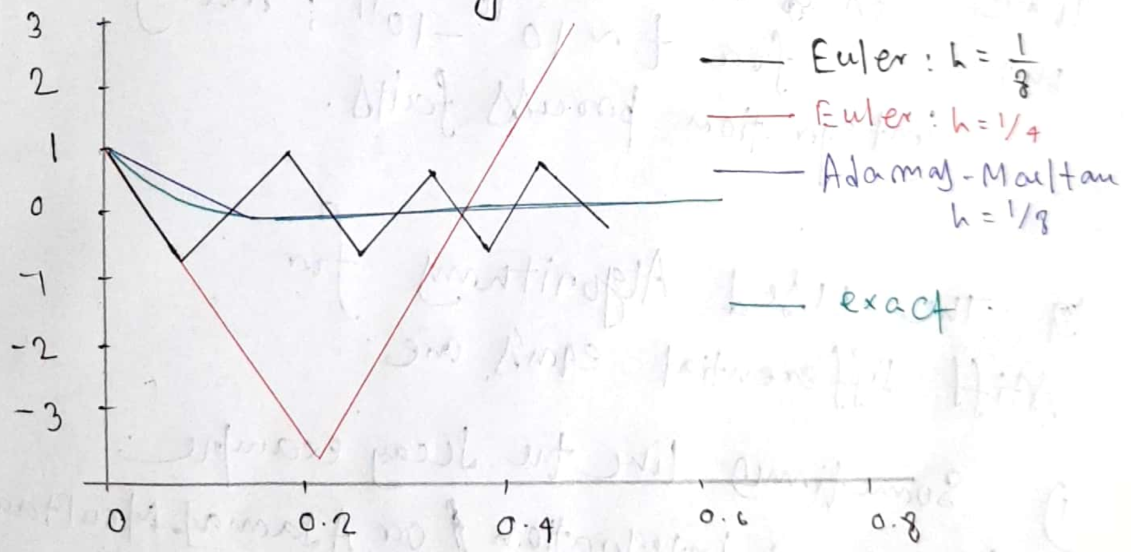
15. Stiff differential eq:—

A Stiff diff eq is certain type of diff eq for which the solutions with regularly uses numerical methods are very unstable. For those types of eqn's; generally some mathematical analysis can finds the particular parameter responsible for the stiffness. But generally there is no way to find the responsible parameter of an arbitrary diff eq.

example: Let $y'(t) = -15\,y(t)$ $t \geq 0$; $y(0) = 1$

∴ the exact sol is: $y(t) = e^{-15t}$.

Now if we do the numerical solution then we get the following result:



i) Euler $(h = \frac{1}{4})$ goes off very fast.

ii) " $(h = \frac{1}{8})$ oscillates and the absolute error is large.

iii) Adamas - Moultan $(h = \frac{1}{8})$ is very close to exact graph.

when the Solution (numerical) is given by:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(t_n, y_n) + f(t_{n+1}, y_{n+1})\right]$$

For this decay eqn the backward or implicit ~~a~~ integral method is also effective

Another example:–

$$\left.\begin{aligned} \dot{x} &= -0.04x + 10^4 yz \\ \dot{y} &= 0.04x - 10^4 yz - 3\times10^7 y^2 \\ \dot{z} &= 3\times10^7 y^2 \end{aligned}\right\} \text{Robertson chemical reaction eqn}$$

for this problem the unstable region is for large t value.

like ~~t~~ $t \sim 10^2$ is fine

but for $t \sim 10^{10} - 10^{11}$; many integration process fails.

The used Algorithms for stiff differential eqn's are:

i) Sometimes like the decay example backward integration / or Adams-Moultan algorithm is effective.

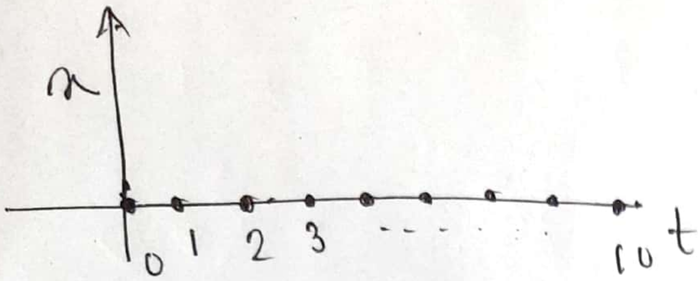
ii) VSVO - BBDF - Method.
(The reference from where I got this are in the file 'references'.)

iii) Sometimes Adaptive step size algo is effective.

In python $\boxed{\text{Scipy.integrate.odeint}}$ can handel stiff and non-stiff eqn's.

However in mathematica it is very easy to solve ~~hugely~~ ~~to~~ hugely stiff diff eq. There is no need to find a special module.

6. $\dfrac{d^2\phi}{dx^2} = \dfrac{\phi(x+h) + \phi(x-h) - 2\phi(x)}{h^2}$



given $x''(t) = -g = -10$

$x(0) = x(10) = 0.$

$$\left.\begin{aligned}
x_0 - 2x_1 + x_2 &= -10 \\
x_1 - 2x_2 + x_3 &= -10 \\
x_2 - 2x_3 + x_4 &= -10 \\
&\dots\dots \\
x_{n-2} - 2x_{n-1} + x_n &= -10.
\end{aligned}\right\} \quad x_0 = x_n = 0. \rightarrow n-1 \text{ eqns}$$

So putting $x_0 = x_n = 0.$

$$\left.\begin{aligned}
-2x_1 + x_2 &= -10 \\
x_1 - 2x_2 + x_3 &= -10 \\
x_2 - 2x_3 + x_4 &= -10 \\
&\dots\dots \\
x_{n-2} - 2x_{n-1} &= -10
\end{aligned}\right\} \quad \begin{aligned}&n-1 \text{ eqns} \\ &n-1 \text{ vars.}\end{aligned}$$

in matrix form:

$$\begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & \cdots & \vdots \\ 0 & 0 & 1 & -2 & 1 & 0 & \cdots & \\ \cdots & & & & & & \cdots & \\ & & & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & \cdots & & & & & \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} = -10 \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$