Dolph Tchebysheff Amplitude Distribution

Q. Design a four element broadside array of $\lambda/2$ spacing between elements. The pattern is to be optimum with a side lobe level 19.1 dB down the main lobe maximum.

Procedure for solving:

Step1: Calculate r.

$$r = \frac{Main\ lobe\ maximum}{Side\ lobe\ level}$$

(or) Side lobe level below main lobe maximum in dB = $20 \log_{10} r$

Step 2: Select the Tchebysheff polynomial $T_m(x)$ of the same degree as the array polynomial.

$$T_{n-1}(x_0) = r$$

then Solve it for x_0

(Or)

$$x_0 = \frac{1}{2}\left[\left\{r + \sqrt{r^2 + 1}\right\}^{\frac{1}{m}} + \left\{r - \sqrt{r^2 - 1}\right\}^{\frac{1}{m}}\right]$$

Where, m = n-1

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Step 3: Choose array polynomial E_t from following equation suitably

(a)

$E_{t0} = a_0 Z + a_1 [4Z^3 - 3Z] + a_2 [16 Z^5 - 20 Z^3 + 5Z] + a_3 [64 Z^7 - 112 Z^5 + 56 Z^3 - 7Z] + \ldots$

(b)

$E_{t0} = a_0 + a_1 [2Z^2 - 1] + a_2 [8 Z^4 - 8 Z^2 + 1] + a_3 [32 Z^6 - 48 Z^4 + 18 Z^2 - 1] + \ldots$

- These are Z - transform based expression.

Step 4: Equate Tchebysheff polynomial with array polynomial E_{t0} i.e.

$$T_{n-1}(x) = E_t$$

Calculate co-efficient and take ratios of relative current amplitude.

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Step 1: $19.1 = 20 \log_{10} r \Rightarrow \log_{10} r = 19.1/20 \Rightarrow r = 9$ (approx.)

Step 2: Tchebysheff polynomial of degree = n-1

Here $n = 4$, $d = \lambda/2$,

$m = n - 1 \Rightarrow 4 - 1 = 3$ i.e. $T_3(x_0)$

$$T_3(x_0) = 9$$

$$(4x_0^3 - 3x_0) = 9$$

For factorization add and subtract $6x_0^2$ and writing $3x_0$ as $6x_0 - 9x_0$, we get

$$4x_0^3 - 6x_0^2 + 6x_0^2 + 6x_0 - 9x_0 - 9 = 0$$

Solving this equation gives

$$x_0 = 1.5$$

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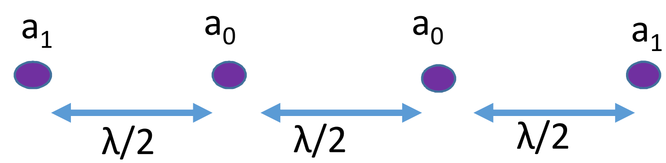
Step 3:



$$E_4 = a_0 Z + a_1\,[4Z^3 - 3Z]$$

$$E_4 = T_3\,(x)$$

Step 4:

$$a_0 Z + a_1\,[4Z^3 - 3Z] = (4x^3 - 3x)$$

$$a_0\left(\frac{x}{x_0}\right) + a_1\,[4\left(\frac{x}{x_0}\right)^3 - 3\left(\frac{x}{x_0}\right)] = (4x^3 - 3x)$$

Equating like terms

$$x\left[\frac{a_0 - 3a_1}{x_0}\right] = -3\,x$$

Substituting x_0=1.5

$$a_0 - 3\,a_1 = -\,4.5 \qquad \text{(A)}$$

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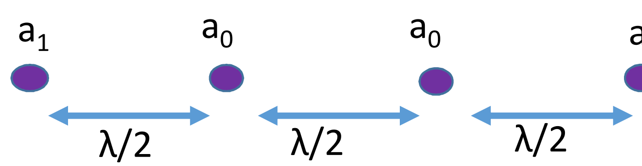
$$4a_1\left(\frac{x}{x_0}\right)^3 = 4x^3$$

$$a_1 = x_0^3 = (1.5)^3 = 3.375$$

- From (A) we get,

$$a_0 = 5.625$$

- Thus the broadside array of 4 sources is shown below:



3.375 5.625 5.625 3.375

1 1.66 :: 1.66 : 1

Thus relative amplitudes are 1: 1.66 :: 1.66 : 1