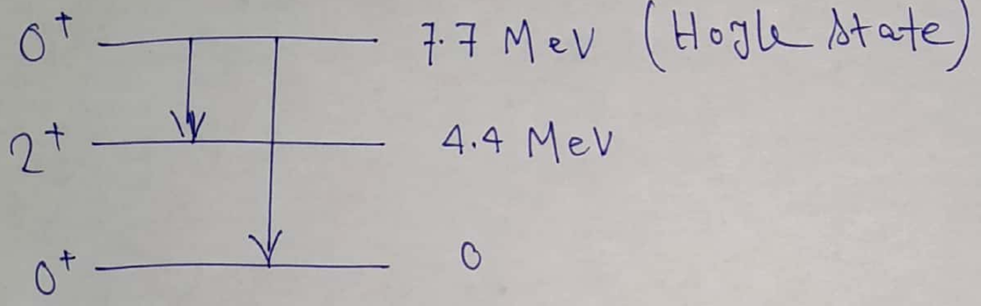
SAGAR DAM

DNAP.

Prob: 1



Electromagnetic decay of Hoyle state is shown above.

Now for $0^+ \rightarrow 2^+$ transition, gamma ray energy is : $\Delta E_\gamma = (7.7 - 4.4)$ MeV $= 3.3$ MeV.

And here $\Delta I = 2$.

This transition is E_2 in nature.

And for $0^+ \rightarrow 0^+$ transition:

this is eather $\beta^\pm$ pair production decay; with $\Delta I = 0$

This is a $E0$ transition. with a very little probability.

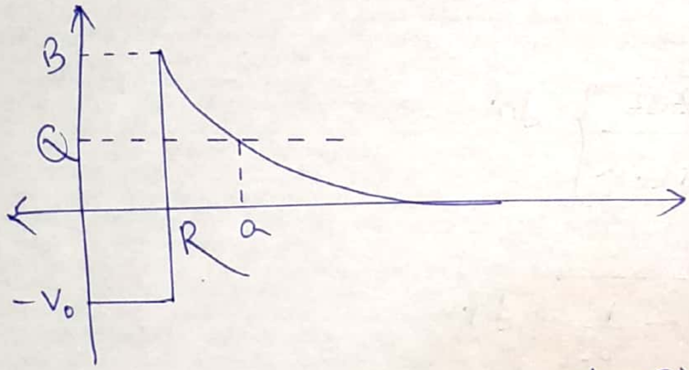
$2^+ \longrightarrow 0^+$ decay:

$\Delta\pi$ = NO.

possible transition is E2 ~~g~~ . gamma ray emission with photon energy = 4.4 MeV.

Prob: 2

In α decay: the α particle is in the strong nuclear potential (assuming to be a finite box potential here) when $r < R$ (R = nuclear radius) and it feels the standard coloumb field when $r > R$ (i.e after leaving the nucleus; when the strong force decays very rapidly)



$$V(r) = \begin{cases} -V_0 \ (V_0 = +ve) & \text{if } r < R \\ \dfrac{2Ze^2}{4\pi\epsilon_0 r} & r > R \end{cases}$$

The coloumb Barrier is given by: $B = \dfrac{2Ze^2}{4\pi\epsilon_0 R}$

And Q value of the reaction (K.E of α at infinity)

$$Q = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{a}$$

i.e a is the radius at outer side when the total energy of α is matching with the coloumb barrier.

(Here Z is the proton number of the daughter nucleus)

So: $\dfrac{B}{Q} = \dfrac{a}{R}$

Now from W.K.B theory; the transmission coefficient for α decay $P = e^{-2G}$

when G is called Gamow factor for α decay which is given by:

$$G \simeq \int_R^a \sqrt{\frac{2m}{\hbar^2}\left(V(r) - Q\right)}\; dr$$

$$= \int_R^a \sqrt{\frac{2m}{\hbar^2}\left\{\frac{2Ze^2}{4\pi\epsilon_0 r} - \frac{2Ze^2}{4\pi\epsilon_0 a}\right\}}\; dr$$

$$= \sqrt{\frac{2m}{\hbar^2} \times \frac{2Ze^2}{4\pi\epsilon_0}} \int_R^a \sqrt{\frac{1}{r} - \frac{1}{a}}\; dr$$

(Substituting $r = a \sin^2\theta$; i.e. $dr = 2a \sin\theta \cos\theta \, d\theta$ the integral becomes:)

$$G \simeq \sqrt{\frac{mZe^2}{\hbar^2 \pi \epsilon_0}} \int_{r=R}^a \sqrt{\frac{1}{a}(\operatorname{cosec}^2\theta - 1)} \times 2a \sin\theta \cos\theta \, d\theta$$

$$\simeq 2\sqrt{\frac{maZe^2}{\hbar^2 \pi \epsilon_0}} \int_{r=R}^a \sin\theta \cos\theta \cdot \cot\theta \, d\theta$$

i.e $G \simeq 2\sqrt{\dfrac{m a Z e^2}{\pi \hbar^2 \epsilon_0}} \displaystyle\int_{r=R}^{a} \cos^2\theta \, d\theta$

$$G \simeq \sqrt{\frac{m a Z e^2}{\pi \hbar^2 \epsilon_0}} \int_{r=R}^{a} (1+\cos 2\theta)\, d\theta$$

$$= \sqrt{\frac{m a Z e^2}{\pi \hbar^2 \epsilon_0}} \left(\theta + \frac{\sin 2\theta}{2}\right)_{r=R}^{a}$$

now $\sin^2\theta = \dfrac{r}{a}$ i.e $\theta = \sin^{-1}\sqrt{\dfrac{r}{a}}$

i.e $G \simeq \sqrt{\dfrac{m a Z e^2}{\pi \hbar^2 \epsilon_0}} \left(\sin^{-1}\sqrt{\dfrac{r}{a}} + \sqrt{\dfrac{r}{a}} \cdot \sqrt{1-\dfrac{r}{a}}\right)_{r=R}^{a}$

$$= \sqrt{\frac{m a Z e^2}{\pi \hbar^2 \epsilon_0}} \left(\sin^{-1} 1 + \sqrt{\frac{a}{a}} \cdot \sqrt{1-\frac{a}{a}} - \sin^{-1}\sqrt{\frac{R}{a}} - \sqrt{\frac{R}{a}} \cdot \sqrt{1-\frac{R}{a}}\right)$$

$$= \sqrt{\frac{m a Z \cdot e^2}{\pi \hbar^2 \epsilon_0}} \left(\cos^{-1}\sqrt{\frac{R}{a}} - \sqrt{\frac{R}{a}} \cdot \sqrt{1-\frac{R}{a}}\right)$$

$\left(\because \sin^{-1} 1 - \sin^{-1}\sqrt{\dfrac{R}{a}} = \dfrac{\pi}{2} - \sin^{-1}\sqrt{\dfrac{R}{a}} = \cos^{-1}\sqrt{\dfrac{R}{a}}\right)$

Now we got $\dfrac{R}{a} = \dfrac{Q}{B}$ (B = Barrier height) i.e. $\boxed{G = \sqrt{\dfrac{m a z e^2}{\pi \epsilon_0 \hbar^2}}\left(\cos^{-1}\sqrt{\dfrac{Q}{B}} - \sqrt{\dfrac{Q}{B}\left(1-\dfrac{Q}{B}\right)}\right)}$ Ans

Clearly if $Q = B$; i.e the α particle had the same energy to ~~overcome~~ the barrier height (i.e classically that is the marginal point when the α particle can just make an escape from the potential well); the tunnelling factor is,

$$G = \sqrt{\frac{m a z e^2}{\pi \hbar^2 \epsilon_0}}\left(\cos^{-1} 1 - \sqrt{\frac{Q}{Q}\left(1-\frac{Q}{Q}\right)}\right)$$

$$= 0$$

i.e. tunneling probability $P = e^{-2G} = 1$

i.e. the α particle can surely get emitted.

In this expression there is still a term of a inside root; which I forgot to change. However using $Q = \dfrac{2 z e^2}{4\pi \epsilon_0 a}$; we get G fully in terms of Q & B as:

$$G = \frac{2 z e^2}{4\pi \epsilon_0}\sqrt{\frac{2m}{\hbar^2 Q}}\left(\cos^{-1}\sqrt{\frac{Q}{B}} - \sqrt{\frac{Q}{B}\left(1-\frac{Q}{B}\right)}\right)$$ Ans

Clearly as $Q \to 0$; i.e α particle has very low energy we get $G \to \infty$ i.e $P = e^{-2G} \to 0$; i.e no escape. which is also obvious.

Prob: 3

by nucleus

The total energy realeased in $\beta^{\pm}$ decay is the total energy shared between $\beta^{\pm}$ & $\nu / \bar{\nu}$ particle.

i.e $E_0 = E_e + E_{\bar{\nu}}$

with $E_e^2 = m_e^2 c^4 + p_e^2 c^2$

$E_{\bar{\nu}}^2 = m_{\bar{\nu}}^2 c^4 + p_{\bar{\nu}}^2 c^2$

Now if we get dn no of final count in β^{-} decay when the e^- & $\bar{\nu}_e$ has the momentum value bound between the range $[p_e, p_e + dp_e]$ & $[p_{\bar{\nu}}, p_{\bar{\nu}} + dp_{\bar{\nu}}]$ then; Using $dp_x dp_y dp_z \propto p^2 dp$ we get:

$$dn \propto p_e^2 p_{\bar{\nu}}^2 dp_e dp_{\bar{\nu}}$$

now: $$p_{\bar{\nu}} = \frac{\sqrt{E_{\bar{\nu}}^2 - m_{\bar{\nu}}^2 c^4}}{c} = \frac{\sqrt{(E_0 - E_e)^2 - m_{\bar{\nu}}^2 c^4}}{c}$$

And $dp_e dp_{\bar{\nu}} = J dp_e dE_0$

J = Jacobian of transform

$$= \begin{vmatrix} \frac{\partial p_e}{\partial p_e} & \frac{\partial p_e}{\partial E_0} \\ \frac{\partial p_{\bar\nu}}{\partial p_e} & \frac{\partial p_\nu}{\partial E_0} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ \frac{\partial p_{\bar\nu}}{\partial p_e} & \frac{E_0 - E_e}{c\sqrt{(E_0-E_e)^2 - m_{\bar\nu}^2 c^4}} \end{vmatrix}$$

$$= \frac{E_0 - E_e}{c\sqrt{(E_0-E_e)^2 - m_{\bar\nu}^2 c^4}}$$

i.e. $dn \propto p_e^2 (E_0 - E_e)\sqrt{(E_0-E_e)^2 - m_{\bar\nu}^2 c^4} \; dp_e \, dE_0$ .... (1)

Now from fermi - Golden rule; transition rate

$$\lambda_{if} = \frac{2\pi}{\hbar^2} |M|^2 \frac{dn_{if}}{dE}$$

($M = \langle i | H' | f \rangle$ = element of H' matrix at i,f place) i.e. $|M|^2$ being a constant related to the perturbing part of the Hamiltonian; we can safely conclude, the rate of β decay with e^- energy in range $[p_e, p_e + dp_e]$ is given by:

$$d\lambda_{if} \propto \frac{dn_f}{dE_0}$$

i.e. $$d\lambda_{if} \propto p_e^2 (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4} \, dp_e$$

(using (1))

So the intensity of the emitted electron as a fn of it's energy

$$I_e(p_e) \propto \frac{d\lambda_{if}}{dp_e} \propto p_e^2 (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4}$$

(excluding the Fermi factor)

i.e. $$I_e(E_e) = I(p_e) \times \left(\frac{dE_e}{dp_e}\right)$$ (by variable transform from $p_e \to E_e$)

i.e. $$I_e(E_e) = p_e^3 \times \left(\frac{E_0}{E_e} - 1\right) \sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4}$$

$$\left(\because \frac{dE_e}{dp_e} = \frac{p_e c^2}{E_e} \text{ from } E_e^2 = p_e^2 c^2 + m_e^2 c^4\right)$$

i.e. $$I_e(E_e) = \left(\frac{E_0 - E_e}{E_e}\right) \times \left(E_e - m_e^2 c^4\right)^{3/2} \sqrt{(E_0 - E_e)^2 - m_\nu^2 c^4}$$

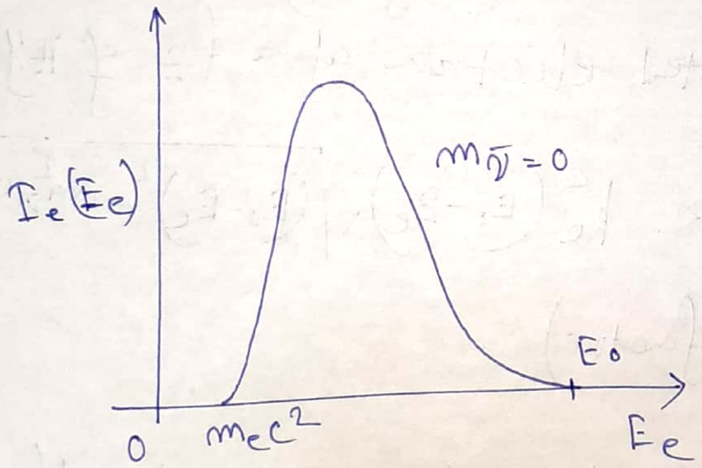
Clearly $I_e(E_e) = 0$ if $E_e = m_e c^2$; i.e. $p_e = 0$ ~~as it also do not conserve~~ And also if $E_0 = E_e$

Now for the given two conditions:

a) $m_{\bar{\nu}} << m_e$:—

when $m_{\bar{\nu}} = 0$; the non-zero $I(E_e)$ starts at $E_e = m_e c^2$ and makes a peak at $\frac{\partial I}{\partial E_e} = 0$ (some point not calculated) and again goes to zero at $E_e = E_0$.

i.e. $(E_0 - E_e) = 0$.

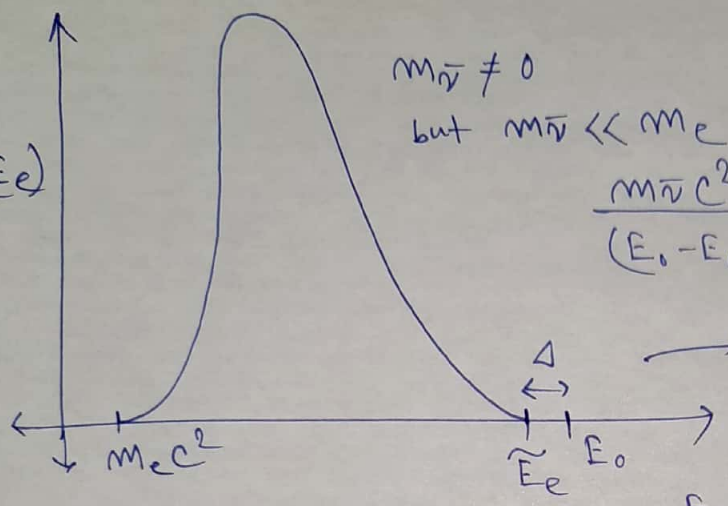


if $m_{\bar{\nu}} \approx 0$ such that $m_{\bar{\nu}} << \frac{(E_0 - E_e)}{c^2}$ then the I_e starts at $E_e = m_e c^2$ but ~~gets to~~ starts to get modified slowly at the tail end.

we expand: (Taylor expand)

$$I(E_e) = \left(\frac{E_0 - E_e}{E_e}\right) \cdot (E_e - m_e c^2)^{3/2} \times (E_0 - E_e) \times \left(1 - \frac{m_{\bar{\nu}}^2 c^4}{2(E_0 - E_e)^2} + \frac{1}{8}\left(\frac{m_{\bar{\nu}}^2 c^4}{(E_0 - E_e)^2}\right)^2 - \cdots\right)$$

Here the tail of $I(E_e)$ comes to zero, slightly before E_0; the plot looks like:



$m_{\bar{\nu}} \neq 0$

but $m_{\bar{\nu}} \ll m_e$ so that $\frac{m_{\bar{\nu}} c^2}{(E_0 - E_e)} \ll 1$

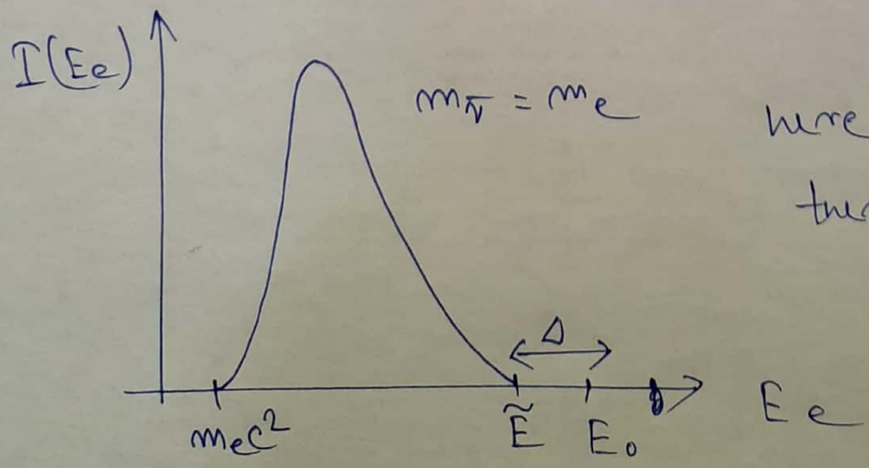
here $\tilde{E}$ is so that $I(\tilde{E}) = 0$

& $\Delta = E_0 - \tilde{E}$

from our calculation $\Delta \sim m_{\bar{\nu}}^2$

b) $\underline{m_{\bar{\nu}} = m_e}$:-

here the tail ~~end~~ end gets shifted towards left and the gap becomes prominent. The plot is like:



here if $\Delta = E_0 - \tilde{E}$

then $\Delta = m_{\bar{\nu}} c^2$
$= m_e c^2$

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4. a

Let's first calculate the decay Q values for ^{57}Co for β^-, β^+ & e^- ~~captu~~ capture schemes.

i) β^- decay:

$$^{57}_{27}Co \longrightarrow {}^{57}_{28}Ni + e^- + \bar{\nu}_e$$

$$Q = \{M(^{57}Co) - M(^{57}Ni)\} c^2$$

$$= (56.936290 - 56.939793) \times 931.5 \text{ MeV}.$$

$$= -3.2630445 \text{ MeV}.$$

ii) β^+ decay:

$$^{57}_{27}Co \longrightarrow {}^{57}_{26}Fe + e^+ + \nu_e$$

$$Q = \{M(^{57}_{27}Co) - M(^{57}_{26}Fe)\} c^2 - 2m_e c^2$$

$$= \{(56.936290 - 56.935392) \times 931.5 - 2 \times 0.54\} \text{ MeV}$$

$$= -0.185513 \text{ MeV}.$$

iii) e^- capture:

$$^{57}_{27}Co + e^- \longrightarrow {}^{57}_{26}Fe + \nu_e$$

$$Q = \{M(^{57}_{27}Co) - M(^{57}_{26}Fe)\} c^2$$

$= 0.8365$ MeV.

So depending on the Q values; we conclude that for $^{57}_{27}Co$ the only possible decay mode is e^- capture. The $\beta^\pm$ decay is not energetically possible.

In reality ^{57}Co mainly decays to excited levels of $^{57}_{26}Fe$ of 136.47 KeV (99.80%) and 706.42 keV (0.183%) by e^- capture process.

The probability of ~~that~~ decay to other states of ^{57}Fe is very small.

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4.b

$$^{96}Zr \longrightarrow {}^{96}Nb + e^- + \bar{\nu}_e$$

The G.S of ^{96}Nb is 6^+; other states are $5^+, 4^+, 3^+$ which has energy respectively 44.19 keV, 146.03 keV, 184.58 keV.

The G.S. of ^{96}Zr is 0^+

Now the Q value for $0^+ \longrightarrow 6^+$ transition of ^{96}Zr to ^{96}Nb is:

$$= (95.9082734 - 95.908101) \times 931.5 \text{ MeV}$$

$$= 160.94 \text{ keV}.$$

Since parity is not conserved in the transition so $L = 6$

As $\Delta I = 6$, $S = 0, 1$ both are allowed. i.e. 6th forbidden Fermi and G.T transitions are allowed.

Q for $0^+ \longrightarrow 5^+$ transition:

$$= (160.94 - 44.19) \text{ keV} \quad \left(\because E(6^+) - E(5^+) = -44.19 \text{ keV for } {}^{96}Nb \right)$$

$$= 116.75 \text{ keV}.$$

here $\Delta I = 5$

as L = even so $L = 4$, $S = 1$

i.e 4 th forbidden G.T transition is allowed.

for $0^+ \rightarrow 4^+$ transition:

$Q = (160.94 - 146.10)$ kev $= 14.84$ kev.

Here $\Delta I = 4$

So $l = 4$, $S = 0, 1$

i.e 4th forbidden Fermi & G.T both are possible.

for $0^+ \longrightarrow 3^+$ transition:

$Q = (160.94 - 184.58)$ kev

$= -23.64$ kev < 0.

i.e $Q < 0$ so this transition is not energitically possible.

$^{48}Ca \longrightarrow {}^{48}Sc + e^- + \bar{\nu}_e$

Gs of ^{48}Ca : 0^+

^{48}Sc : 6^+

other possible states of ^{48}Sc are $5^+, 4^+, 3^+$ which has energies as 130.94 kev, 252.835 kev & ~~652~~ 622.64 kev.

Q value for $0^+ \longrightarrow 6^+$ transition:

$= (47.95252290 - 47.952231) \times 931.5$ MeV.

$= 271.905$ keV.

Here $\Delta I = 6$ i.e. $l = 6$ and $S = 0, 1$

So Both Fermi & G.T modes are possible transition ~~...~~ with 6th degree of forbidden.

for $0^+ \longrightarrow 5^+$:

$Q = (271.905 - 130.94)$ keV.

$= 140.965$ keV.

$\Delta I = 5$ i.e $l = 4$ And $S = 1$

i.e. Only G.T with 4th ~~...~~ forbidden state is possible.

for $0^+ \longrightarrow 4^+$:

$Q = (271.905 - 252.35)$ keV.

$= 19.555$ keV.

$\Delta I = 4$

$\therefore l = 4$ & $S = 0, 1$

i.e Fermi & G.T both decay modes are possible in 4th forbidden state.

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for $0^+ \longrightarrow 3^+$ transition,

$$Q = (271.905 - 622.64)\ \text{Kev}.$$

$$< 0$$

i.e this transition is not possible.

<u>4.C</u> $\;{}^7Be + e^- \longrightarrow {}^7Li + \nu_e$

Assuming $m_\nu << m_e$ here.

Now $Q = \{M({}^7Be) - M(\underbrace{{}^7Li}_{\text{in u}})\} \times 931.5$ Mev

$$= (7.0169297 - 7.016004) \times 931.5\ \text{Mev.} \quad \left(\text{in G.S of } {}^7Li\right)$$

$$= 0.86134\ \text{Mev}$$

Now, from the ${}^7Be$ atom's frame of reference

$$\vec{p}_i = 0$$

So from C.O.M principle: $\vec{p}_f = \vec{p}_i = \vec{p}_{Li} + \vec{p}_\nu = 0$

i.e. $\vec{p}_{Li} = -\vec{p}_\nu$

And $Q = p_\nu c + \sqrt{m_{Li}^2 c^4 + p_{Li}^2 c^2} - m_{Li} c^2$

at non relativistic limit $m_{Li} c^2 >> p_{Li} c$

i.e. $Q \approx p_\nu c + m_{Li} c^2 \left\{1 + \dfrac{p_{Li}^2}{2 m_{Li}^2 c^2}\right\} - m_{Li} c^2$

i.e $Q \approx p_\nu c + \dfrac{p_{Li}^2}{2 m_{Li}}$

i.e. $Q = p_\nu c + \dfrac{p_\nu^2}{2 m_{Li}}$

i.e. $p_\nu^2 + 2 m_{Li} c\, p_\nu - 2 m_{Li} Q = 0$

i.e. $p_\nu = \dfrac{-2 m_{Li} c \pm \sqrt{4 m_{Li}^2 c^2 + 8 m_{Li} Q}}{2}$

$= \dfrac{-m_{Li} c \pm \sqrt{m_{Li}^2 c^2 + 2 Q m_{Li}}}{1}$

i.e. $p_\nu c = -m_{Li} c^2 + \sqrt{m_{Li}^2 c^4 + 2 Q m_{Li} c^2}$.

Now $m_{Li} c^2 = 7.016004 \times 931.5$ MeV

$= 6535.407726$ MeV.

i.e. $p_\nu c = 0.861283$ MeV (using values)

= recoil energy of ν

And $E_{Li} = Q - p_\nu c$

$= (0.86\cancel{8}134 - 0.861283)$ MeV.

≈ 57 eV

= recoil energy of ^7Li

<u>Ans</u>

Prob: 5

a) $n \longrightarrow p$

G.S: $\frac{1}{2}^+ \qquad \frac{1}{2}^+$

So $\Delta I = 0$; i.e the allowed transition $l = 0$ is possible. as parity change $\Delta\pi$ = no.

i.e. $\hat{I}_i = \hat{I}_f + \hat{S}$

$\frac{1}{2} \qquad \frac{1}{2}$

So $S = 0, 1$ both are possible

$\therefore$ Fermi & G.T both allowed. i.e mixed

b) $^3H \longrightarrow {}^3He$

$\frac{1}{2}^+ \qquad \frac{1}{2}^+$

The explanation is same as part (a)

possible modes: Fermi + G.T. i.e. mixed.

c) $^{60}Co \longrightarrow {}^{60}Ni$

$5^+ \qquad 4^+$

$\Delta\pi$ = no change.

$\Delta I = 1$

for allowed: $l = 0$ transition $I_i = I_f + S$

gives $S = 1$ is possible.

i.e. G.T for allowed transition can occure.

d) $^{117}In \longrightarrow {}^{117}Sn$

$\frac{9}{2}^+$ $\qquad$ $\frac{7}{2}^+$

$\Delta I = 1$

So for $l=0$ (as $\Delta\pi$ = no) i.e allowed transition we must have $S=1$.

i.e Allowed G.T is possible.

e) $^{152}Eu \longrightarrow {}^{152}Gd$.

G.S: 0^- $\qquad$ 0^+

$\Delta I = 0$; π = yes.

clearly l can't be 0, 2, ...

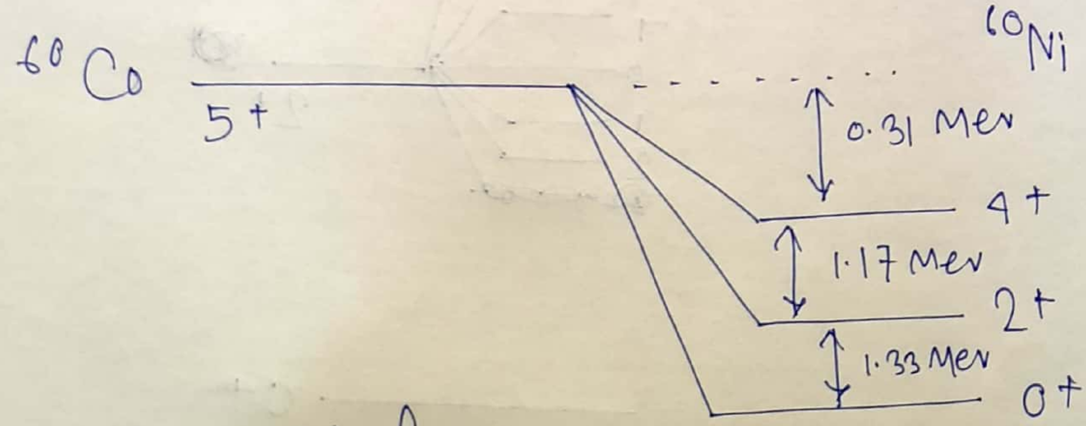
for $l=1$ i.e. first forbidden we ~~must~~ must have $S=1$ for angular momentum conservation.

i.e the transition is first forbidden G.T type.

8. The gamma ray is emitted from the nucleus due to its transition from excited state to lower state. Now in this process the angular momentum of the nucleus is being changed. The orbital angular momentum, being for its θ, ϕ ~~dependance~~ dependance the emitted photon should emit at some particular direction to conserve the angular momentum & hence we should see the directionality.

But without any external $\vec{B}$ field; the sample of radiating particles are oriented along random direction in space. As the number of atoms in the sample is huge so we get the statistical effect; i.e some averaged over angular distribution i.e which means approximately equal no of atoms are oriented for each (θ, ϕ) direction making the radiation isotropic at room temperature.

(iv)



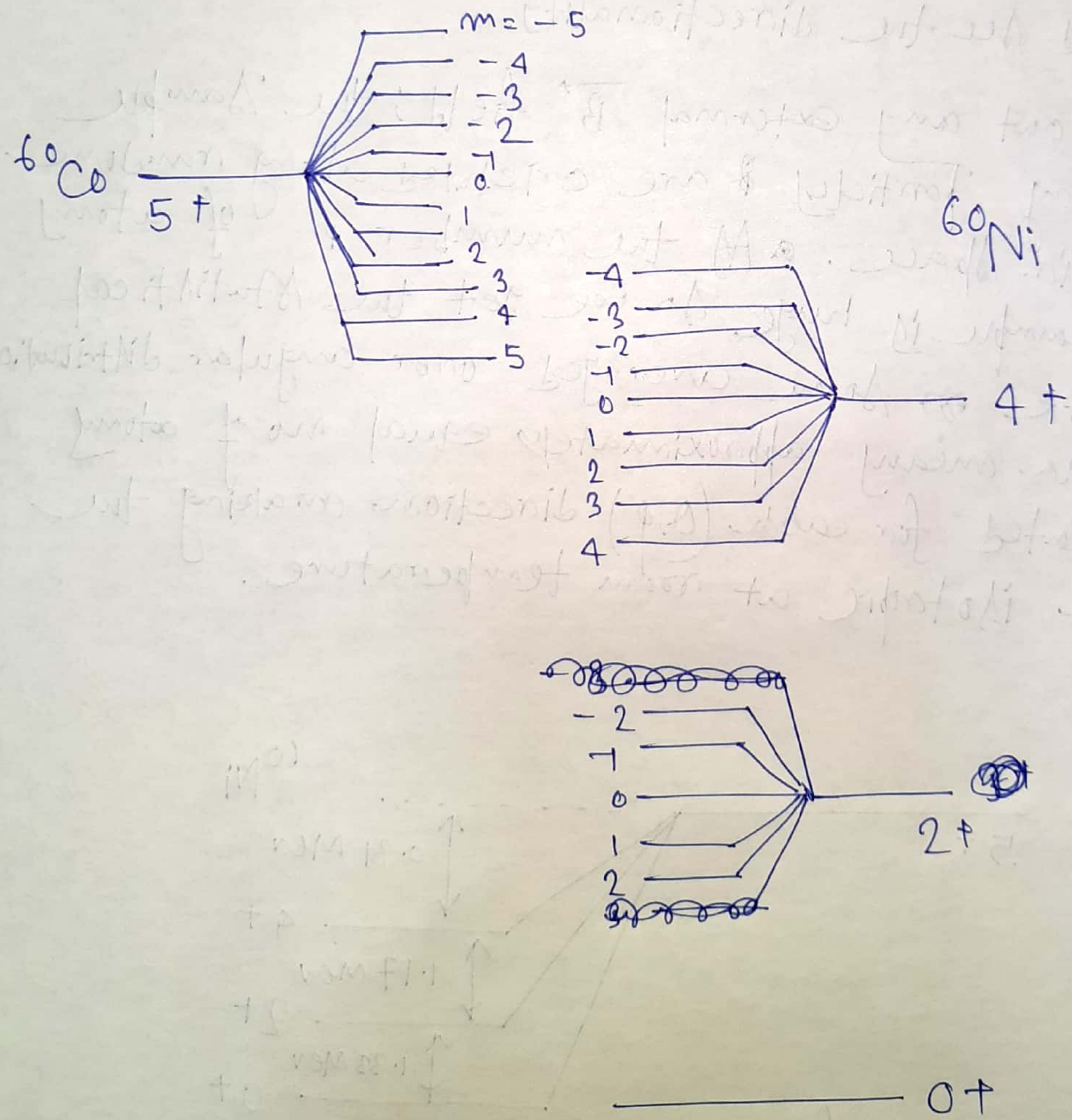
without $\vec{B}$ field

Now in presence of $\vec{B}$ field: each I level splits into different level with energy gap from the parent state as:

$$\Delta H = -\langle \mu_N \vec{B} \rangle = -\mu_N m B_z$$

taking $\hat{z} \parallel \vec{B}$: $\Delta H = -\mu_N m B$.

i.e After the field the splitted energy diagram looks;



Now the energy gap between the ^{60}Co levels are

$$\Delta E = \mu_N B$$

The statistical distribution will follow

$$W(E) \sim e^{-\beta \Delta E} \sim e^{-\frac{\mu_N B}{KT}}$$

At $T \to \infty$; $W(E) \to 1$ giving the ~~homogeneous~~ isotropic distn and hence isotropic radiation.

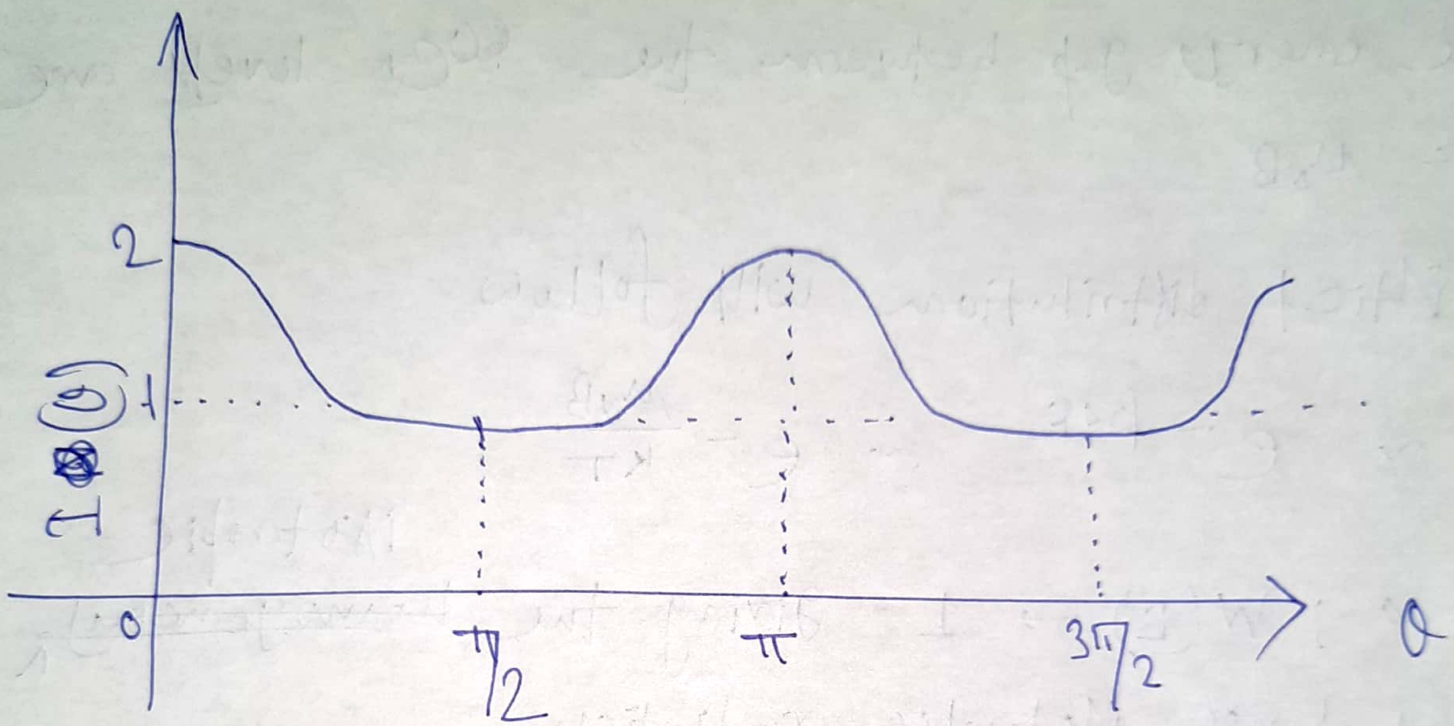
At lower temp: the states will be unevenly populated (e.g for ^{60}Co the lower energy state will have more population)

we take that at $\frac{\mu_N B}{KT} \leq 1$ the distribution will be ~~[illegible]~~ significantly anisotropic.

i.e $T \leq \frac{\mu_N B}{K}$ (K = Boltzman const)

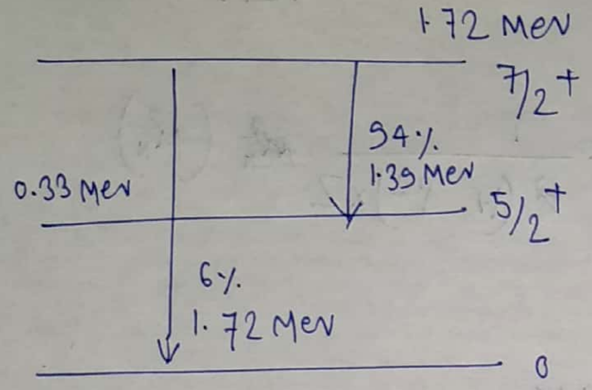
(iii) for E_2 transition the Gamma distn will follow $(1 + \cos^4\theta)$ form

(θ = angle between magnetic field & detection direction)



2
1
I(θ)
0
π/2
π
3π/2
θ

Prob: 7



for $7/2^+ \rightarrow 5/2^+$ transition we get: $\Delta I = \left(\frac{7}{2}+\frac{5}{2}\right)$ to $\left(\frac{7}{2}-\frac{5}{2}\right)$

i.e. $\Delta I = 6, 5, 4, 3, 2, 1$

And $\Delta\pi = $ no

So the possible transition is E_2 & M_1

for $\frac{7}{2}^+ \longrightarrow \frac{3}{2}$ transition: $\Delta I = \frac{7+3}{2}$ to $\frac{7-3}{2}$

i.e. $\Delta I = 5, 4, 3, 2$

i.e. Lowest order possible transition $= E2$

Now; $m_{fi}(\sigma_L) = \int \psi_f^* \, m\sigma \, \psi_i \, d\tau$

where m_{fi} is the (f,i)th element of multipole operator.

The mixing rate $\delta = \frac{m_{fi}(E2)}{m_{fi}(M1)} = 0.14 \pm 0.12$.

Now $B(M_1) = 0.0568 \times E_\gamma^{-3}\, B_r \times \frac{1}{(1+\delta^2) \times \tau} \; (\mu_N^2)$

(where B_r = Branching ratio)

Here $E_\gamma = 1.39$ MeV ; $\delta = 0.14$

$B_r = 0.94$; $\tau = 0.048$ ps. = lifetime

i.e. $B(M_1) = 0.0568 \times \frac{1.39^{-3} \times 0.94}{(1+0.14^2) \times 0.048} \mu_N^2$

$= 0.4062 \; \mu_N^2$.

And ~~B(M1)~~ $B(E_2) = \frac{0.0816\,(1-B_r)}{\tau E_\gamma^5}$ (in e^2b^2)

$= 6.776 \times 10^{-3} \; e^2b^2$.

Ans

Prob 6

a) for ^{17}N the shell model configurations are:

State	config
$1/2^-$	$\pi[p_{1/2}]^{-1}$
$3/2^-$	$\nu[p_{3/2}]^{-1}$

(Using NNDC)

b) for $\frac{3}{2}^-$ to ground state $\left(\frac{1}{2}^-\right)$

$I_i = 3/2$, $I_f = 1/2$ And $\Delta\pi$ = unchanged

i.e. $\Delta I = (I_i + I_f)$ to $|I_i - I_f|$

$= 1, 2.$

So the transitions are M_1 & E_2.

c) $\langle f|M_1|i\rangle = g_l D^l_{if} + g_s D^s_{if}$

Now the transition probability (Fermi's rule)

$= |\langle\psi_f|M_1|\psi_i\rangle|^2.$

with $|\psi_f\rangle = \frac{1}{2}^-$ i.e $^1p_{1/2}$

& $|\psi_i\rangle = 3/2^-$ i.e. $^1p_{3/2}$

$$\therefore \left\langle {}^1p_{3/2} \middle| M_1 \middle| {}^1p_{1/2} \right\rangle = g_l \times (-0.564) + g_s \times (0.564)$$

$$= 0.564\,(g_s - g_l)\frac{\mu_N}{c}$$

i.e $\left|\left\langle {}^1p_{3/2} \middle| M_1 \middle| p_{1/2} \right\rangle\right|^2$ = transition rate = P

$$= 0.318\,(g_s - g_l)^2 \cdot \frac{\mu_N^2}{c^2}.$$

Neglecting E_2 transition contribution:

lifetime $\tau = \frac{6\hbar \times 9}{2 \times 2P} \times \left(\frac{c}{\omega}\right)^3$

$$= \frac{9\,6\hbar\,c^5}{4 \times 0.318\,(g_s - g_l)^2 \mu_N^2}.$$

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