)}.$$

② For $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$; $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

the eq in $F^{\alpha\beta}$ be:

$$\partial_\alpha F^{\alpha\beta} = \mu_0 j^\beta$$

for $\beta = 0$: $\partial_0 F^{00} + \partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = \rho/c\epsilon_0$

i.e $\quad 0 + \partial_x E_x + \partial_y E_y + \partial_z E_z = \rho/\epsilon_0$

i.e $\quad \nabla \cdot \vec{E} = \rho/\epsilon_0 . \longrightarrow$ ME : 3

for $\beta = 1$: $\partial_0 F^{01} + \partial_1 F^{11} + \partial_2 F^{21} + \partial_3 F^{31} = \frac{j_1}{(\mu_0)7}$

i.e $\frac{1}{c^2} \partial_t E_x + 0 + \partial_y B_z - \partial_z B_y = \frac{j_1}{(\mu_0)7}$

i.e $(\vec{\nabla} \times \vec{B})_x = \mu_0 j_x + \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$

or $\quad \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \longrightarrow$ M.E.(4)

The Lorentz invariant quantities from $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ must be some scalar. As scalars are unchanged under L.T.

The scalars constructed from $F, \tilde{F}$ be

i) $F_{\mu\nu} F^{\mu\nu} = 2\left(\frac{E^2}{c^2} - B^2\right)$

(derived in problem 1.8)

ii) $F_{\mu\nu} \tilde{F}^{\mu\nu} = \left(0 - \frac{E_x B_x}{c} - \frac{E_y B_y}{c} - \frac{E_z B_z}{c} - \frac{E_x B_x}{c} - \frac{E_y B_y}{c} - \frac{E_z B_z}{c} + 0 - \frac{E_z B_z}{c} - \frac{E_y B_y}{c} - \frac{E_z B_z}{c} + 0 - \frac{E_x B_x}{c} - \frac{E_y B_y}{c} - \frac{E_x B_x}{c}\right)$

i.e $F_{\mu\nu} \tilde{F}^{\mu\nu} = -4 \frac{\vec{E} \cdot \vec{B}}{c}.$

$\therefore$ Two Lorentz invariant quantities:

$$\left.\begin{aligned} &\text{i) } F_{\mu\nu} F^{\mu\nu} = F_{\mu\nu} g^{\mu\alpha} F_{\alpha\beta} g^{\beta\nu} = 2\left(\frac{E^2}{c^2} - B^2\right) \\ &\text{ii) } F_{\mu\nu} \tilde{F}^{\mu\nu} = F_{\mu\nu} g^{\mu\alpha} \tilde{F}_{\alpha\beta} g^{\beta\nu} = -4 \frac{\vec{E} \cdot \vec{B}}{c} \end{aligned}\right\}$$

Answer

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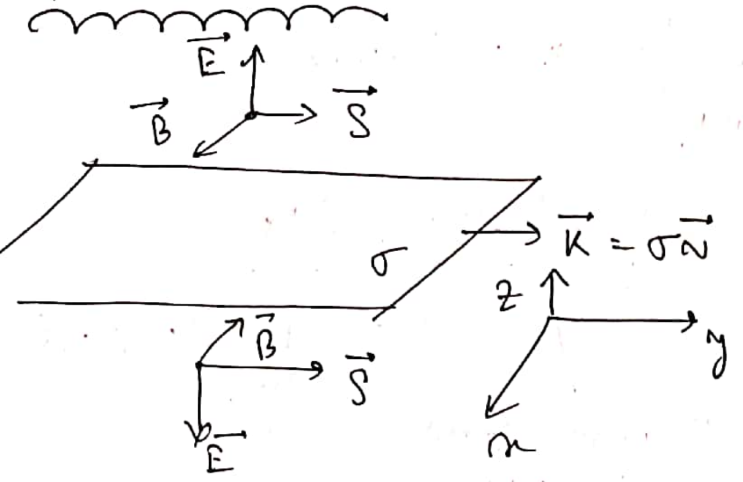
Problem: 1·12



At any point not on the sheet with $z > 0$ ($z = 0$ is the plate)

The electric field & magnetic field be given by: (as $\vec{v} \parallel \hat{y}$)

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z} \; ; \; \vec{B} = \frac{\mu_0(\vec{K}\times\hat{n})}{2} = \frac{\mu_0 K}{2}\hat{x} = \frac{\mu_0\sigma v}{2}\hat{x}$$

And for $z < 0$ i.e below the sheet:

$$\vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{z} \; ; \; \vec{B} = -\frac{\sigma\mu_0 v}{2}\hat{x}$$

i.e the poynting vector is constant everywhere and given by:

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\cancel{\mu_0}}\,\frac{\cancel{\mu_0}\sigma v}{2}\times\frac{\sigma}{2\epsilon_0}(\hat{z}\times\hat{x})$$

$$= \frac{\sigma^2 v}{4\epsilon_0}\hat{y}.$$

So that's the energy transmitted by ~~the~~ the field per unit time per unit area.

Clearly any long stripe with area vector along $\hat{y}$ & perpendicular to the sheet would give an infinite ~~amo~~ amount of energy passing through.

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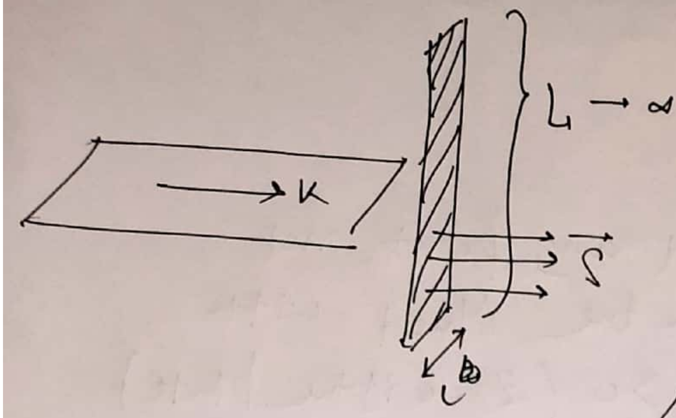


The poynting theorem tells:

$$\frac{dw}{dt} = -\frac{d}{dt}\int \frac{\left(\epsilon_0 E^2 + B^2/\mu_0\right)}{2} d\tau - \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\cdot d\vec{S}$$

work done on charges

Energy stored in field

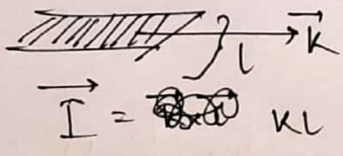
Energy transmitted by field.

The integral of $\left(\frac{\vec{E}\times\vec{B}}{\mu_0}\right)$ i.e $\vec{S}$ on the sheet would give an infinite result.

Ans, E^2, B^2 = constant; hence $\frac{d}{dt}\int\left(\frac{E^2\epsilon_0 + B^2/\mu_0}{2}\right)d\tau = 0$

This is because as the sheet is infinite; the electrical energy / workdone to move the charges is infinite here.

Better to say for any small cross section perpendicular to $\vec{K}$ carries a current (Kl) (l = length of section)



But from Ohms law the required potential for the current is:

$V = IR = KlR \to \infty$

as $R \to \infty$ due to the fact that the length parallel to ~~the~~ $\vec{K}$ (of the stripe) is $\to \infty$.

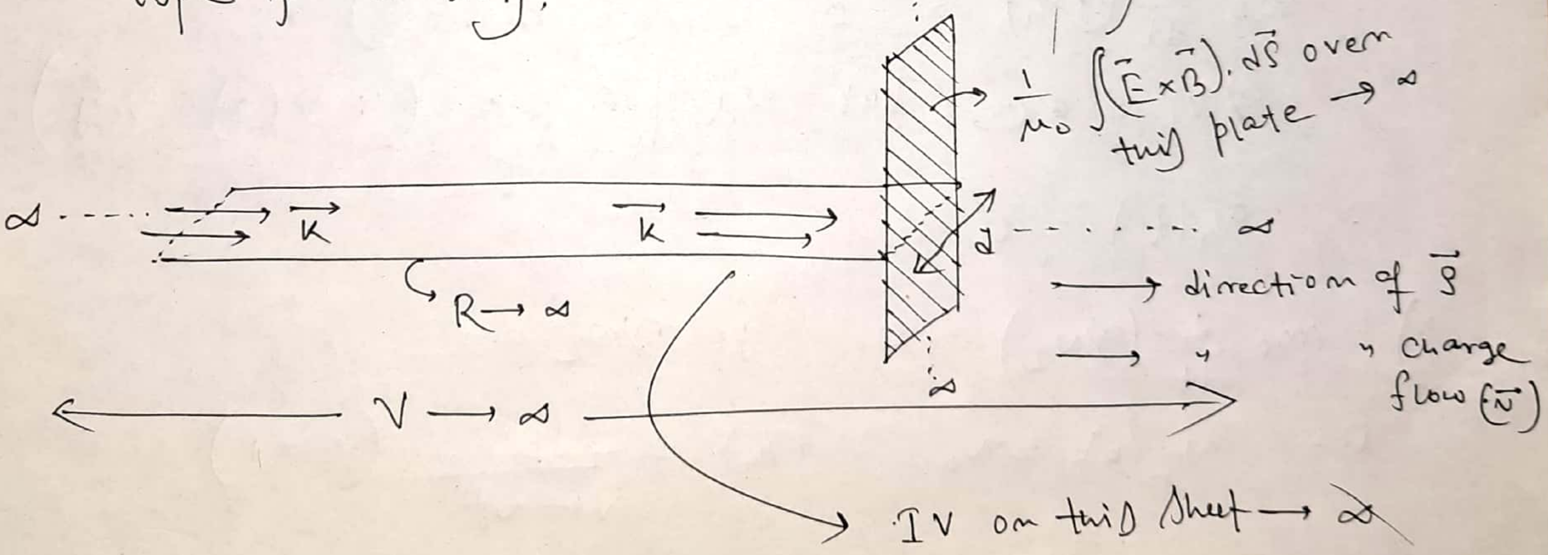
But the workdone is done to move the charges along $\vec{K}$ & the energy is being transmitted by the field along $\vec{K}$. i.e Poynting's theorem gurantees the direction of energy flow. But both the transmitted energy by the field along the direction of flow & the total work done to move the charges for any strip of ~~length~~ width d (perpendicular to $\vec{K}$) are infinite.

So unlike the finite current; here this shows an

infinity = infinity

coming from $\frac{dW}{dt}$ → coming from $\frac{1}{\mu_0}\int(\vec{E}\times\vec{B})\cdot d\vec{S}$

type of equality.



Answer

Problem:- 1.13.

The relativistic Lorentz force law is given by,

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \rightarrow \left(\text{excluding } p^0\right)$$ coming from $\left(m\frac{\partial p^\mu}{\partial \tau} = q u_\nu F^{\mu\nu}\right)$

but $\vec{p} = m_0(\gamma\vec{v})$ (excluding p^0)

i.e $m_0 \frac{d}{dt}(\gamma\vec{v}) = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$

$\vec{v}\times\vec{B} = 0$ $(\because \vec{B} = 0)$

i) for $v = v_1$:

$$m_0 \frac{d}{dt}(\gamma v_1) = 0 \quad (\because E_x = 0)$$

$\therefore \gamma v_1 = c$; but $v_1(t)\Big|_{t=0} = 0$ $\left(\because \vec{v}(0) = v_0\hat{y}\right)$

$\therefore v_1(t) = 0.$

ii) $m_0 \frac{d}{dt}(\gamma v_2) = 0 \quad \therefore \gamma v_2 = c$

but $v_2(0) = v_0$; $\gamma(0) = \frac{1}{\sqrt{1-v_0^2}}$ $(c=1)$

$$\therefore \quad c = \frac{v_0}{\sqrt{1-v_0^2}}.$$

i.e $\gamma v_2 = \frac{v_0}{\sqrt{1-v_0^2}}$

iii) $m\frac{d}{dt}(\gamma v_3) = qE$

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$\therefore \quad \gamma v_3 = \frac{qEt}{m_0} + c$

at $t=0$; $v_3 = 0$ i.e $c = 0$

$\therefore \quad \gamma v_3 = \frac{qEt}{m_0}$ i.e $v_3(t) = \frac{qEt}{m_0 \gamma}$.

Now from the Lorentz eq of p^0 we get:

$$\left(m_0 \frac{dp^0}{d\tau} = q u_\nu F^{0\nu} \right)$$

$$m_0 \frac{d}{dt}(\gamma) . \frac{dt}{d\tau} = q \gamma v_3 E .$$

i.e $m_0 \frac{d\gamma}{dt} . \gamma = q \gamma v_3 E .$

i.e $\frac{d\gamma}{dt} = \frac{q}{m} E v_3 . = \frac{q}{m} \left(\frac{E . qEt}{m_0 \gamma} \right)$

i.e $\gamma \frac{d\gamma}{dt} = \frac{q^2 E^2 t}{m_0} = \frac{1}{2} \frac{d}{dt} (\gamma^2)$

$\Rightarrow \gamma^2 = \frac{2 q^2 E^2 t^2}{m_0} + k$

at $t = 0$; $\gamma = \frac{1}{\sqrt{1 - v_0^2}}$. i.e $k = \frac{1}{1 - v_0^2}$

$\therefore \gamma^2 = \frac{2 q^2 E^2 t^2}{m_0} + \frac{1}{1 - v_0^2} \quad (c = 1)$

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$\therefore$ The velocities be given by:

$$\left.\begin{aligned} v_1(t) &= 0 \\ v_2(t) &= \frac{v_0}{\gamma\sqrt{1-v_0^2}} \\ v_3(t) &= \frac{qEt}{m_0 \gamma} \end{aligned}\right\} \quad (c=1)$$

<u>Answer</u>

when

$$\gamma = \gamma(t) = \left(\frac{2q^2E^2t^2}{m_0} + \frac{1}{1-v_0^2} \right)^{1/2}$$

(Here I've used the form of relativistic Lorentz law $m\,\partial_\tau p^\mu = q U_\nu F^{\mu\nu}$. But have not derived the law as it's not told to derive.)

[III] positions

i) $\frac{dx}{dt} = v_1 = 0 \Rightarrow x = x_0 = \text{const}.$

ii) $\frac{dy}{dt} = v_2 = \frac{v_0}{\sqrt{1-v_0^2}} \cdot \frac{1}{\sqrt{a^2t^2 + b^2}}$

$$(\gamma^2 \sim a^2t^2 + b^2)$$

on integrating: $y = \frac{v_0 b}{a} \sinh^{-1}\left(\frac{at}{b}\right) + y_0.$

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iii) $\frac{dz}{dt} = v_3 = \frac{qEat}{m_0 \gamma} = \frac{qEt}{m_0 \sqrt{a^2t^2+b^2}}$.

integrating (from mathematica):

$$Z = \frac{qE}{2m_0 a} \int \frac{d(a^2t^2+b^2)}{(a^2t^2+b^2)^{1/2}}$$

$$= \frac{qE}{2m_0 a} \cdot \sqrt{a^2t^2+b^2} + z_0 .$$

Now; if $y_0 = 0$ then we get:

$y = \frac{v_0 b}{a} \sinh^{-1}\left(\frac{at}{b}\right)$ i.e $t = \frac{b}{a} \sinh\left(\frac{ay}{bv_0}\right)$

$$\therefore Z = \frac{qE}{2m_0 a} \sqrt{b^2(1+\sinh^2(\cdot\cdot))} + z_0 .$$

$$= \frac{qEb}{2m_0 a} \cosh\left(\frac{ay}{bv_0}\right)$$

If initial energy & momentum be E_0 & P_0 then

$p_0 = m\gamma(0) v_0$; $E_0 = m\gamma(0)$ $(c=1)$

$$\therefore Z = \frac{E_0}{qE} \cosh\left(\frac{qEy}{p_0}\right) + z_0 \quad (c=1)$$

$\longrightarrow$ proved

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(b) At $v << 1$ or $v_0 << 1$ i.e non relativistic limit;

$\gamma \to 1$;

$v_1 = 0$ i.e $x(t) = x_0$

$v_2(t) = \dfrac{v_0 \times 1}{\sqrt{1-v_0^2}} \approx v_0$ i.e $y = v_0 t + y_0$

$v_3(t) = \dfrac{qEt}{m\gamma} \approx \dfrac{qEt}{m}$ i.e $z = \dfrac{qEt^2}{2m} + z_0$

(classical non relativistic result) $\longrightarrow$ Answer

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Problem 1.15.

1) $-\frac{1}{2}F^{\mu\nu}\cdot(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

$= -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}.$

i.e $\mathcal{L} = \frac{1}{4\pi}\cdot\left(-\frac{1}{4}\right)F^{\mu\nu}F_{\mu\nu} + j^\mu A_\mu$

(And $\partial_\mu j^\mu = 0$; continuity eq)

The eq'ns we get from varying $\mathcal{L}$ i.e, by stationary condition for S are Maxwell's eqns. which were obtained in problem 1.8.

$$\mathcal{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + j^\mu A_\mu$$

$$= -\frac{1}{16\pi}g_{\lambda\mu}g_{\nu\sigma}(\partial^\mu A^\sigma - \partial^\sigma A^\mu) + j_\mu A^\mu$$

$$\therefore \frac{\partial\mathcal{L}}{\partial(\partial^\beta A^\alpha)} = -\frac{1}{16\pi}g_{\lambda\mu}g_{\nu\sigma}\left\{\begin{array}{l}\delta^\mu_\beta\delta^\sigma_\alpha F^{\lambda\nu} - \delta^\sigma_\beta\delta^\mu_\alpha F^{\lambda\nu} \\ +\delta^\lambda_\beta\delta^\nu_\alpha F^{\mu\sigma} - \delta^\nu_\beta\delta^\lambda_\alpha F^{\mu\sigma}\end{array}\right\}$$

(derivation was done in 1.8. So not done again here)

i.e $\frac{\partial\mathcal{L}}{\partial(\partial^\beta A^\alpha)} = -\frac{1}{4\pi}F_{\beta\alpha} = \frac{1}{4\pi}F_{\alpha\beta}.$

And $\frac{\partial\mathcal{L}}{\partial A^\alpha} = J^\mu.$

$\therefore$ E.L. eq gives: $\frac{1}{4\pi}\partial^\beta F_{\beta\alpha} = J_\mu$

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2. given transformation: $\delta\phi(x) = -\delta x_\mu \partial^\mu \phi$

i.e $x_\mu \longrightarrow x_\mu - \delta x_\mu.$

$\therefore A \longrightarrow A'(x - \delta x)$

i.e $\delta A_\mu = -(\partial_\nu A_\mu)\,\delta x^\nu - (A_\nu)\cdot\partial_\mu(\delta x^\nu)$

$= -(\partial^\alpha g_{\alpha\nu} A_\mu)\,\delta x^\nu - A^\alpha g_{\alpha\nu}(\partial_\mu(\delta x^\nu))$

$= -(\partial^\alpha A_\mu)\,g_{\alpha\nu}(\delta x^\nu) - A^\alpha \partial_\mu(g_{\alpha\nu}\,\delta x^\nu)$

$= -(\delta x_\alpha)\cdot\partial^\alpha A_\mu - A^\alpha \partial_\mu(\delta x_\alpha)$

$\longrightarrow$ proved

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4. $t^{00} = \frac{1}{4\pi}\left(F^{0i}F^{0}_{i} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\,g^{00}\right)$

$= \frac{1}{4\pi}\left(E^2 - \frac{1}{4}\,2(+E^2 - B^2)\times(+1)\right)$

$= \frac{1}{4\pi}\left(E^2 + \frac{B^2-E^2}{2}\right) = \frac{1}{4\pi}\,\frac{E^2+B^2}{2}.$

$t^{0i} = \frac{1}{4\pi}\left(F^{0j}F_{j}^{\ i} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\,\underbrace{g^{0i}}_{\to 0}\right)$

$= \frac{1}{4\pi}\left(F^{0j}F^{ik}g_{jk}\right)$

$= \frac{1}{4\pi}\left(\epsilon_{ikl}E_jB_l\,\delta_{jk}\right) = \frac{1}{4\pi}\left(\epsilon_{ikl}E_kB_l\right)$

i.e. $t^{0i} = \frac{1}{4\pi}(\vec{E}\times\vec{B})_i$

$t^{ij} = \frac{1}{4\pi}\left(F^{i\mu}F^{j\nu}g_{\mu\nu} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\,\delta^{ij}\right)$

$= \frac{1}{4\pi}\left(F^{i0}F^{j0}\cdot(1) + F^{ik}F^{jm}(-\delta_{km}) - \frac{(B^2-E^2)}{2}\delta^{ij}\right)$

$\cancel{= \frac{1}{4\pi}\left(E_iE_j - \frac{(B^2-E^2)}{2}\delta^{ij} - \delta_{km}\epsilon_{ikl}\epsilon_{jmn}B_lB_n\right)}$

$\cancel{= \frac{1}{4\pi}\left(\quad " \quad\quad " \quad - (\delta_{ij}\delta_{mm} - \delta_{im}\delta_{mj})B_lB_n\right)}$

$= \frac{1}{4\pi}\left(E_iE_j + \frac{B^2-E^2}{2}\delta^{ij} - \epsilon_{ikl}\epsilon_{jkm}B_lB_m\right)$

$= \frac{1}{4\pi}\left(E_iE_j + \frac{B^2-E^2}{2}\delta^{ij} - (\delta_{ij}\delta_{lm} - \delta_{im}\delta_{lj})B_lB_m\right)$

$= \frac{1}{4\pi}\left(\quad " \quad\quad " \quad - (B_iB_j\,\delta_{ij} - B_iB_j)\right)$

i.e. $t^{ij} = \frac{1}{4\pi}\left(E^iE^j + \frac{B^2-E^2}{2}\delta^{ij} - B^2 + B_iB_j\right)$ $\longrightarrow$ Ans

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$\boxed{///}$ $Tr(t) = -t_{00} + t_{ii} = -\frac{1}{4\pi}(E^2+B^2) + \frac{1}{4\pi}\left(\frac{3}{2}(B^2+E^2) - (B^2+E^2)\right)$

$= -\frac{1}{8\pi}(E^2+B^2) - \frac{1}{4\pi} \times \frac{1}{2}(E^2+B^2)$

$= 0 \longrightarrow$ Answer

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