

~~scribble~~

$$(r_1 e^{i\theta_1}) r_2 e^{i\theta_2}$$

$$r_1 r_2 e^{i\theta_2} \cdot (e^{i\theta_1}) r_2 e^{i\theta_2}$$

$$(r_1 r_2) e^{i\theta_2}$$

$$\parallel$$

$$R e^{i\theta_2}$$

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$$R (\cos \theta_2 + i \sin \theta_2)$$

$$R \cos \theta_2 \quad (R \sin \theta_2)^i$$

$$\parallel$$

$$R_c \cdot R_s^i$$

$$\parallel$$

$$R_c \{ \cos(\ln R_s) + i \sin(\ln R_s) \}$$

$$\text{or } R_c e^{i \ln R_s}$$

~~scribble~~

$$y = x^i \quad (x \in \mathbb{R})$$

$$\ln y = i \ln x$$

$$= e^{i \frac{\pi}{2} \ln x}$$

$$\ln x = e^{-i \frac{\pi}{2} \ln y}$$

$$= \ln y^{-i}$$

$$x = y^{-i}$$

$$y = (y^{-i})^i$$

~~scribble~~

~~$$y = x^i = (e^{\ln x})^i$$~~

$$y = x^i = (e^{\ln x})^i$$

$$= e^{i \ln x}$$

$$= \cos(\ln x)$$

$$+ i \sin(\ln x)$$

$$\text{Ans } (e^{i\theta})^{r_1 e^{i\theta}} = (e^{r_1 i\theta})^{e^{i\theta}}$$

$$= e^{i r_1 \theta e^{i\theta}}$$

$$\text{Ans } (e^{i\theta_1})^{r_2 e^{i\theta_2}} = (e^{r_2 i\theta_1})^{(\cos\theta_2 + i\sin\theta_2)}$$

$$= e^{r_2 i\theta_1 \cos\theta_2} \cdot e^{-r_2 \theta_1 \sin\theta_2}$$

$$\text{i.e. } (r_1 e^{i\theta_1})^{(r_2 e^{i\theta_2})}$$

$$= R_1 e^{i \ln R_1} \times e^{i r_2 \theta_1 \cos\theta_2} \cdot e^{-r_2 \theta_1 \sin\theta_2}$$

$$= (R_1 e^{-r_2 \theta_1 \sin\theta_2}) \times e^{i(\ln R_1 + r_2 \theta_1 \cos\theta_2)}$$

$$= (r_1^{r_2 \cos\theta_2} e^{-r_2 \theta_1 \sin\theta_2}) e^{i(\ln(r_1^{r_2 \sin\theta_2}) + r_2 \theta_1 \cos\theta_2)}$$